Gapless Dirac spectrum at high temperature

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Background

- Just above T_c $\langle\bar{\psi}\psi\rangle \neq 0$ if P-loop is complex

  (Chandrasekharan and Christ, hep-lat/9509095)

  $\Rightarrow$ Chiral symmetry is restored at T_c only if P-loop real

- Random matrix model $\Rightarrow$ Chiral symmetry restoration occurs
  - at higher T if P-loop complex for SU(3)
  - never if P-loop<0 for SU(2) (Stephanov, PLB375 (1996) 249)

- Lattice:
  - SU(3): in all P-loop sectors spectral gap appears at the same $T = T_c$ (Gattringer et al. PRD66 (2002) 054502)
  - SU(2): $\rho(0) \neq 0$ up to $T = 2T_c$ (Bornyakov et al. arXiv:0807.1980)

Qualitative picture

- In quenched SU(N) YM Polyakov-loop Z(N) symmetry spontaneously broken above T_c (deconfined phase).

- Chiral symmetry restoration above T_c depends strongly on the Polyakov-loop sector

- Banks-Casher:

$$\langle \bar{\psi}\psi \rangle = \pi\rho(0)$$

  chiral symmetry breaking $\Leftrightarrow$ Dirac operator spectral density at 0

- Experience:

  - $(-1) \times P$ closer to 1 $\Rightarrow$ more low Dirac modes

  - $(-1) \times P$ effective boundary condition for quarks

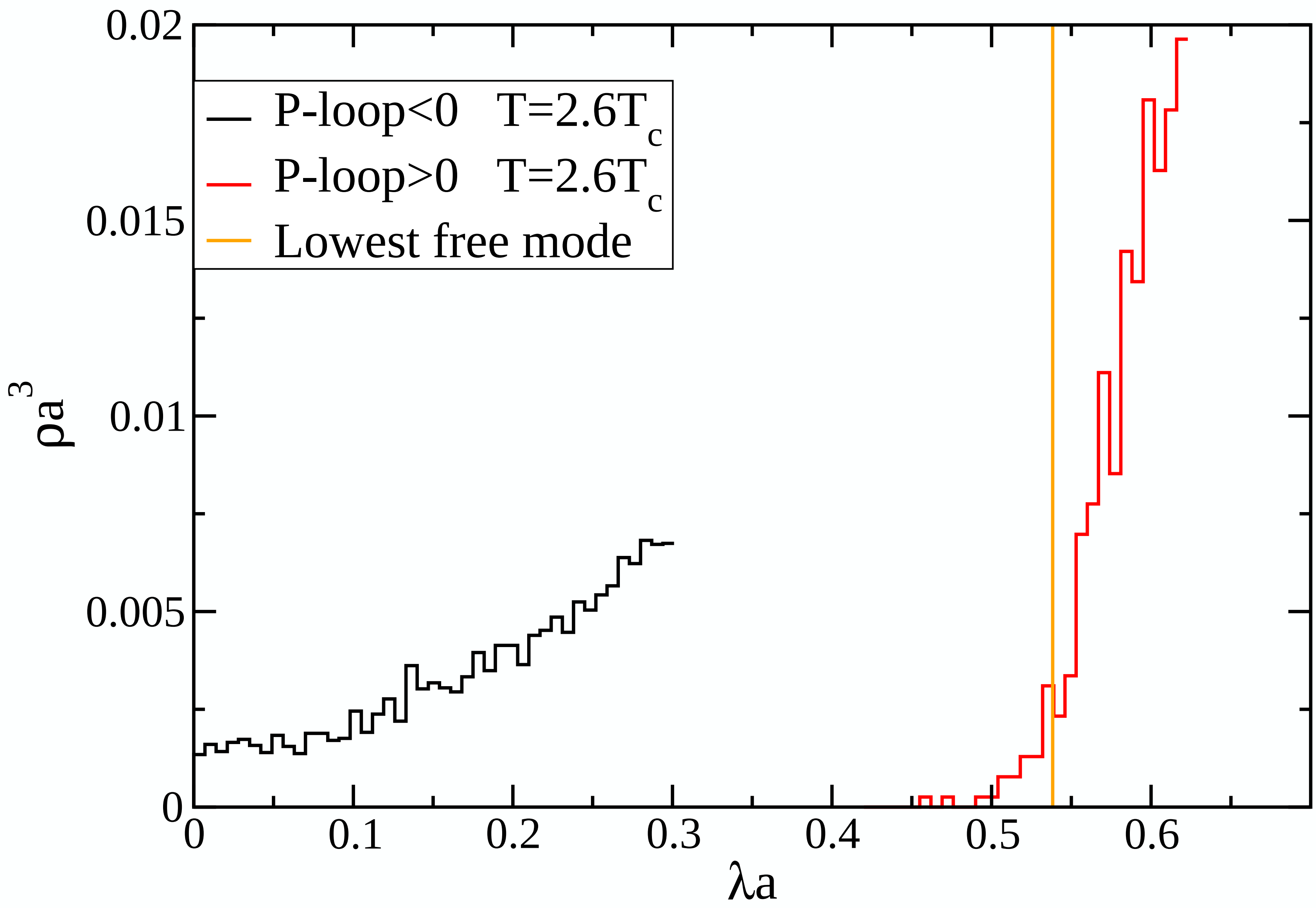
SU(2) further questions

- Does $\rho(0) \neq 0$ persist at arbitrarily high T in the P-loop<0 sector?
- Comparison of Dirac spectrum with random matrix theory (around and above T_c)
- Instantons $\Leftrightarrow$ $\rho(0) \neq 0$?
- How do dynamical fermions select the correct P-loop sector?

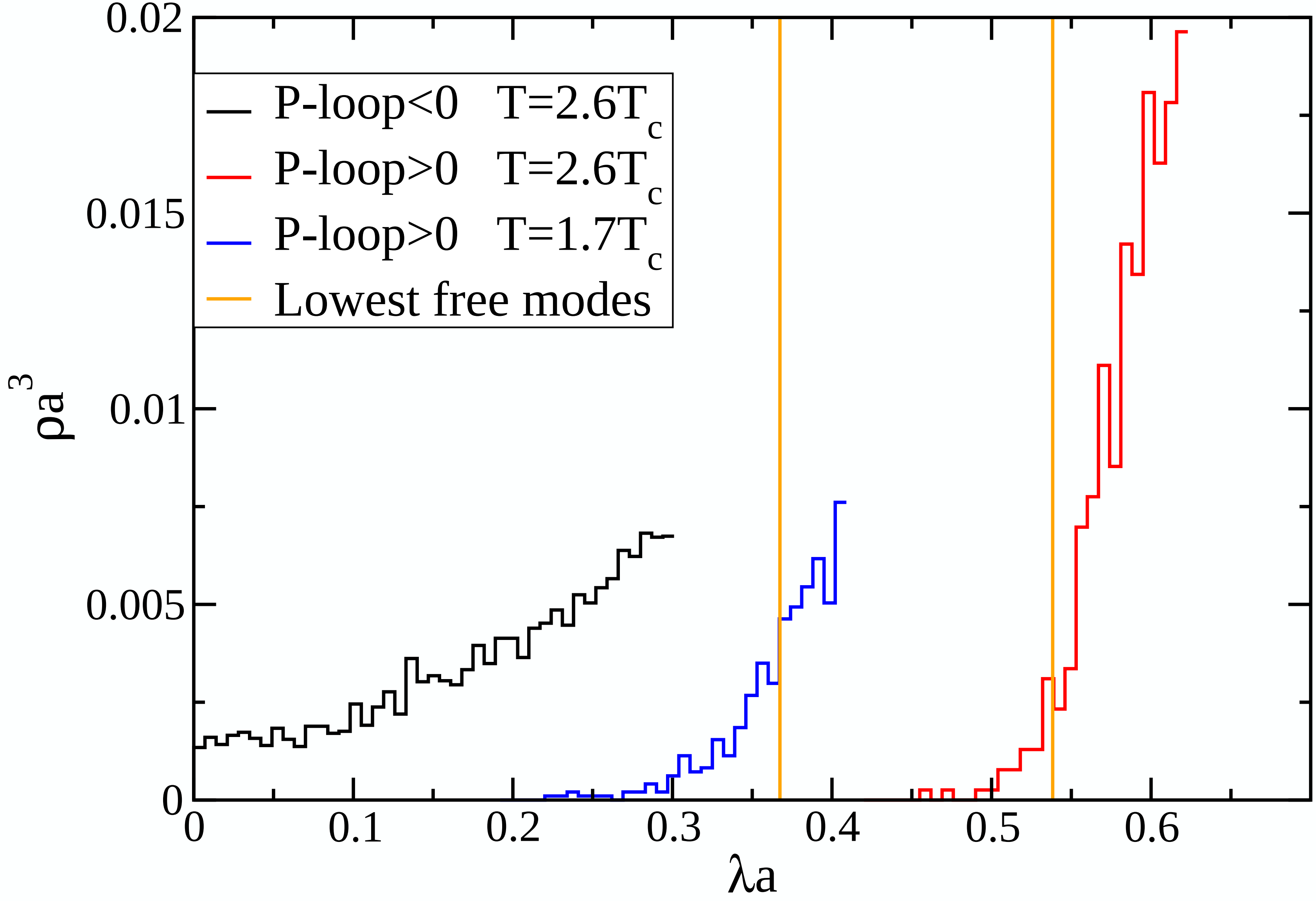
SU(2) simulation parameters

- All runs at quenched $\beta = 2.6$ (β_c for N_T=10.4)
- Vary N_T to change temperature
- $T = 2.6T_c \ (N_T = 4)$, $T = 1.7T_c \ (N_T = 6)$
- Spatial sizes: $N_S = 8, 10, 12, 16, 20$: $N_{T_c}/N_S = 0.52 - 1.30$
- Overlap Dirac operator
- Antiperiodic quark boundary condition in time

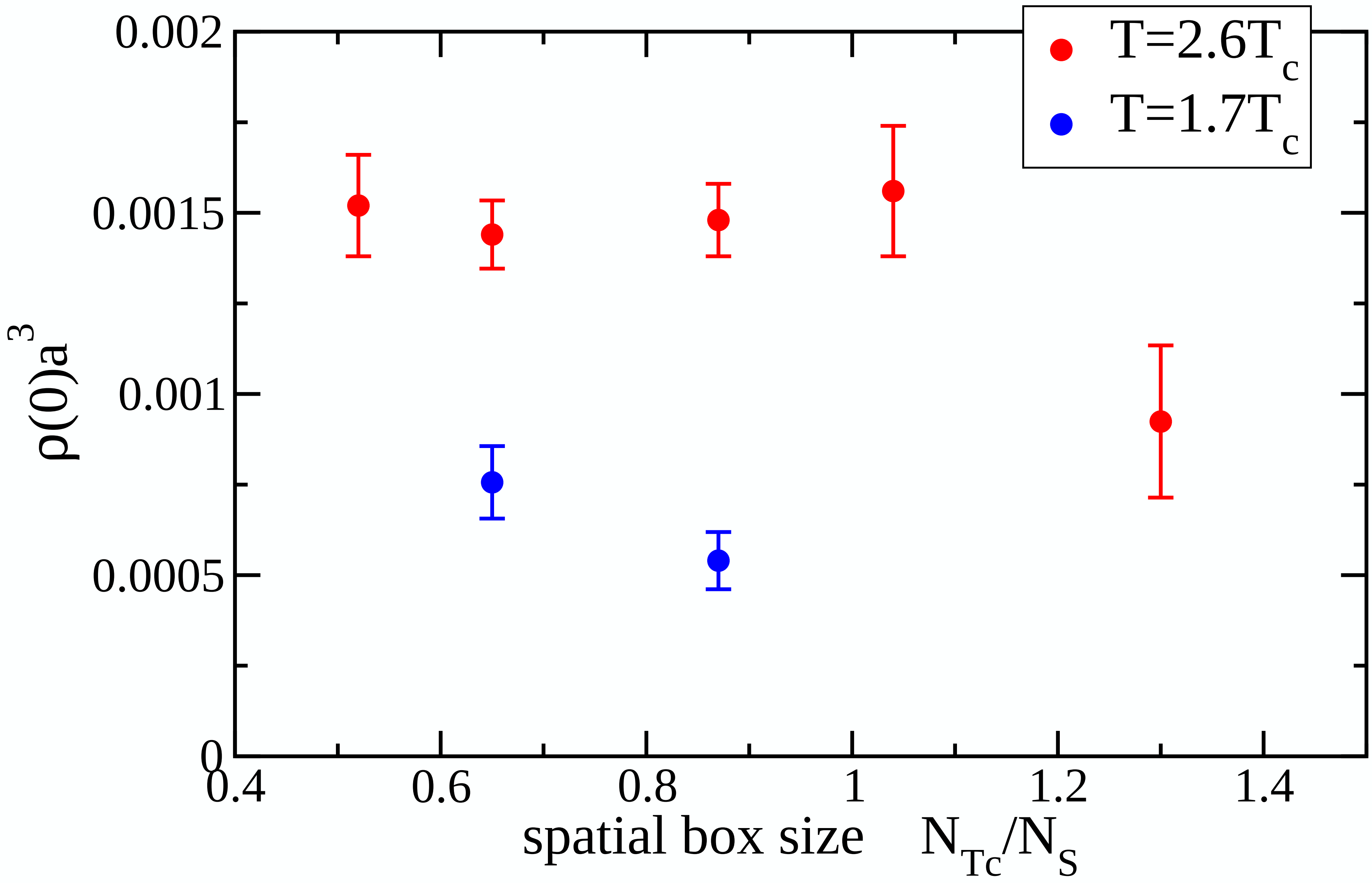
Density of low modes for different Polyakov loop sectors



Density of low modes for different Polyakov loop sectors

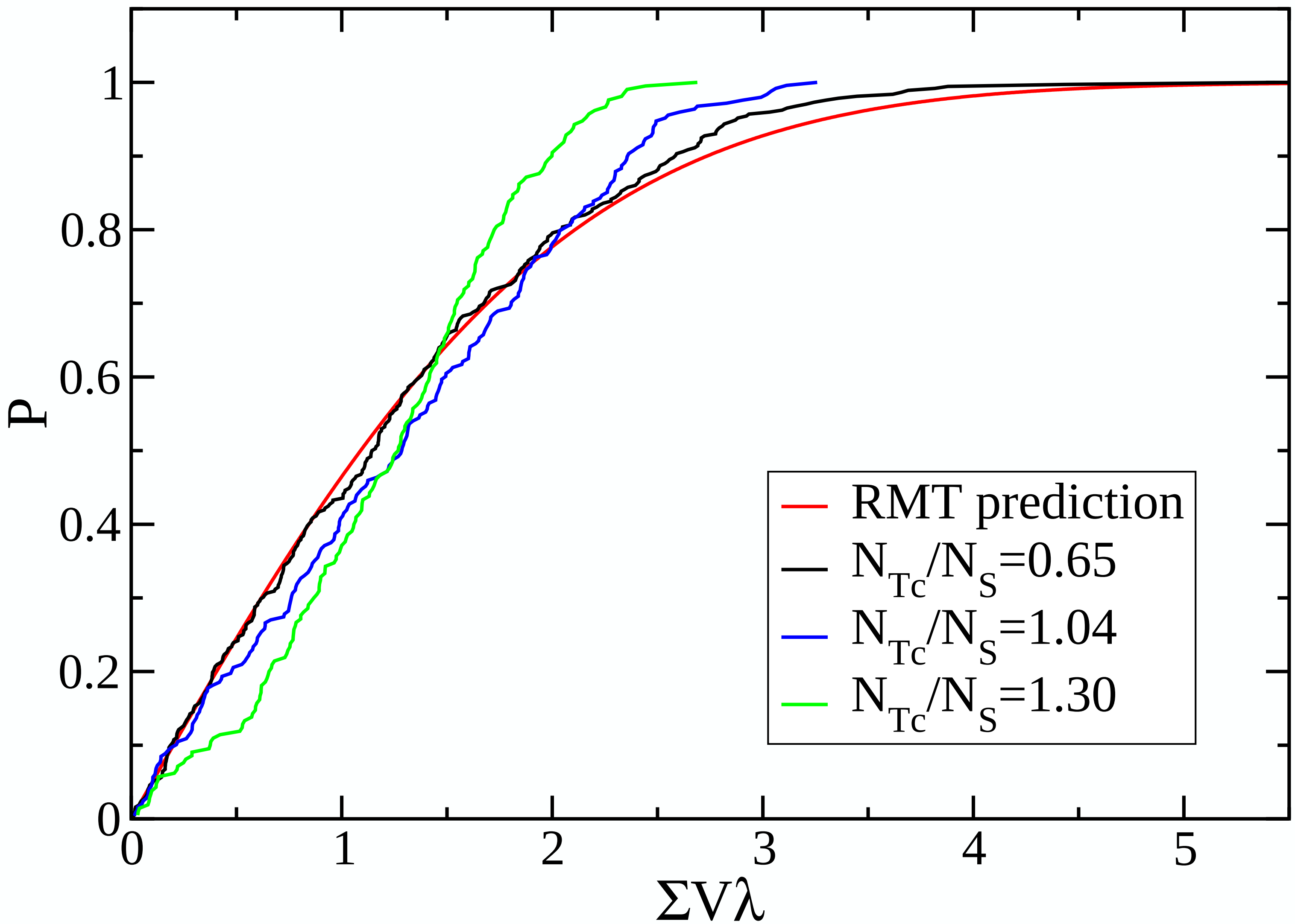


Density of modes at zero



Cumulative distribution of scaled smallest eigenvalues for Q=0

$T = 2.6T_c \quad \Sigma = \langle\bar{\psi}\psi\rangle$: best one-parameter fit to random matrix prediction



Possible role of instantons?

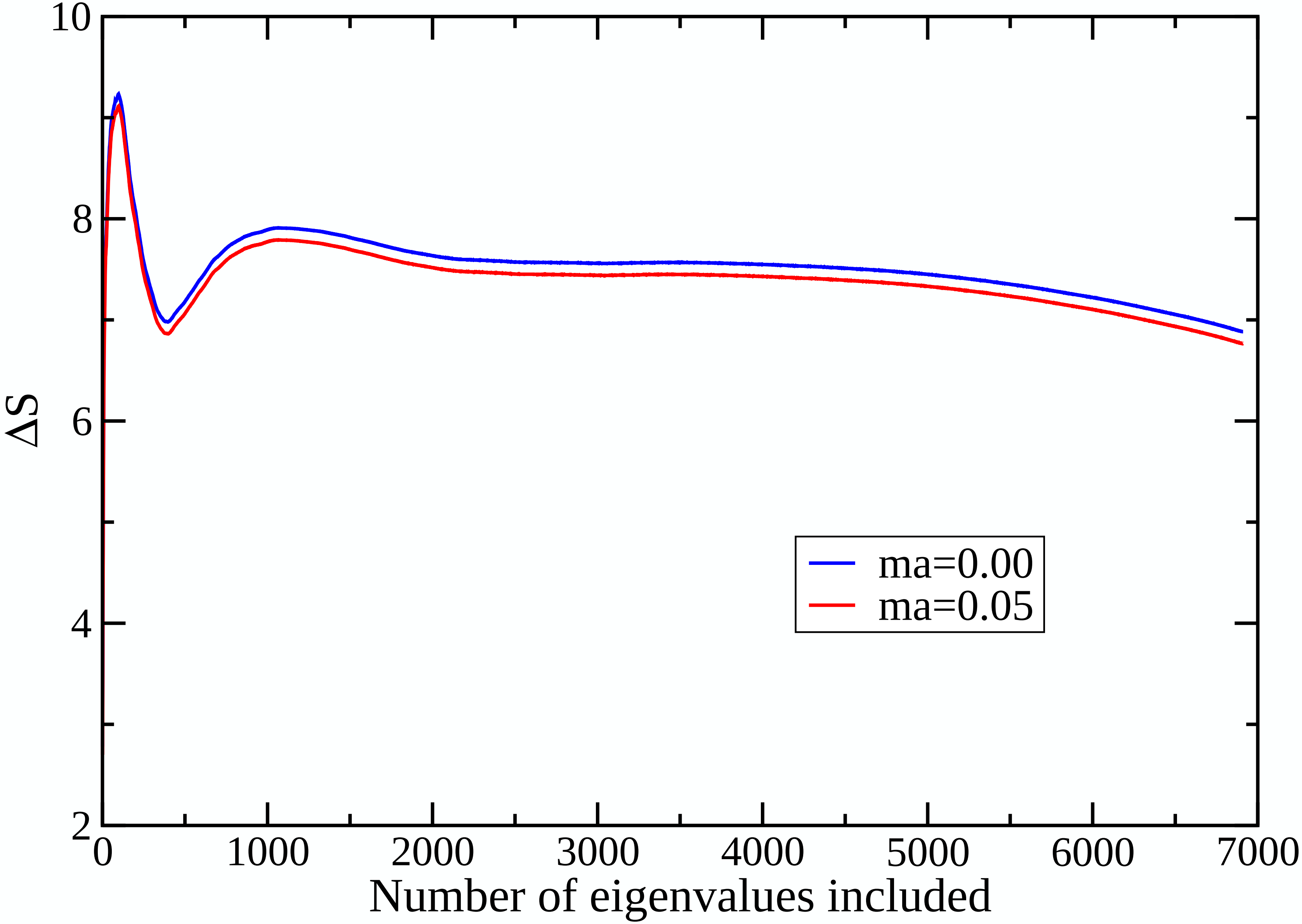
- Common wisdom: instanton-antiinstanton 0-modes $\Rightarrow \quad \rho(0) \neq 0$
- As temperature goes up:
  - Topological susceptibility drops (instantons "squeezed out")
  - $\rho(0) \approx \langle \bar{\psi}\psi \rangle$ increases
- $\Rightarrow$ At high T instantons cannot be responsible for $\rho(0) \neq 0$

Why is $\langle\bar{\psi}\psi\rangle = 0$ above T_c in the real world?

- Fermion determinant breaks P-loop $Z(N)$ symmetry
- Favors sector with the least number of low modes
- Effective boundary condition as far from periodic as possible
  - P-loop real for $SU(3)$
  - P-loop<0 for $SU(2)$
- Is it really only the low modes that matter?

Difference in fermion action between P-loop sectors

one quark flavor of mass m



Conclusions

- In quenched SU(2) above T_c chiral condensate has strong dependence on the P-loop average
  - If $\langle P \rangle > 0$ condensate vanishes at T_c
  - If $\langle P \rangle < 0$ condensate increases with T
- In the $\langle P \rangle < 0$ sector with chiral symmetry broken above T_c
  - Good agreement with random matrix theory
  - Topological charge fluctuations cannot account for low Dirac modes
- In the real world:
  - Fermion determinant suppresses “wrong” P-loop sector
  - Small fraction of lowest Dirac modes $(< 1\%)$ responsible for that
- Picture should be qualitatively similar for other Dirac operators and $SU(3)$