Electromagnetic splittings of hadrons from Staggered quarks in full QCD

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Outline of this talk

- ⋆ Electromagnetic effects in QCD: introduction and motivation
- ⋆ Methodology employed in the present work
- ⋆ Numerical results and analysis
- ⋆ Summary and tasks ahead

References:

1. A. Duncan, E. Eichten, H. Thacker, PRL 76, 3894 [*Duncan(1996)*]
2. T. Blum, T. Doi, M. Hayakawa, T. Izubuchi, N. Yamada, PRD 76, 114506 [*Blum(2007)*]
3. J. Bijnens, N. Danielsson, PRD 75, 014505 [*Bijnens(2007)*]
4. C. Aubin, C. Bernard, C. DeTar, J. Osborn, S. Gottlieb, E.B. Gregory, D. Toussaint, U.M. Heller, J.E. Hetrick, R. Sugar, PRD 70, 114501 [*MILC(2004)*]

Hadron Isomultiplets

n^0	(udd)	939.6 MeV		$\pi^\pm$	$(u\bar{d})$	139.6 MeV
p^+	(uud)	938.3 MeV		π^0	$(q\bar{q})$	135.0 MeV
Σ^-	(dds)	1197.4 MeV		$K^\pm$	$(u\bar{s})$	493.7 MeV
Σ^0	(uds)	1192.5 MeV		K^0	$(d\bar{s})$	497.7 MeV
Σ^+	(uus)	1189.4 MeV				

Two sources of mass splittings

- Isospin breaking contributions due to the difference in masses of the u and d quarks
- Different electromagnetic charges between the meson and baryon isospin members

Different contributions to mass splittings

$$
\begin{aligned}
M_{\pi^\pm} - M_{\pi^0} &= 4.594 \text{ MeV (Expt)} \\
M_{\pi^\pm} - M_{\pi^0}\big|^{d-u} &\approx 0.164(30) \text{ MeV} \\
M_{\pi^\pm} - M_{\pi^0}\big|^{\text{em}} &\approx 4.43(30) \text{ MeV}
\end{aligned}
$$

$$
\begin{aligned}
M_{K^0} - M_{K^\pm} &= 3.971(30) \text{ MeV (Expt)} \\
M_{K^0} - M_{K^\pm}\big|^{d-u} &\approx 3.973(30) \text{ MeV} \\
M_{K^0} - M_{K^\pm}\big|^{\text{em}} &\approx -0.044 \text{ MeV}
\end{aligned}
$$

In the chiral limit and to $\mathcal{O}(e^2)$,

$$\text{Dashen's theorem :} \quad \left(M^2_{\pi^\pm} - M^2_{\pi^0}\right)_{\text{e.m.}} = \left(M^2_{K^\pm} - M^2_{K^0}\right)_{\text{e.m.}}$$

electromagnetic interactions modifying LO psuedoscalar masses

$$M^2_{\pi^\pm} = 2\hat{m}B_0 + \frac{2Ce^2}{F_0^2}(q_u - q_d)^2, \quad M^2_{K^\pm} = (\hat{m} + m_s)B_0 + \frac{2Ce^2}{F_0^2}(q_u - q_s)^2$$

Corrections to Dashen's theorem

At NLO, *i.e.* $\mathcal{O}(e^2 m)$, Dashen's theorem receives correction, denoted as (for $m_u = m_d = \hat{m}$),

$$\Delta M_D^2 = \Delta M_K^2 - \Delta M_\pi^2 = \left(M_{K^\pm}^2 - M_{K^0}^2\right)_{\text{e.m.}} - \left(M_{\pi^\pm}^2 - M_{\pi^0}^2\right)_{\text{e.m.}}$$

or parametrize as in *MILC(2004), Blum(2007)*

$$\Delta M_K^2 = (1 + \Delta_E)\Delta M_\pi^2$$

- ⋆ NLO correction to Dashen's theorem can be large [*Donoghue et.al. PRD 47 (1993) 2089; Urech NPB 433 (1995) 234; Bijnens et.al. NPB 490 (1997) 239*].
- ⋆ Estimation of electromagnetic contributions to M_π and especially $M_{K^{\pm,0}}$ has large uncertainties.
- ⋆ Determination of quark masses, particularly m_u/m_d, relies for electromagnetic contributions on continuum phenomenology which results in $\sim 20\%$ error in quark mass ratio [*MILC (2204)*].

Lattice calculation of meson splittings

Mass splittings within isomultiplets from EM $\sim m_d - m_u$

- Lattice computation of hadron spectra and pattern of isomultiplet mass splittings by including EM fields in QCD $\rightarrow$ understanding isospin symmetry breaking
- Precision determination of quark masses (m_u, m_d, m_s) and their ratios.
- Phenomenologically relevant quantities like ΔM^2 can be determined from pQQCD where the photons are only coupled to the valence quarks.
- pQχPT for $N_f = 2+1$ at NLO has been worked out by *Bijnens et.al (2007)* $\longrightarrow$ to be used for extrapolation to chiral limit, requires determination of EM *LEC*s.

Pure electromagnetic correction from lattice

Pure electromagnetic correction (relevant for determining corrections to Dashen's theorem) is computed from [*Bijnens et.al (2007)*],

$$\begin{aligned}\Delta M^2 &= M^2(\chi_1,\chi_3,q_1,q_3) - M^2(\chi_1,\chi_3,q_3,q_3)\\ &\quad -M^2(\chi_1,\chi_1,q_1,q_3) + M^2(\chi_1,\chi_1,q_3,q_3)\end{aligned}$$

where $\chi_i \equiv 2B_0 m_i$ and $M^2(\chi_1,\chi_3,q_1,q_3)$ denotes mass of the meson with valence quark masses $\chi_1 = \chi_2,\ \chi_3$ and valence charges $q_1,\ q_3$. Lattice data ΔM^2 is fitted to

$$\begin{aligned}\Delta M^2 &= \mathcal{A}_1(\chi_{13}-\chi_{11}) + \mathcal{A}_2\left[\chi_{13}\log\left(\frac{\chi_{13}}{\mu^2}\right) - \chi_{11}\log\left(\frac{\chi_{11}}{\mu^2}\right)\right]\\ &\quad +\mathcal{A}_3\left[\chi_{1s}\log\left(\frac{\chi_{1s}}{\mu^2}\right) - \chi_{3s}\log\left(\frac{\chi_{3s}}{\mu^2}\right)\right]\\ &\quad +\mathcal{A}_4\left[\chi_{13}\left\{1-\frac{1}{2}\log\left(\frac{\chi_{13}}{\mu^2}\right)\right\} - \chi_{11}\left\{1-\frac{1}{2}\log\left(\frac{\chi_{11}}{\mu^2}\right)\right\}\right]\end{aligned}$$

$SU(3)_{\mathrm{color}} \times U(1)_{\mathrm{em}}$ fields on lattice

- Gluon configurations generated with $N_f = 2+1$ dynamical Asqtad staggered quarks (by MILC).

- Quenched photon configurations generated with non-compact $U(1)$ action [*Duncan et.al. (1996), Blum et.al. (2007)*],

$$S_{\mathrm{em}} = \frac{1}{4e^2} \sum_{n,\mu,\nu}' \left(\partial_\mu^{\mathrm{R}} A_\nu(n) - \partial_\nu^{\mathrm{R}} A_\mu(n)\right)^2 , \qquad \nabla \cdot \vec{A} = 0$$

  subjected to Coulomb gauge fixing.

- Photon fields are obtained in momentum space from Gaussian random numbers and coordinate space Coulomb gauge fields $\{A_\mu(n)\}$ are recovered by FFT $\Rightarrow U_\mu^{\mathrm{em}}(n) = e^{iqeA_\mu(n)}$.

- Charged quark propagators are computed for $U_\mu^{\mathrm{qcd}}(n) \times U_\mu^{\mathrm{em}}(n)$.

First attempt with staggered quarks

Partially quenched $N_f = 2 + 1$ calculations

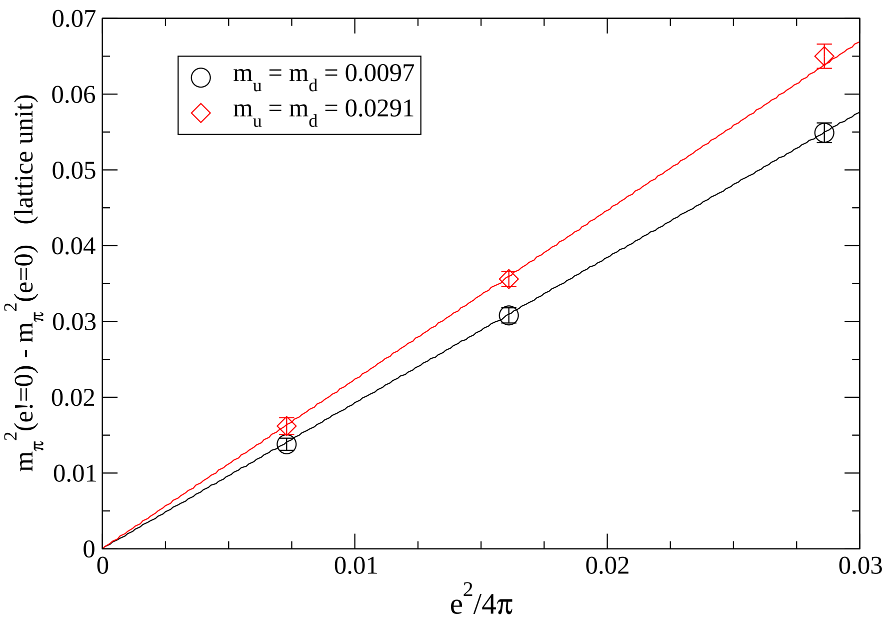
lattice	β	a (fm)	m_l/m_s	cfgs
$20^3 \times 48$	6.566	0.15	0.00484/0.0484	200
$16^3 \times 48$	6.572	0.15	0.0097/0.0484	400
$16^3 \times 48$	6.586	0.15	0.0194/0.0484	400
$16^3 \times 48$	6.600	0.15	0.0290/0.0484	400

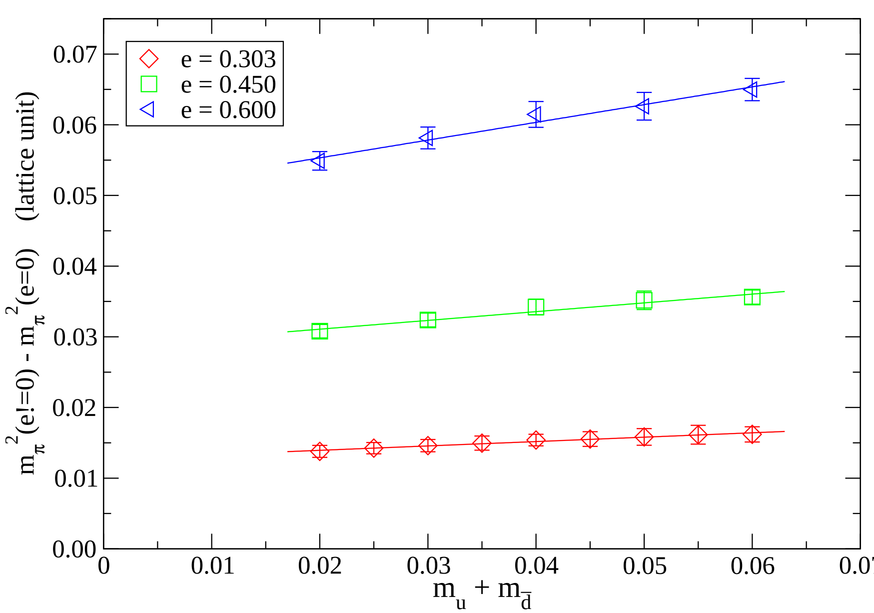
- Asqtad improved staggered valence quark propagators for m_q^{v} = 0.0048, 0.0097, 0.0145, 0.0194, 0.0242, 0.0291, 0.0388, 0.0484
- Wall sources for charged quark propagators. Fits reported here are all correlated χ^2 fits and jackknife errors.
- Masses are extracted from exponential fall-off of meson propagators in the time range 9–24, taking into account the correlations among time slices.

First attempt with staggered quarks

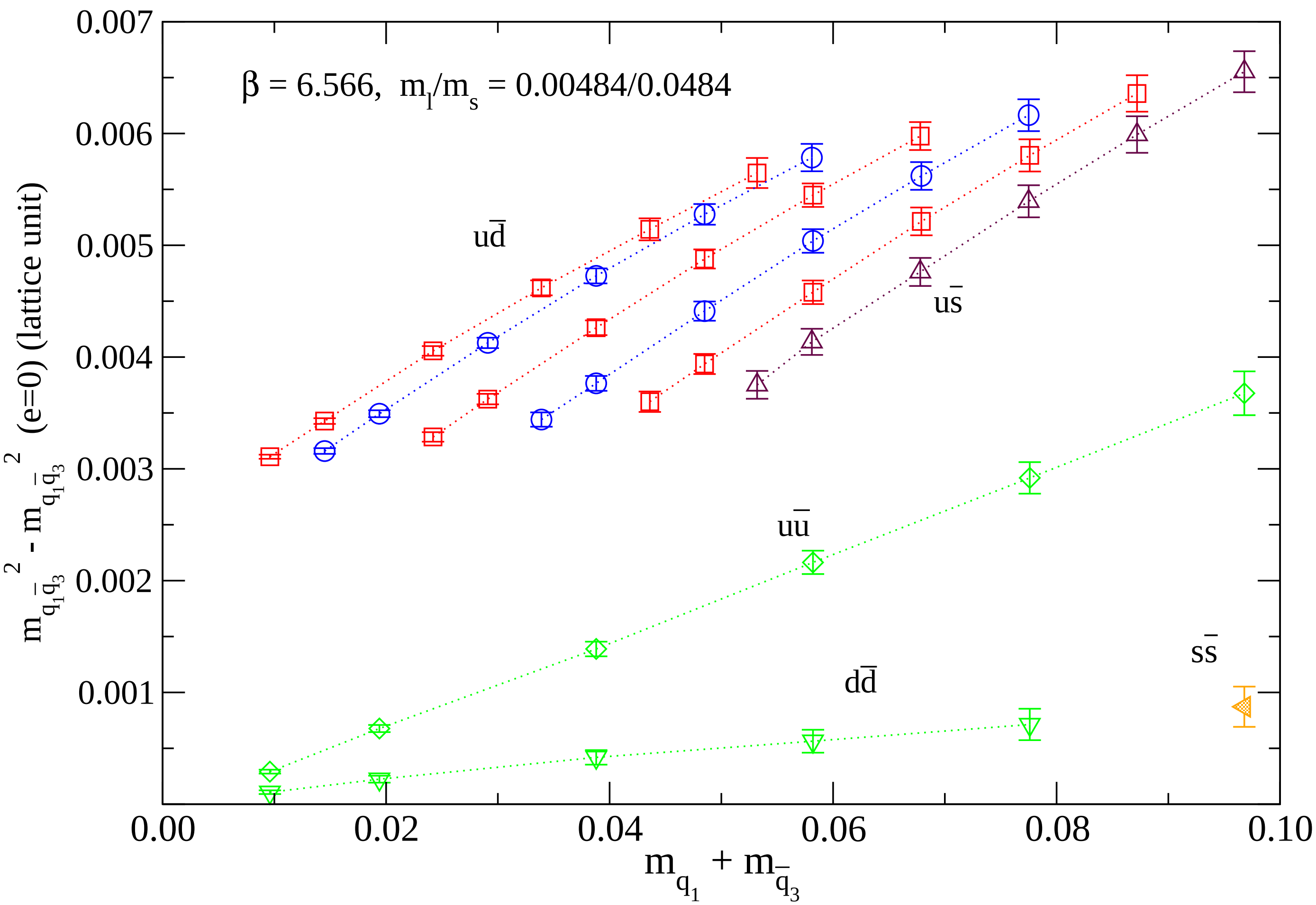
Estimate of $\mathcal{O}(e^2)$ and $\mathcal{O}(e^2 m)$ contribution to pseudoscalar masses

$$m_\pi^2(e \neq 0) - m_\pi^2(e = 0) = \mathcal{A}_0 (q_u - q_d)^2 + \mathcal{O}(e^2 m)$$

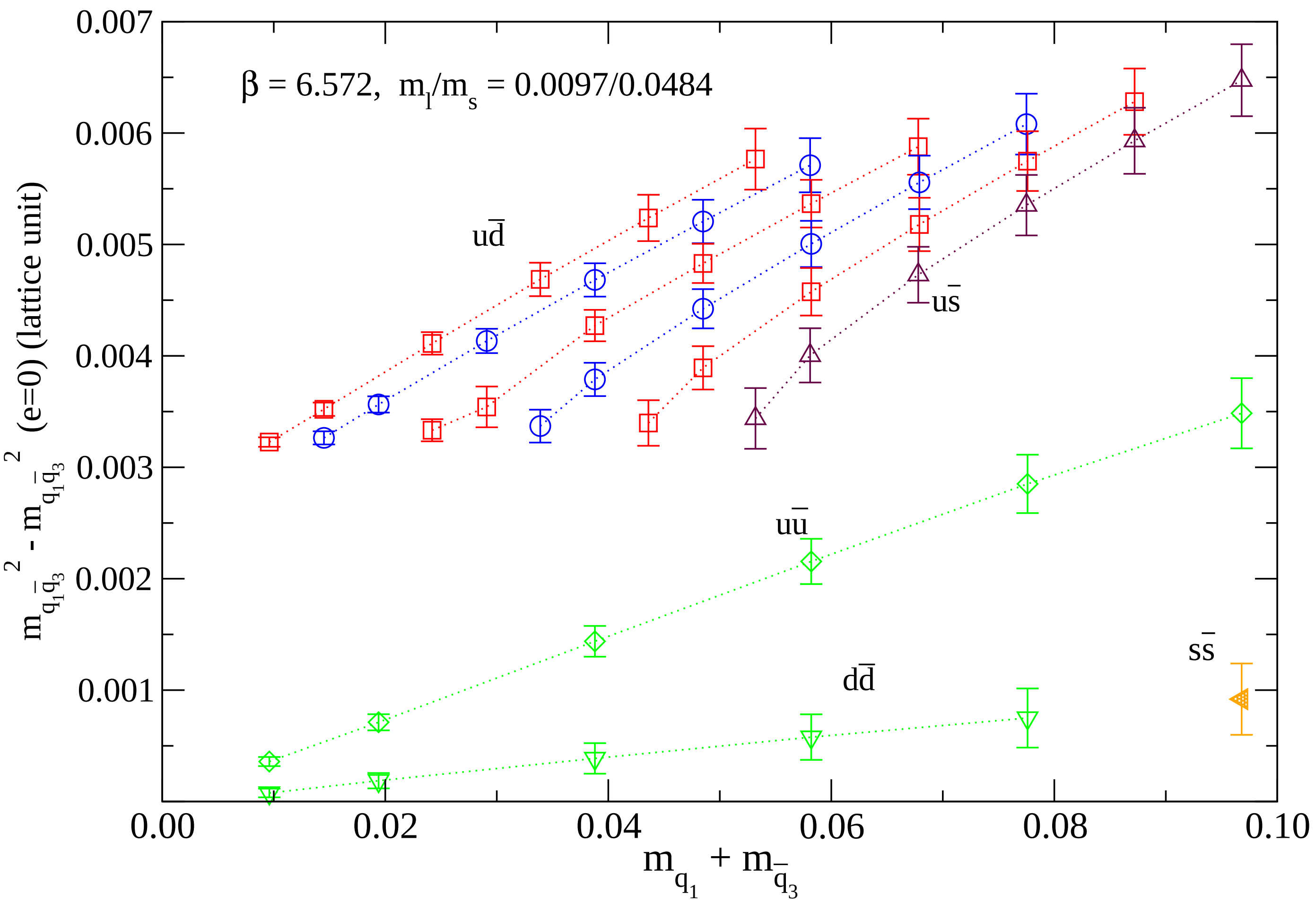




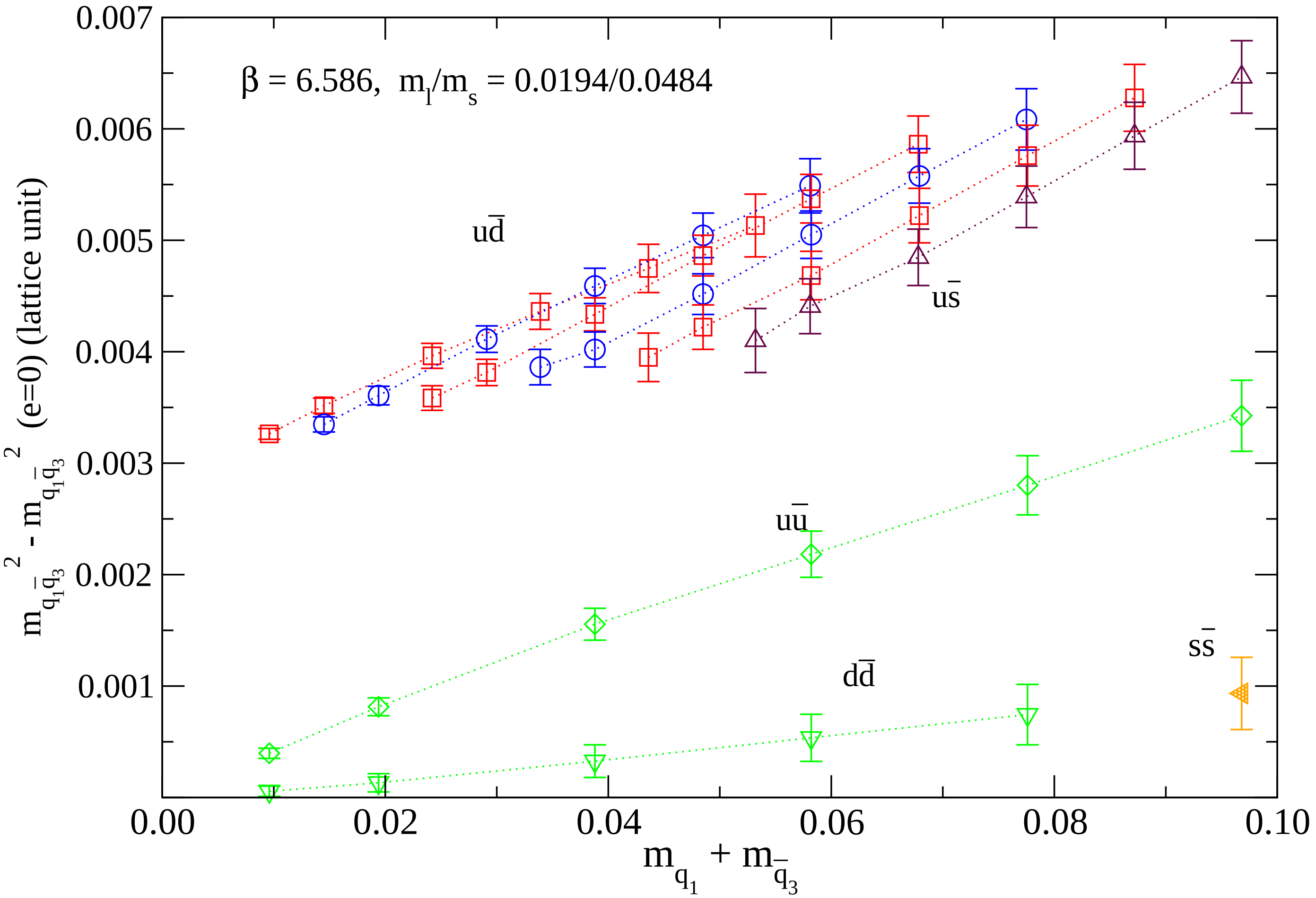
Meson splittings: preliminary result



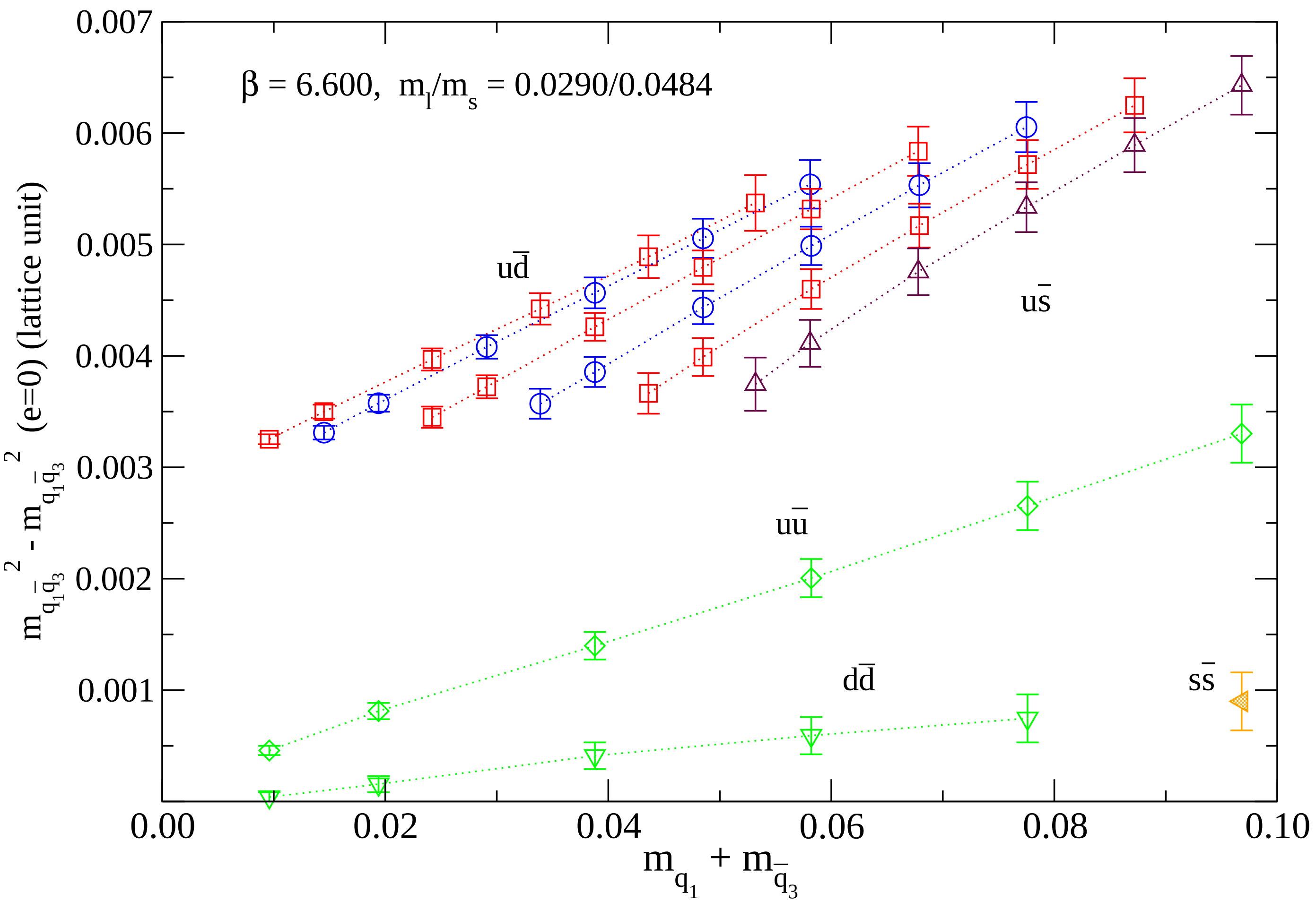
Meson splittings: preliminary result



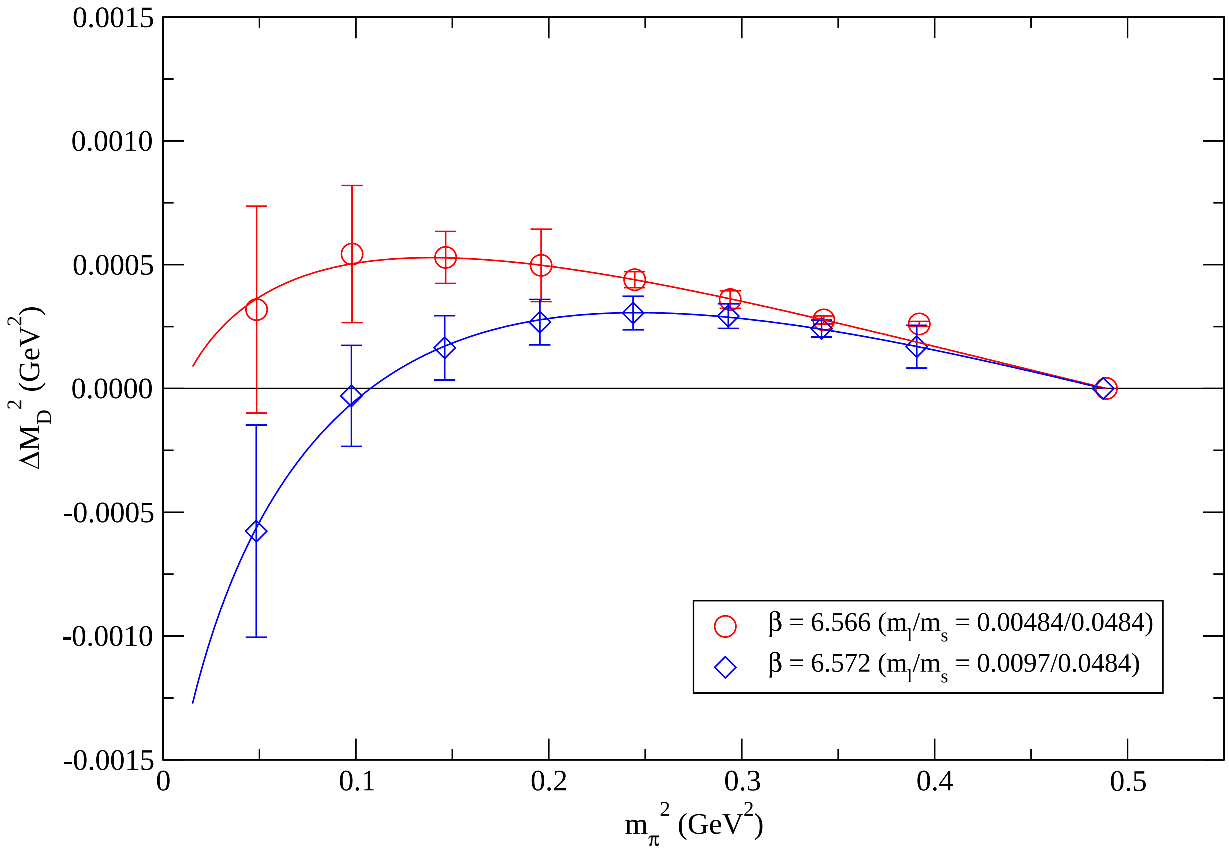
Meson splittings: preliminary result



Meson splittings: preliminary result



Dashen correction: preliminary result



$$7.0\times 10^{-4} < \Delta M_D^2\ (\mathrm{GeV}^2)\ < 1.8\times 10^{-3}\quad (1.07\times 10^{-3})$$

Summary and tasks ahead

- ⋆ Inclusion of electromagnetic corrections in Lattice QCD is important to understand isospin symmetry breaking and precision calculation of quark masses and their ratios.
- ⋆ As a simple first step, EM fields included in QCD are in quenched approximation and NLO pQχPT 2+1 flavor for such QED + QCD is available for use with lattice results.
- ⋆ We have analyzed four MILC $N_f = 2+1$, $a \approx 0.15$fm ensembles and initial results show good meson splittings, which will soon be analyzed with NLO pQχPT.
- ⋆ Preliminary estimation of Dashen correction has been carried out and found to be in the correct range as predicted from continuum.
- ⋆ Extension of the present study in larger volume and smaller lattice spacing ($a = 0.12$fm, 0.09fm) is under way.