Pion vector and scalar form factors with dynamical overlap quarks

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1. introduction

pion vector form factor $F_V(q^2)$

- expr't + ChPT $\Rightarrow$ $\langle r^2 \rangle_V$, l_6
- LQCD $\Rightarrow$ deep understanding of q^2 dependence / chiral behavior
  $\Rightarrow$ K, D, B decays

pion scalar form factor $F_S(q^2)$

- $\langle r^2 \rangle_S$ $\Rightarrow$ l_4 $\Leftrightarrow$ l_4 from F_π
- $\langle r^2 \rangle_S$: enhanced chiral log $-6/(4\pi F)^2 \ln[M_\pi^2/\mu^2]$
  $\Leftrightarrow$ $\langle r^2 \rangle_V$: $-1/(4\pi F)^2 \ln[M_\pi^2/\mu^2]$
- direct determination in LQCD $\Leftarrow$ needs disconnected 3-pt. functions

1. introduction

this work

calculate pion form factors in $N_f = 2$ QCD

- overlap quarks + small m_{ud} $\Rightarrow$ straightforward comparison w/ ChPT
- all-to-all quark propagators
  $\Rightarrow$ precise determination of connected / disconnected 3-pt. functions

outline

- simulation method
- determination of $F_V(q^2)$ and $F_S(q^2)$
- parametrization of q^2 dependence
- chiral extrapolation of $\langle r^2 \rangle_V$, $\langle r^2 \rangle_S$, ...

2.1 simulation method: configuration generation

production run

- $N_f = 2$ QCD w/ degenerate u and d quarks
- Iwasaki gauge + overlap quarks w/ std. Wilson kernel
- determinant to suppress zero modes: $\mathrm{det}[H_W^2]/\mathrm{det}[H_W^2 + \mu^2]$ $(\mu = 0.2)$
- $\beta = 2.30$: $a = 0.1184(3)(21)$ fm $\Leftarrow$ $r_0 = 0.49$ fm
- $16^3 \times 32$: $L \sim 1.9$ fm

for pion form factors

- 4 m_{ud}: $m_{ud} \simeq m_{s,\mathrm{phys}}/6 - m_{s,\mathrm{phys}}/2$, $M_\pi \simeq 290 - 520$ MeV
- 100 conf $\times$ 100 HMC traj. = 10,000 traj.
- in $Q = 0$ sector need to study effects of fixed topology *(Aoki et al., 2007)*
- local and smeared operators : $\phi_l(|\mathbf{r}|) = \delta_{\mathbf{r},\mathbf{0}}$, $\phi_s(|\mathbf{r}|) = \exp[-0.4|\mathbf{r}|]$
- $|\mathbf{p}| \le \sqrt{3}$ (in units of $2\pi/L$) $\Rightarrow$ $|q^2| \lesssim 1.7$ GeV^2

overview of our dynamical overlap project $\Rightarrow$ plenary talk by S.Hashimoto

2.2 simulation method: measurements

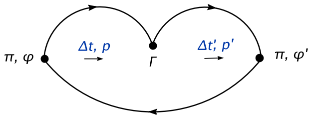
all-to-all quark propagators *(TrinLat, 2005)*

- low-mode projection : $D\,u^{(k)} = \lambda^{(k)} u^{(k)}$ $\quad (k \le N_{\rm ep} = 100)$
- noise method : $D\,x^{(r,d)} = \eta^{(r,d)}$ $\quad (r \le N_r = 1)$ w/ dilution for color/spinor/t

$$D^{-1} \;=\; \sum_{k=1}^{N_{\rm ep}} \frac{u^{(k)}}{\lambda^{(k)}} u^{(k)\dagger} + (1 - P_{\rm low}) \sum_{r=1}^{N_r} \sum_{d=1}^{N_d} \frac{x^{(r,d)}}{N_r} \eta^{(r,d)\dagger} \;=\; \sum_{k=1}^{N_{\rm vec}=N_{\rm ep}+N_r N_d} v^{(k)}\, w^{(k)\,\dagger}$$

$$v^{(k)} = \{u^{(1)}/\lambda^{(1)}, \ldots, x^{(1,1)}/N_r, \ldots\}, \quad w^{(k)} = \{u^{(1)}, \ldots, \eta^{(1,1)}, \ldots\}$$

connected 3-pt. functions



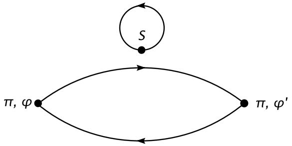
- Δt : temporal separation src $\Leftrightarrow$ opr
- $\Delta t'$: temporal separation opr $\Leftrightarrow$ snk
- $\mathbf{p}$: initial meson momentum
- $\mathbf{p}'$: final meson momentum

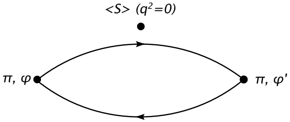
$$\mathcal{M}^{(k,l)}_{\Gamma,\phi}(t;\mathbf{p}) \;=\; \sum_{\mathbf{x},\mathbf{r}} \phi(\mathbf{r})\, w(\mathbf{x}+\mathbf{r},t)^{(k)\dagger}\, \Gamma\, v^{(l)}(\mathbf{x},t)\, \exp[-i\mathbf{p}\mathbf{x}]$$

$$C^{\rm conn}_{\pi\Gamma\pi,\phi\phi'}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') \;=\; \frac{1}{N_t} \sum_{t=1}^{N_t} \sum_{k=1}^{N_{\rm vec}} \sum_{l=1}^{N_{\rm vec}} \sum_{m=1}^{N_{\rm vec}} \mathcal{M}^{(m,l)}_{\pi,\phi'}(t+\Delta t+\Delta t'; \mathbf{p}') \times \mathcal{M}^{(l,k)}_{\Gamma,\phi}(t+\Delta t; \mathbf{p}-\mathbf{p}')\, \mathcal{M}^{(k,m)}_{\pi,\phi}(t; -\mathbf{p})$$

2.2 simulation method: measurements

disconnected 3-pt. functions





$$C^{\rm disc}_{\pi S\pi,\phi\phi'}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{1}{N_t}\sum_{t=1}^{N_t}\sum_{k=1}^{N_{\rm vec}}\sum_{l=1}^{N_{\rm vec}} \mathcal{M}^{(k,l)}_{\pi,\phi'}(t+\Delta t+\Delta t'; \mathbf{p}')\,\mathcal{M}^{(l,k)}_{\pi,\phi}(t; -\mathbf{p}) \times \sum_{m=1}^{N_{\rm vec}} \mathcal{M}^{(m,m)}_{S,\phi_{\rm l}}(t+\Delta t; \mathbf{p}-\mathbf{p}')$$

$$C^{\rm VEV}_{\pi S\pi,\phi\phi'}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{1}{N_t}\sum_{t=1}^{N_t}\sum_{k=1}^{N_{\rm vec}}\sum_{l=1}^{N_{\rm vec}} \mathcal{M}^{(k,l)}_{\pi,\phi'}(t+\Delta t+\Delta t'; \mathbf{p}')\,\mathcal{M}^{(l,k)}_{\pi,\phi}(t; -\mathbf{p}) \times \left\langle \frac{1}{N_t}\sum_{t=1}^{N_t}\sum_{m=1}^{N_{\rm vec}} \mathcal{M}^{(m,m)}_{S,\phi_{\rm l}}(t+\Delta t; \mathbf{p}-\mathbf{p}') \right\rangle_{\rm conf}$$

$$C^{\rm sngl}_{\pi S\pi,\phi\phi'} = C^{\rm conn}_{\pi S\pi,\phi\phi'} - C^{\rm disc}_{\pi S\pi,\phi\phi'} + C^{\rm VEV}_{\pi S\pi,\phi\phi'}$$

3.1 determination of form factors : $F_V(q^2)$

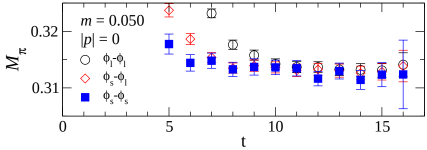
ratio method (*S. Hashimoto, et al., 2000*)

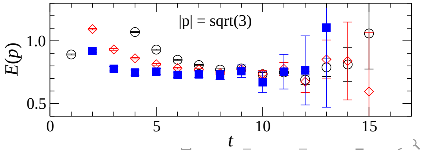
$$C^{\rm conn}_{\pi V_4 \pi,\phi_s\phi_s}(\Delta t,\Delta t';\mathbf{p},\mathbf{p}') \quad \to \quad \frac{\sqrt{Z_{\pi,\phi}(|\mathbf{p}|)\, Z_{\pi,\phi}(|\mathbf{p}'|)}}{4E(p)E(p')\, Z_V}\, e^{-E(p)\Delta t}\, e^{-E(p')\Delta t'}\, \langle \pi(p')\,|V_4|\,\pi(p)\rangle$$

$$C_{\pi\pi,\phi\phi'}(\Delta t;\mathbf{p}) \to \frac{\sqrt{Z_{\pi,\phi}(|\mathbf{p}|)\, Z_{\pi,\phi'}(|\mathbf{p}'|)}}{2E(p)}\, e^{-E(p)\Delta t}, \qquad \sqrt{Z_{\pi,\phi}(|\mathbf{p}|)} = \langle \pi(p)|O_{\pi,\phi}(\mathbf{p})^\dagger\rangle$$

$$R_4(\Delta t,\Delta t';\mathbf{p},\mathbf{p}') \quad = \quad \frac{C_{\pi V_4 \pi,\phi_s\phi_s}(\Delta t,\Delta t';\mathbf{p},\mathbf{p}')}{C_{\pi\pi,\phi_s\phi_l}(\Delta t;\mathbf{p})\, C_{\pi\pi,\phi_l\phi_s}(\Delta t';\mathbf{p}')} = \frac{\langle \pi(p')\,|V_4|\,\pi(p)\rangle}{\sqrt{Z_{\pi,{\rm lcl}}\, Z_{\pi,{\rm lcl}}}\, Z_V}$$

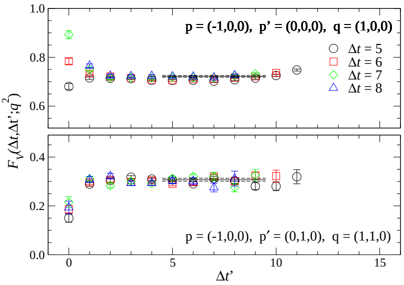
$$F_V(\Delta t,\Delta t';q^2) \quad = \quad \frac{2M_\pi}{E(p)+E(p')}\, \frac{R_4(\Delta t,\Delta t';\mathbf{p},\mathbf{p}')}{R_4(\Delta t,\Delta t';\mathbf{0},\mathbf{0})}$$

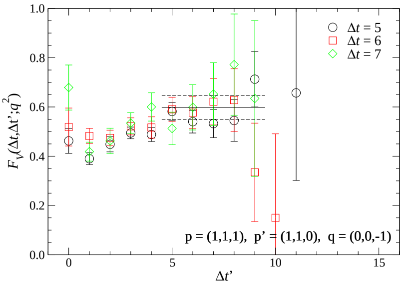




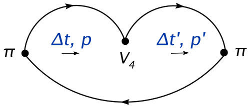
3.1 determination of form factors : $F_V(q^2)$

effective value $F_V(\Delta t, \Delta t'; q^2)$ at $m = 0.050$





- conventional : $\Delta t + \Delta t'$ fixed
- all-to-all : can take any combination of $(\Delta t, \Delta t')$
- accurate when $|\mathbf{p}|$, $|\mathbf{p}'|$ are not large



- $F_V(q^2) \Leftarrow$ constant fit + leading finite V correction *(Borasoy-Lewis, 2005)*

3.2 determination of form factors : $F_S(q^2)$

ratio method

$$\langle \pi(p') \, |S| \, \pi(p) \rangle \quad = \quad F_S(q^2)$$

$$R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') \quad = \quad \frac{C^{\text{sngl}}_{\pi S \pi, \phi_{\text{s}} \phi_{\text{s}}}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C_{\pi\pi, \phi_{\text{s}} \phi_{\text{l}}}(\Delta t; \mathbf{p}) \, C_{\pi\pi, \phi_{\text{l}} \phi_{\text{s}}}(\Delta t'; \mathbf{p}')} = \frac{\langle \pi(p') \, |S| \, \pi(p) \rangle}{\sqrt{Z_{\pi,\text{lcl}} \, Z_{\pi,\text{lcl}}} \, Z_S}$$

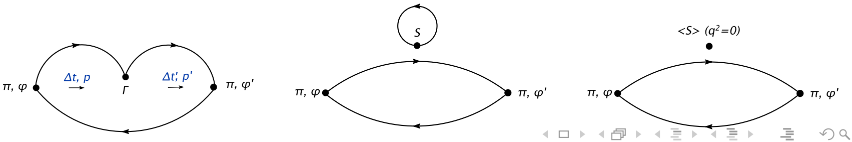
$$\frac{F_S(\Delta t, \Delta t'; q^2)}{F_S(\Delta t, \Delta t'; 0)} \quad = \quad \frac{R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_S(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})}$$

(no kinematical factor)

$$F_S(\Delta t, \Delta t'; 0) \quad \Leftarrow \quad C^{\text{sngl}}_{\pi S \pi}(\mathbf{q}=\mathbf{0}) \; = \; C^{\text{conn}}_{\pi S \pi}(\mathbf{q}=\mathbf{0}) - \left(C^{\text{disc}}_{\pi S \pi}(\mathbf{q}=\mathbf{0}) - C^{\text{vev}}_{\pi S \pi}(\mathbf{q}=\mathbf{0}) \right)$$

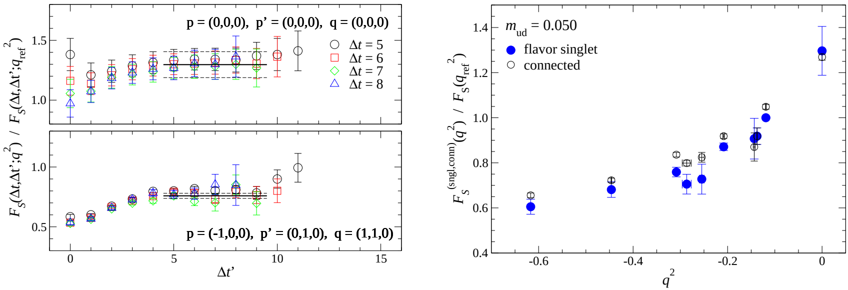
$$\frac{F_S(\Delta t, \Delta t'; q^2)}{F_S(\Delta t, \Delta t'; q^2_{\text{ref}})} \quad = \quad \frac{R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_S(\Delta t, \Delta t'; \mathbf{1}, \mathbf{0})}$$

(normalized @ $|\mathbf{p}| = 1, |\mathbf{p}'| = 0$)



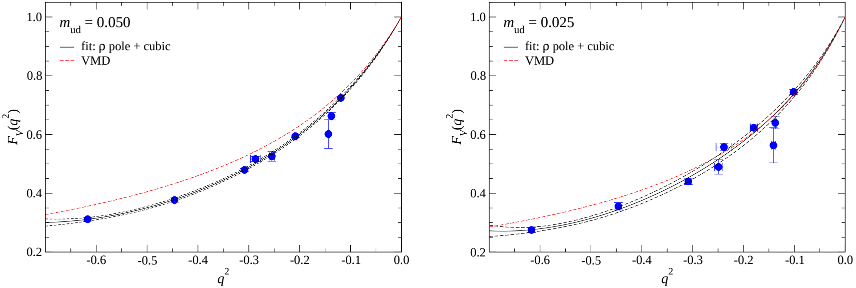
3.2 determination of form factors : $F_S(q^2)$

$F_S(\Delta t, \Delta t'; q^2)/F_S(\Delta t, \Delta t'; q^2_{\rm ref})$ $\quad$ $F_S(q^2)/F_S(q^2_{\rm ref})$, $\quad F_S{}^{\rm conn}(q^2)/F_S(q^2_{\rm ref})$



- large error @ $q^2=0$ $\Leftarrow$ subtraction $C^{\rm disc}_{\pi S\pi} - C^{\rm VEV}_{\pi S\pi}$
- constant fit $\Rightarrow$ $F_S(q^2)/F_S(q^2_{\rm ref})$
- disconnected diagram $\Rightarrow$ small correction to $F_S(q^2)/F_S(q^2_{\rm ref})$

4.1 q^2 dependence : $F_V(q^2)$

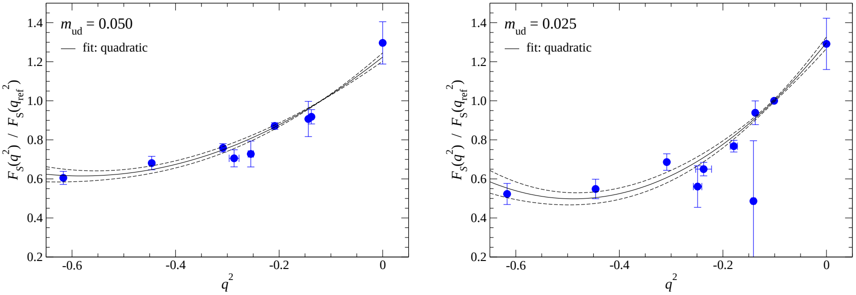


- close to VMD near $q^2 = 0$ $\Rightarrow$ include ρ meson pole w/ measured mass

$$F_V(q^2) = \frac{1}{1 - q^2/M_\rho^2} + c_1\, q^2 + c_2\, (q^2)^2 + c_3\, (q^2)^3 = 1 + \frac{\langle r^2 \rangle_V}{6}\, q^2 + c_V\, (q^2)^2 + \ldots$$

- w/ quad. / cubic correction $\Rightarrow$ reasonable $\chi^2/\text{dof} \sim 1$
- simple polynomial, single pole forms $1/(1 - q^2/M_{\text{pole}}^2)$ $\Rightarrow$ $\chi^2/\text{dof} \sim 2-5$

4.2 q^2 dependence : $F_S(q^2)$



with our statistical accuracy ...

- can be fitted to the quadratic form

$$F_S(q^2) = 1 + \frac{\langle r^2 \rangle_S}{6} q^2 + c_S (q^2)^2$$

- cubic, single pole forms $\Rightarrow$ consistent $\langle r^2 \rangle_S$ w/ larger error
- c_S : ill-determined (strongly depends on the parametrization form)

5.1 chiral extrapolation : w/ NLO ChPT formulae

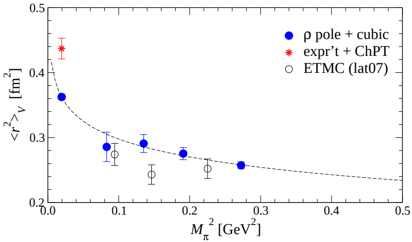
vector charge radius $\langle r^2 \rangle_V$

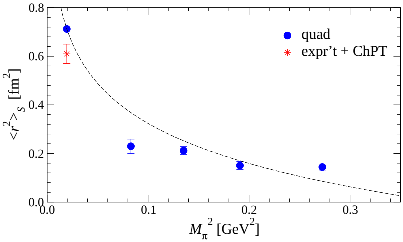
$$\langle r^2 \rangle_V = -(1/NF^2)(1 + N l_6^r) - (1/NF^2) \ln[M_\pi^2/\mu^2]$$

scalar charge radius $\langle r^2 \rangle_S$

$$\langle r^2 \rangle_S = (1/NF^2)(-13/2 + 6N l_4^r) - (6/NF^2) \ln[M_\pi^2/\mu^2]$$

$(N=(4\pi)^2$; use $F = 78.8$ MeV from M_π, F_π *(JLQCD+TWQCD,2008)*; set $\mu = 4\pi F)$





- acceptable $\chi^2/\text{dof} \sim 0.3$
- $\langle r^2 \rangle_V = 0.362(4)$ fm^2 at $m_{ud,\text{phys}}$

  $\Leftrightarrow$ expr't+ChPT : 0.437(16) fm^2

  (Bijnens et al., 1998)

- unacceptable $\chi^2/\text{dof} \sim 17$
- $\langle r^2 \rangle_S = 0.712(8)$ fm^2 at $m_{ud,\text{phys}}$

  $\Leftrightarrow$ ChPT : $\langle r^2 \rangle_S = 0.61(4)$ fm^2

  (Colangelo et al., 2001)

5.1 chiral extrapolation : w/ NLO ChPT formulae

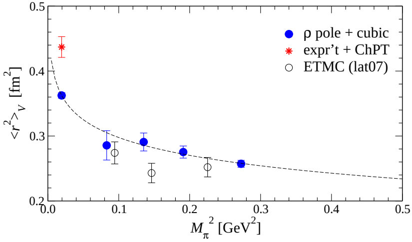
vector charge radius $\langle r^2 \rangle_V$

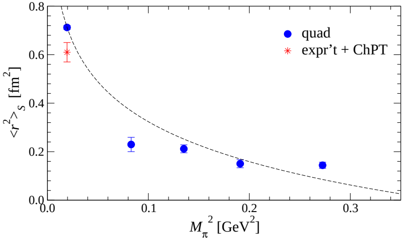
$$\langle r^2 \rangle_V = -(1/NF^2)(1 + N l_6^r) - (1/NF^2) \ln[M_\pi^2/\mu^2]$$

scalar charge radius $\langle r^2 \rangle_S$

$$\langle r^2 \rangle_S = (1/NF^2)(-13/2 + 6 N l_4^r) - (6/NF^2) \ln[M_\pi^2/\mu^2]$$

($N = (4\pi)^2$; use $F = 78.8$ MeV from M_π, F_π *(JLQCD+TWQCD,2008)*; set $\mu = 4\pi F$)





- comparison w/ ETMC's result (twisted mass, $a = 0.09$ fm, $L = 2.2$ fm, @lat07)
  - $\Rightarrow$ not due to finite V corrections, effects of fixed Q, finite a, ...
- $F_V(q^2) \sim$ VMD : resonance exchange $\Rightarrow m_q$ dep @ NNLO

5.2 chiral extrapolation : w/ NNLO ChPT formulae

NNLO formulae *(Bijnens-Colangelo-Talavera, 1998)*

$$\langle r^2\rangle_V = -\frac{1}{NF^2}\left(1+6Nl_6{}^r\right) - \frac{1}{NF^2}\ln\left[\frac{M_\pi^2}{\mu^2}\right] + \frac{1}{N^2F^4}\left(\frac{13N}{192} - \frac{181}{48} + 6N^2 r_{V,1}\right)M_\pi^2 + \frac{1}{N^2F^4}\left(\frac{19}{6} - 6Nl_{1,2}^r\right)M_\pi^2 \ln\left[\frac{M_\pi^2}{\mu^2}\right]$$

$$\langle r^2\rangle_S = \frac{1}{NF^2}\left(-\frac{13}{2} + 6Nl_4^r\right) - \frac{6}{NF^2}\ln\left[\frac{M_\pi^2}{\mu^2}\right] + \frac{1}{N^2F^4}\left(-\frac{23N}{192} + \frac{869}{108} + 88Nl_{1,2}^r + 80Nl_2^r + 5Nl_3 - 24N^2 l_3^r l_4^r + 6N^2 r_{S,1}\right)M_\pi^2 + \frac{1}{N^2F^4}\left(-\frac{323}{36} - 124Nl_{1,2}^r + 130Nl_2^r\right)M_\pi^2\ln\left[\frac{M_\pi^2}{\mu^2}\right] - \frac{65}{3N^2F^4}M_\pi^2\ln\left[\frac{M_\pi^2}{\mu^2}\right]^2$$

$$c_V = \frac{1}{60NF^2}\frac{1}{M_\pi^2} + \frac{1}{N^2F^4}\left(\frac{N}{720} - \frac{8429}{25920} + \frac{N}{3}l_{1,2}^r + \frac{N}{6}l_6^r + N^2 r_{V,2}\right) + \frac{1}{N^2F^4}\left(\frac{1}{108} + \frac{N}{3}l_{1,2}^r + \frac{N}{6}l_6^r\right)\ln\left[\frac{M_\pi^2}{\mu^2}\right] + \frac{1}{72N^2F^4}\ln\left[\frac{M_\pi^2}{\mu^2}\right]^2$$

$$c_S = \ldots$$

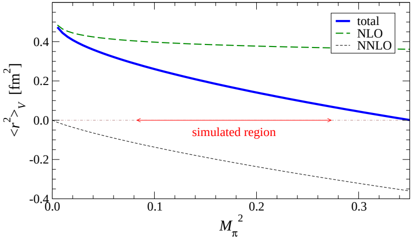
$$l_{1,2}^r = l_1^r - l_2^r/2$$

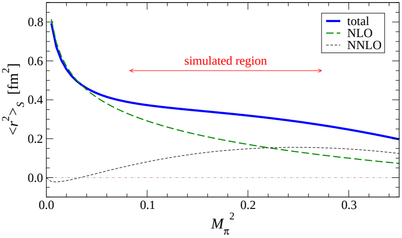
5.2 chiral extrapolation : w/ NNLO ChPT formulae

Is NNLO necessary?

$\langle r^2 \rangle_{V,S}$ from NNLO formulae w/ phenomenological estimates of LECs

- $F = F_\pi/1.069$ from *Bijnens et al., 1998*
- NLO LECs l_i^r from *Bijnens et al., 1998*, or *Colangelo et al., 2001*
  $\bar{l}_6 = 16.0, \quad \bar{l}_4 = 4.4, \quad \bar{l}_1 = -0.36, \quad \bar{l}_2 = 4.31, \quad \bar{l}_3 = 2.9$
- NNLO LECs $r_{X,i}$ from *Bijnens et al., 1998* ($\Leftarrow$ resonance saturation)
  $r_{V,1} = 2.5 \times 10^{-4}, \quad r_{V,2} = 2.6 \times 10^{-4}, \quad r_{S,1} = -3.0 \times 10^{-5}$



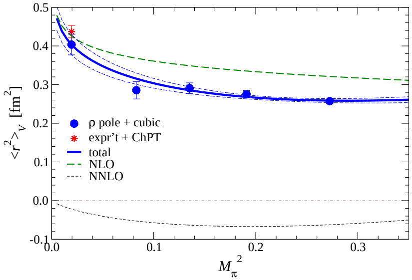


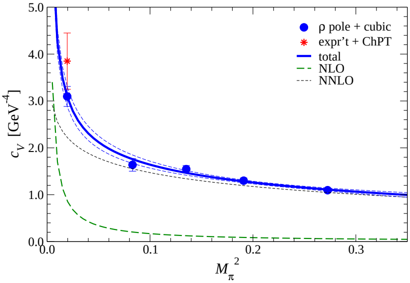
NNLO contribution may modify M_π^2 dependence significantly

5.2 chiral extrapolation : w/ NNLO ChPT formulae

simultaneous fit to $\langle r^2\rangle_V$ and c_V

$\langle r^2\rangle_V$, c_V : 8 data with l_6^r, $l_{1,2}^r$, $r_{V,1}$, $r_{V,2}$ $\quad (l_{1,2}^r = l_1^r - l_2^r/2)$





- dof = 4, $\chi^2/\mathrm{dof} = 1.2$
- consistent with expr't (with larger errors than NLO...)

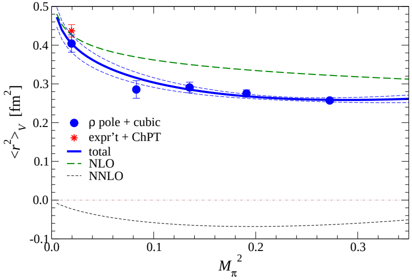
$$\langle r^2\rangle_V = 0.404(27)\ \mathrm{fm}^2, \quad c_V = 3.10(21)\mathrm{GeV}^{-4}$$

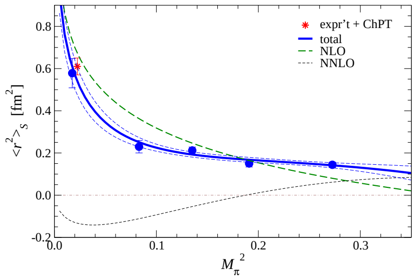
$$\Leftrightarrow \langle r^2\rangle_V = 0.437(16)\ \mathrm{fm}^2, \quad c_V = 3.85(60)\ \mathrm{GeV}^{-4}$$

5.2 chiral extrapolation : w/ NNLO ChPT formulae

simultaneous fit to $\langle r^2\rangle_V$, $\langle r^2\rangle_S$ and c_V

- $\langle r^2\rangle_S \ni l_4^r, l_1^r, l_2^r, l_3^r, r_{S,1}$
- fix $\bar{l}_2 = +4.31$ *(Colangelo et al., 2001)*, $\bar{l}_3 = +3.44$ *(JLQCD+TWQCD's analysis of M_π, F_π)*
- fit parameters : $l_6^r, l_4^r, l_{1,2}^r, r_{V,1}, r_{V,2}\ r_{S,1}$

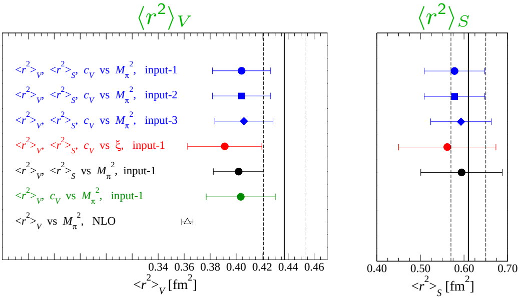




- dof = 6, $\chi^2/\text{dof} = 1.3$
- reasonable agreement w/ expriment

5.2 chiral extrapolation : w/ NNLO ChPT formulae

varying fitting method / input



fitting method

- blue = $\langle r^2 \rangle_{V,S}, c_V$ vs M_π^2
- red = $\langle r^2 \rangle_{V,S}, c_V$ vs $\xi = M_\pi^2/(4\pi F_\pi)^2$
- black = $\langle r^2 \rangle_{V,S}$ vs M_π^2
- green = $\langle r^2 \rangle_{V}, c_V$ vs M_π^2

input

- circle = input-1 in previous slide
- square = input-2
  l_3 from *Colangelo et al., 2001*
- diamond = input-3
  l_2, l_3 from *Bijnens et al., 1998*

$\langle r^2 \rangle_V = 0.404(22)(22)$ fm^2, $\langle r^2 \rangle_S = 0.578(69)(46)$ fm^2, $c_V = 3.11(14)(86)$ GeV^{-4}

$\bar{l}_6 = 11.8(0.7)(1.3) \quad \Leftrightarrow \quad \bar{l}_6 = 16.0(0.9)$ *(Bijnens et al., 1998)*

$\bar{l}_4 = 4.06(44)(99) \quad \Leftrightarrow \quad \bar{l}_4 = 4.14(64)$ *(JLQCD+TWQCD, 2008)*, $4.39(22)$ *(Colangelo et al., 2001)*

$l_1^r - l_2^r/2 = -2.9(0.8)(2.4) \times 10^3 \quad \Leftrightarrow \quad l_1^r - l_2^r/2 = -4.9(0.6) \times 10^3$ *(Colangelo et al., 2001)*

$r_{V,1} \simeq -1.1 \times 10^{-5}$, $r_{V,2} \simeq 4.0 \times 10^{-5}$, $r_{S,1} \simeq 1.3 \times 10^{-4}$ with 50 – 100% error

6. summary

pion form factors in $N_f=2$ QCD with overlap quarks

- w/ all-to-all propagators
  - $F_V(q^2)$: accurate data for connected correlators $\Delta F_V(q^2) \approx 2$ %,
  - $F_S(q^2)$: disconnected diagrams are taken into account $\Delta F_S(q^2) \approx 6$ %
- q^2 dependence
  - $F_V(q^2)$: ρ pole + small correction
  - $F_S(q^2)$: pole contributions are not clear with our accuracy ...
- chiral fit
  - NLO ChPT fails to reproduce $\langle r^2 \rangle_S$ at $\gtrsim m_{s,\mathrm{phys}}/6$ $\Rightarrow$ use NNLO ChPT
  - $\langle r^2 \rangle_V = 0.404(22)(22)$ fm^2, $\langle r^2 \rangle_S = 0.578(69)(46)$ fm^2
- need further studies of systematics
  - FSE on $F_S(q^2)$, effects due to fixed Q
- future directions
  - extension to $N_f=3$
  - $K \to \pi$ decays
  - physics involving disconnected diagrams : η', ...