QCD Equation of State at Non-zero Chemical Potential

S. Basak, A. Bazavov, C. Bernard, C. DeTar, W. Freeman, S. G., U.M. Heller, J.E. Hetrick,

J. Laiho, L. Levkova, J. Osborn, R. Sugar, D. Toussaint

[MILC Collaboration]

`sg at indiana.edu`

Outline

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- Methodology
- Results
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Motivation

- Experiments at RHIC start with a baryon rich environment; hence they naturally have a non-zero chemical potential.
- Finite temperature field theory formalism easily admits a chemical potential, but we are left with a complex action and can no longer use importance sampling.
- If the chemical potential is small, we can employ the Taylor expansion method:
  - C.R. Allton *et al.*, Phys. Rev. D 66 (2002) 074507
  - R.V. Gavai and S. Gupta, Phys. Rev. D 68 (2003) 034506

Methodology

Physical quantities of interest are Taylor expanded in the chemical potentials for light and strange quarks. For example:

$$\frac{p}{T^4} = \frac{\ln Z}{VT^3} = \sum_{n,m=0}^{\infty} c_{nm}(T) \left(\frac{\bar{\mu}_l}{T}\right)^n \left(\frac{\bar{\mu}_h}{T}\right)^m . \qquad (1)$$

Only terms with $n+m$ even appear due to CP symmetry.

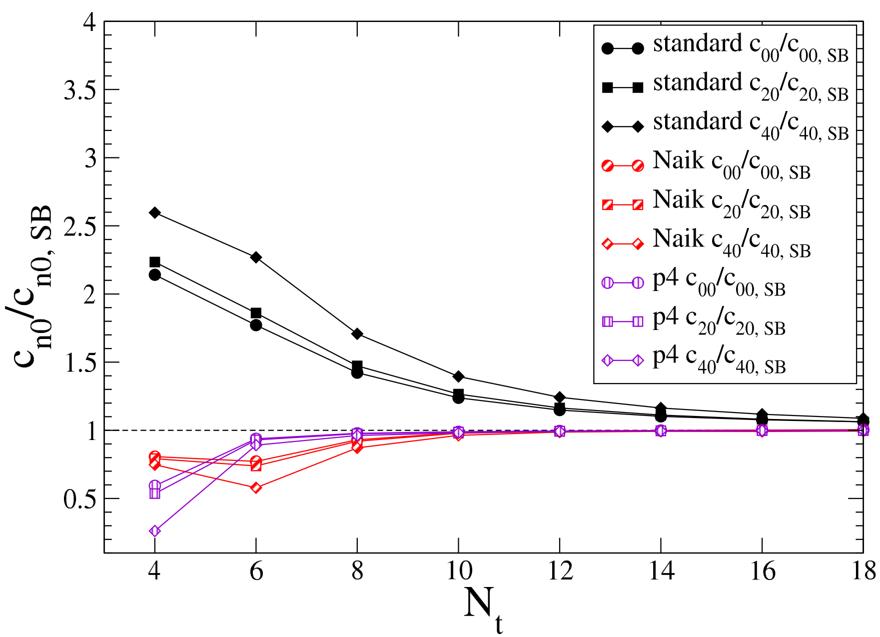
$$c_{nm}(T) = \frac{1}{n!}\frac{1}{m!}\frac{N_\tau^3}{N_\sigma^3}\left.\frac{\partial^{n+m}\ln Z}{\partial(\mu_l N_\tau)^n \partial(\mu_h N_\tau)^m}\right|_{\mu_{l,h}=0} . \qquad (2)$$

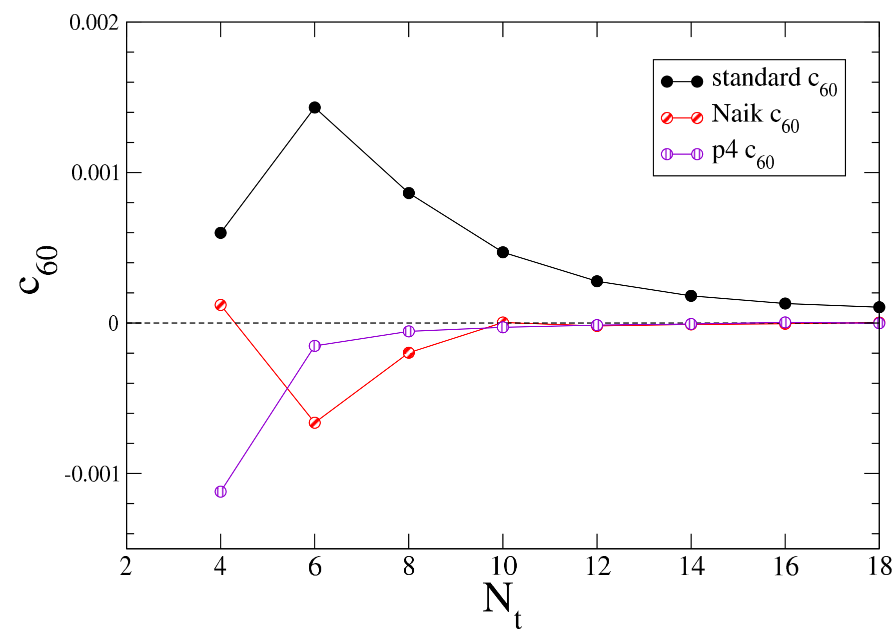
For the interaction measure,

$$\frac{I}{T^4} = -\frac{N_t^3}{N_s^3}\frac{d\ln Z}{d\ln a} = \sum_{n,m}^{\infty} b_{nm}(T) \left(\frac{\bar{\mu}_l}{T}\right)^n \left(\frac{\bar{\mu}_h}{T}\right)^m . \qquad (3)$$

- Temperature dependendent coefficients $c_{nm}(T)$ and $b_{nm}(T)$ are combinations of observables that can be calculated on non-zero T ensembles, but with zero chemical potential.
- We Taylor expand up to 6th order.
- 40 fermionic observables have to be determined using stochastic estimators, as well as several gluonic observables.
- Details can be found in C. Bernard *et al.*, Phys. Rev. D 77,014503 (2008); arXiv:0710.1330 [hep-lat].
- Ensembles are generated on a line of constant physics with $m_l = 0.1 m_s$ and m_s approximately the physical strange quark mass.

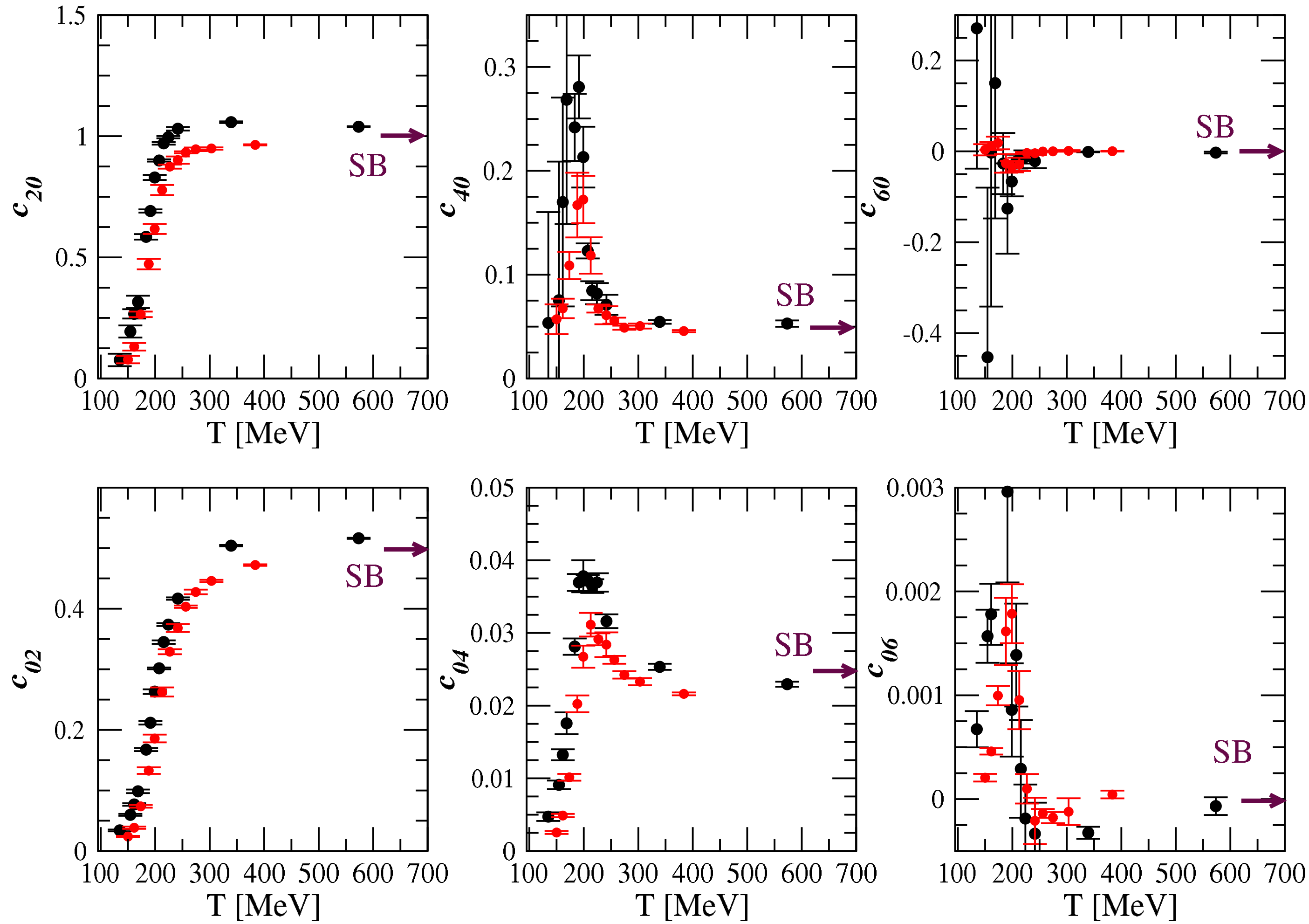
- Our previous work used lattices with $N_t = 4$. We now use $N_t = 6$ and compare with coarser lattices.
- It is interesting to compare the free theory for different N_t to see how the continuum limit is approached.





- Next let's compare $N_t = 4$ (black) and 6 (red) for the interacting theory.

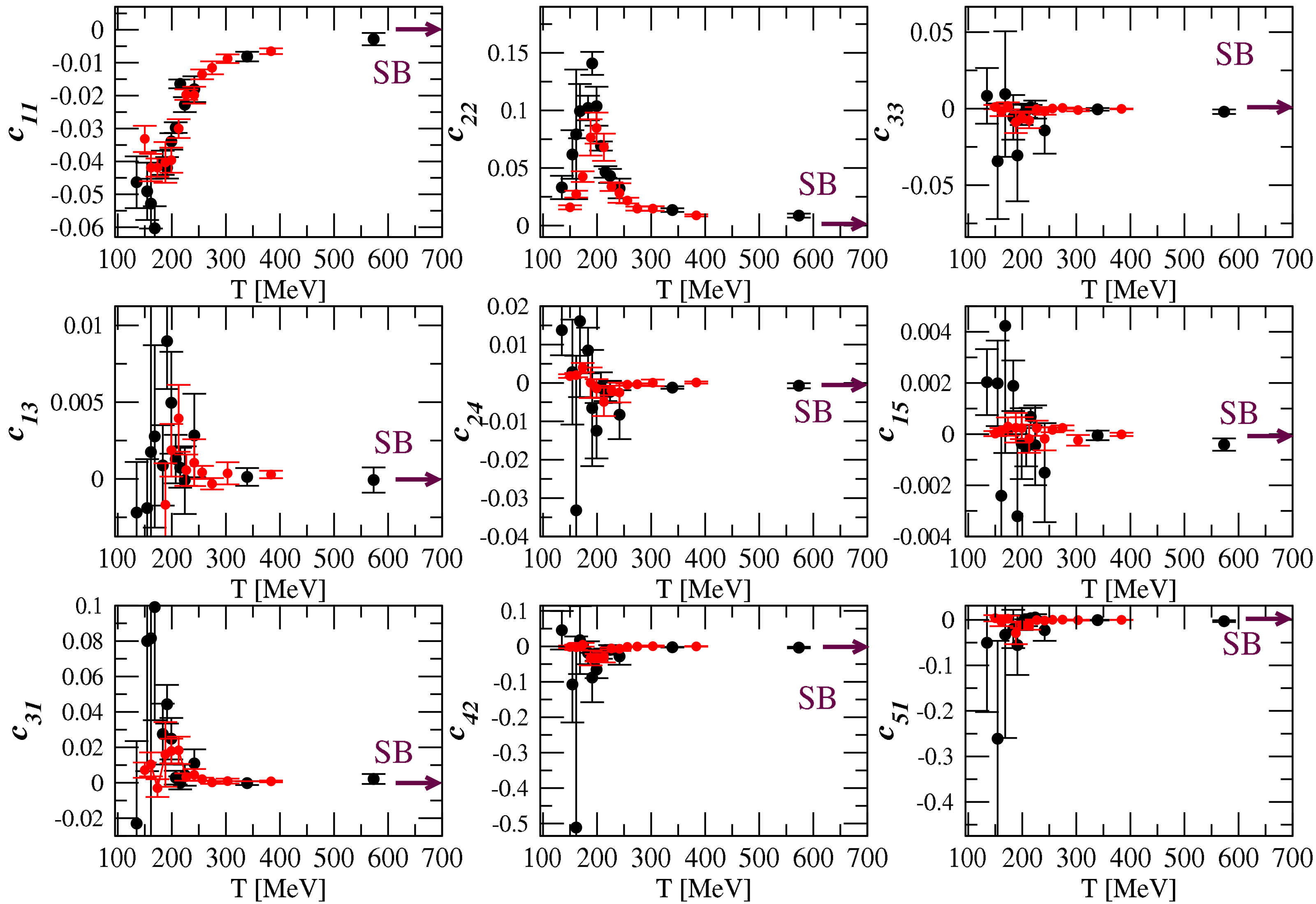
Unmixed coefficients for pressure



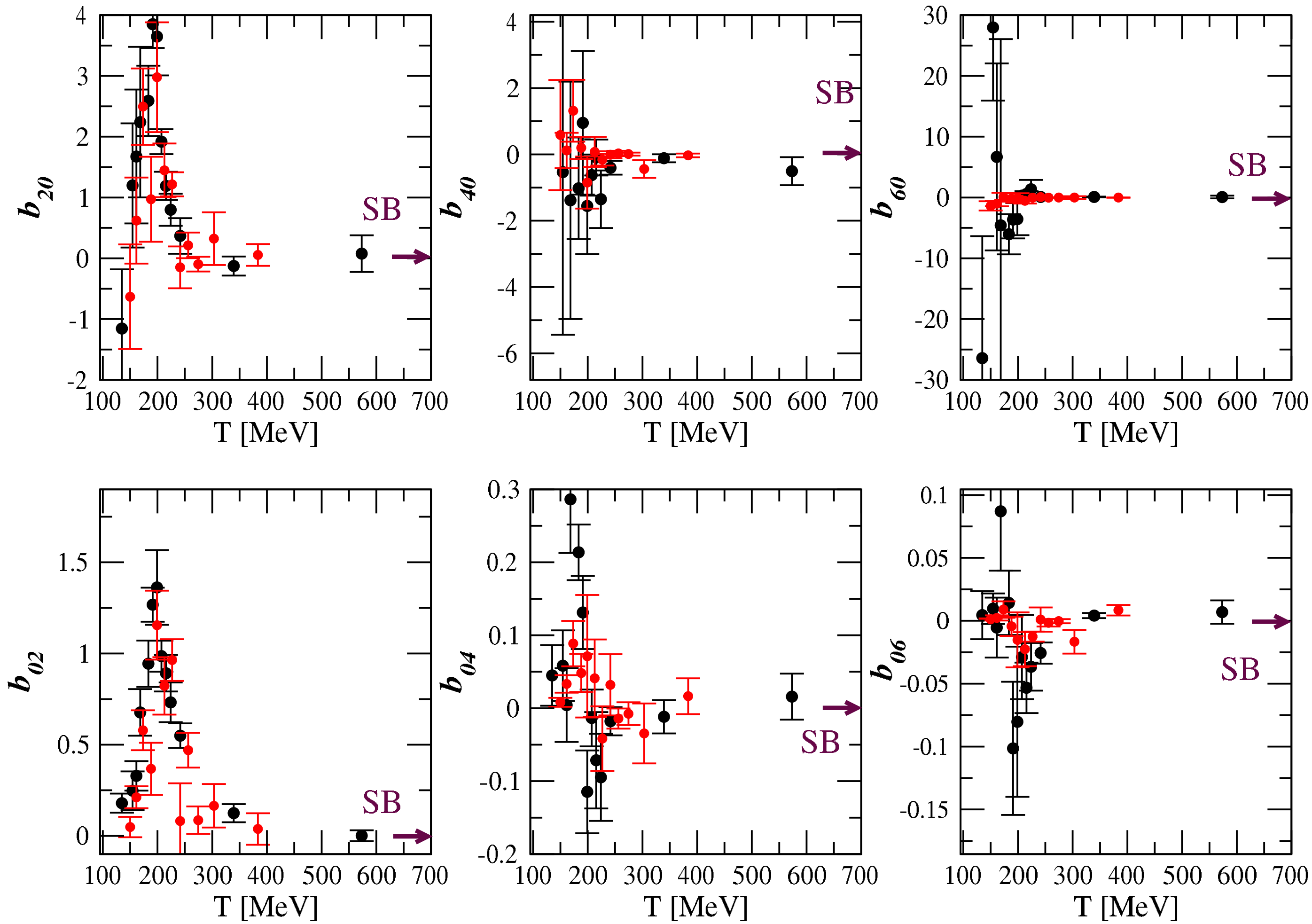
Comments on coefficients

- There is considerable structure at low T and then an approach to SB limit above the cross-over temperature.
- Higher coefficients are small.
- Errors grow rapidly for higher order terms.
- Errors are better controlled for $N_t = 6$.

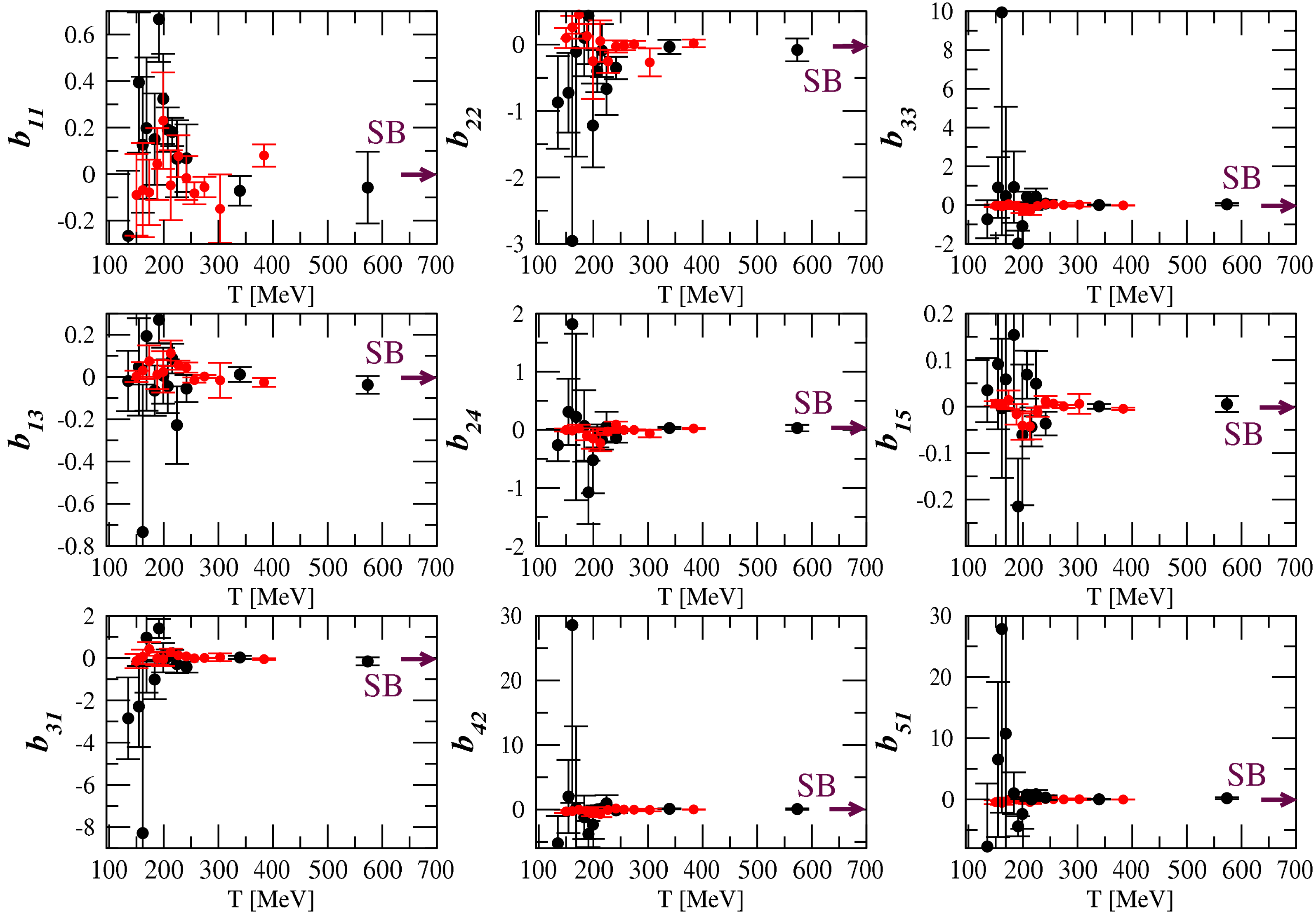
Mixed coefficients for pressure



Unmixed coefficients for I



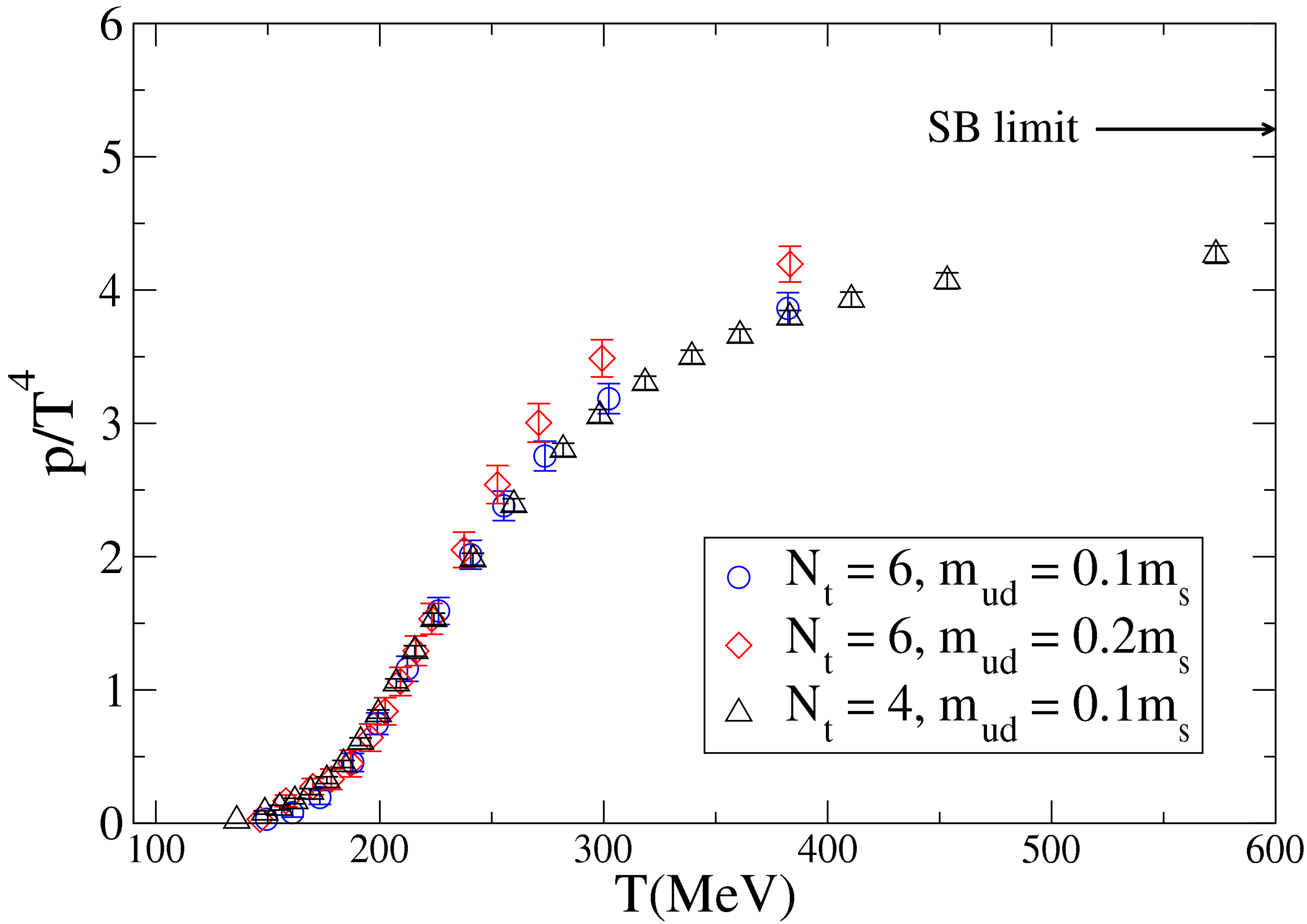
Mixed coefficients for I



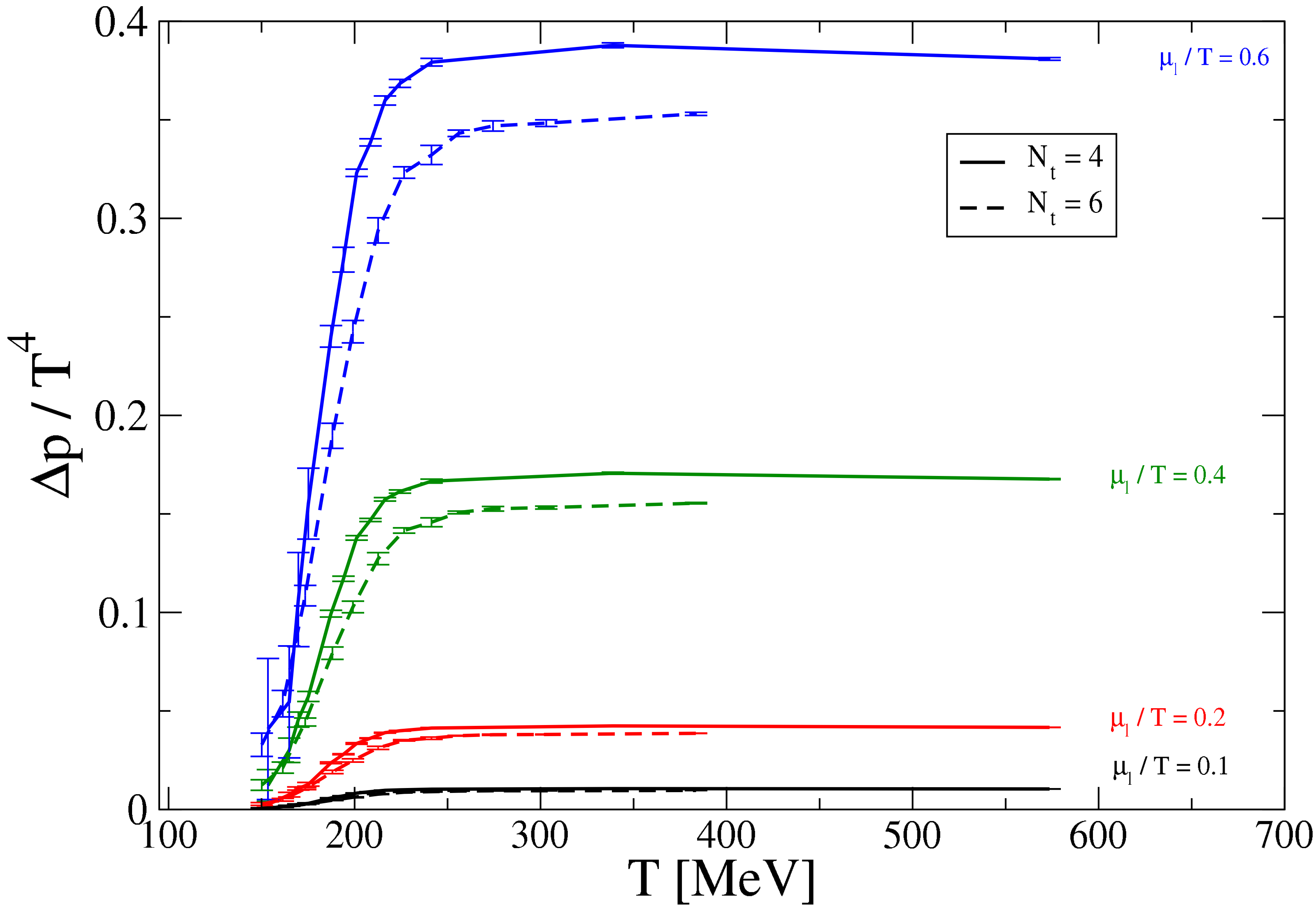
Results

- With the coefficients in hand, we can calculate interesting quantities, such as
  - pressure
  - interaction measure
  - energy density
  - quark number density
  - quark number susceptibility
- Due to non-zero $C_{n1}(T)$ terms a non-zero strange quark density is induced even with $\mu_h = 0$. To study the $n_s = 0$ plasma, we must tune μ_h as a function of μ_l and T.
- We show change in pressure, interaction measure and energy density for various μ_l as a function of T.

Pressure with $\mu = 0$

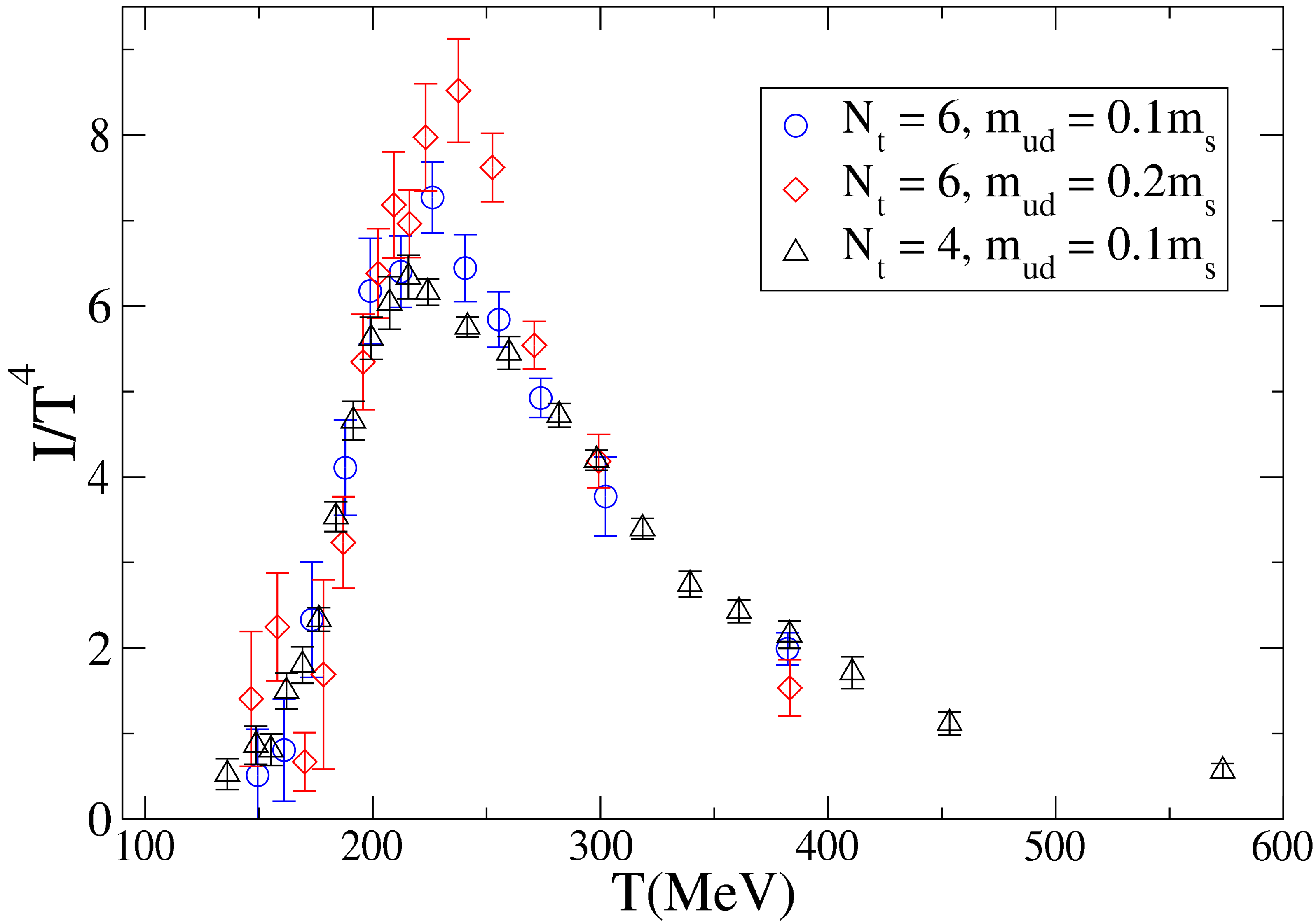


Δ Pressure

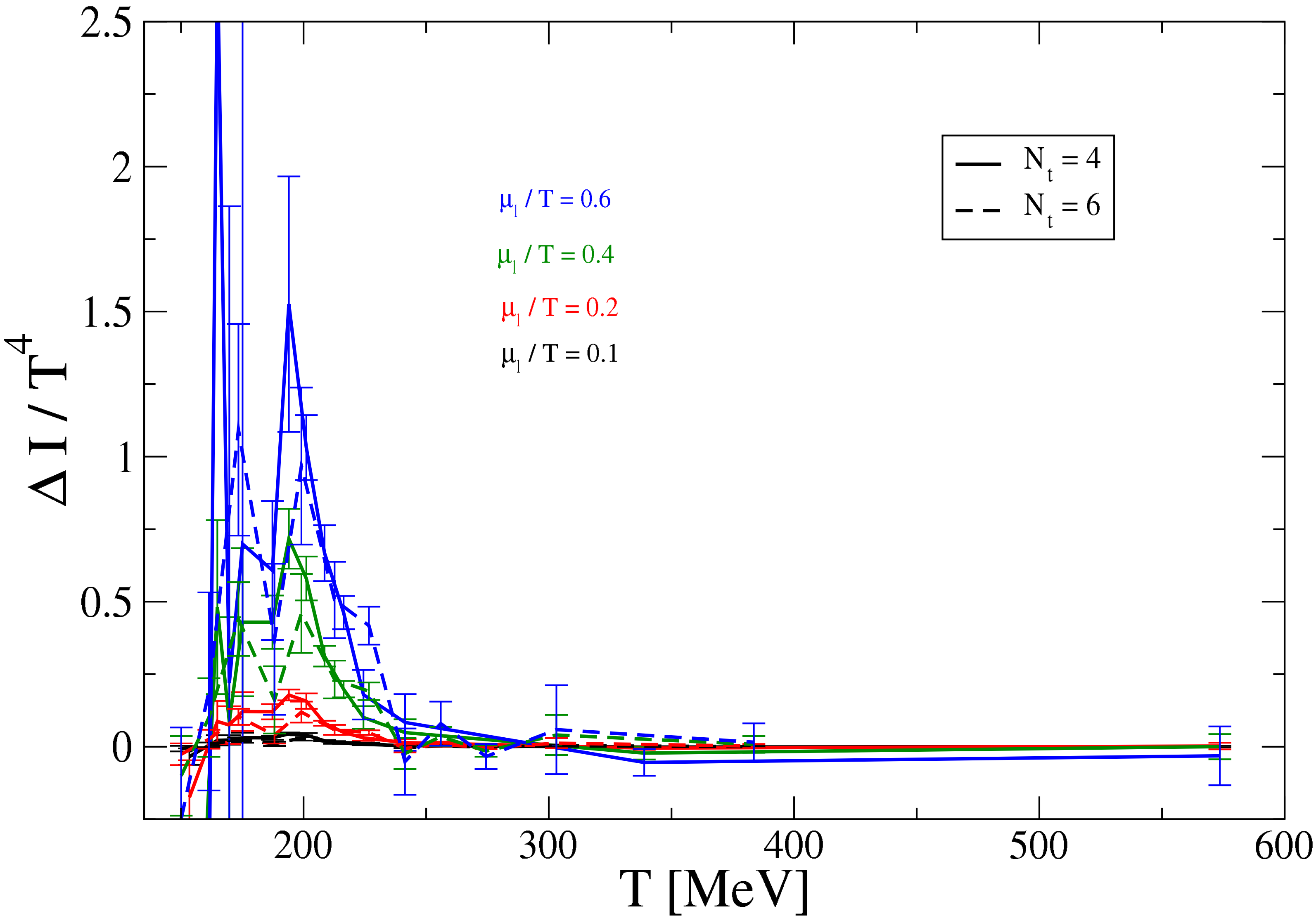


0.4
0.3
0.2
0.1
0
100
200
300
400
500
600
700
T [MeV]
Δp / T⁴
$N_t = 4$
$N_t = 6$
$\mu_l / T = 0.6$
$\mu_l / T = 0.4$
$\mu_l / T = 0.2$
$\mu_l / T = 0.1$

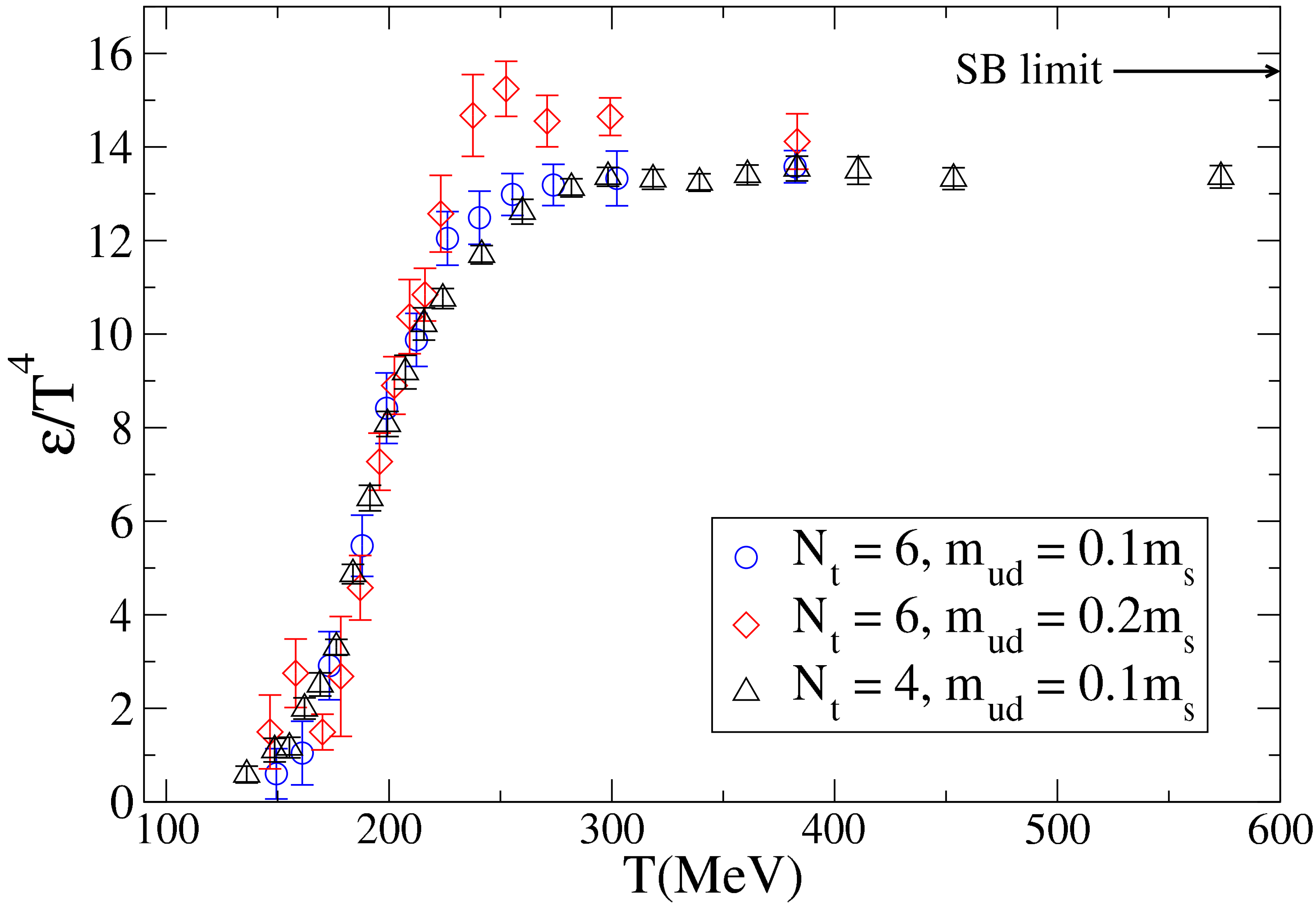
Interaction measure with $\mu = 0$



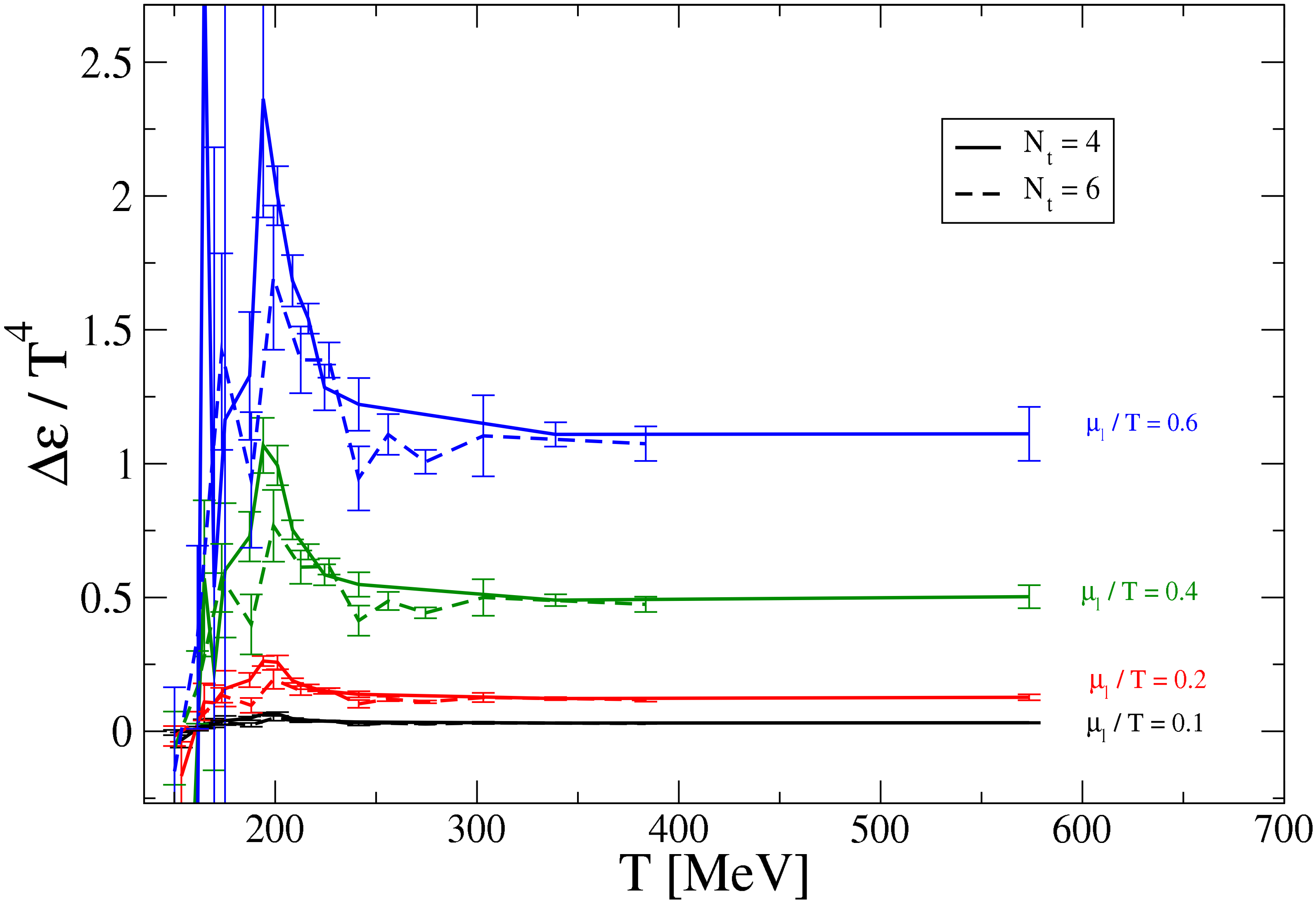
Δ Interaction Measure



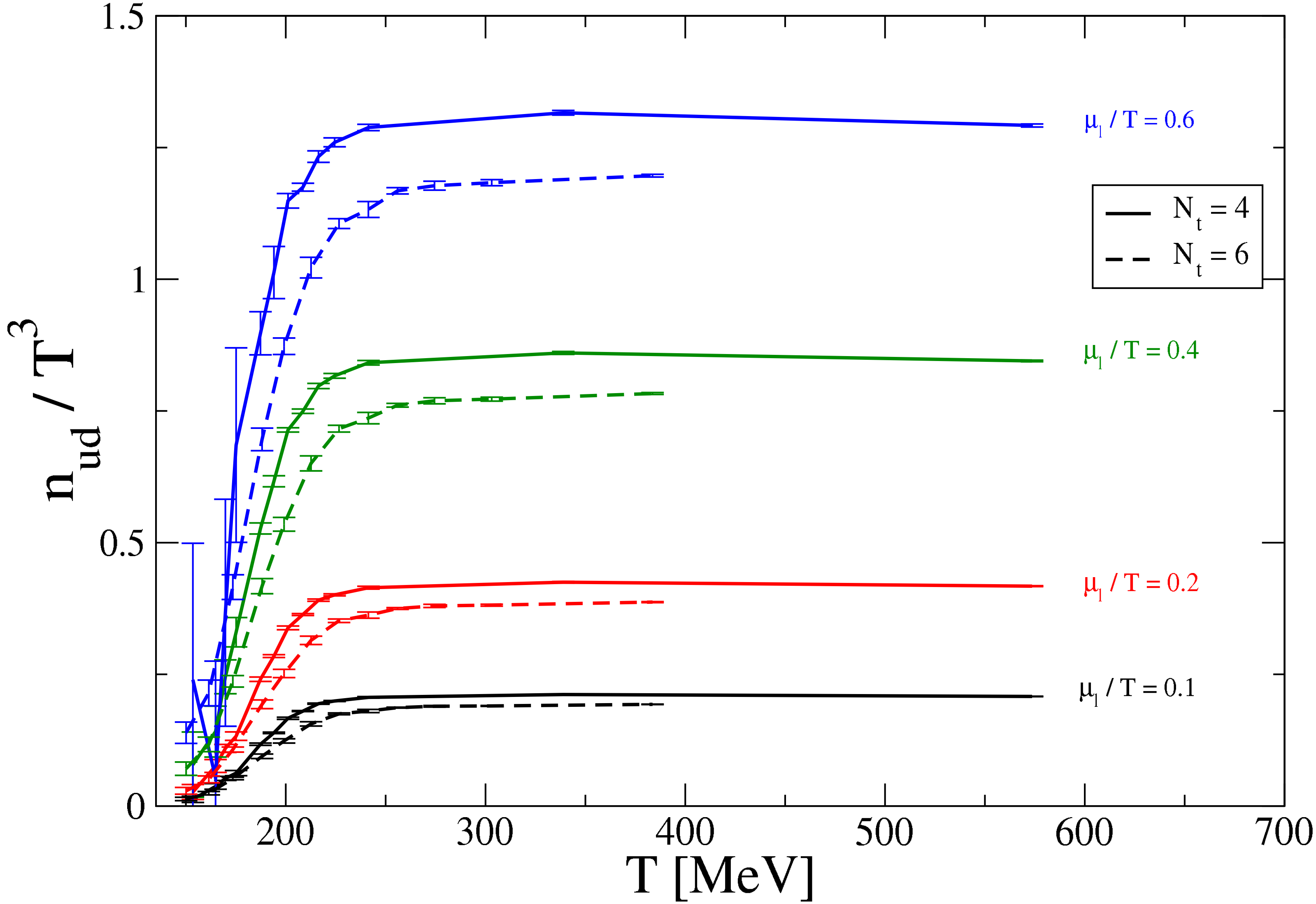
Energy density with $\mu = 0$



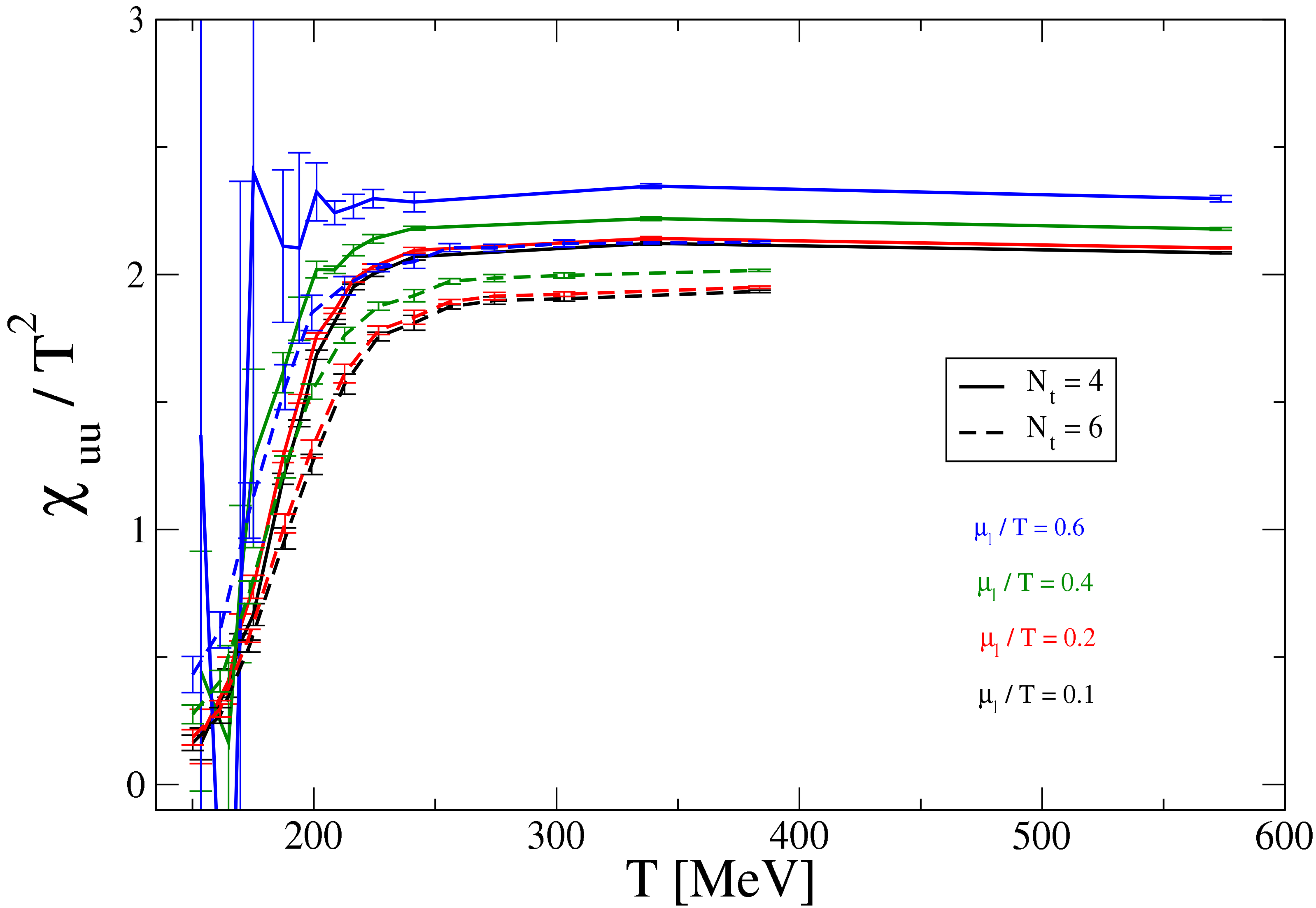
△ Energy Density



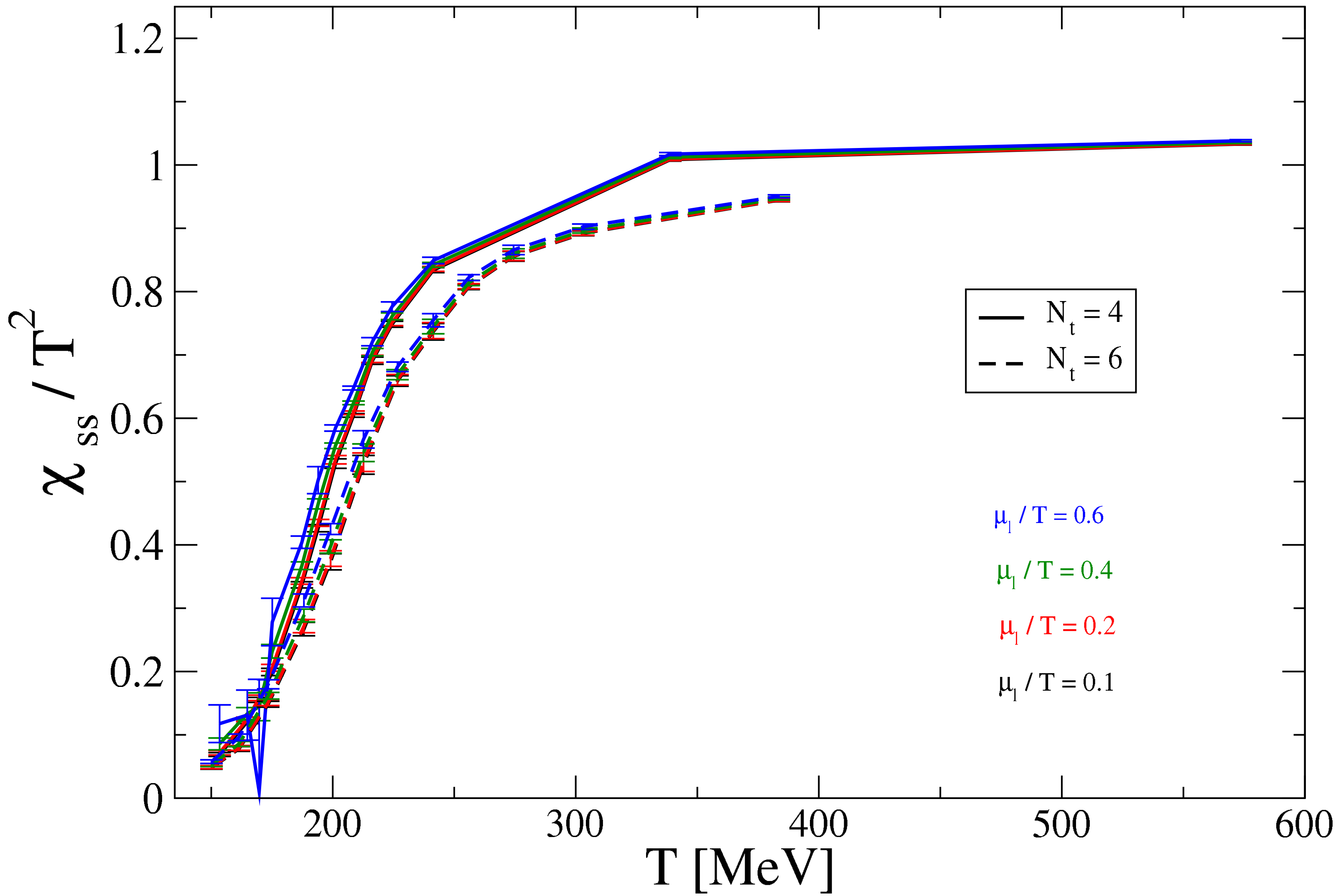
Light quark density



Light quark susceptibility



Strange quark susceptibility



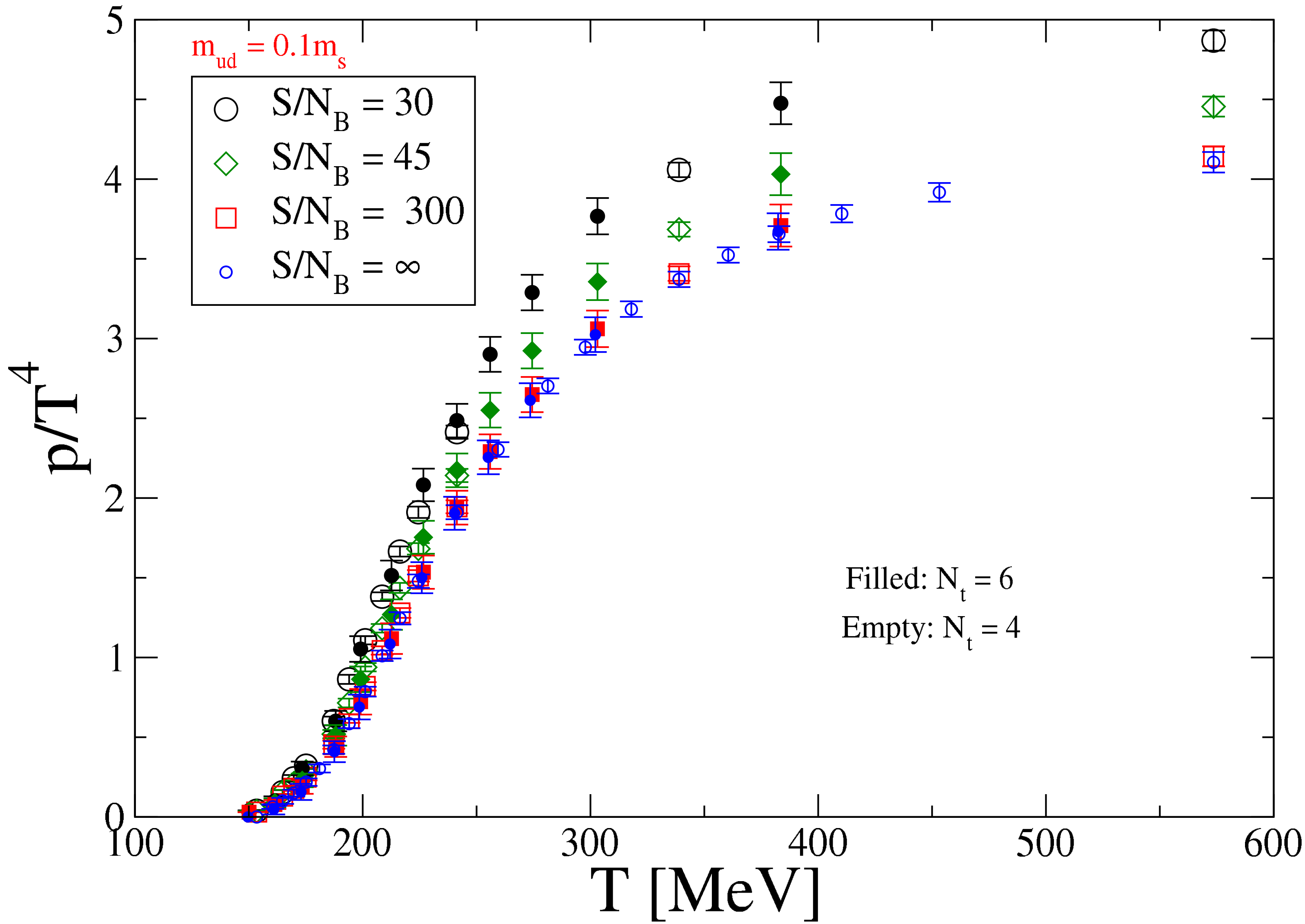
Isentropic EOS

- In a heavy-ion collision, after thermalization the system expands and cools with constant entropy.
- We would like to find the EOS with fixed ratio of entropy to baryon number.
- We calculate EOS with appropriate ratios for AGS, SPP, RHIC:

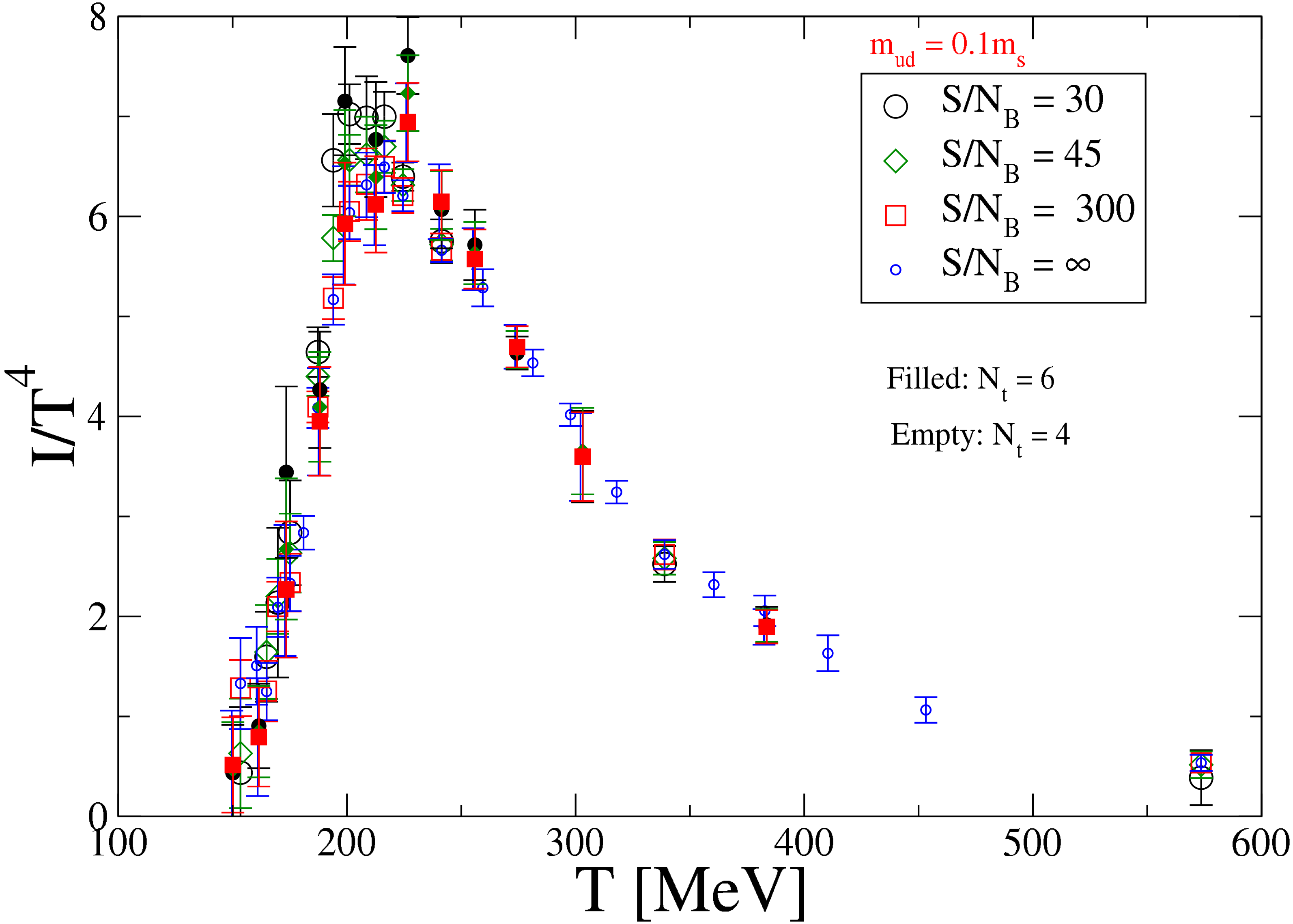
Expt	s/n_B
AGS	30
SPS	45
RHIC	300

- This requires finding trajectories in the (μ_l,μ_s,T) space with $n_s = 0$ and s/n_B as in the table.

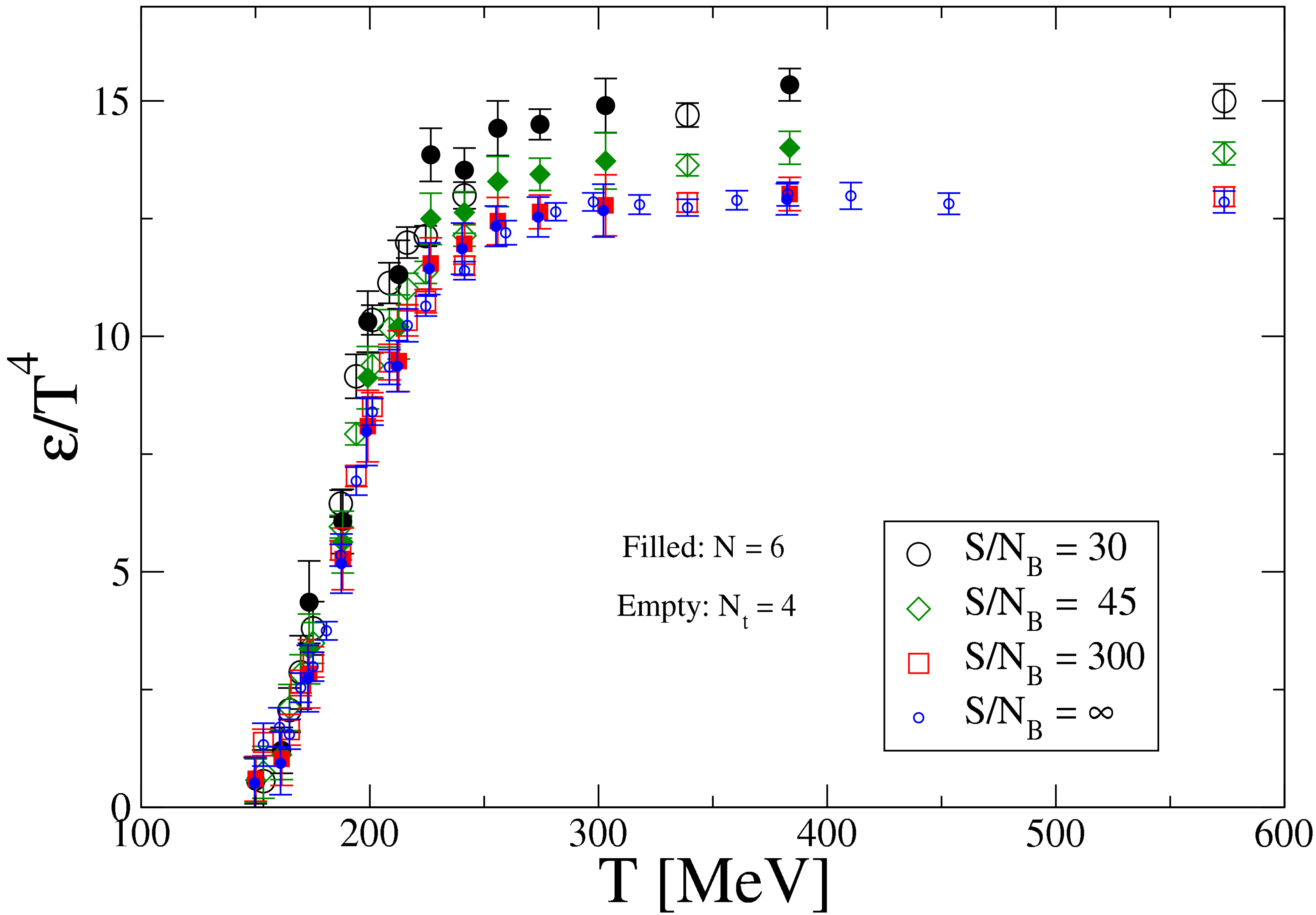
Isentropic Pressure



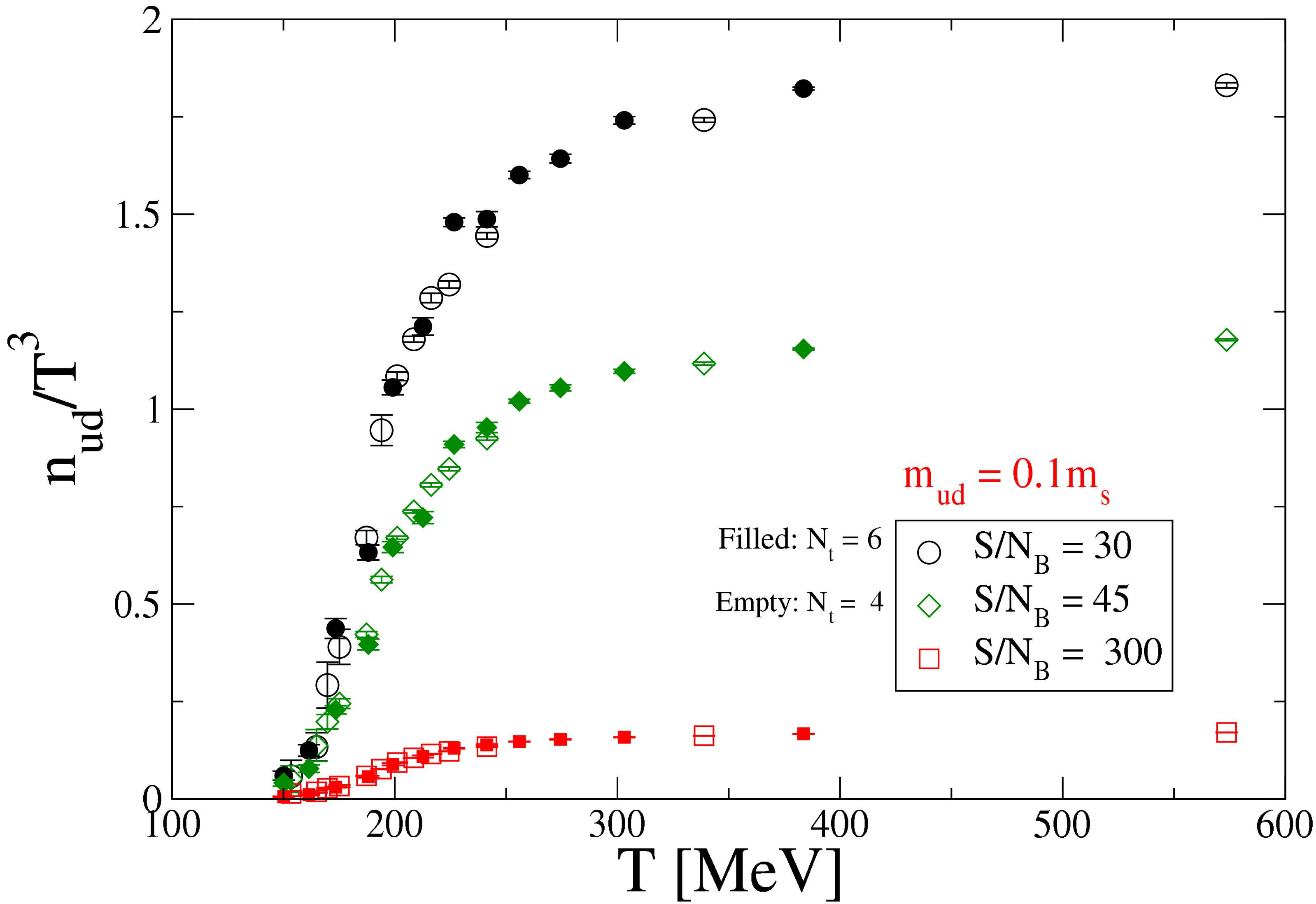
Isentropic interaction measure



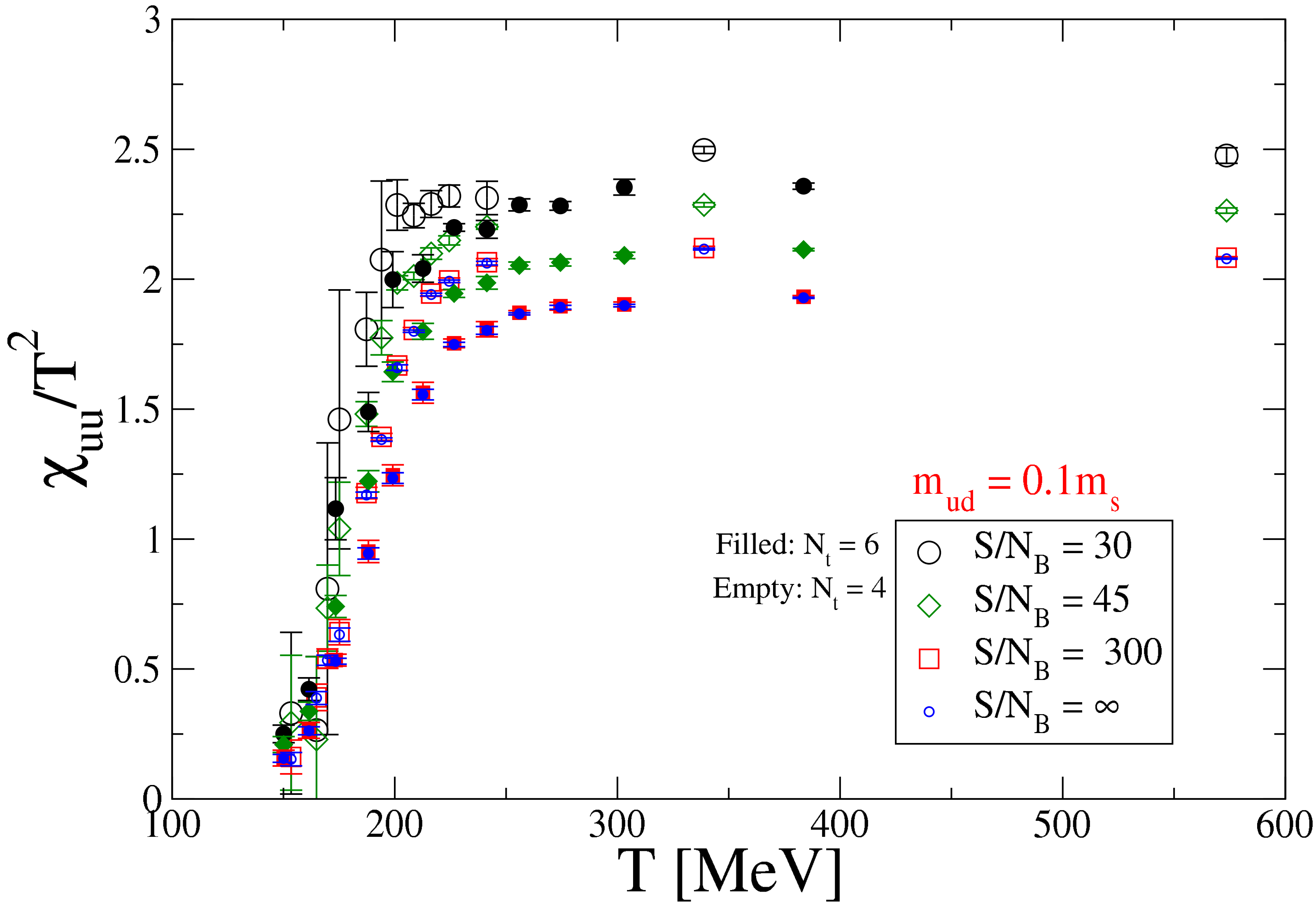
Isentropic energy density



Isentropic light quark density



Isentropic light quark susceptibility



Absence of a peak indicates that we are far from a critical endpoint.

Conclusions

- We have extended our sixth order Taylor expansion study of thermodynamics with chemical potential toward the continuum limit by going from $N_t = 4$ to 6.
- We compute the coefficients relevant for both pressure p and interaction measure I.
- We observe modest lattice spacing effects, with the effect of chemical potential smaller on the pressure and quark densities and susceptibilities at the smaller lattice spacing.
- We have calculated the isentropic equation of state, which is interesting for phenomenology.
- It would be interesting to extend this work to yet smaller lattice spacing and to go to lighter quark mass.
- Happy birthday, Carleton!