Scaling behavior and sea quark dependence of pion spectrum

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Outline

Introduction to staggered pion spectrum

Scaling behavior of pion spectrum

Sea quark dependence of pion spectrum

Cubic wall sources and Cubic U(1) sources

Summary

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Summary

Pion spectrum in staggered fermion formalism

$$\mathrm{SU}(4) \xrightarrow{\mathscr{O}(a^2)} \mathrm{SO}(4) \xrightarrow{\mathscr{O}(a^2p^2)} \mathrm{SW}_4$$

- Pion tastes $(\gamma_5 \otimes \xi_T)$, $\xi_T \in \{I, \xi_5, \xi_\mu, \xi_{\mu 5}, \xi_{\mu\nu}\}$.
- In continuum limit, they repect SU(4).
- SχPT : $\mathscr{O}(a^2)$ terms break SU(4) down to SO(4).
- SχPT : $\mathscr{O}(a^2p^2)$ terms break SO(4) down to SW_4.

Improved staggered fermions

- The taste-breaking comes from high momentum gluon exchange.
- Fat-links reduce the taste-breaking by suppressing these interactions.
- We have found that HYP staggered fermions reduce the taste symmetry breaking more efficiently than asqtad staggered fermions.

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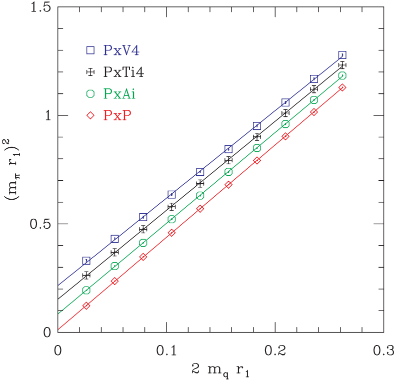
Parameters of the MILC fine lattices and coarse lattices

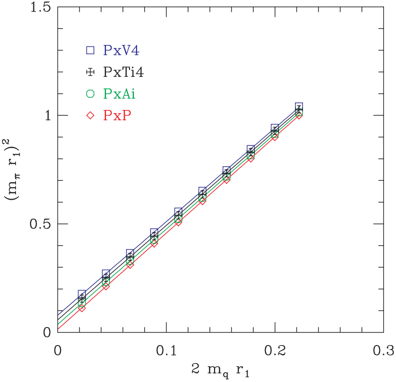
1-loop tadpole-improved Symanzik gauge action,
$2+1$ flavors of Asqtad staggered sea quarks;
Coulomb gauge fixing;
HYP smeared staggered valence quarks

parameters	MILC fine lattices	MILC coarse lattices
sea quark masses	$am_l = 0.0062$,	$am_l = 0.01$,
	$am_s = 0.031$	$am_s = 0.05$
β	7.09	6.76
a	0.09fm	0.125fm
geometry	$28^3 \times 96$	$20^3 \times 64$
# of confs	995	671
valence quark masses	$0.003, 0.006, \ldots, 0.030$	$0.01, 0.02, \ldots, 0.05$

Comparison between coarse lattices and fine lattices (I)



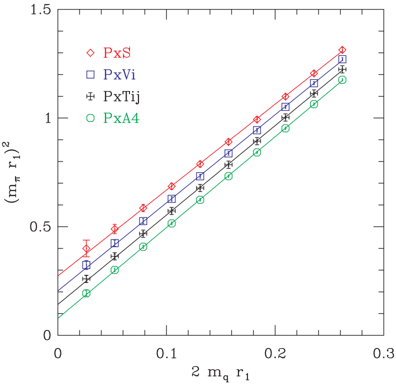
(a) Coarse lattices



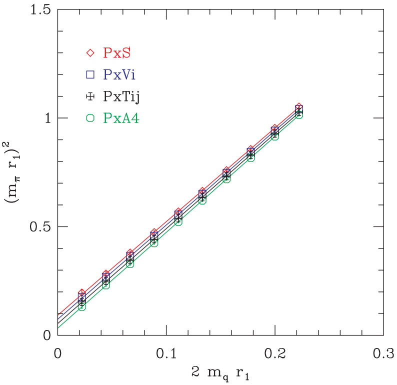
(b) Fine lattices

- Coarse : $\mathcal{O}(a^2) \approx \mathcal{O}(p^2)$. (S$\chi$PT)
- Fine : $\mathcal{O}(a^2) \approx \mathcal{O}(p^4) \ll \mathcal{O}(p^2)$. (S$\chi$PT)

Comparison between coarse lattices and fine lattices (II)



(c) Coarse lattices



(d) Fine lattices

- Coarse : $\mathscr{O}(a^2) \approx \mathscr{O}(p^2)$. (S$\chi$PT)
- Fine : $\mathscr{O}(a^2) \approx \mathscr{O}(p^4) \ll \mathscr{O}(p^2)$. (S$\chi$PT)

Scaling behavior of pion spectrum

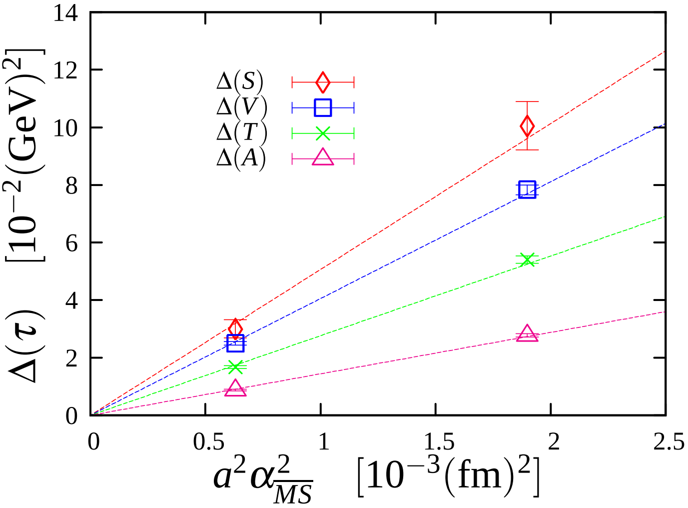
Splittings of pion multiplet spectrum : $\Delta(\tau)$

$$m_\pi^2(\tau) = m_\pi^2(P) + \Delta(\tau)$$

Taste[1] τ	$\Delta(\tau)$ $[(\mathrm{GeV})^2]$		$\Delta(\mathrm{Fine})/\Delta(\mathrm{Coarse})$
	$a = 0.125\mathrm{fm}$	$a = 0.09\mathrm{fm}$	
A	0.0278(6)	0.0087(3)	0.314(12)
T	0.0540(13)	0.0168(4)	0.310(11)
V	0.0783(17)	0.0250(6)	0.319(11)
S	0.1005(84)	0.0300(31)	0.299(40)

[1] P : Pseudo-scalar(ξ_5), A : Axial vector($\xi_{\mu 5}$), T : Tensor($\xi_{\mu\nu}$), V : Vector(ξ_μ), S : Scalar(I)

Scaling behavior of pion spectrum



- $\Delta(\tau)$ behave linearly as a function of $a^2\alpha^2_{\overline{MS}}$.

Outline

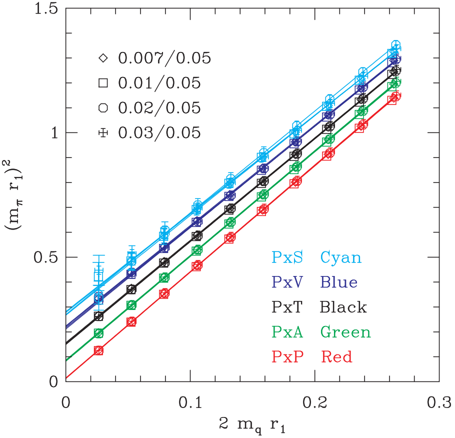
Lattice ensembles

- We used MILC coarse lattice ensembles ($a = 0.125$fm).

Geometry	$m_l a$(light quark mass)	$m_s a$(strange quark mass)
$24^3 \times 64$	0.005	0.05
$20^3 \times 64$	0.007	0.05
$20^3 \times 64$	0.010	0.05
$20^3 \times 64$	0.020	0.05
$20^3 \times 64$	0.030	0.05

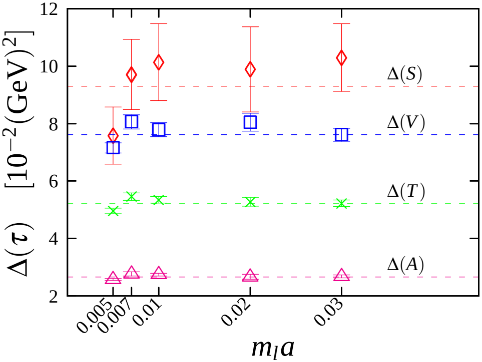
- We study the dependence of pion spectrum on light sea quark masses.

Sea quark dependence of pion spectrum



- Various light quark mass data points are on top of each other.
- Slopes are parallel to each other.
- So there is no dependence of pion spectrum on light sea quark mass.

Comparing splittings in the chiral limit



- Except for $m_l a = 0.005$, $\Delta(\tau)$ does not depend on the sea quark mass within statistical uncertainty.
- Note that the ensemble for $m_l a = 0.005$ has a larger volume of 24^3 compare to other ensembles.
- This could come from a finite volume effect but is also consistent with others within two σ.

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Summary

Sources

- In order to select a specific pion taste we must choose sources and sinks that belong to a specific irrep of the time slice group.
- Propagators are obtained by solving the Dirac equation with source h

$$(D+m)\chi = h$$

$$\Rightarrow \chi(x,a;t;\vec{A}) = \sum_{y,b} G(x,a;y,b)h(y,b;\vec{A})$$

- G is the point-to-point quark propagator, x, y label lattice sites, and a, b are color indices.

Cubic wall sources and Cubic U(1) sources

Cubic wall sources

$$h(y,b;t;\vec{A}) = \delta_{y_4,t} \sum_{\vec{n}} \delta^3_{\vec{y},2\vec{n}+\vec{A}} \eta(b)$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{\eta} \eta(c)\eta^*(c') = \delta_{c,c'}$$

Cubic U(1) sources

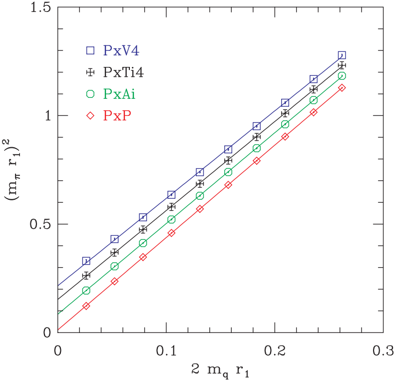
$$h(y,b;t;\vec{A}) = \delta_{y_4,t} \sum_{\vec{n}} \delta^3_{\vec{y},2\vec{n}+\vec{A}} \eta(\vec{n},b)$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{\eta} \eta(\vec{n},c)\eta^*(\vec{n}',c') = \delta_{\vec{n},\vec{n}'}\delta_{c,c'}$$

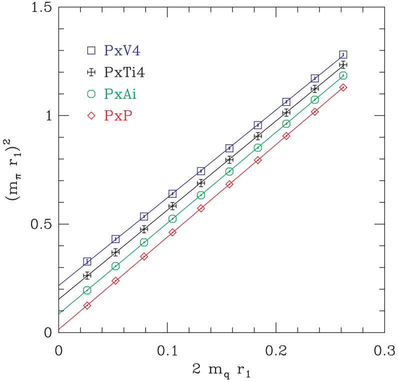
- $\vec{n}$ is a vector labeling 2^3 cubes in the time slice.
- $\vec{A}$ labels points within the cubes.
- η's are U(1) noise vectors normalized as in the above formulae.

Comparison between CW and CU1 sources (I)

$20^3 \times 64$, $m_l a = 0.01, m_s a = 0.05$



(e) Cubic wall [CW]

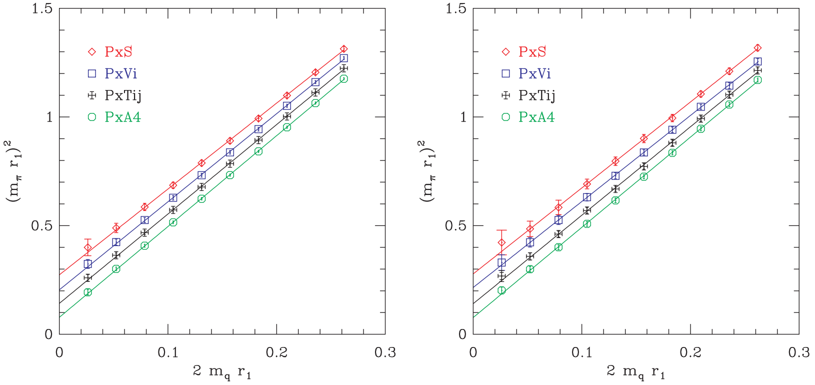


(f) Cubic U(1) [CU(1)]

- There is no difference between CW and CU(1) for LT tastes.

Comparison between CW and CU1 sources (II)

$20^3 \times 64$, $m_l a = 0.01, m_s a = 0.05$



(g) Cubic wall (CW) (h) Cubic U(1) (CU(1))

- In the case of NLT tastes, statistical uncertainties for CW are smaller than those for CU(1).
- We prefer CW to CU(1) for our future numerical study.

Comparison between CW and CU1 sources (III)

Taste	$\Delta(\tau)$ [(GeV)^2]	
	Cubic wall [CW]	Cubic U(1) [CU(1)]
$\xi_i\xi_5$	0.0278(6)	0.0274(5)
$\xi_4\xi_i$	0.0540(13)	0.0535(11)
ξ_4	0.0783(17)	0.0779(24)
$\xi_4\xi_5$	0.0253(33)	0.0240(49)
$\xi_i\xi_j$	0.0500(43)	0.0486(61)
ξ_i	0.0740(57)	0.0773(101)
I	0.1005(84)	0.1014(134)

- As predicted by SχPT, the data respect SO(4) symmetry.
- Statistical gains for CW are about twice compared to CU(1) for NLT tastes.
- Therefore we prefer CW to CU(1).

Outline

Summary

- With HYP staggered valence quarks, the taste breaking is reduced by factor of 0.3 on fine lattices ($a = 0.09$fm) than coarse lattices ($a = 0.125$fm).
- There is no dependence of pion spectrum on sea quark masses.
- We prefer using cubic wall sources since statistical errors are smaller for cubic wall sources than for cubic U(1) sources.