# Nucleon form factors from dynamical $N_f$=2+1 domain wall fermions


Takeshi YAMAZAKI

Yukawa Institute for Theoretical Physics

Y. Aoki, T. Blum, H-W. Lin, M. F. Lin, S. Ohta, S. Sasaki, R. J. Tweedie, and J. M. Zanotti

for RBC-UKQCD Collaborations


Refs. PoS(LAT2007):165(2007) and PRL100:171602(2008)


Lattice2008 @ College of William and Mary, Williamsburg, USA

July 14, 2008

# Outline

1. Introduction

2. Simulation parameters

3. Results

   - $g_A/g_V$
   - Axial vector and induced pseudoscalar form factors
   - Vector and induced tensor form factors

4. Summary

Related talk : S. Ohta, nucleon structure functions, 7/18(Fri) 17:20

# 1. Introduction

Motivation :

understand nucleon physics from first principle

(lattice) QCD

We calculate matrix elements related to isovector form factors and moments of structure functions of nucleon on $N_f = 2 + 1$ domain wall fermion (DWF) configuration

(generated by RBC-UKQCD collaborations on QCDOC)

## Isovector form factors

- Vector and induced tensor form factors

(elastic proton-electron scattering)

$$\langle N,p|V_\mu(q)|N,p'\rangle \;=\; \overline{u}_N(p)\left(F_1(q^2)\gamma_\mu + i\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2M_N}\right)u_N(p')$$

$$q_\nu = p'_\nu - p_\nu$$

$F_1(q^2), F_2(q^2) \rightarrow$ $F_1(0) = F_1^p(0) - F_1^n(0) = 1$

$F_2(0) = \mu_p - \mu_n - 1$ ($\mu_i$ : magnetic moment)

$\langle r_1^2\rangle, \langle r_2^2\rangle$ related to charge radii $\langle r_p^2\rangle, \langle r_n^2\rangle$

- Axial vector and induced pseudoscalar form factors

($\beta$ decay; muon capture on proton; neutrino-nucleon scattering; pion electroproduction)

$$\langle N,p|A_\mu(q)|N,p'\rangle \;=\; \overline{u}_N(p)\left(G_A(q^2)i\gamma_5\gamma_\mu + i\gamma_5 q_\mu G_P(q^2)\right)u_N(p')$$

$G_A(q^2), G_P(q^2) \rightarrow$ $G_A(0)$ : axial charge, $\langle r_A^2\rangle$

$g_{\pi NN}$ : pion-nucleon coupling

$g_P$ : pseudoscalar coupling for muon capture

## Isovector form factors

Recent works: Alexandrou *et al* PRD74:034508; PRD76:094511 ($N_f = 0, 2$ Wilson)

Göckeler *et al* PRD71:034508; PoS(LAT2007)161($N_f = 0, 2$ Wilson)

Hägler *et al* arXiv:0705.4295 ($N_f = 2 + 1$ Mixed action)

Sasaki and TY arXiv:0709.3150 ($N_f = 0$ DWF)

Lin *et al* arXiv:0802.0863 ($N_f = 2$ DWF)

| | valence | sea | $N_f$ | L[fm] | $m_\pi >$[GeV] |
|---|---|---|---|---|---|
| This work | DWF | DWF | 2+1 | 2.7(1.8) | 0.33 |
| Alexandrou | Wilson | — | 0 | 3.0 | 0.41 |
| Alexandrou | Wilson | Wilson | 2 | 1.9 | 0.38 |
| Göckeler | Clover | — | 0 | 1.7 | 0.55 |
| Göckeler | Clover | Clover | 2 | 2.0 | 0.35 |
| Hägler | DWF | Imp. staggered | 2+1 | 2.5(3.5) | 0.35 |
| Lin | DWF | DWF | 2 | 1.9 | 0.49 |
| Sasaki | DWF | — | 0 | 3.6 | 0.39 |

DWF has good chiral symmetry on lattice. It is advantage to calculate nucleon matrix element, especially axial charge.

Our calculation is carried out with $N_f = 2 + 1$ dynamical quark effect on relatively larger volume at lighter pion mass.

# 2. Simulation parameters

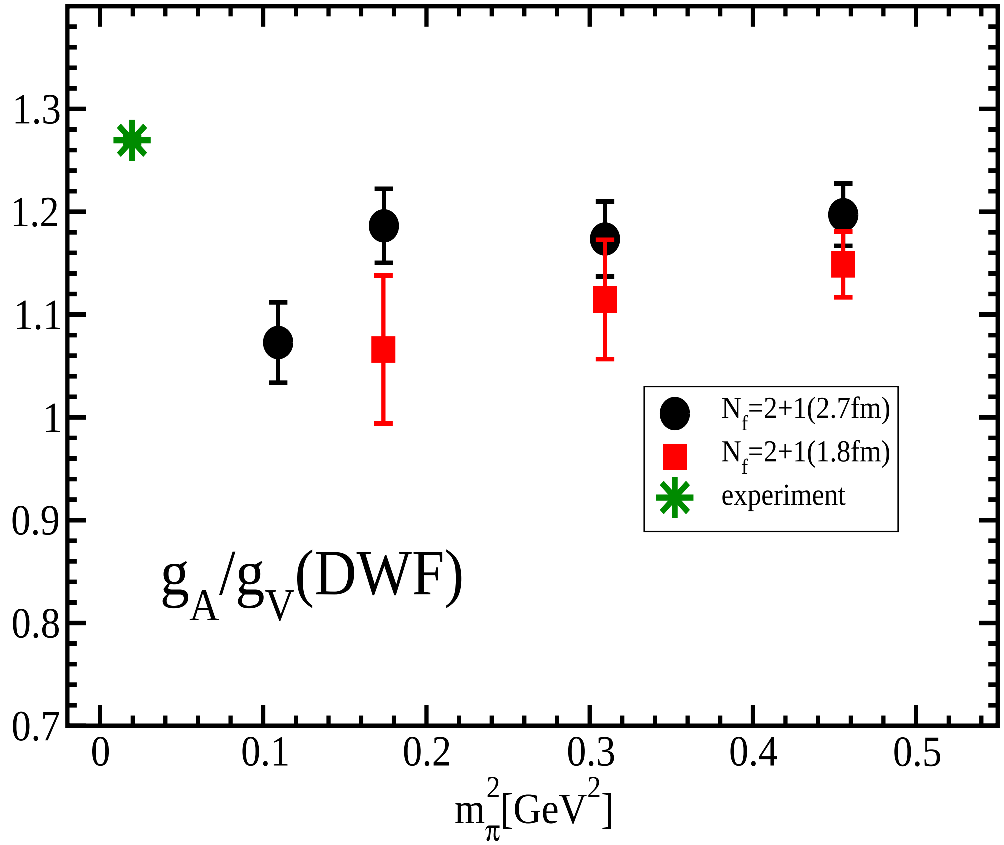
- $N_f = 2 + 1$ Iwasaki gauge + Domain Wall fermion actions
- $\beta = 2.13$ $a^{-1} = 1.73$ GeV $M_5 = 1.8$ $m_{\rm res} \approx 0.003$
- Lattice size $24^3 \times 64 \times 16$ ($La \approx 2.7$ fm)
  $16^3 \times 32 \times 16$ ($La \approx 1.8$ fm) for $g_A/g_V$
- $m_s = 0.04$ fixed (close to $m_s^{phys}$)
- quark masses $m_f = m_{sea} = m_{val}$ and confs.

| $m_f$ | $m_\pi$[MeV] | $m_N$[GeV] | # of confs. | $N_{meas}$ |
|---|---|---|---|---|
| 0.005 | 330 | 1.15 | 932 | 4 |
| 0.01 | 420 | 1.22 | 356 | 4 |
| 0.02 | 560 | 1.39 | 98 | 4 |
| 0.03 | 670 | 1.55 | 106 | 4 |

- We focus only on isovector quantities. (no disconnected diagram)
- Four different non-zero $q^2$ with $(pL/2\pi)^2 = 1, 2, 3, 4$
- Matrix elements are evaluated by ratio of 3- and 2-point functions.
- $t_{snk} - t_{src} = 12 \approx 1.37$ fm

# 3. Results

## 3.1. Axial charge $g_A/g_V = G_A(0)/F_1(0)$

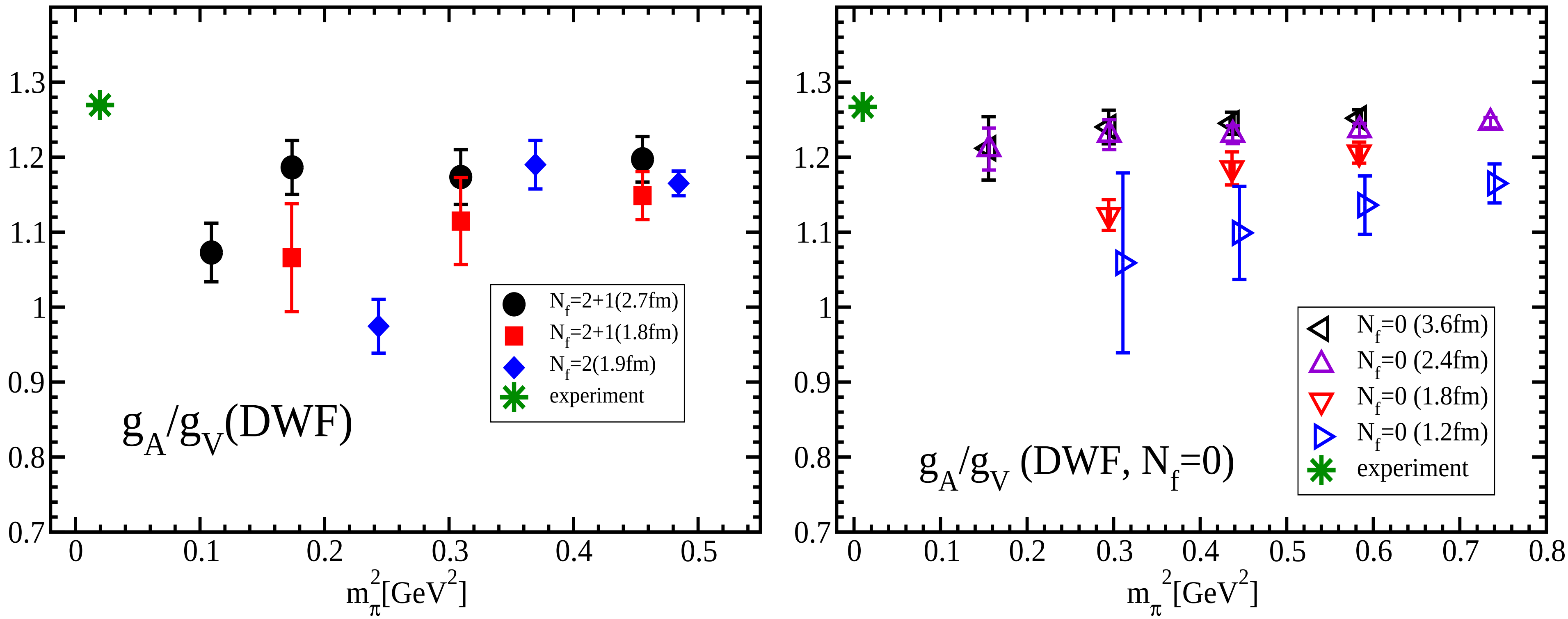



Heavier three data are almost independent of $m_\pi^2$, while lightest data is 9% smaller than other masses.

Smaller volume data are systematically below larger volume data.

(1.8 fm) data are calculated on $16^3 \times 32 \times 16$ with heavier three quark masses

# $g_A/g_V$ in DWF




Refs. $N_f = 0$ DWF PRD68:054509(RBCK), arXiv:0709.3150;

$N_f = 2$ DWF arXiv:0802.0863(RBC)

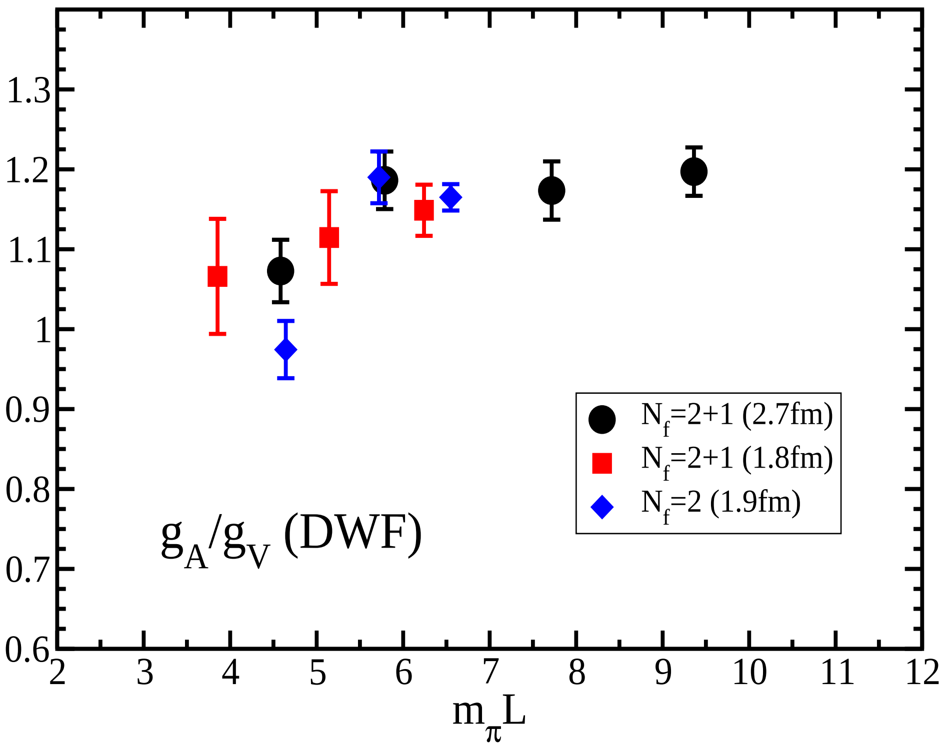
Similar behavior was seen in $N_f = 2$, but it sets in at heavier pion mass.

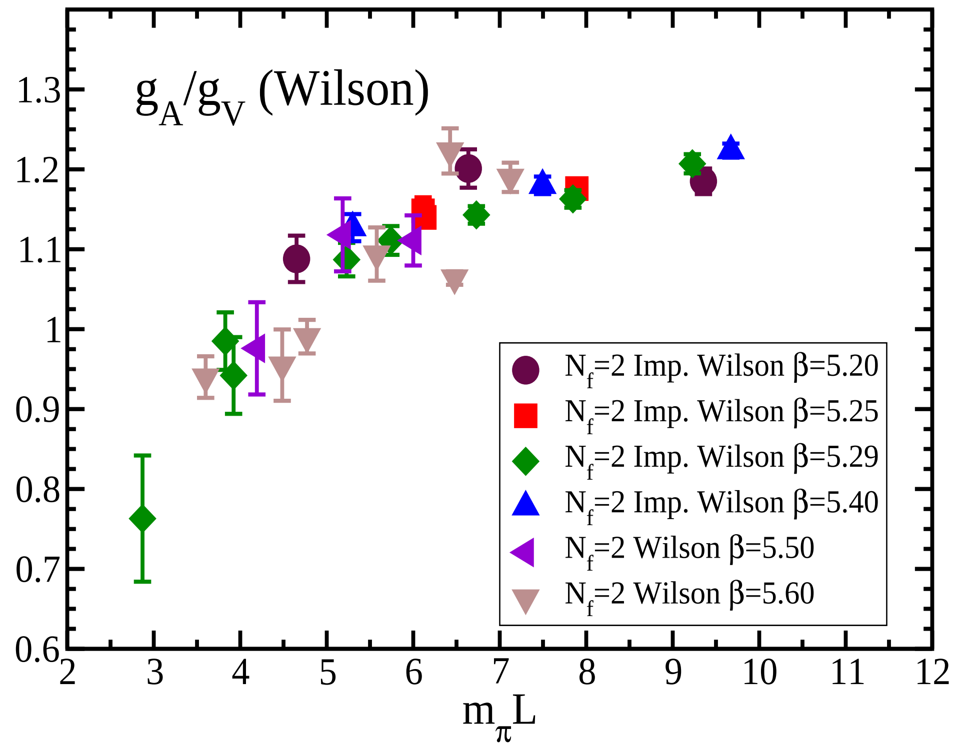
We suspect that downward behavior is caused by finite volume effect.

$m_\pi$ dependence of $N_f = 2+1$ is similar to $N_f = 0$ on $L < 2.4$ fm, which is caused by finite volume.

In $N_f = 0$ such a dependence disappears when $L > 2.4$ fm. Large finite volume effect is not expected on 2.7 fm.

# $m_\pi L$ scaling of $g_A/g_V$ in dynamical calculations




Wilson PRD66:034506(LHPC and SESAM); Wilson PRD76:094511

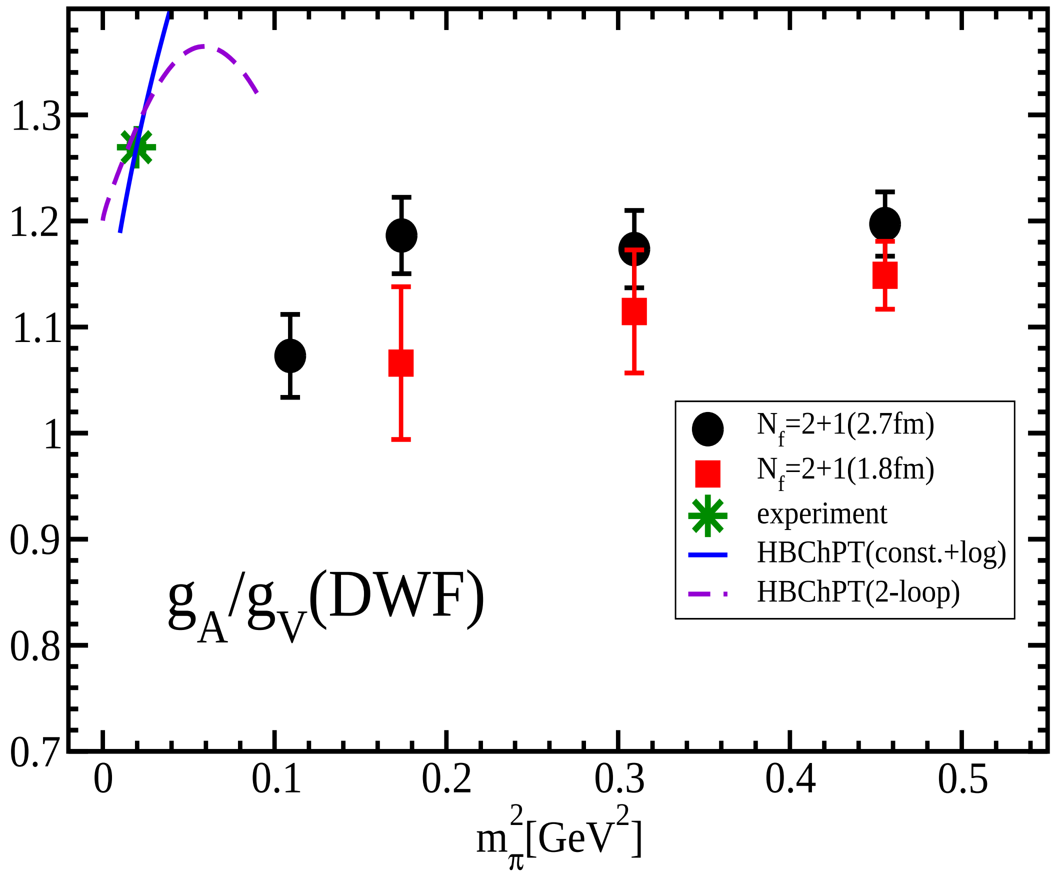
Refs. Imp. Wilson PRD74:094508(QCDSF)

$N_f = 2 + 1$ data on two volumes scale in $m_\pi L$.

Similar scaling is seen in two-flavor (Imp.) Wilson fermion calculations with various $m_\pi = 0.38$–$1.18$ GeV, $V = (0.95$–$2.0$ fm$)^3$, and $\beta$.

This observation is used for chiral extrapolation of $g_A/g_V$.

# Chiral extrapolations of $g_A/g_V$



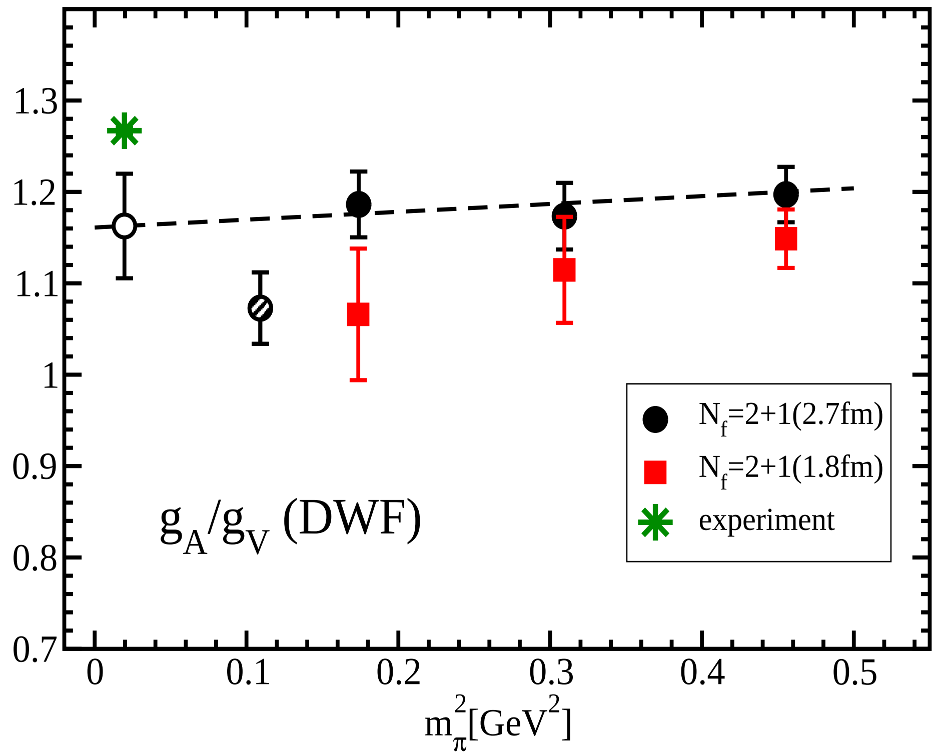

We did not use heavy baryon chiral perturbation theory(HBChPT) formula for chiral extrapolation.

Our $m_\pi$ is beyond the region where HBChPT is valid even at two-loop order, $m_\pi < 300$ MeV. Bernard and Meißner PLB639:278

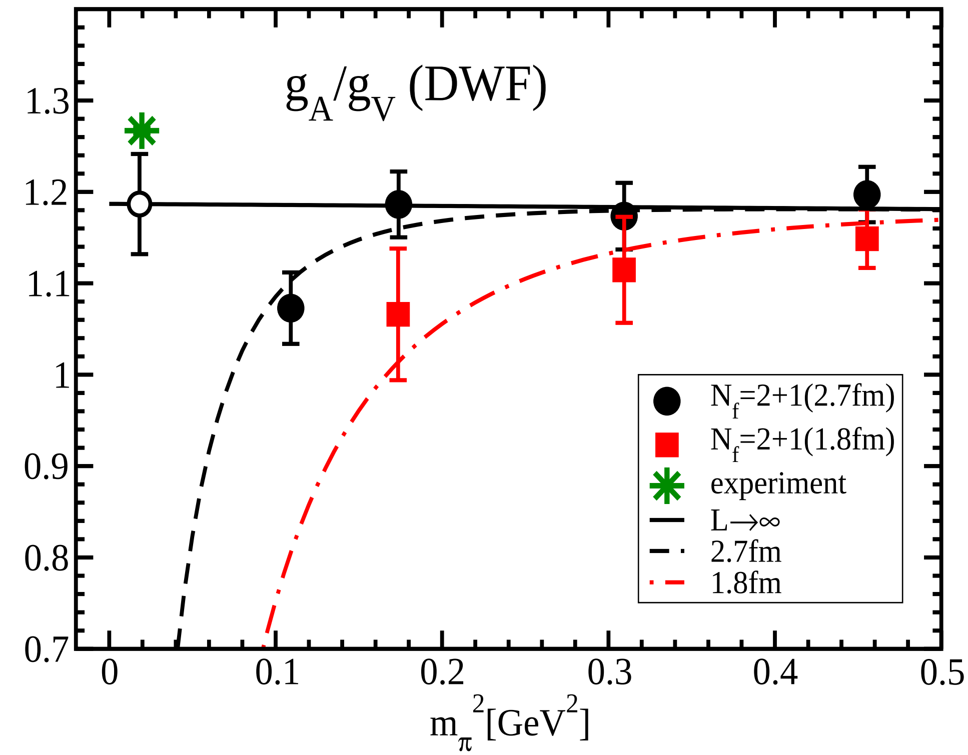
Most works employed HBChPT with $\Delta$ baryon for chiral extrapolation, but estimated finite volume effect is less than 1% at lightest point.

# Chiral extrapolations of $g_A/g_V$ (cont'd)

linear




linear$+f_V(m_\pi L)(=e^{-m_\pi L})$



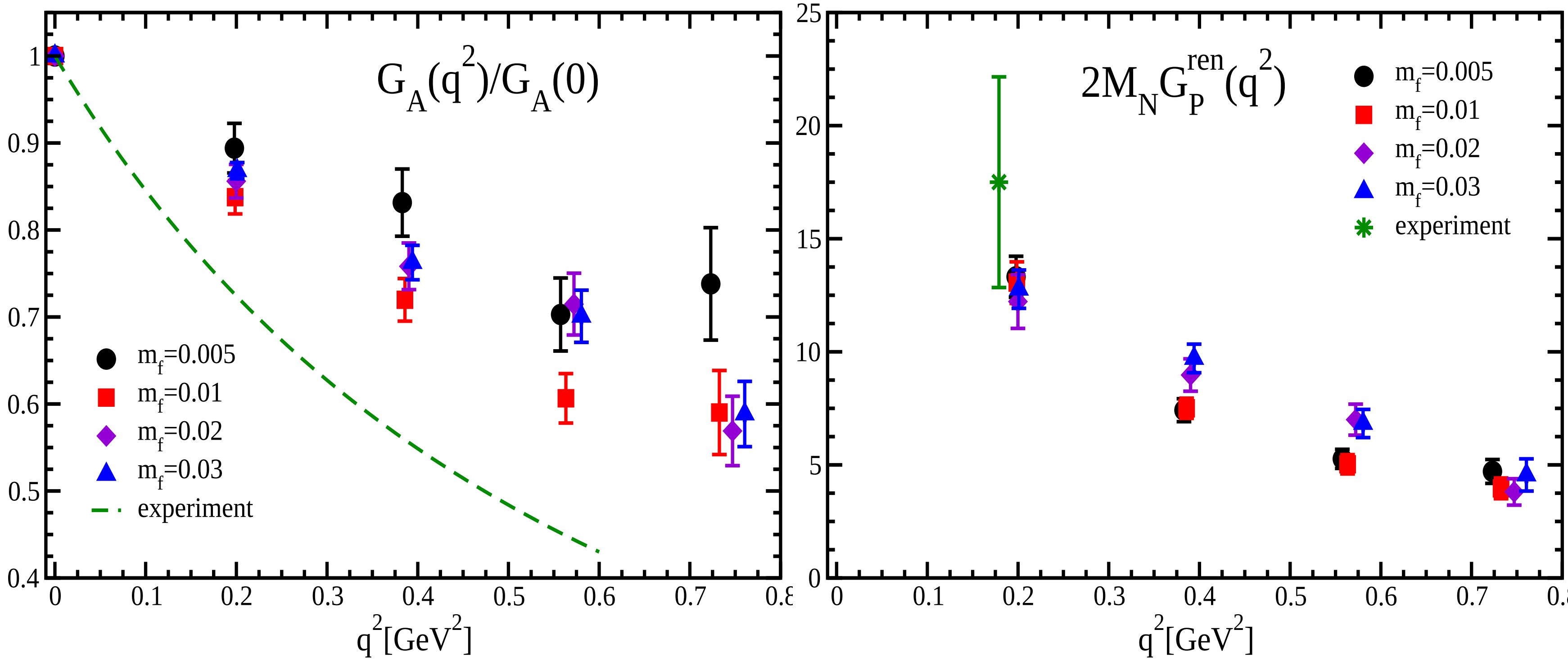

1.17(6) at $m_\pi = 135$ MeV

1.20(6)(4) at $m_\pi = 135$ MeV

Second error is systematic determined from different choice of $f_V$, such as $x^{-3}$, $x^{1/2}e^{-x}$, and $m_\pi^2 e^{-x}/x^{1/2}$ with $x = m_\pi L$.

From fit with $f_V$, we estimate that one needs $L \approx$ 3.5–4.5 fm ($m_\pi L \approx$ 6–8) to aim finite volume effect being below 1% at $m_\pi = 330$ MeV.

# 3.2. Axial vector and induced pseudoscalar form factors




$G_P(q^2)$ is renormalized by $Z_V = 1/F_1(0) \approx Z_A$.

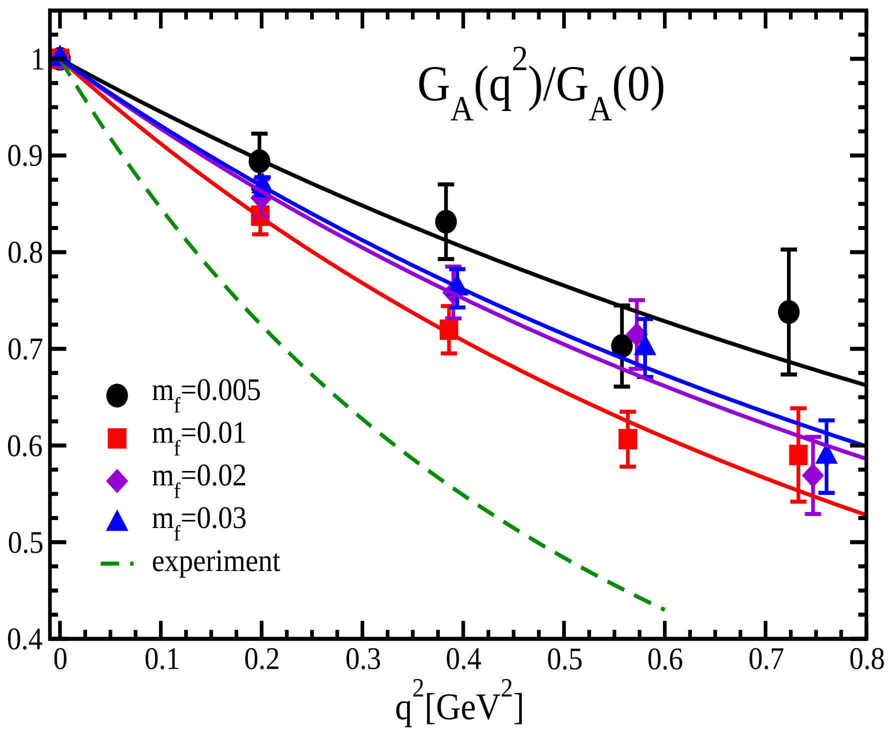
$m_f$ dependence of $G_A$ is strange, and not monotonic function of $m_f$.
Lightest data is larger than heavier mass data.

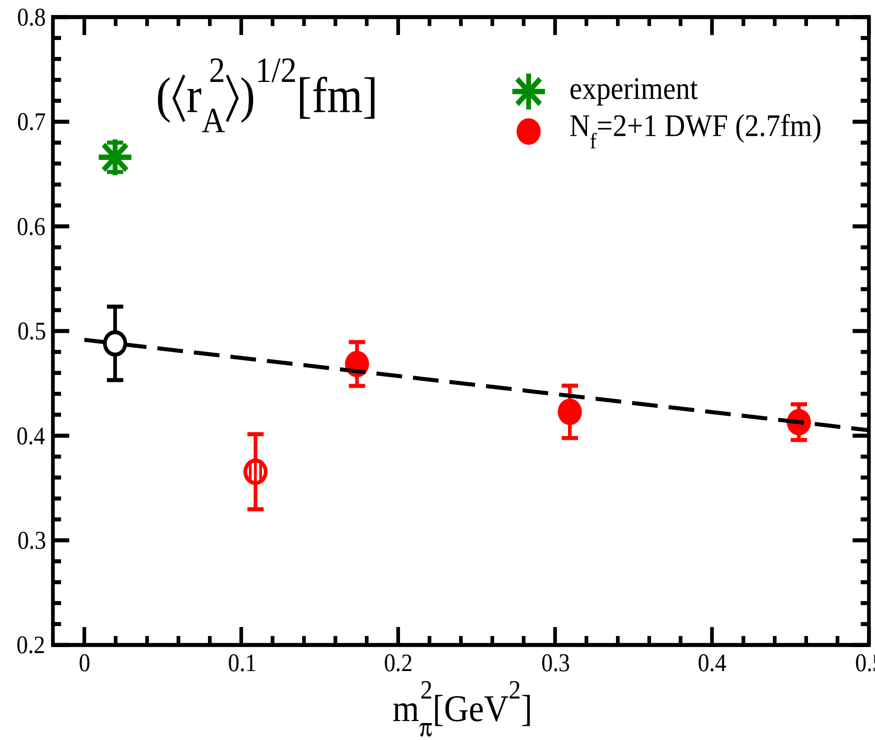
# Dipole fit of $G_A(q^2)$ and Axial charge rms radius $\sqrt{\langle r_A^2 \rangle}$

1 parameter fit $\langle r_A^2 \rangle$

$$\frac{G_A(q^2)}{G_A(0)} = \frac{1}{(1 + \langle r_A^2 \rangle q^2/12)^2}$$

$$\sqrt{\langle r_A^2 \rangle}$$

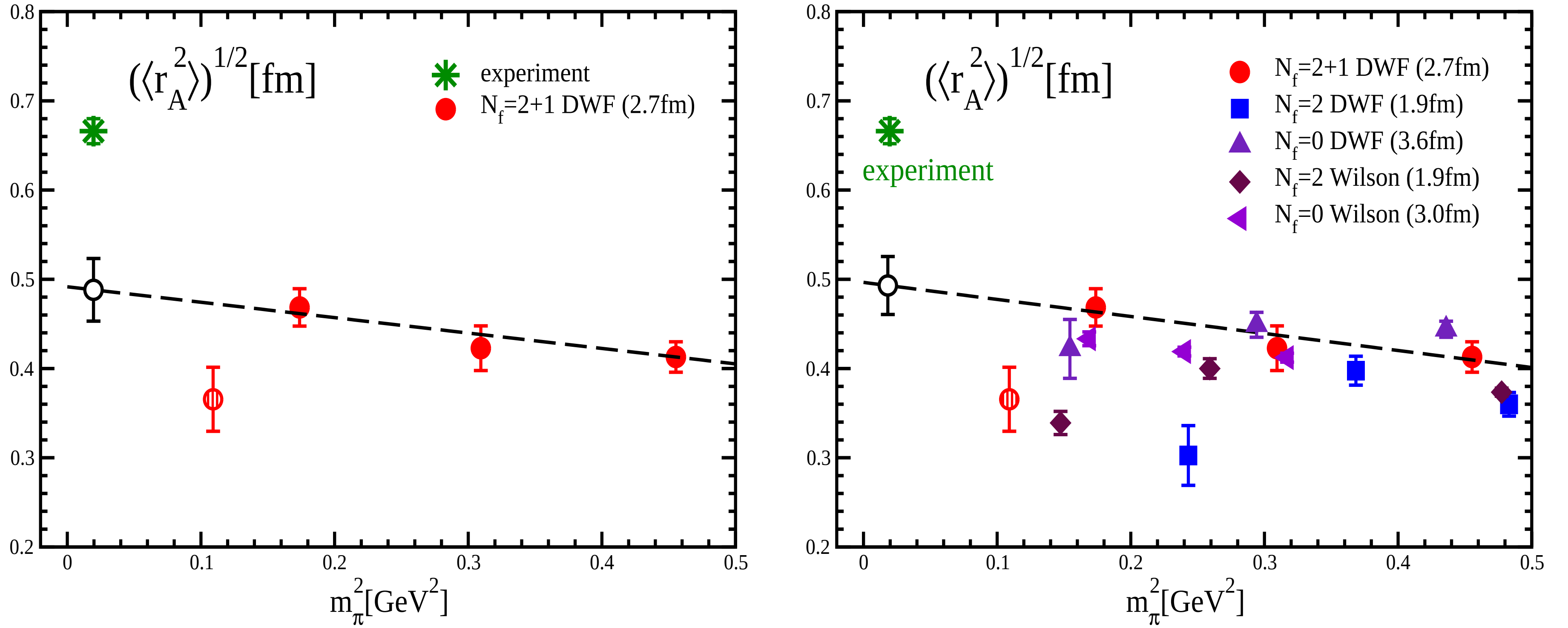



Lightest result of $\sqrt{\langle r_A^2 \rangle}$ is smaller than other mass points, and goes away from experiment.

This $m_\pi^2$ dependence is similar to one in $g_A/g_V$

# Axial charge rms radius $\sqrt{\langle r_A^2 \rangle}$

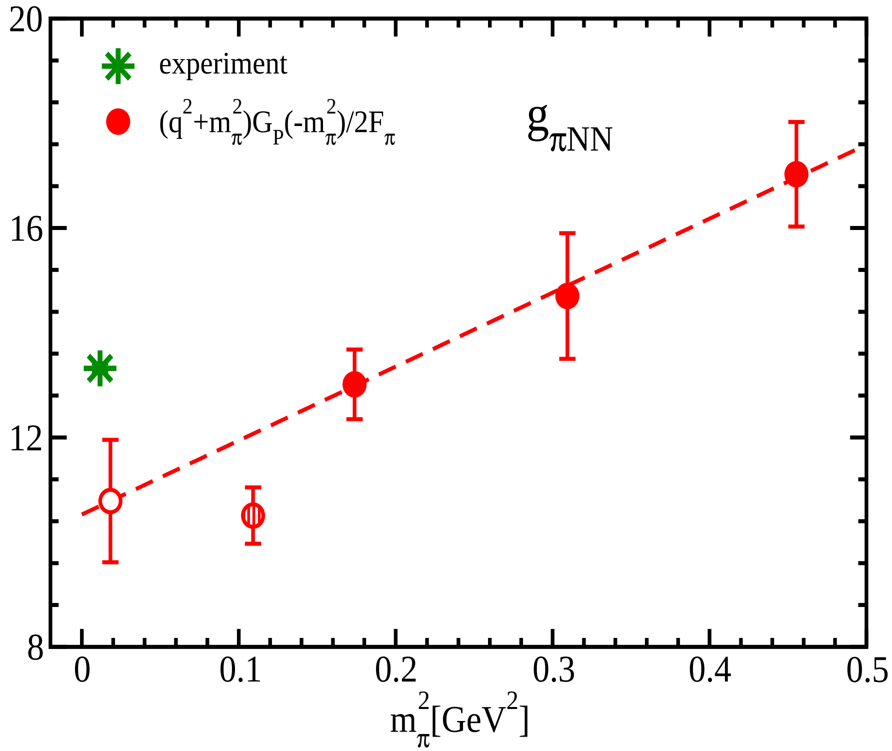


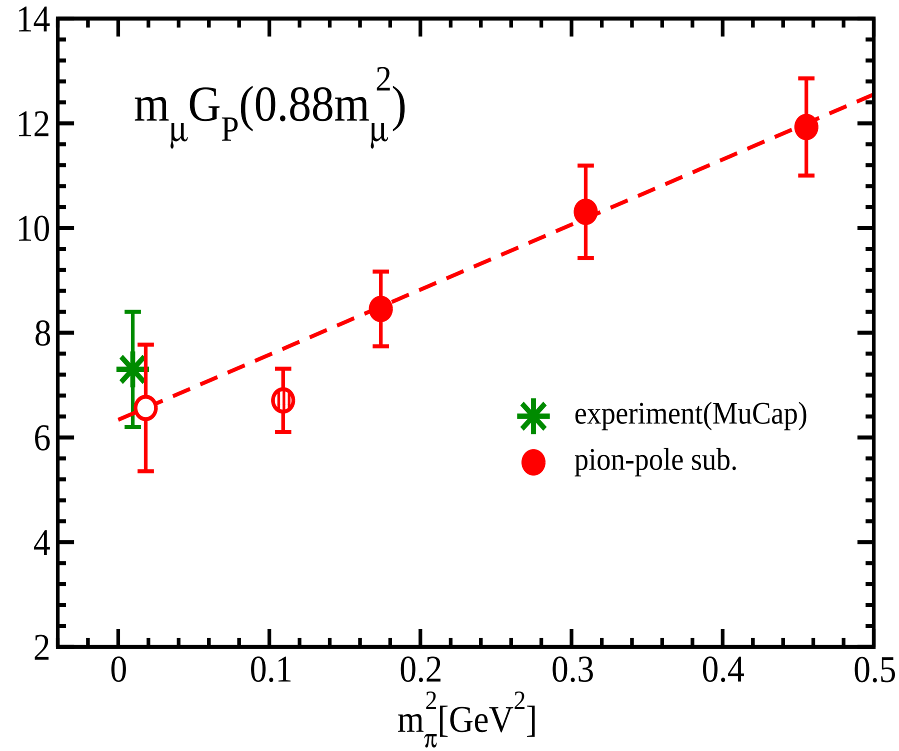

$N_f = 0, 2$ Wilson PRD74:034508; $N_f = 0$ DWF arXiv:0709.3150

Similar trend is seen in $N_f = 2$ data on (1.9 fm)$^3$ volume, but the downward behavior sets in at heavier $m_\pi$ as in $g_A/g_V$.

This would indicate large finite volume effect at lightest point.

## $g_{\pi NN}$ coupling and $g_P$ for muon capture




$g_{\pi NN}$ is evaluated by definition of $g_{\pi NN}$ with $G_P(q^2)$.

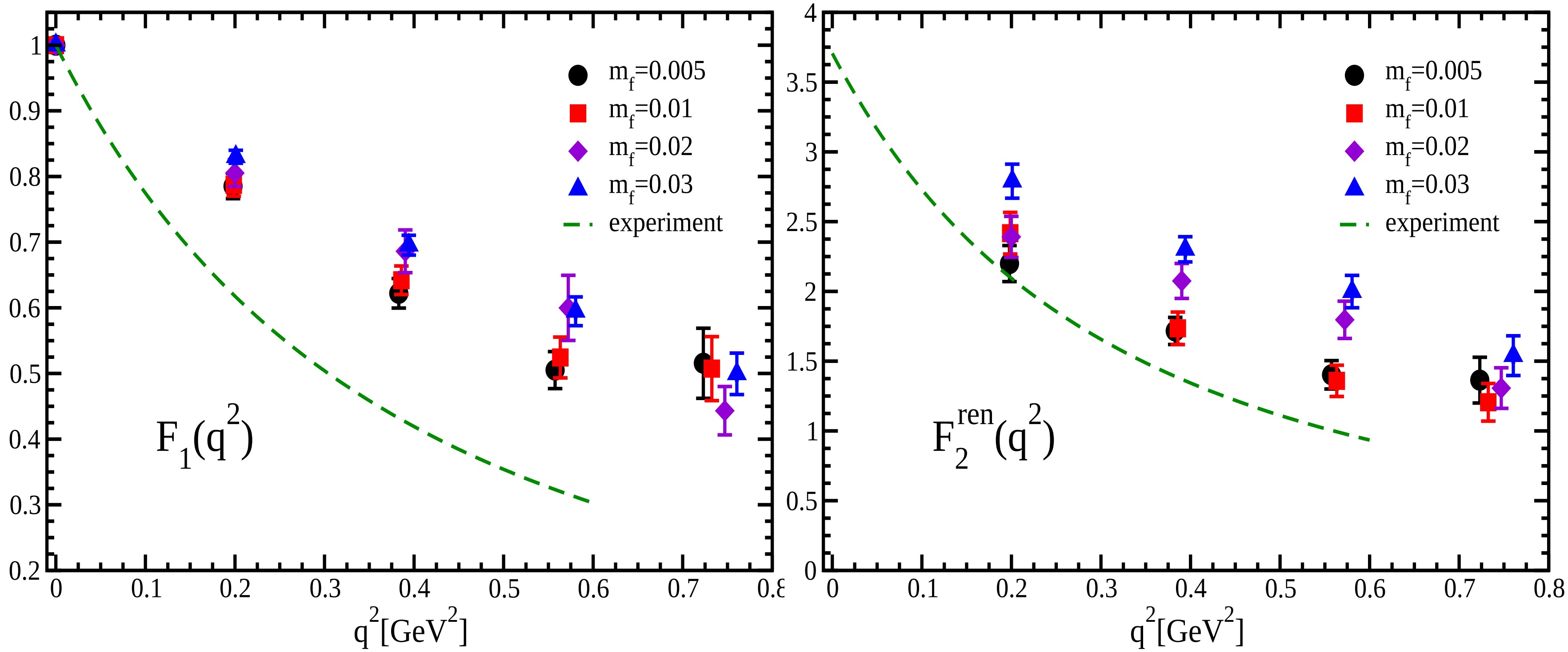
$g_P$ is determined with assumption that $G_P(q^2)$ has pion-pole.

Results at $m_\pi^{\mathrm{phys}}$ reasonably agree with experiments.

Lightest data are smaller than linear fit line using heavier data.

$G_A$ and $G_P$ are sensitive to finite volume.

## 3.3 Vector and induced tensor form factors



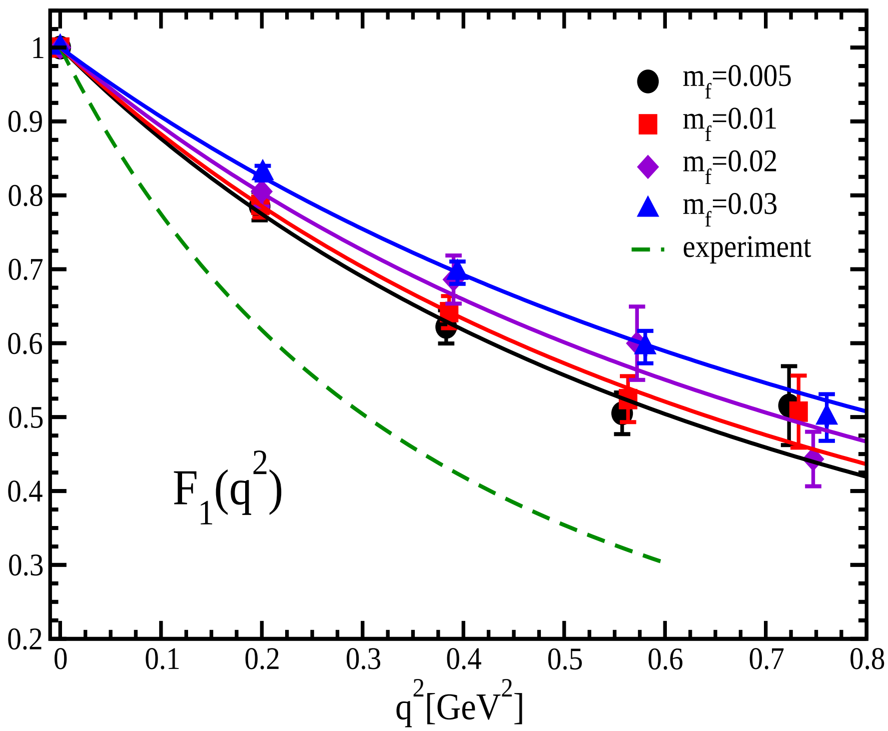

renormalized by $Z_V = 1/F_1(0)$

There is no strange, non-monotonic behavior in the form factors.

# Dipole fit of form factors

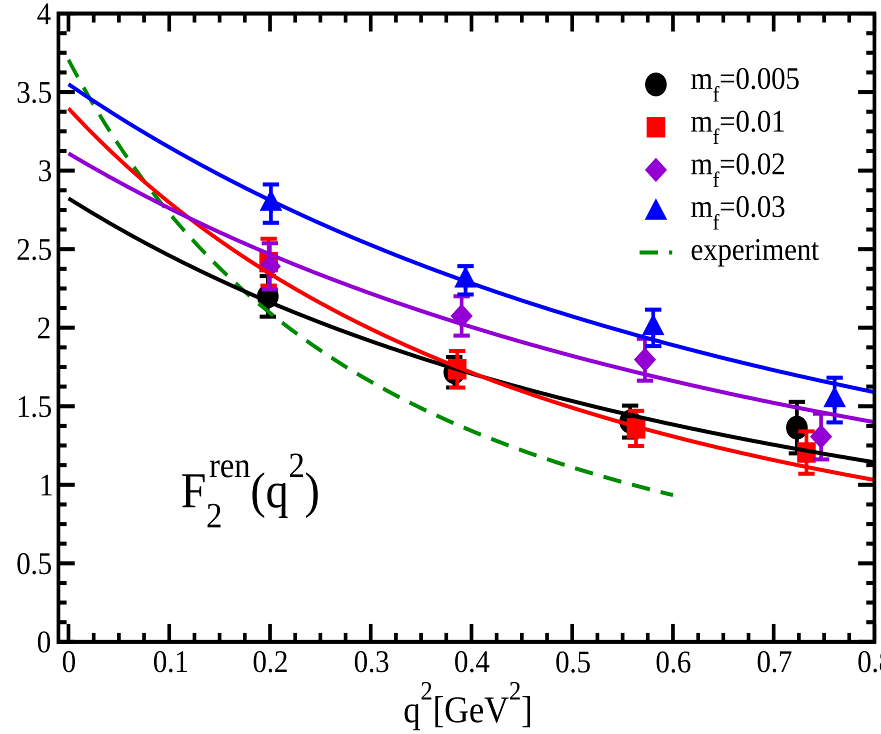
1 parameter fit $\langle r_1^2 \rangle$

$$\frac{1}{(1+\langle r_1^2\rangle q^2/12)^2}$$




2 parameters fit $F_2(0)$ and $\langle r_2^2 \rangle$

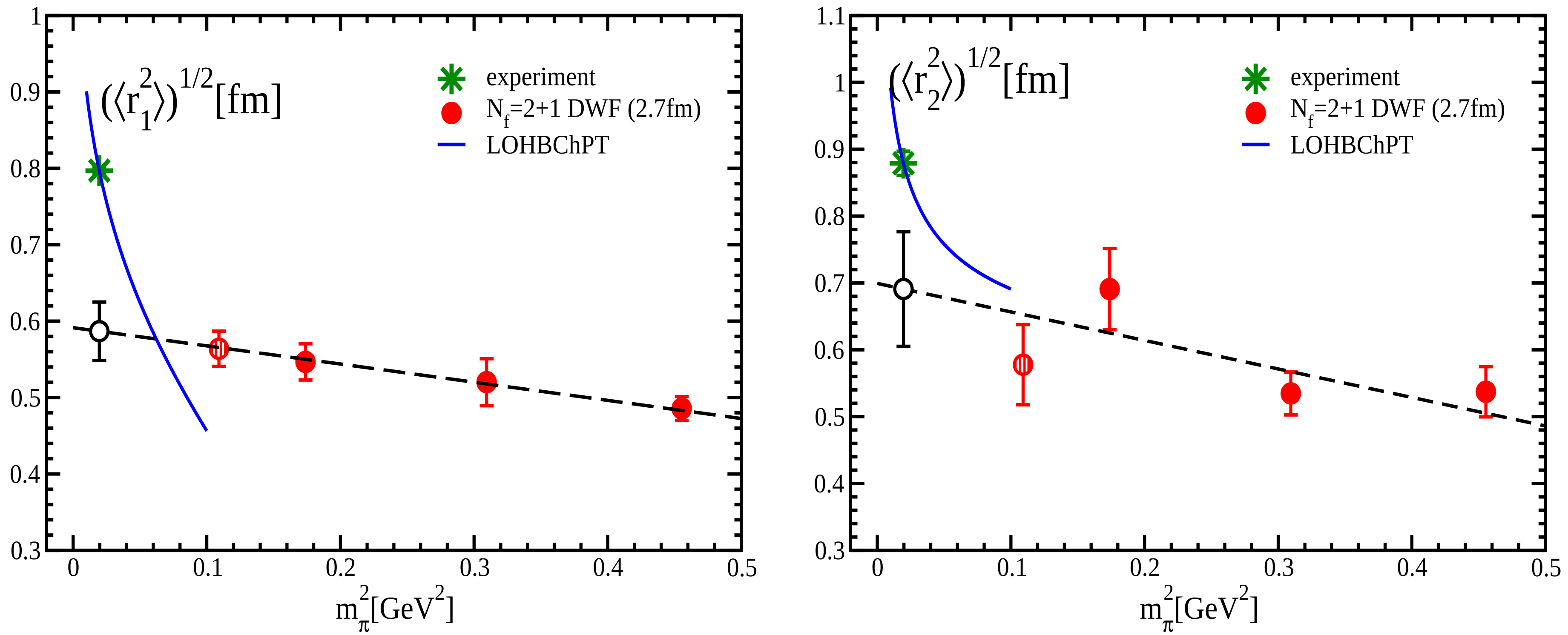
$$\frac{F_2(0)}{(1+\langle r_2^2\rangle q^2/12)^2}$$




$$F_2(0) = \mu_p - \mu_n - 1$$

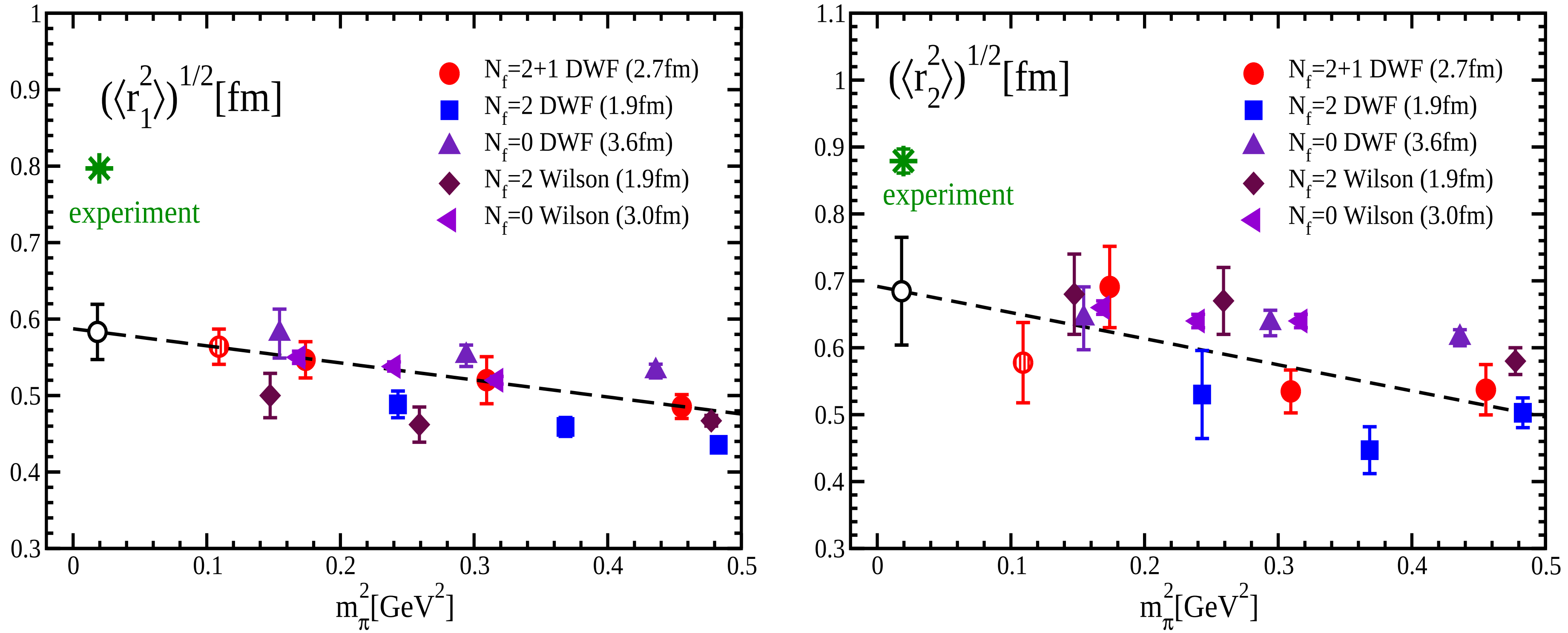
$F_2(0)$ cannot be calculated directly, so that $F_2(0)$ is a free parameter in $F_2(q^2)$ fit.

# Dirac and Pauli rms radius $\sqrt{\langle r_1^2 \rangle}$, $\sqrt{\langle r_2^2 \rangle}$




Result increases as $m_\pi$ decreases, but are smaller than experiments.
Lightest results are consistent with linear extrapolations with other data.
In HBChPT both radii diverge at chiral limit, while such a behavior is not seen.
Lighter quark mass calculation, *e.g.*, $m_\pi \approx 200$ MeV, would be necessary to observe divergent behavior.

# Dirac and Pauli rms radius $\sqrt{\langle r_1^2 \rangle}$, $\sqrt{\langle r_2^2 \rangle}$

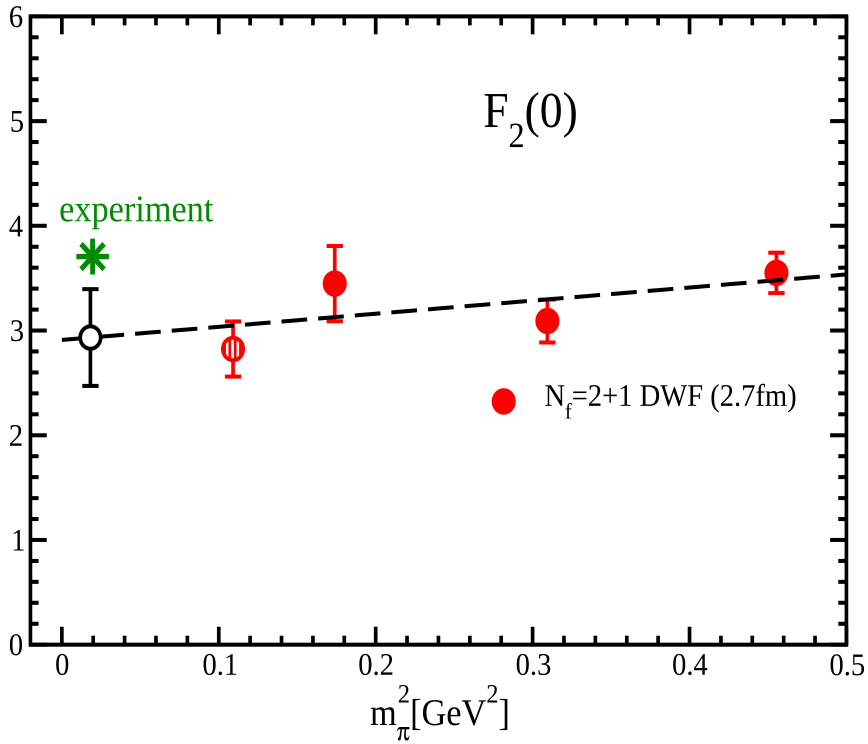


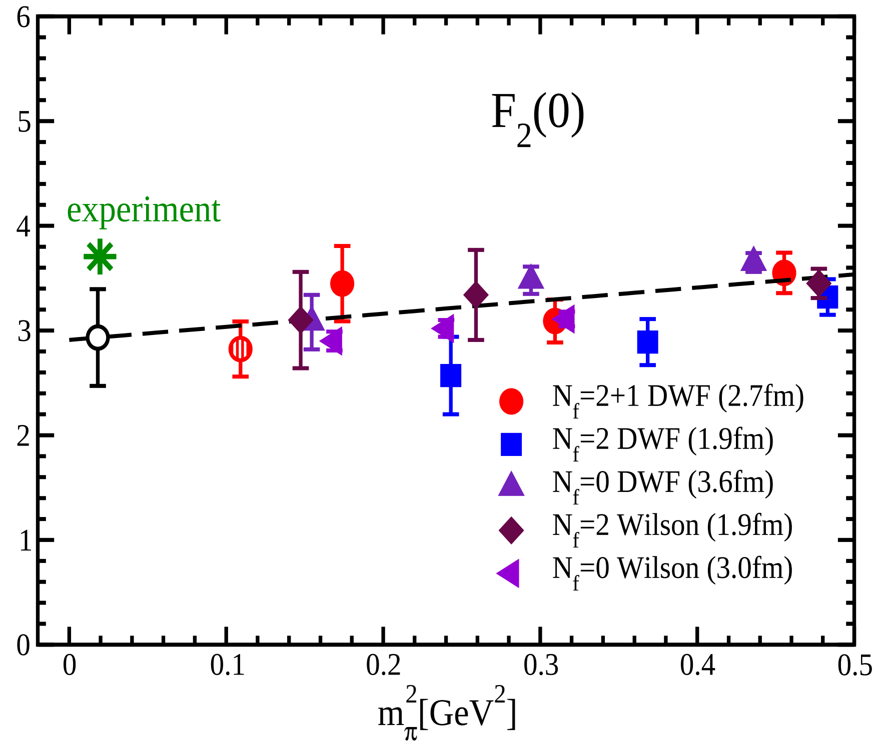

$N_f = 0, 2$ Wilson PRD74:034508; $N_f = 0$ DWF arXiv:0709.3150

Linear $m_\pi^2$ dependences are consistent with previous results in quenched and dynamical calculations.

All the results do not have divergent behavior.

# $F_2(0) = \mu_p - \mu_n + 1$




$N_f = 0, 2$ Wilson PRD74:034508; $N_f = 0$ DWF arXiv:0709.3150

$F_2(0)$ has mild $m_\pi$ dependence, and reasonably agrees with experiment in physical pion mass. $m_\pi$ dependence is similar to previous results.

Results obtained from $F_1(q^2)$ and $F_2(q^2)$ have no strange $m_\pi$ dependence in contrast to $G_A(q^2)$ and $G_P(q^2)$. (less sensitive to finite volume effect)

# 4. Summary

- We calculated nucleon form factors with $N_f = 2 + 1$ dynamical domain wall fermions at four quark masses.
- Axial charge $g_A/g_V = G_A(0)/F_1(0)$
  - Our $g_A/g_V$ at $m_\pi = 330$ MeV is affected by large finite volume effect even on $(2.7\text{ fm})^3$ volume. $g_A/g_V$ scales in single variable $m_\pi L$.
  - Our data on $(2.7\text{ fm})^3$ and $(1.8\text{ fm})^3$ volumes are described well by linear with finite volume correction.

    $$g_A/g_V = 1.20(6)(4) \text{ at physical pion mass}$$
  - We estimate that we need $L > 3.5$fm to obtain $g_A/g_V$ for less than 1% finite volume effect at $m_\pi = 330$ MeV.

# 4. Summary (cont'd)

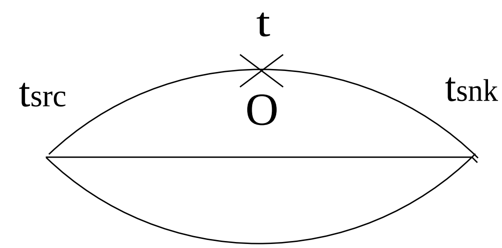
- Isovector $G_A$ and $G_P$
  - Results obtained from $G_A$ and $G_P$ at $m_\pi = 330$ MeV would include large finite volume effect. (similar to $g_A/g_V$)
  - Matrix element of axial vector current is sensitive to finite volume.

- Isovector $F_1$ and $F_2$
  - Results obtained from $F_1$ and $F_2$ have mild linear $m_\pi$ dependence in all pion masses.
  - Matrix element of vector current is less sensitive to finite volume than one with axial vector current.
  - $m_\pi \approx 200$ MeV might be necessary to observe divergent behavior in $\sqrt{\langle r_1^2 \rangle}, \sqrt{\langle r_2^2 \rangle}$.

## Future work

- Larger volume (with auxiliary determinants)
- Lighter quark mass for comparison with ChPT
- Finite volume study (difference finite volume effect in matrix elements for vector and axial vector currents)

# Backup Slides




## Matrix elements

$$R_{\vec{p}}^{\mathcal{P}\mathcal{O}}(t, t_{snk}, t_{src}) = \frac{G_{\vec{p}}^{\mathcal{P}\mathcal{O}}(t)}{G_{\vec{0}}^{G}(t_{snk})} \left[ \frac{G_{\vec{p}}^{L}(t_{snk} - t + t_{src}) G_{\vec{0}}^{G}(t) G_{\vec{0}}^{L}(t_{snk})}{G_{\vec{0}}^{L}(t_{snk} - t + t_{src}) G_{\vec{p}}^{G}(t) G_{\vec{p}}^{L}(t_{snk})} \right]^{1/2}$$

$$\propto \langle N(0) | \mathcal{O}(q) | N(p) \rangle \quad (t_{src} \ll t \ll t_{snk})$$

Normalization of nucleon operator is canceled.

$G_{\vec{p}}^{\mathcal{P}\mathcal{O}}$ : 3-point function of $\mathcal{O}$ with $\vec{p}$ and projector $\mathcal{P}$
gauge invariant Gauss smearing source is employed.

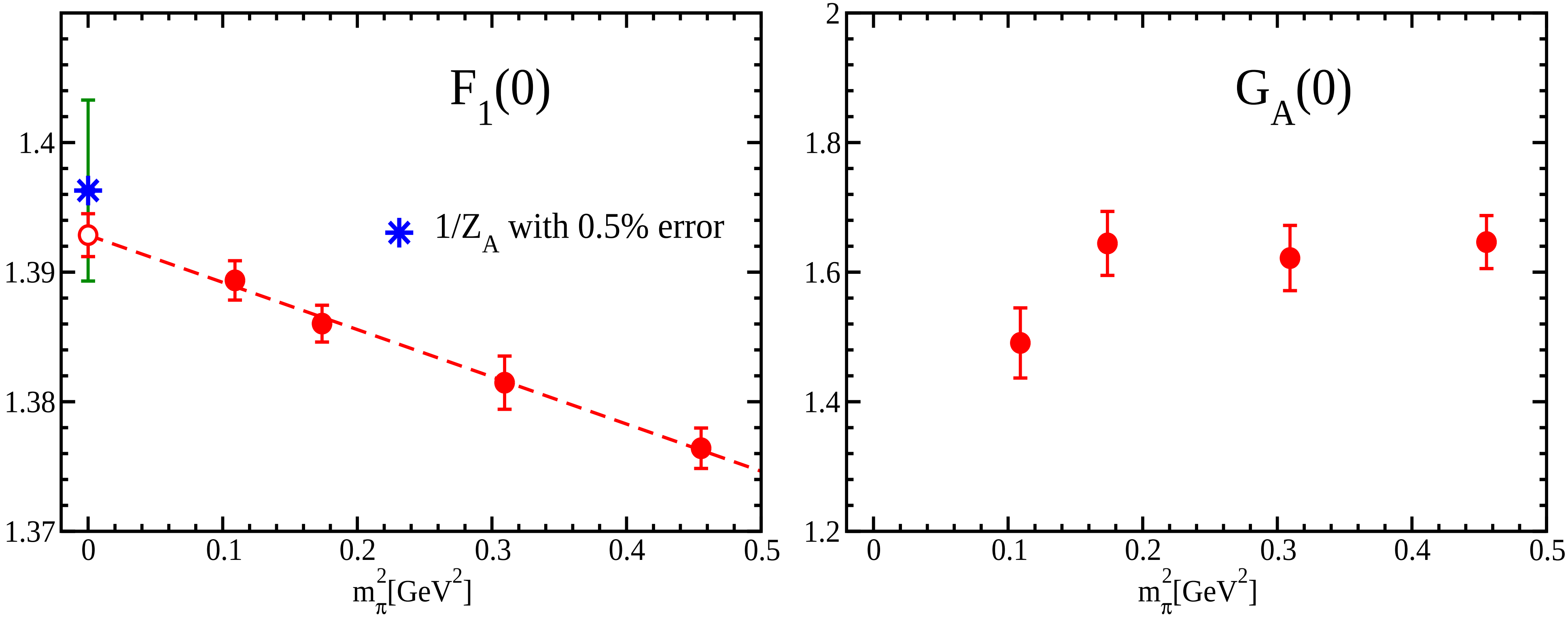
$G_{\vec{p}}^{G,L}$ : 2-point function with $\vec{p}$ and gauss smearing(G) or local(L) sink
gauss smearing source

$$\mathcal{P}\mathcal{O} = P_t V_t \quad R_{\vec{p}}^{\mathcal{P}\mathcal{O}} \propto F_E = F_1(q^2) - \frac{q^2}{(2M)^2} F_2(q^2) \quad P_t = (1 + \gamma_t)/2$$

$$\mathcal{P}\mathcal{O} = P_{xy} V_x \quad R_{\vec{p}}^{\mathcal{P}\mathcal{O}} \propto F_M = F_1(q^2) + F_2(q^2) \quad P_{xy} = (1 + \gamma_t)\gamma_x \gamma_y / 2$$

$F_1$ and $F_2$ are obtained by solving linear equations.
$G_A$ and $G_P$ are obtained by a similar way.

# Each part of $g_A/g_V = G_A(0)/F_1(0)$




Strange behavior is not seen in $F_1(0)$, because it has reasonable $m_\pi$ dependence and extrapolated value at chiral limit which is consistent with $1/Z_A$ obtained from conserved axial vector current in 0.5%.

$G_A(0)$ has similar $m_\pi$ dependence to $g_A/g_V$.

$G_A(0)$ is affected by finite volume.

# 3. Results

## 3.1. Axial charge $g_A/g_V$

Isovector axial charge $g_A/g_V = G_A(0)/F_1(0)$ is related to neutron $\beta$ decay, and spontaneous chiral symmetry breaking of strong interaction, and well determined in experiment

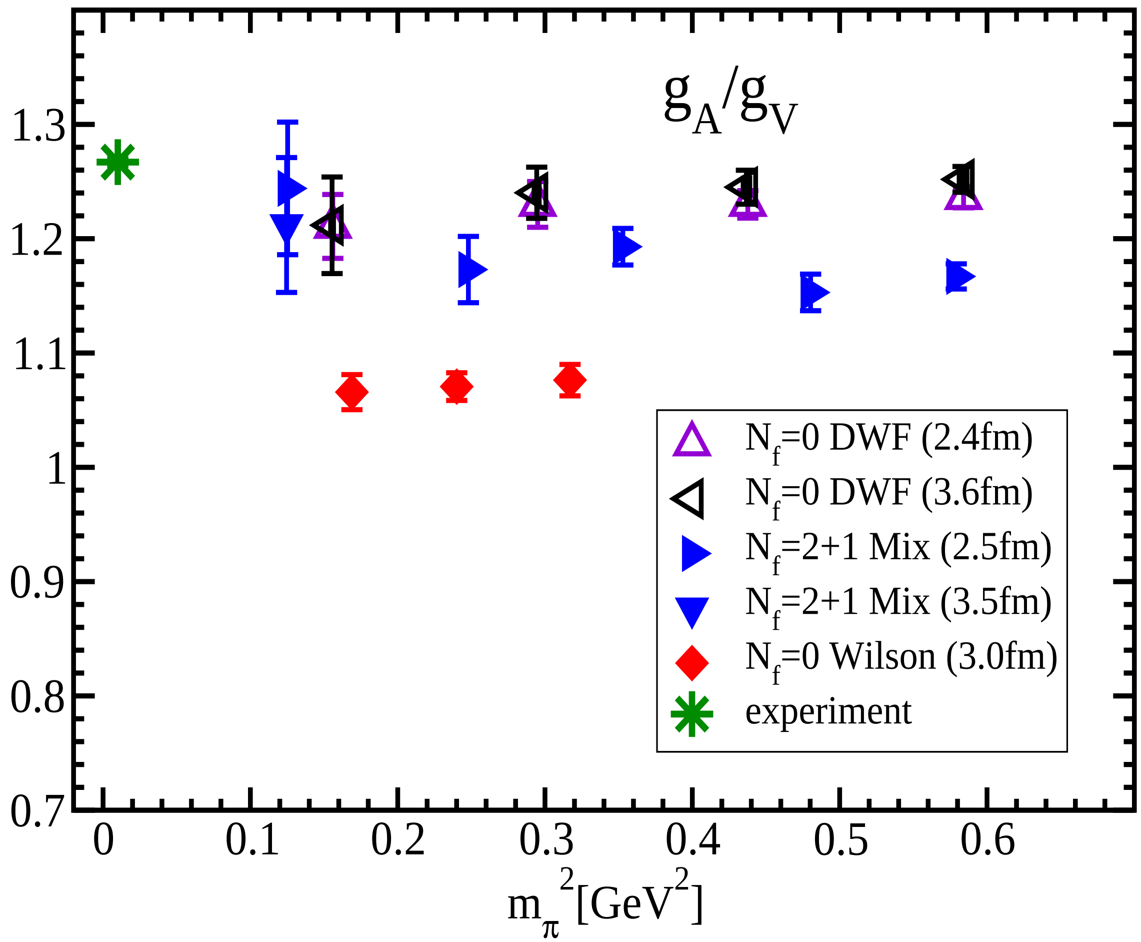
$$g_A/g_V = 1.2695(29) \text{ (PDG)}$$

$g_A/g_V$ is relatively easy to calculate with lattice QCD.

- We need matrix elements at $q^2 = 0$ without disconnected quark diagram.
- Renormalization is simple when we use a lattice chiral fermion action, *e.g.*, DWF. $$\frac{\langle N|A_\mu(q=0)|N\rangle}{\langle N|V_\mu(q=0)|N\rangle} = \frac{Z_A g_A}{Z_V g_V} = \frac{g_A}{g_V} \text{ with } Z_A \approx Z_V$$

Calculation of $g_A/g_V$ is a precision test of (lattice) QCD. However ....

## $g_A/g_V$ in previous calculations

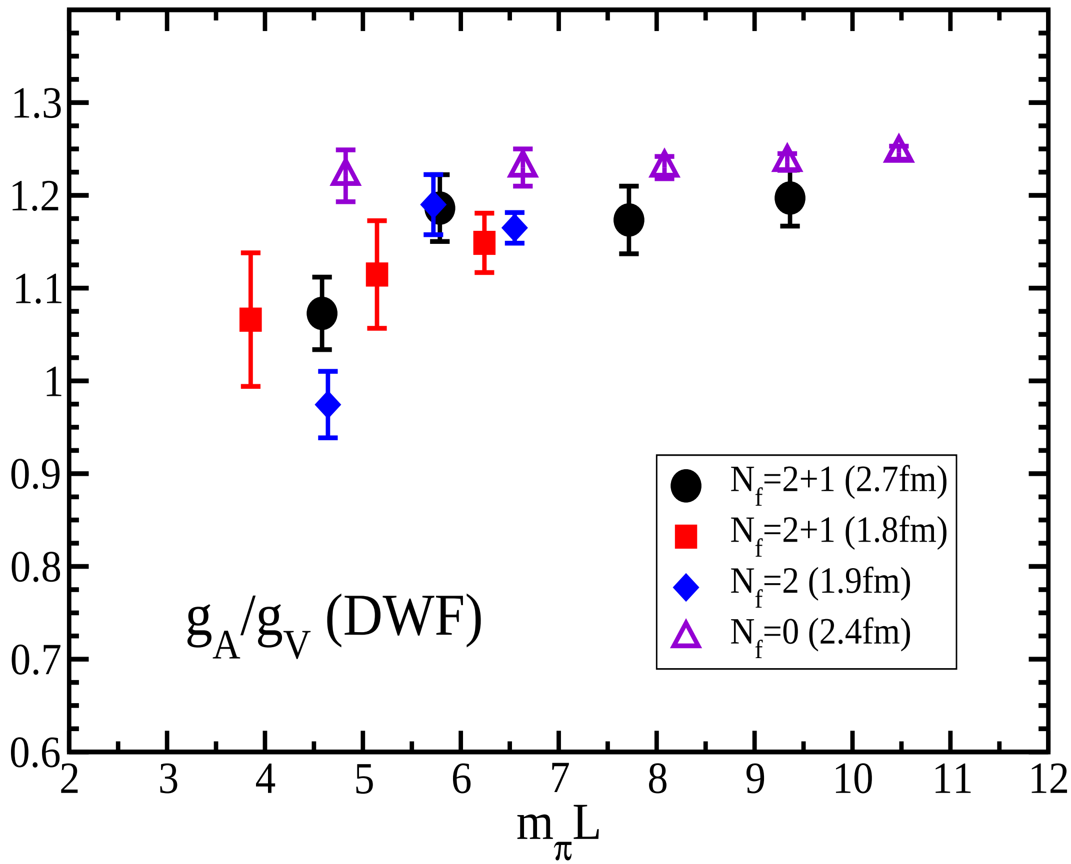



Refs. Mixed action PRL96:052001(LHPC); quenched Wilson PRD76:094511

Finite volume effect seems negligible on $V > (2.4\text{ fm})^3$ in DWF calculation.

Wilson calculation seems to have other systematic uncertainties.

## $m_\pi L$ scaling of $g_A/g_V$ in DWF




Quenched result on 2.4 fm is almost flat in $m_\pi L$ in contrast to dynamical one.

Quenched calculation might be less sensitive to finite volume effect due to lack of dynamical fermions.

## Scaling of $g_A/g_V$ in DWF (cont'd)

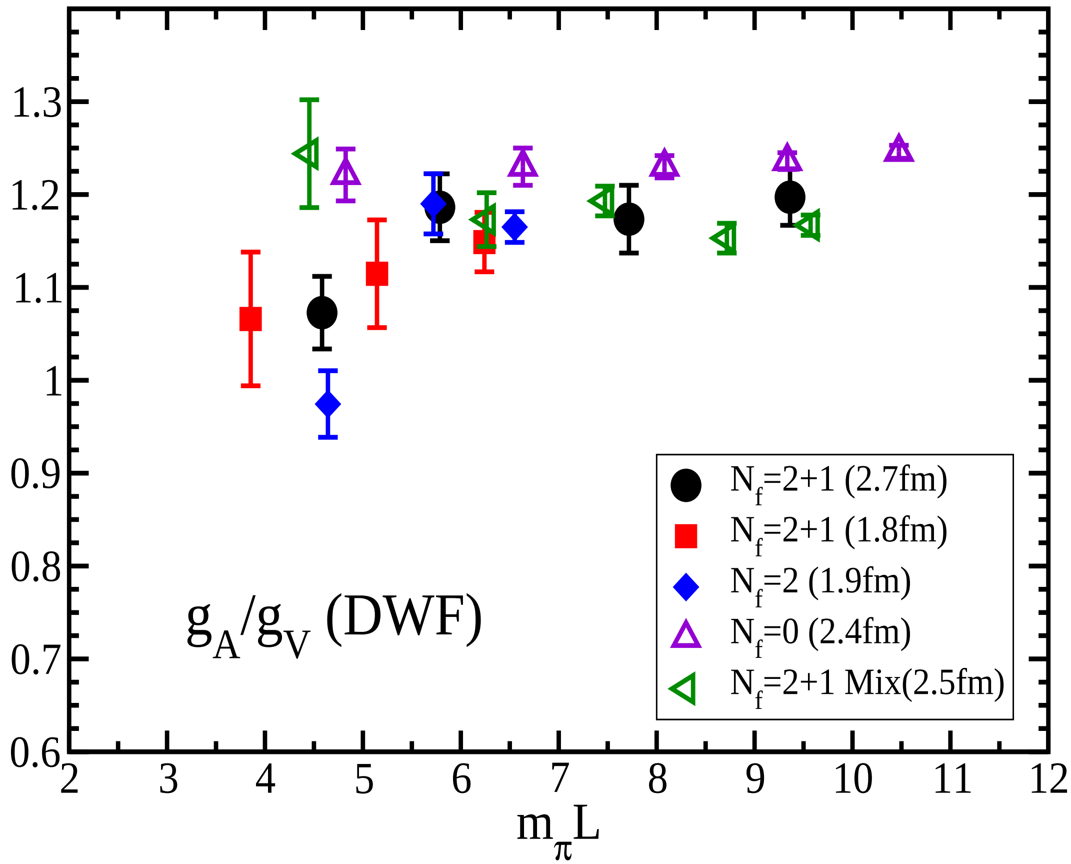
- Mixed action calculation is partially quenched; valence DWF and improved staggered sea fermions.
- Matching between valence and sea fermions is done by tuning of valence DWF $m_\pi$ to be lightest $m_\pi$ in staggered fermion.
- However, there is an ambiguity in choosing a pseudoscalar meson in staggered fermion, because there are several.

Valence $m_\pi$ is tuned to be heaviest $m_\pi$ in staggered fermion.

Refs. Bar *et al* PRD72:054502; Prelovsek PRD73:014506

If valence fermion is much lighter than sea fermion, mixed action calculation effectively becomes quenched calculation.

# Scaling of $g_A/g_V$ in DWF (cont'd)

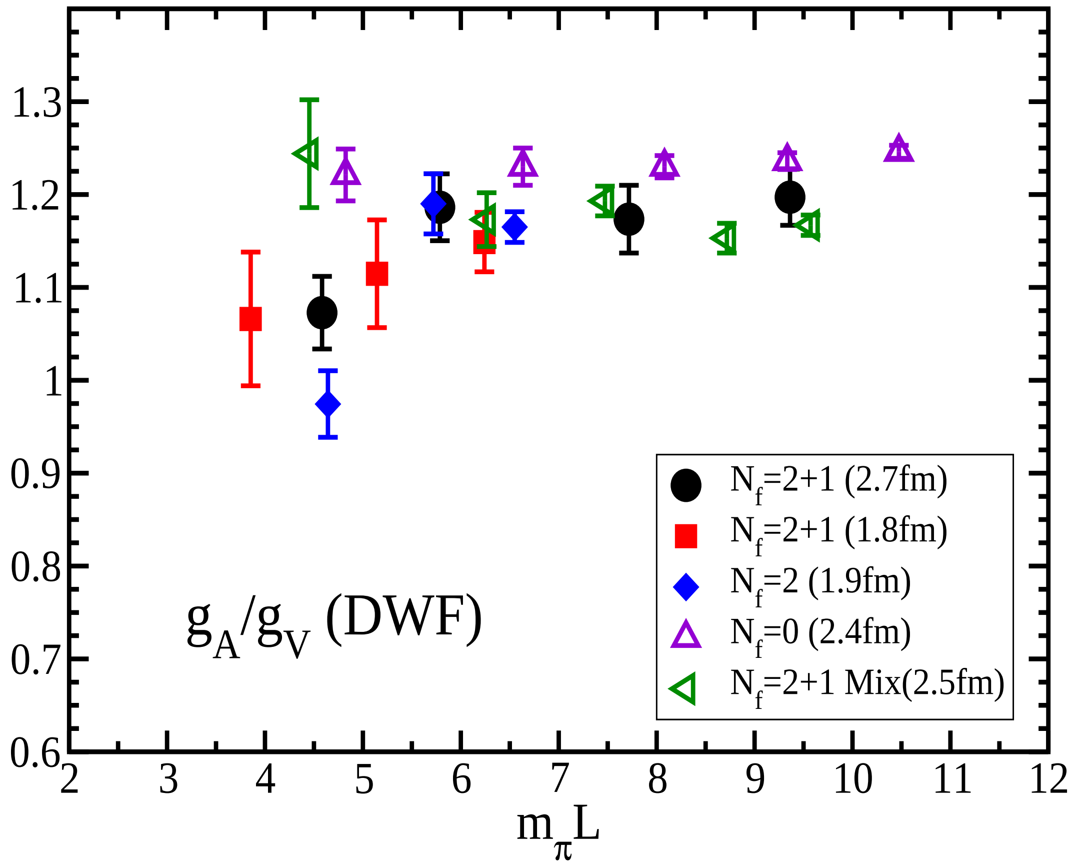



Ref. Mixed action PRL96:052001(LHPC)

If $m_f^{\text{valence}} \ll m_f^{\text{sea}}$, mixed action calculation effectively becomes quenched calculation. This may lead to deviation from our unitary calculation, and consistency with quenched calculation.

(Partially-)quenched log term predicted in ChPT might explain this dependence. PRD58:074509, hep-lat/0703012, arXiv:0706.0035

# Scaling of $g_A/g_V$ in DWF




Ref. Mixed action PRL96:052001(LHPC)

At heavier $m_\pi$ on $L = 2.7$ fm, $N_f = 2 + 1$ and mixed action data are consistent.

However, they are different at lightest point, $m_\pi L \sim 4.5$ in 2.1 $\sigma$.

Mixed action data at $m_\pi L \sim 4.5$ is closer to quenched data.

A possible explanation of difference between our data and mixed action data is simply dynamical fermion effect.

# Chiral extrapolations of $g_A/g_V$ (cont'd)

To connect lattice data and HBChPT, one introduces degree of freedom of $\Delta$ baryon.

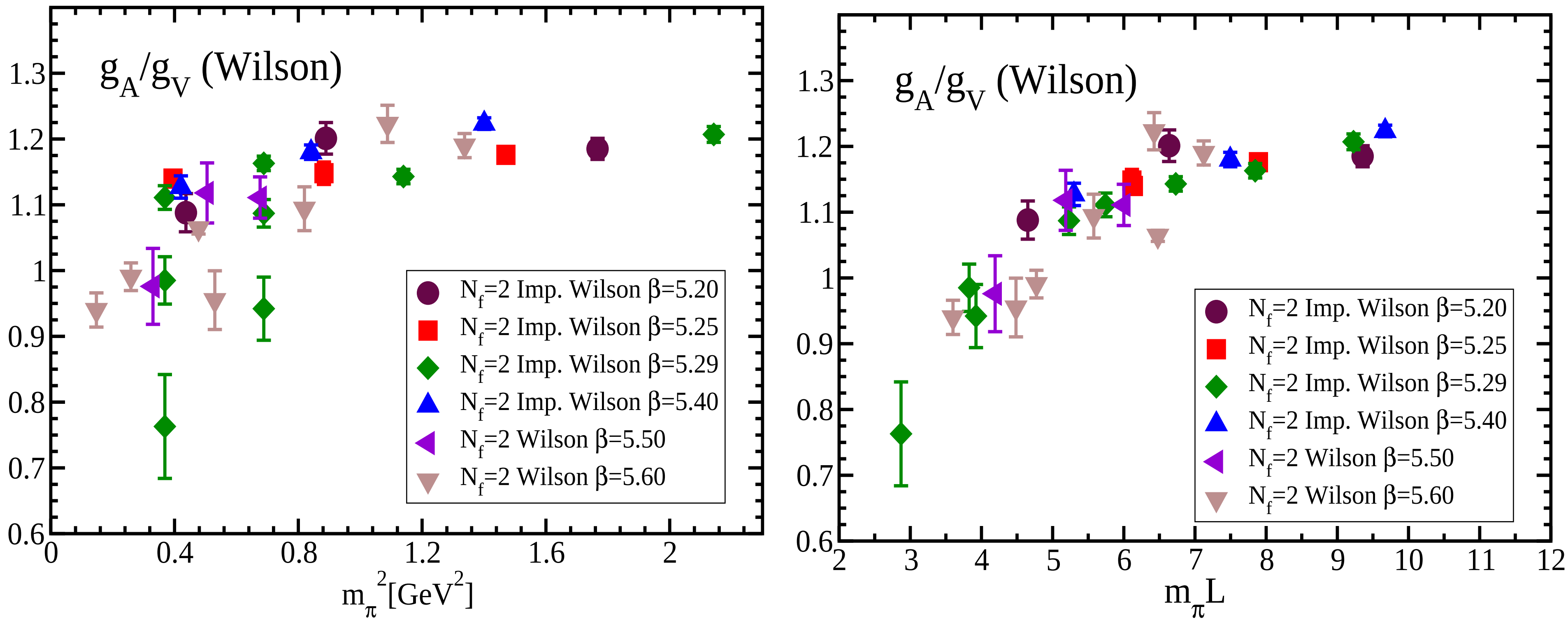
Six unknown parameters are in formula, so that we cannot fit without fixing some parameters.

Even if some of them are fixed by experimental inputs, estimated finite volume effect from HBChPT is less than 1% at 2.7 fm and $m_\pi = 330$ MeV.

This estimation is much smaller than our observation.

1. Linear extrapolation with larger volume data except lightest point
2. Simultaneous fit with all data on both volumes
   Assuming simple formula;
   $$A + Bm_\pi^2 + Cf_V(m_\pi L)$$
   $A + Bm_\pi^2$ : physical $m_\pi$ dependence in infinite volume limit
   $f_V$ : finite volume effect which is a function of $m_\pi L$ only, and vanishes rapidly towards $L \to \infty$.(consistent with observed effect)

# $m_\pi L$ scaling of $g_A/g_V$ in dynamical Wilson calculations



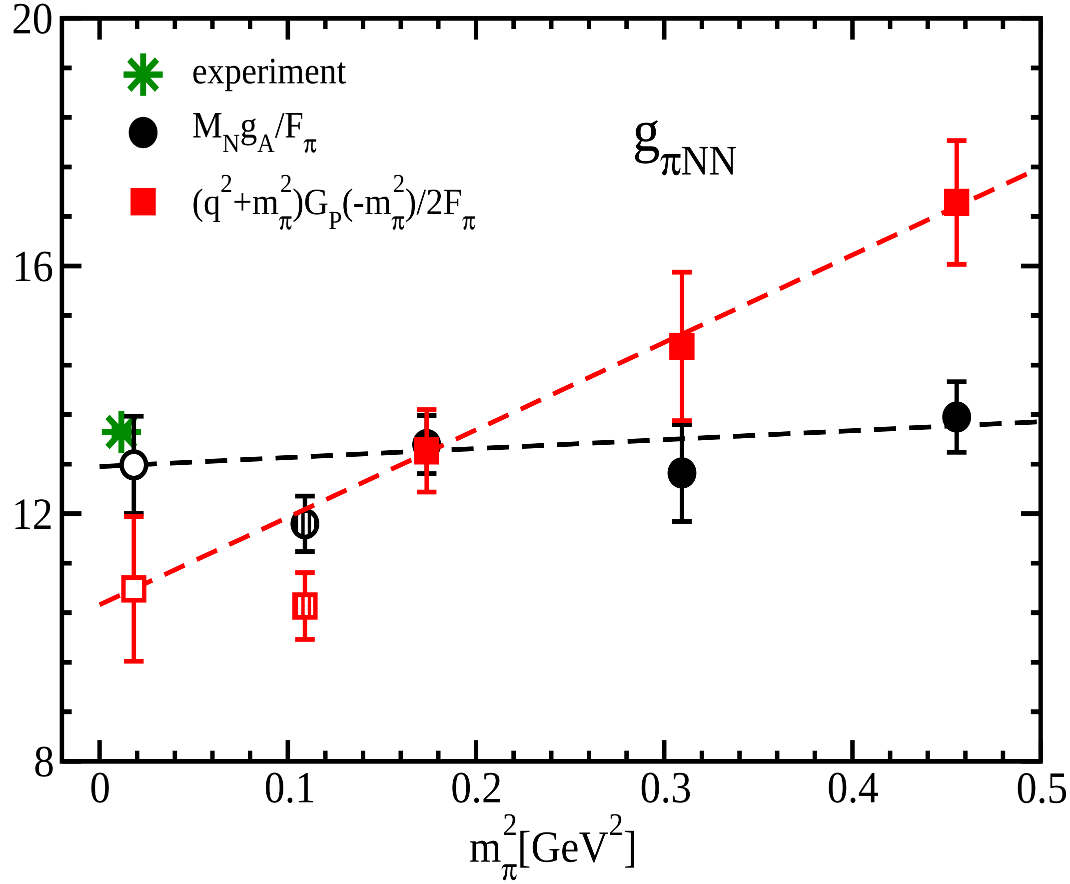

Refs. Imp. Wilson PRD74(2006)(QCDSF); Wilson PRD66(2002)(LHPC and SESAM); Wilson arXiv:0706.3011

Two-flavor (Imp.) Wilson fermion calculations with various $m_\pi = 0.38$–1.18 GeV, $V = (0.95\text{–}2.0\ \text{fm})^3$, and $\beta$, at $\kappa_{sea} = \kappa_{val}$.

# $g_{\pi NN}$ coupling

$g_{\pi NN} \approx m_N g_A / f_\pi$ (GT relation at $m_\pi = 0$)

$g_{\pi NN} = \lim_{q^2 \to -m_\pi^2} (q^2 + m_\pi^2) G_P(q^2)/2f_\pi = \alpha_P \times m_N G_A(-m_\pi^2)/f_\pi$

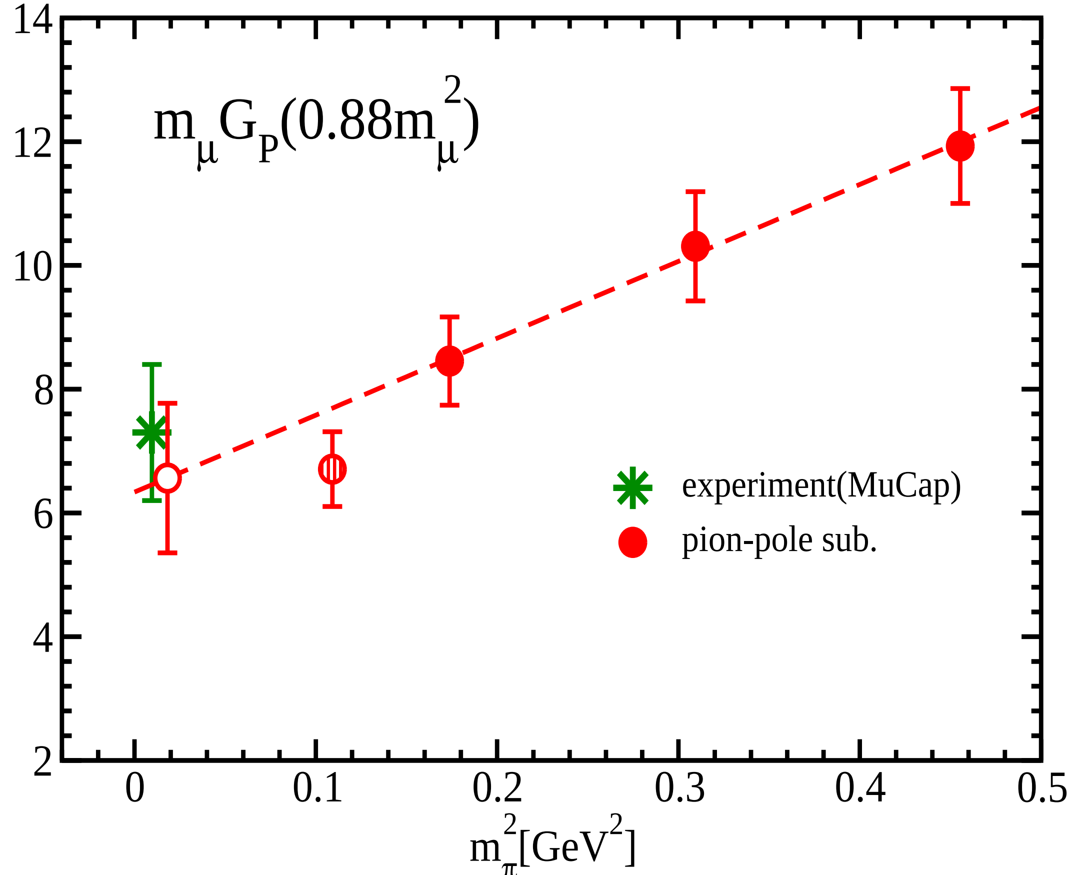



$m_N g_A / f_\pi$ gives mild $m_\pi$ dependence, and extrapolated value is comparable with experiment.

$g_{\pi NN}$ has large slope, and is below experiment.

## $g_P$ for muon capture

$$g_p = m_\mu G_P(q_0^2) \text{ with } q_0^2 = 0.88 m_\mu^2$$




pion-pole sub. : dipole fit of $A(q^2) = G_P(q^2)(q^2 + m_\pi^2)$

$$g_p = m_\mu A(q_0^2)/(q_0^2 + (m_\pi^{\text{phys}})^2)$$

Results obtained from extrapolation agree with experiments.