B meson decay constant in static approximation with dmain wall fermion and perturbative $O(\alpha_s a)$ matching

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Contents

- Static heavy - DW light quark system
- Perturbative $O(\alpha_s a)$ matching of heavy - light quark bilinear and four-fermion operators
- Effects of the improvement for f_B, B_B
  - Very rough estimation of the effect using last year's RBC/UKQCD data.

Wennekersis et.al. (LATTICE2007).

$B_q^0 - \overline{B}_q^0$ *with HQET*

$B_q^0 - \overline{B}_q^0$ mixing

$$\Delta m_{q(=d,s)} = \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} S_0 \left| V_{tq}^* V_{tb} \right|^2 \eta_B \mathcal{M}_q$$

$$\mathcal{M}_q = \langle \overline{B}_q^0 | [\bar{b}\gamma_\mu P_L q][\bar{b}\gamma_\mu P_L q] | B_q^0 \rangle$$

- Precise lattice input of $\mathcal{M}_q$ is needed.

HQET

- Static limit of b quark: $m_b \longrightarrow \infty$
- Action: $$S_{\text{static}} = \sum_{\vec{x},t} \bar{h}(\vec{x}, t+a) \left[h(\vec{x}, t+a) - U_0^\dagger(\vec{x}, t) h(\vec{x}, t) \right]$$
- Good approximation of b quark.
- Useful as a reference point of $1/m_b$ expansion, relativistic heavy quark theory and so on, for precision calculation.

$O(a)$ *improvement of HQET operator*

O(a)

- O(a) improvement is needed to reduce lattice cutoff effects
  - Morningstar and Shigemitsu [1998] (NRQCD+clover Wilson)
  - Ishikawa, Onogi and Yamada [1998] (HQET+clover Wilson)
- Additional operator is mixed at $O(\alpha_s a)$.
  - O(a) improvement of Heavy-Light quark bilinear

$$J_\Gamma^{(0)\mathrm{cont}} = Z_\Gamma \left(J_\Gamma^{(0)\mathrm{latt}} + c_\Gamma J_\Gamma^{(1)\mathrm{latt}} \right)$$

    - dim 3 operator: $J_\Gamma^{(0)} = \bar{h}\Gamma q$
    - dim 4 operator: $J_\Gamma^{(1)} = \bar{h}\Gamma(a\gamma_i \vec{D}_i)q$ (EOMs are used.)
- Significant effect for f_B .

$O(a)$ improvement of HQET operator

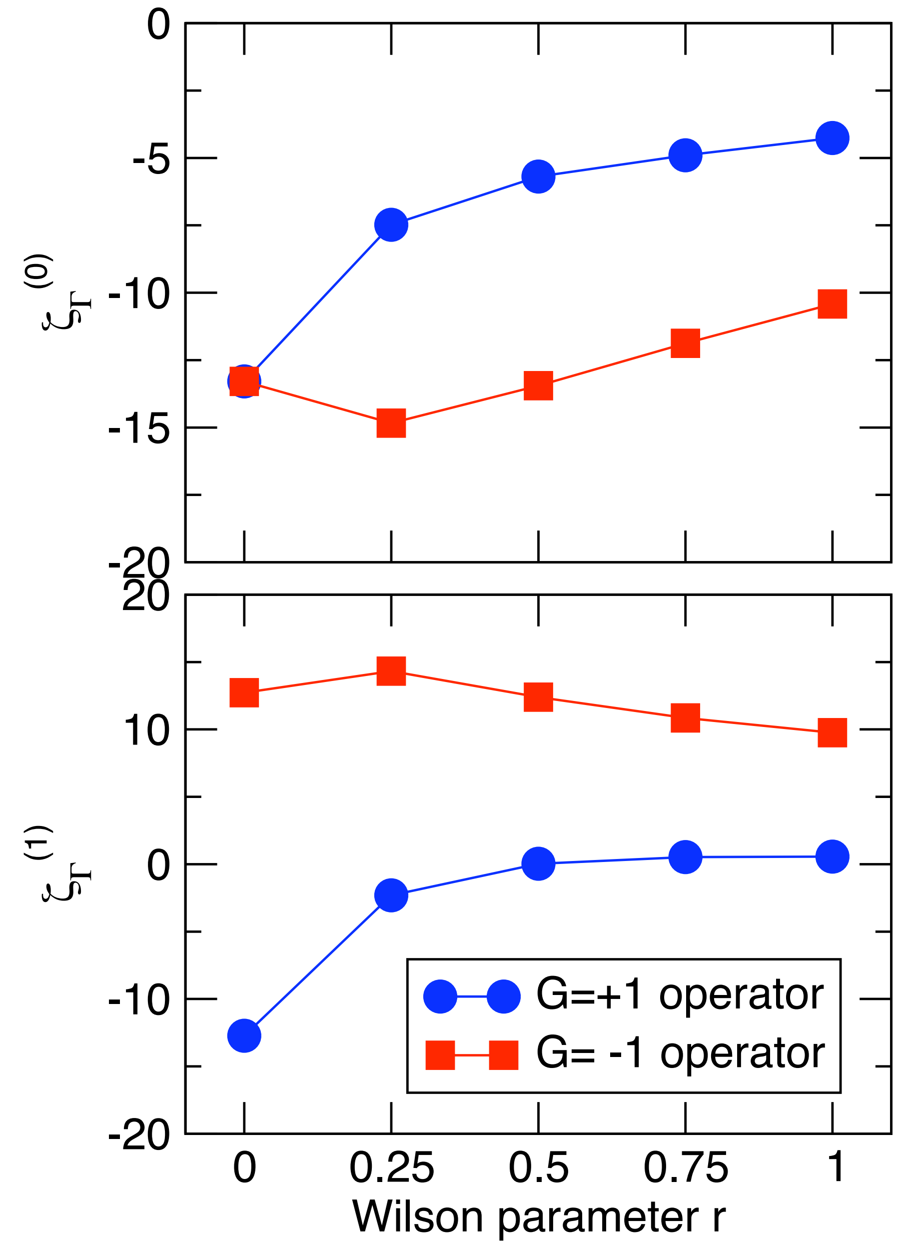
- $O(\alpha_s a)$ matching (tree level improved clover case)

continuum HQET ⟷ lattice HQET

$$J_\Gamma^{(0)\mathrm{cont}} = Z_\Gamma \left(J_\Gamma^{(0)\mathrm{latt}} + c_\Gamma J_\Gamma^{(1)\mathrm{latt}} \right)$$

$$Z_\Gamma = 1 + \frac{\alpha_s}{4\pi} C_F \zeta_\Gamma^{(0)}, \quad c_\Gamma = \frac{\alpha_s}{4\pi} C_F \zeta_\Gamma^{(1)}$$

Ishikawa et.al. ('98)



$$\Gamma\gamma_0 = G\gamma_0\Gamma$$

- For $J_\Gamma^{(0)} = A_0 \ (G=-1)$, O(a) is large at any r .

⇨ Effect for f_B is large.

- Even for chirally symmetric case ($r=0$), O(a) of operator exists.

⟷ light-light case

Our action setup

- HQET(static) for b quark
- Smeared link in HQET action
  - Noise reduction ——— reduction of tadpole contribution [ALPHA]
  - Choice of the smearing: APE, HYP1 and HYP2
- DWF for light quarks
  - 5 dimensional formulation
  - controllable approximated chiral symmetry
  - Operator mixing is quite reduced.
- Iwasaki for gauge action
- dynamical Nf=2+1

On-shell matching of quark bilinear

Matching procedure

continuum full QCD theory (CFQCD)

$O(\alpha_s)$ (Eichiten and Hill (1990))

continuum HQET (CHQET)

$O(\alpha_s)$ DWF+Iwasaki (Loktik and Izubuchi (2007))
DWF+Iwasaki+APE, HYP
(Dumitrescu and Flynn (2007))

$O(\alpha_s a)$ DWF+Iwasaki+APE, HYP

lattice HQET (LHQET)

- Light quarks are assumed as massless quarks.
- Matching is performed by comparing momentum expansion of on-shell scattering amplitude on continuum HQET and lattice HQET.
- MF improvement

On-shell matching of quark bilinear

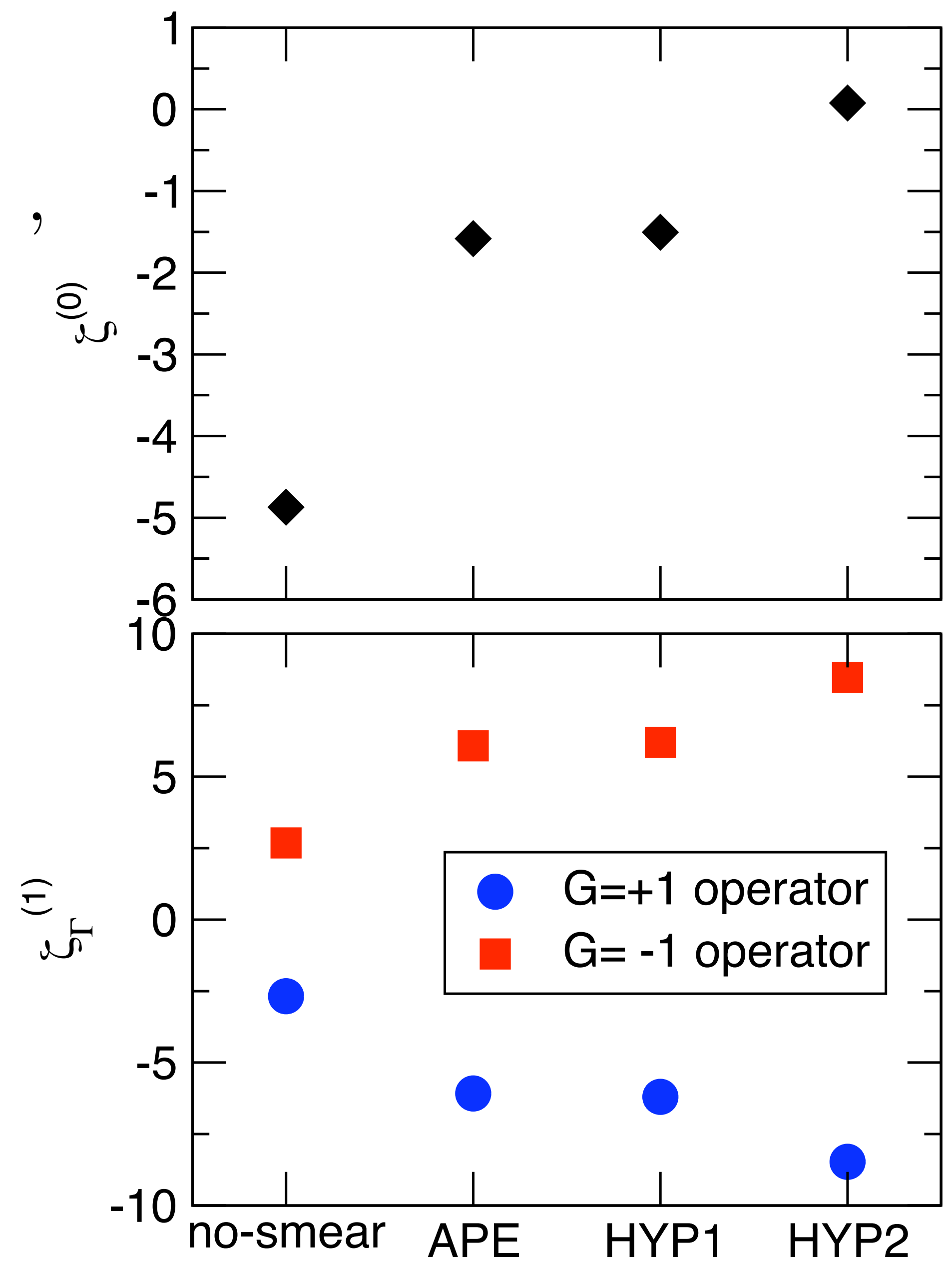
$$J_\Gamma^{(0)\mathrm{CHQET}} = \underbrace{(1-w_0^2)^{-1/2} Z_w^{-1/2}}_{\text{DW specific factor}} Z_\Gamma \left(J_\Gamma^{(0)\mathrm{LHQET}} + c_\Gamma J_\Gamma^{(0)\mathrm{LHQET}} \right)$$

$$Z_\Gamma = 1 + \frac{\alpha_s}{4\pi} C_F \left[\frac{3}{2} \ln\left(a^2\mu^2\right) + \zeta^{(0)} \right],$$

$$c_\Gamma = \frac{\alpha_s}{4\pi} C_F \zeta_\Gamma^{(1)}$$

- $\zeta^{(0)}$ becomes significantly close to 0 by smearing.

- O(a) improve coefficient is increased by smearing.



Effects for f_B

Another form of improvement terms

- We can rewrite the dim 4 operator using EOMs.

$$J_\Gamma^{(1)} = \bar{h}\Gamma(a\gamma_i \vec{D}_i)q \xrightarrow[\text{EOMs}]{} -Ga\partial_0\left(\bar{h}\Gamma q\right) = -Ga\partial_0 J_\Gamma^{(0)}$$

- 2-pt correlator for O(a) improved operators

$$\langle J_\Gamma^{\text{imp}}(t) J_\Gamma^{(0)}(0)\rangle \longrightarrow (1 + c_\Gamma G m_B^* a)\langle J_\Gamma^{(0)}(t) J_\Gamma^{(0)}(0)\rangle$$

f_B

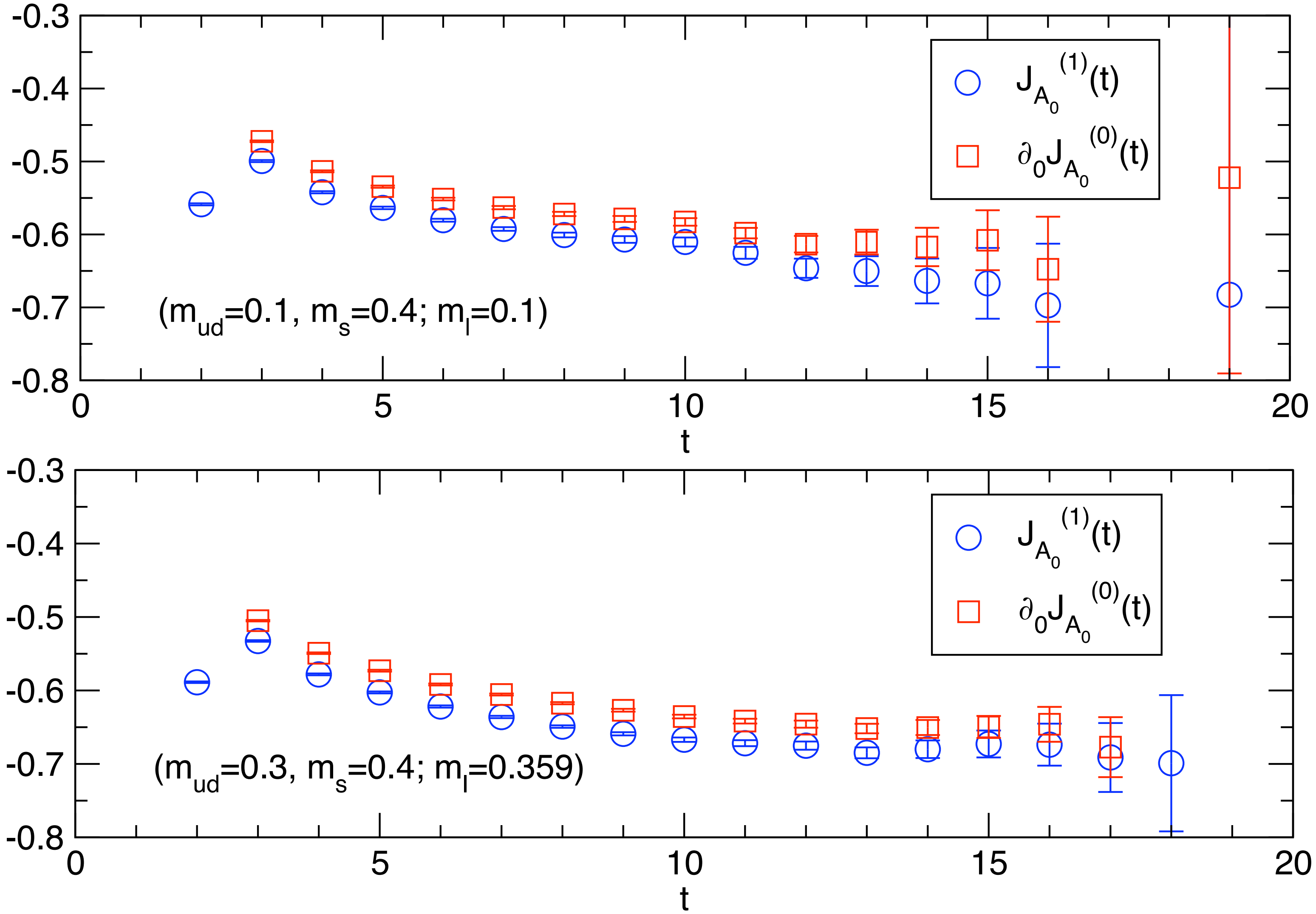
$$f_B \longrightarrow (1 - c_A \hat{m}_B^*) f_B$$

- O(a) improvement is established using unphysical mass $\hat{m}_B^*$

Effects for f_B

$$\frac{\langle J^{(1)}_{A_0}(t) J^{(0)}_{A_0}(0)\rangle}{\langle J^{(0)}_{A_0}(t) J^{(0)}_{A_0}(0)\rangle} \quad \text{VS} \quad \frac{\langle \hat{\partial}_0 J^{(0)}_{A_0}(t) J^{(0)}_{A_0}(0)\rangle}{\langle J^{(0)}_{A_0}(t) J^{(0)}_{A_0}(0)\rangle}$$

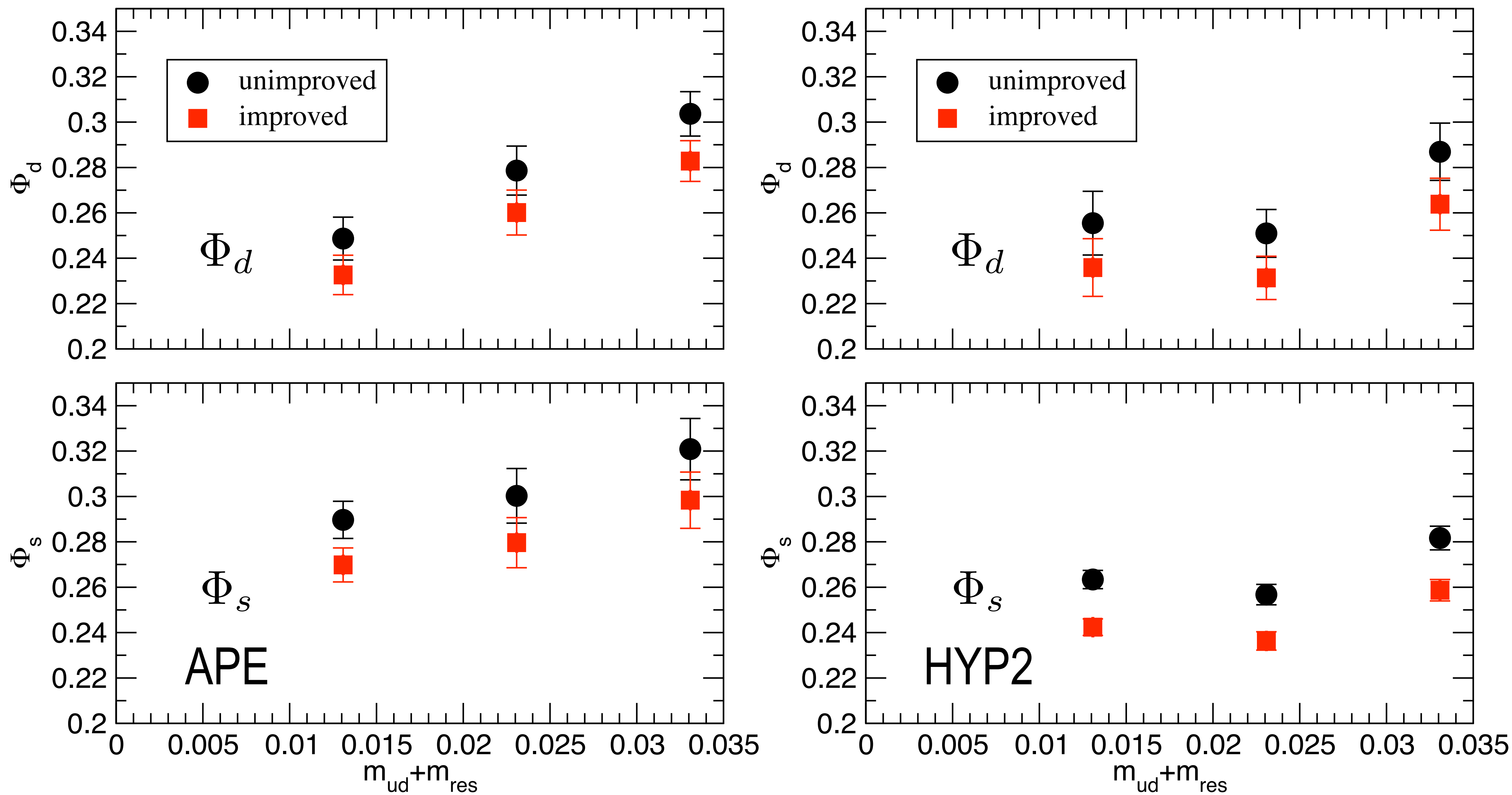
$L^3 \times T \times L_s = 16^3 \times 32 \times 16$ APE



Around 5% discrepancy

Effects for f_B

$$\Phi_q = f_{B_q}\sqrt{m_{B_q}}$$



Improved f_B is about 10% lower than unimoroved one.

On-shell matching of $\Delta B = 2$ operator

$$O_L^{CHQET} = \underbrace{(1-w_0^2)^{-1} Z_w^{-1}}_{\text{DW specific factor}} \; Z_L \left(O_L^{\rm LHQET} + c_L O_{ND}^{\rm LHQET} \right)$$

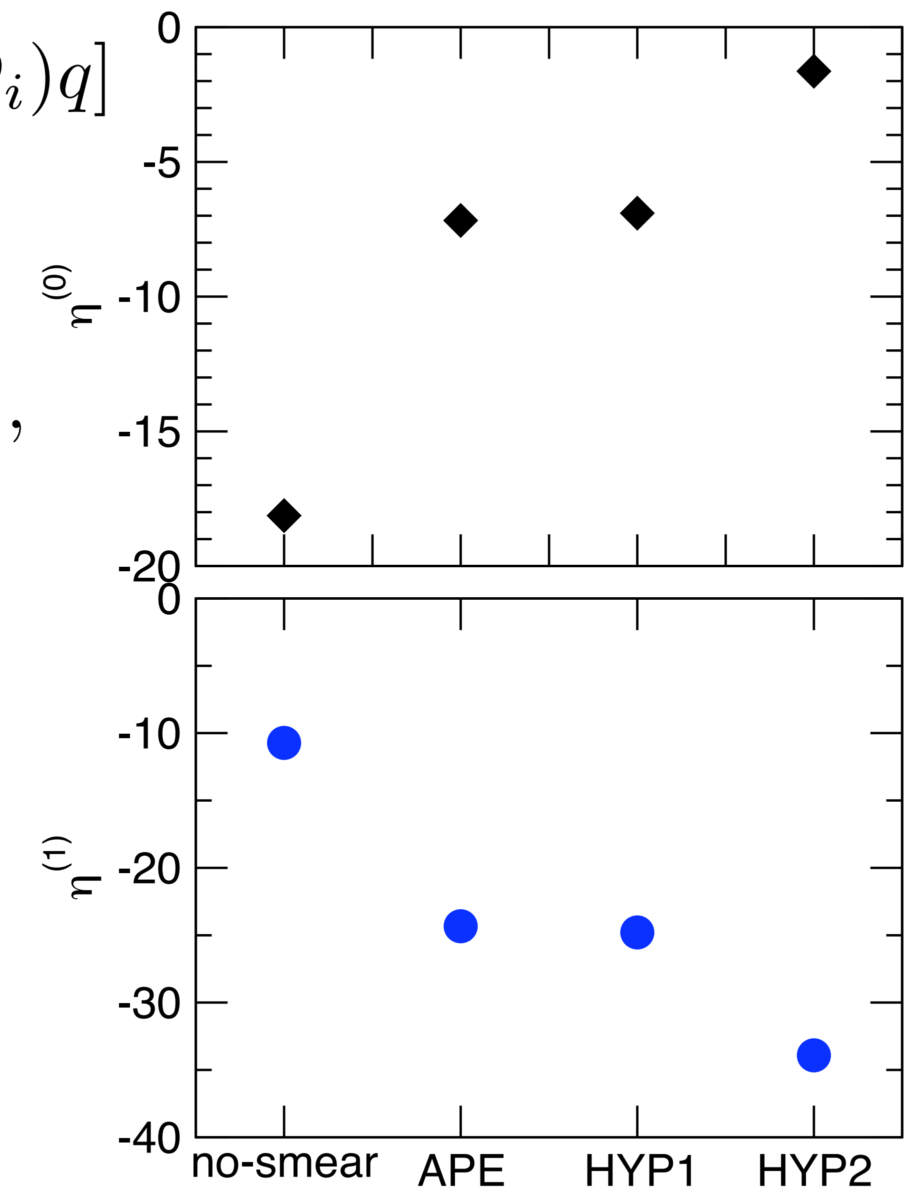
$$O_L^{\rm HQET} = 2[\bar{h}^{(+)}\gamma_\mu P_L q][\bar{h}^{(-)}\gamma_\mu P_L q]$$

$$O_{ND}^{\rm HQET} = 2[\bar{h}^{(+)}\gamma_\mu P_L q][\bar{h}^{(-)}\gamma_\mu P_R (a\gamma_i \vec{D}_i) q]$$
$$+4[\bar{h}^{(+)} P_L q][\bar{h}^{(-)} P_R (a\gamma_i \vec{D}_i) q]$$

$$Z_L = 1 + \frac{\alpha_s}{4\pi} C_F \left[4\ln\left(a^2\mu^2\right) + \eta^{(0)} \right],$$
$$c_L = \frac{\alpha_s}{4\pi} C_F \eta^{(1)}$$

- $\eta^{(0)}$ becomes significantly close to zero by smearing.

- O(a) improve coefficient is increased by smearing.



Effects for B_B

Another form of improve terms

- We can rewrite the operator O_{ND} using EOMs.

$$O_{ND} \xrightarrow[\text{EOMs}]{} -\frac{1}{2}a\partial_0 O_L$$

- 3-pt fincton for O(a) improved operator

$$\begin{aligned} & \langle \bar{A}_0(t_1) O_L^{\text{imp}}(t) A_0(0)^\dagger \rangle \\ \longrightarrow \quad & \langle \bar{A}_0(t_1) O_L(t) A_0(0)^\dagger \rangle - \frac{1}{2} c_L a \underbrace{\langle \bar{A}_0(t_1)(\partial_0 O_L(t)) A_0(0)^\dagger \rangle}_{=0} \end{aligned}$$

B_B

$$B_B = \frac{\langle \bar{B}|O_L|B\rangle}{\frac{8}{3}m_B^2 f_B^2} \longrightarrow (1 + 2c_A \hat{m}_B^*) B_B$$

Summary and Future plans

O(a) improvement of HQET operators

- MF improved 1-loop perturbation
- O(a) improvement is not negligible even for static heavy - DW light.
- If we use the link smearing, the improvement coefficient is larger.

Effects of the improvement for f_B, B_B

- f_B moves about 10% downward by the improvement.
- There is no effect for the matrix element.
- B_B moves about 20% upward by the improvement.

Future plans

- Precise analysis of f_B and B_B using $O(a)$ improvement
- NPT matching and $O(a)$ improvement coefficient (hopefully)