f_K/f_π in full QCD

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in collaboration with

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Introduction: why f_K/f_π ?

W.J. Marciano, PRL 93 231803 (2004) [hep-ph/0402299]:

- $|V_{\rm ud}|$ is known, from super-allowed nuclear β-decays, with 0.03% precision.

- $|V_{\rm us}|$ is much less precisely known, but can be linked to $|V_{\rm ud}|$ via a relation involving f_K/f_π, with everything else known rather accurately:

$$\frac{\Gamma(K\to l\bar\nu_l)}{\Gamma(\pi\to l\bar\nu_l)}=\frac{|V_{\rm us}|^2}{|V_{\rm ud}|^2}\frac{f_K^2}{f_\pi^2}\frac{M_K(1-m_l^2/M_K^2)^2}{M_\pi(1-m_l^2/M_\pi^2)^2}\{1+\frac{\alpha}{\pi}(C_K-C_\pi)\}$$

- CKM unitarity $|V_{\rm ud}|^2+|V_{\rm us}|^2+|V_{\rm ub}|^2=1$ (with $|V_{\rm ub}|$ being negligibly small) is genuine to the SM; any deviation is a *model-independent* signal of BSM physics.

$\Longrightarrow$ calculate f_K/f_π in $N_f=2+1$ QCD (with quark masses extrapolated to the physical point) on the lattice; the precision attained gives the precision of $|V_{\rm us}|$.

Setup: dominant sources of uncertainty

What we do to control systematic uncertainties ("full QCD shopping list"):

(a) $N_f = 2{+}1$ with exact algorithm (and universality class of QCD)

(b) complete baryon octet/decuplet spectrum to set the scale

(c) large spatial volumes ($M_\pi L \geq 4$) to assure small finite volume effects

(d) sufficiently chiral data ($M_\pi \simeq 190\,\mathrm{MeV}$) for small extrapolation range

(e) no less than 3 lattice spacings to assure controlled continuum extrapolation

Setup: action, algorithm, resources

- action:

  tree-level $O(a^2)$ improved Symanzik glue and tree-level $O(a)$ improved fat-clover quarks [6 levels of $\alpha = 0.11$ stout smearing, both in covariant derivative and in $F_{\mu\nu}$]

- algorithm:

  HMC/RHMC with even-odd preconditioning, multiple time-scale Omelyan integration, Hasenbusch acceleration and mixed precision solver [Clark et al '06, Sexton Weingarten '92, Omelyan et al '03, Hasenbusch '01, Urbach et al '06, BMW '08]

- resources:

  BG/L @ FZJ: 2005-2008, $8*2048=16384$ processors PowerPC440 @ 700MHz (2.8 GFlops each), 1GB per node, 3D torus network, 46/37 Tflops peak/sustained

  BG/P @ FZJ: since 2008, $16\cdot4096=65536$ processors PowerPC450 @ 850MHz (3.4 GFlops each), 2GB per node, 3D torus network, 223/180 Tflops peak/sustained

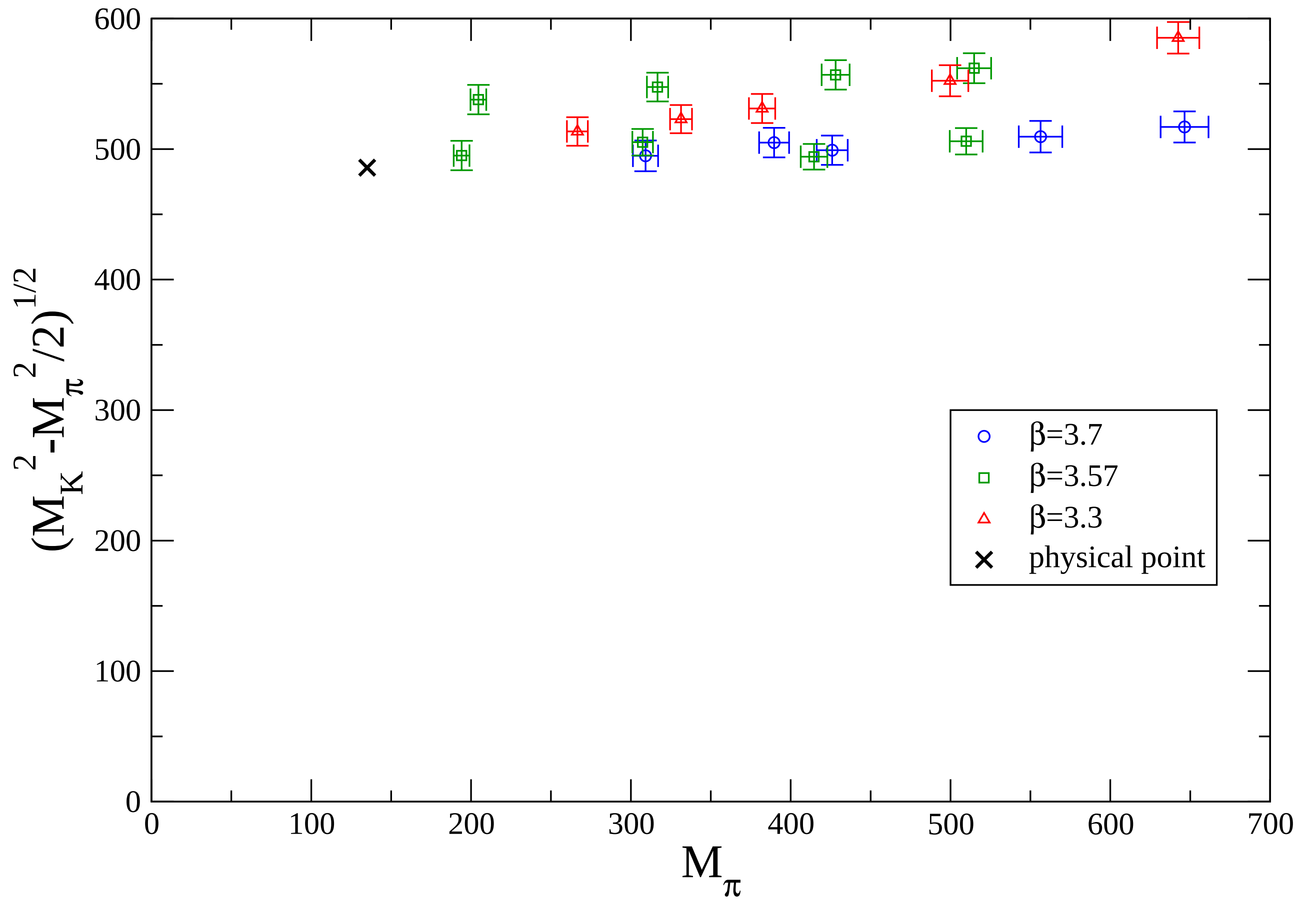
Setup: setting m_{ud}, m_s and a^{-1}

We set m_{ud}, m_s, a^{-1} through M_π, M_K, M_Ξ (or M_Ω).

$\longrightarrow$ S. Krieg: *The hadron spectrum in full QCD: setup and parameter selection*

$\longrightarrow$ Ch. Hoelbling: *The hadron spectrum in full QCD: analysis details and final result*

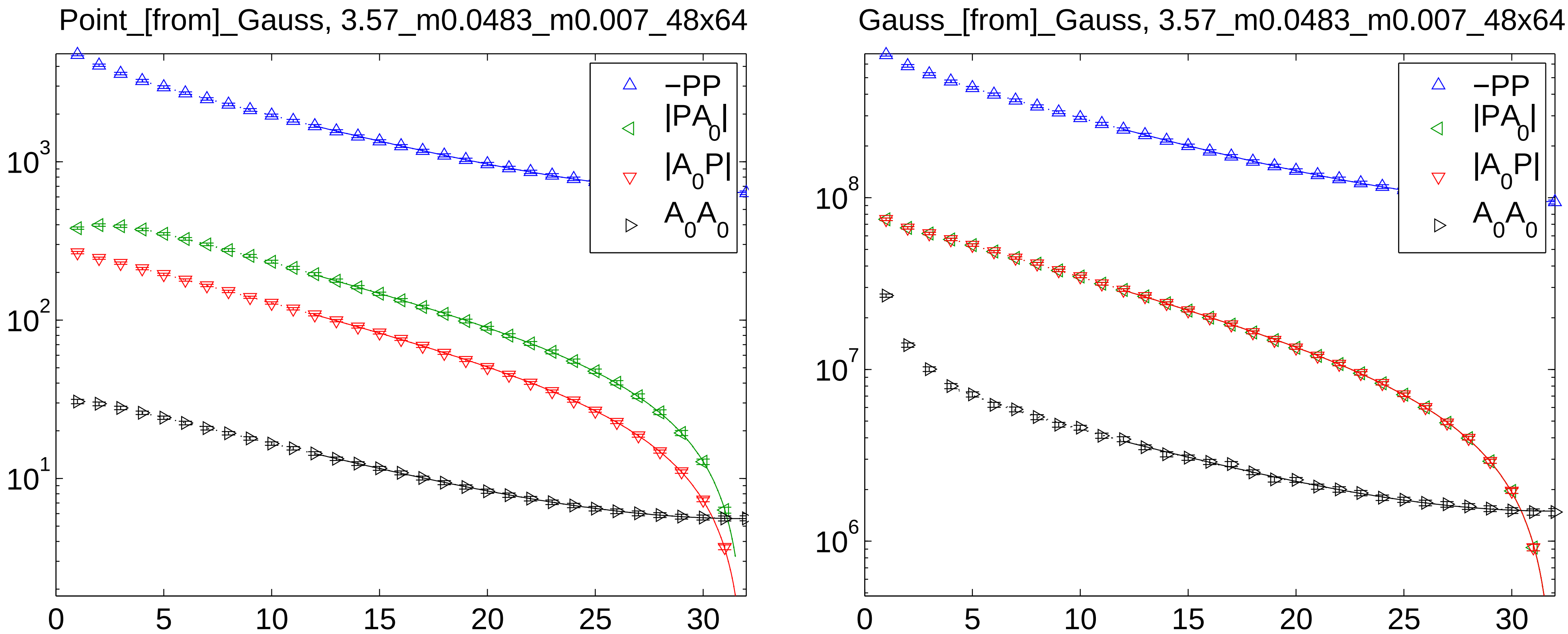
We fix bare strange mass such that renormalized m_s is correct at physical m_{ud} point:



$\Longrightarrow$ extract f_K/f_π from "unitary" data and extrapolate to the physical mass point !

Analysis: combined fits

Excellent data quality even on our lightest ensemble ($M_\pi \simeq 190\,\mathrm{MeV}$ and $L \simeq 4.0\,\mathrm{fm}$):



$\cosh(.)/\sinh(.)$ for $-PP, |PA_0|, |A_0P|, A_0A_0$ with Gauss source and local/Gauss sink

$C_{Xx,Yy}(t) = c_0 e^{-M_0 t} \pm c_0 e^{-M_0(T-t)} + \ldots$ with $X,Y \in \{P, A_0\}$ and $x,y \in \{\mathrm{loc}, \mathrm{gau}\}$

$\longrightarrow$ $c_0 = G\tilde{G}, G\tilde{F}, F\tilde{G}, F\tilde{F}$ (left) and $c_0 = \tilde{G}\tilde{G}, \tilde{G}\tilde{F}, \tilde{F}\tilde{G}, \tilde{F}\tilde{F}$ (right)

$\longrightarrow$ combined 1-state fit of 8 correlators with 5 parameters yields $M_\pi, F_\pi, m_{\mathrm{PCAC}}$

Analysis: chiral extrapolation

- Chiral $SU(3)$ formula:

$$\frac{F_K}{F_\pi} = 1+\frac{1}{32\pi^2F_0^2}\Big\{\frac{5}{4}M_\pi^2\log(\frac{M_\pi^2}{\mu^2})-\frac{1}{2}M_K^2\log(\frac{M_K^2}{\mu^2}) -[M_K^2-\frac{1}{4}M_\pi^2]\log(\frac{4M_K^2-M_\pi^2}{3\mu^2})\Big\}+\frac{4}{F_0^2}[M_K^2-M_\pi^2]L_5$$

- Chiral $SU(2)$_plus_strange formula [arXiv:0804.0473 by RBC/UKQCD, simplified]:

$$\frac{F_K}{F_\pi}=\frac{F_K}{F_\pi}\Big|_{m_{ud}=0}\Big\{1+\frac{5}{8}\frac{M_\pi^2}{(4\pi F)^2}\log\Big(\frac{M_\pi^2}{\Lambda^2}\Big)\Big\}$$

- Polynomial expansion $F_\pi/F_K=d_0+d_1M_\pi+d_2M_\pi^2$ (e.g. around $300\,\mathrm{MeV}$) at fixed physical m_s, together with constraint $F_K=F_\pi$ at $M_K=M_\pi$, suggests:

$$\frac{F_K}{F_\pi}=\frac{c_0+c_1M_K+c_2M_K^2}{c_0+c_1M_\pi+c_2M_\pi^2}$$

$\longrightarrow$ Use all of them and treat spread as indicative of systematic uncertainty !

Analysis: continuum extrapolation

Q: Should we use dedicated version of Chiral Perturbation Theory (XPT) ?

A: Aoki Bär Takeda Ishikawa PRD73, 014511 (2006) for clover fermions:

$$
\begin{aligned}
M_\pi^2 &= B_0 2m\Big\{1+\mu_\pi-\frac{1}{3}\mu_\eta+2mK_3+K_4-\frac{H''}{F_0^2}\Big\}\\
M_K^2 &= B_0(m{+}m_s)\Big\{1+\frac{2}{3}\mu_\eta+(m{+}m_s)K_3+K_4-\frac{H''}{F_0^2}\Big\}\\
F_\pi^2 &= F_0\Big\{1-2\mu_\pi-\mu_K+2mK_6+K_7-\frac{H'}{F_0^2}\Big\}\\
M_K^2 &= F_0\Big\{1-\frac{3}{4}\mu_\pi-\frac{3}{2}\mu_K-\frac{3}{4}\mu_\eta+(m{+}m_s)K_6+K_7-\frac{H'}{F_0^2}\Big\}
\end{aligned}
$$

$\longrightarrow$ Redefine $B_0\{1-H''/F_0^2\}\to B_0'$ and terms with H'' will disappear

$\longrightarrow$ Redefine $F_0\,\{1-H'/F_0^2\}\to F_0'$ and terms with H' will disappear

Conjecture: dedicated WXPT calculations for M_P, F_P not needed; with $m\equiv m_{\mathrm{PCAC}}$ effects of finite lattice spacing are taken into account via augmenting all low-energy constants by factors of $(1{+}\mathrm{const}\,a^2)$ to any chiral order. (not needed below)

Analysis: infinite volume extrapolation

- Finite volume effects on F_K, F_π are known at the 2-loop level [Colangelo et al. '05]

$$\frac{F_\pi(L)}{F_\pi} = 1+\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\frac{1}{M_\pi L}\;1\;\frac{M_\pi^2}{(4\pi F_\pi)^2}\left[I_{F_\pi}^{(2)}+\frac{M_\pi^2}{(4\pi F_\pi)^2}I_{F_\pi}^{(4)}+...\right]$$

$$\frac{F_K(L)}{F_K} = 1+\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\frac{1}{M_\pi L}\frac{F_\pi}{F_K}\frac{M_\pi^2}{(4\pi F_\pi)^2}\left[I_{F_K}^{(2)}+\frac{M_K^2}{(4\pi F_\pi)^2}I_{F_K}^{(4)}+...\right]$$

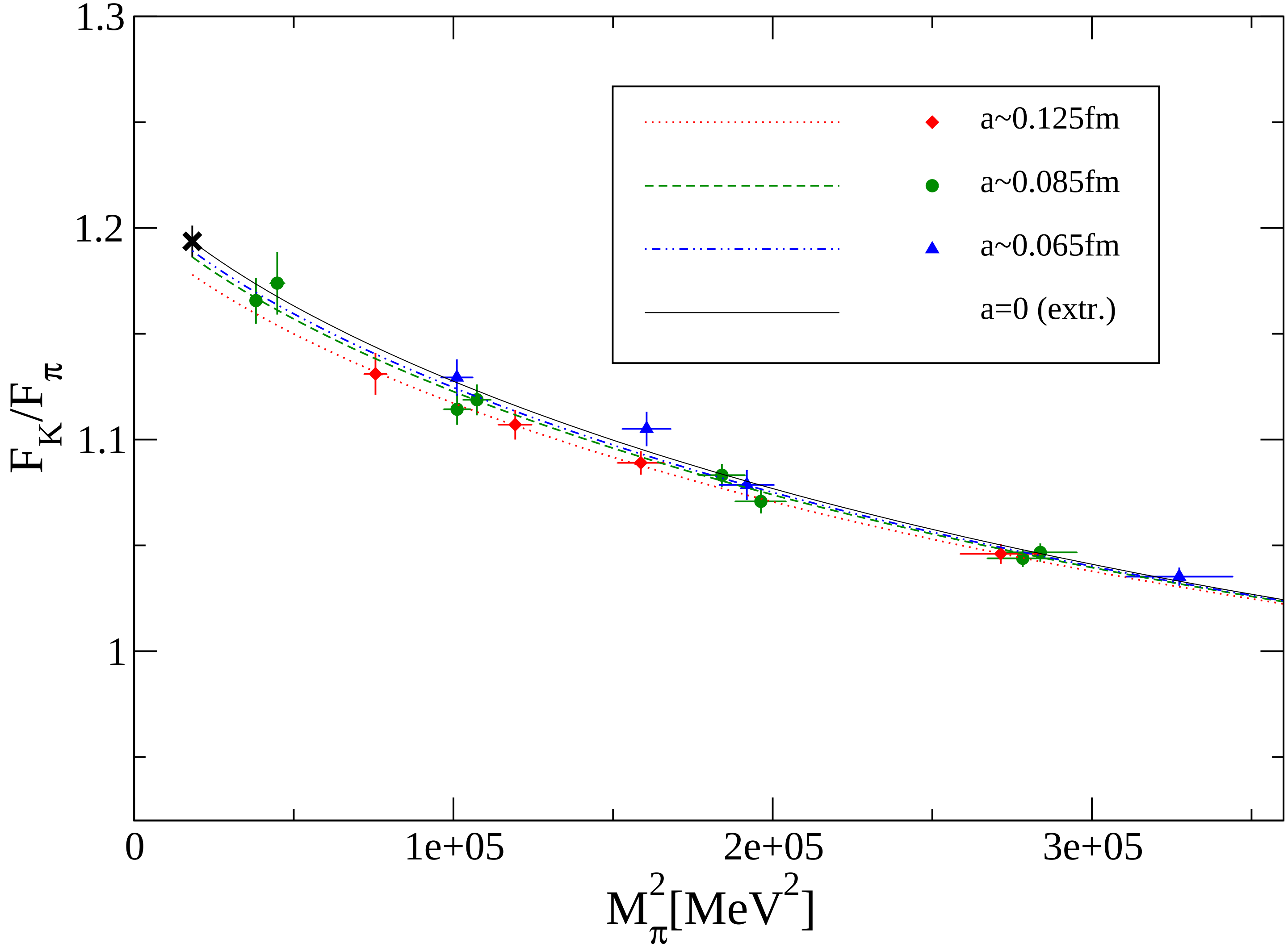
with $I_{F_\pi}^{(2)}=-4K_1(\sqrt{n}\,M_\pi L)$ and $I_{F_K}^{(2)}=-\frac{3}{2}K_1(\sqrt{n}\,M_\pi L)$, where $K_1(.)$ is a Bessel function of the second kind, and lengthy expressions for $I_{F_\pi}^{(4)}, I_{F_K}^{(4)}$.

- In the ratio finite volume effects cancel partly; evident from the 1-loop formula

$$\frac{F_K(L)}{F_\pi(L)} = \frac{F_K}{F_\pi}\left\{1+\sum_{n=1}^{\infty}\frac{m(n)}{\sqrt{n}}\frac{1}{M_\pi L}\frac{M_\pi^2}{(4\pi F_\pi)^2}\left[\frac{F_\pi}{F_K}I_{F_K}^{(2)}-I_{F_\pi}^{(2)}\right]\right\}\;.$$

- We calculate $\frac{F_K(L)}{F_\pi(L)}/\frac{F_K}{F_\pi}$ at 1-loop and 2-loop level, and $F_\pi(L)/F_\pi$ at 2-loop level.

Result: combined fits

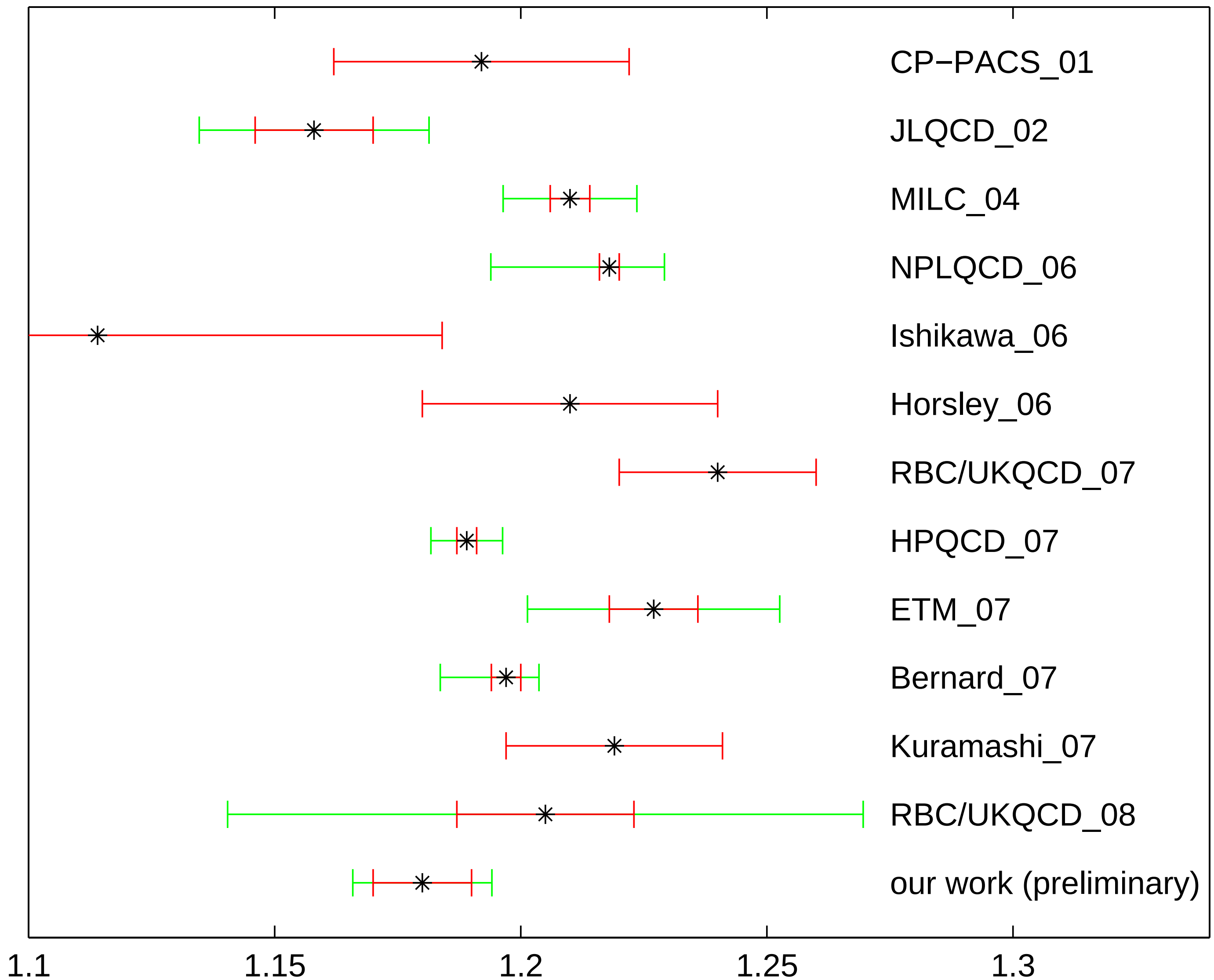


⟶ plot shows $\mathrm{data}(M_\pi^2, 2M_K^2{-}M_\pi^2) - \mathrm{fit}(M_\pi^2, 2M_K^2{-}M_\pi^2) + \mathrm{fit}(M_\pi^2, [2M_K^2{-}M_\pi^2]_{\mathrm{phys}})$.

⟶ f_K/f_π scales rather nicely [we have $a^2/\mathrm{fm}^2 = 0.0042, 0.0072, 0.0156$].

⟹ $f_K/f_\pi = 1.18(1)(1)$ at the physical m_{ud}, in the continuum, for infinite volume.

Result: errorbars over time



Finish: update on $|V_{\mathrm{us}}|$

- Average $|V_{\mathrm{ud}}| = 0.97377(27)$ [PDG'06] and $0.97418(26)$ [Towner'07] to give

$$|V_{\mathrm{ud}}| = 0.97398(18)(20) = 0.97398(27) \; .$$

- Plug experimental information $\Gamma(K \to \mu\bar{\nu})/\Gamma(\pi \to \mu\bar{\nu}) = 1.3337(39)$ [PDG'06] and $C_K - C_\pi = -3.0 \pm 1.5$ [Marciano] into Marciano's equation (first slide); this yields

$$\frac{|V_{\mathrm{us}}|}{|V_{\mathrm{ud}}|} \frac{f_K}{f_\pi} = 0.2757(6) \; .$$

- Upon combining the previous one/two points and our value for f_K/f_π we obtain

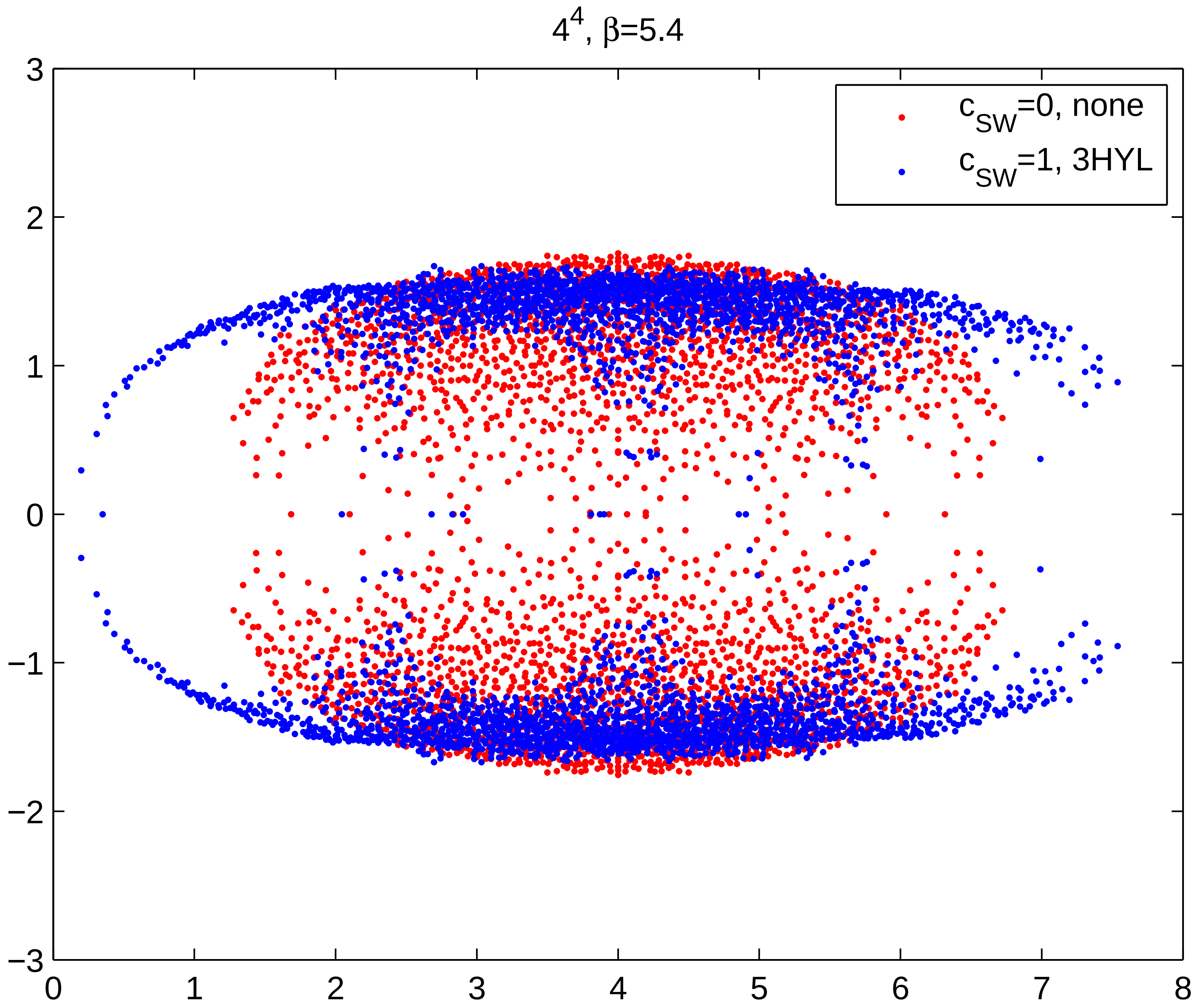
$$|V_{\mathrm{us}}|/|V_{\mathrm{ud}}| = 0.2336(28) \quad \text{and} \quad |V_{\mathrm{us}}| = 0.2276(27) \; .$$

- Upon including $|V_{\mathrm{ub}}| = (4.31 \pm 0.30)\, 10^{-3}$ [PDG'06] we end up with

$$|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + |V_{\mathrm{ub}}|^2 = 1.0004(14) \; .$$

BACKUP SLIDES

Backup: why smear?



Backup: action locality

- locality in position space:

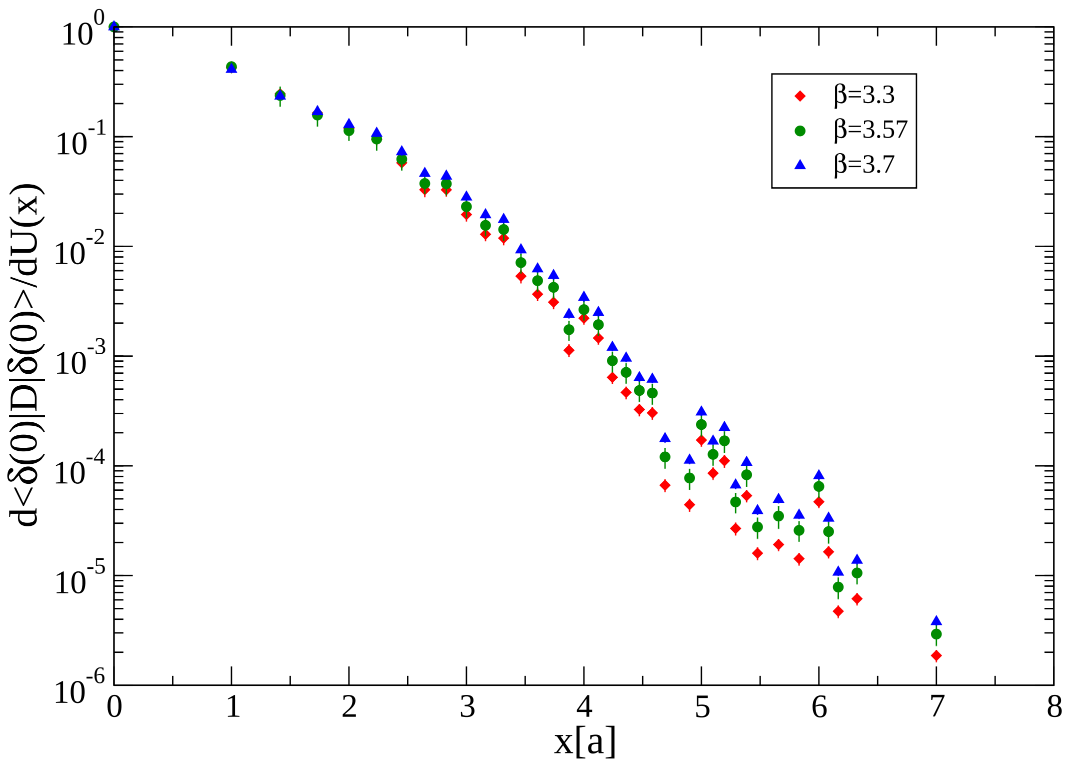
  $|D(x,y)| < \text{const}\, e^{-\lambda|x-y|}$ with $\lambda = O(a^{-1})$ for all couplings.

  Our case: $D(x,y) = 0$ as soon as $|x-y| > 1$ (despite 6 smearings).

- locality in space of gauge fields:

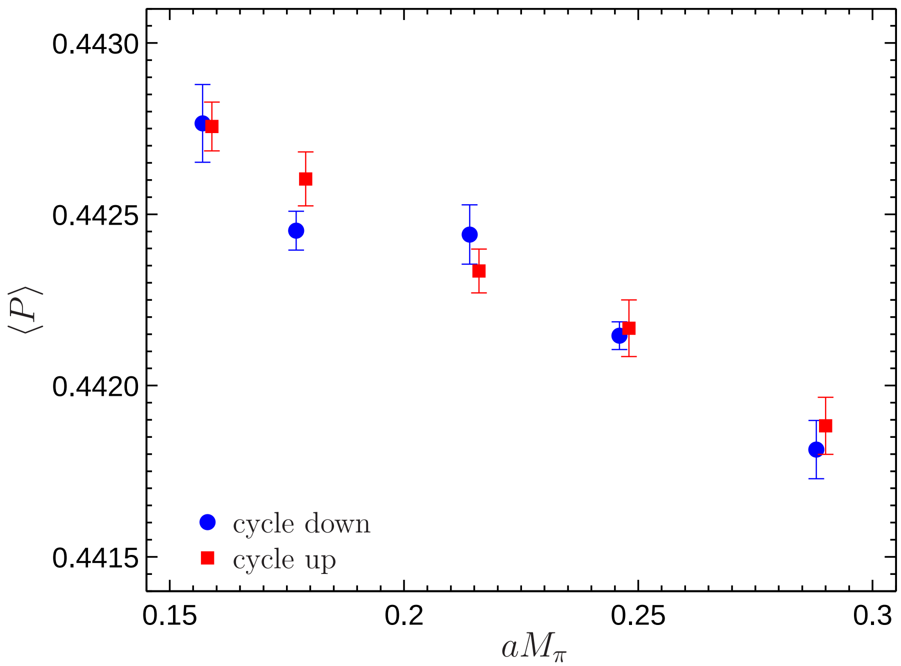
  $|\delta D(x,y)/\delta A(z)| < \text{const}\, e^{-\lambda|(x+y)/2-z|}$ with $\lambda = O(a^{-1})$ for all couplings.

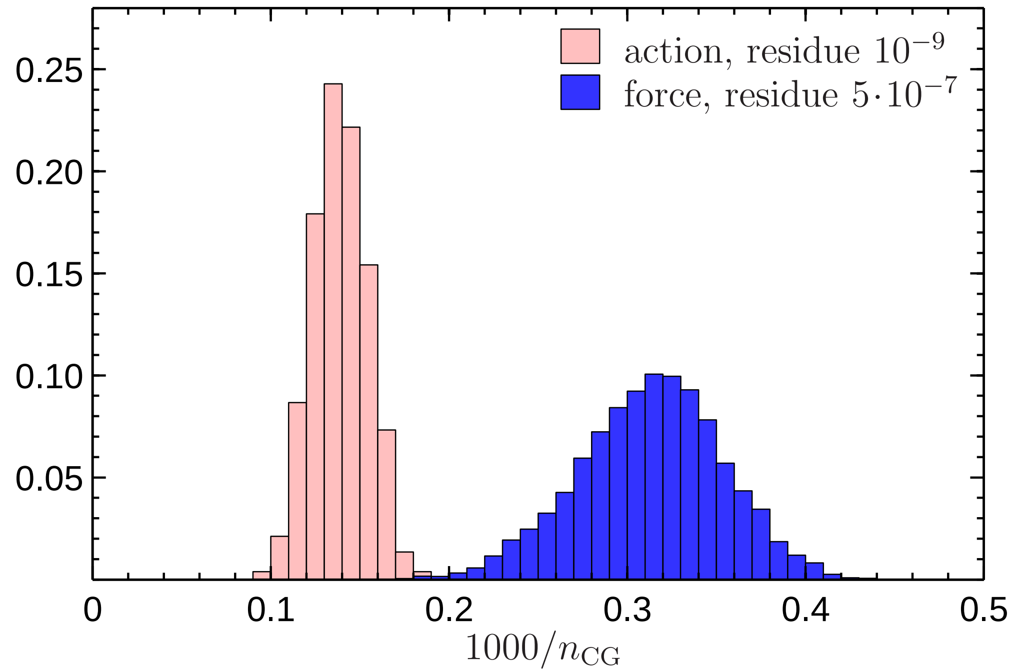
  Our case: $\delta D(x,x)/\delta A(z) < \text{const}\, e^{-\lambda|x-z|}$ with $\lambda \simeq 2.2a^{-1}$ for $2 \le |x-z| \le 6$.



Backup: algorithmic stability

No hysteresis effects and 5σ-stability in light quark mass production runs:





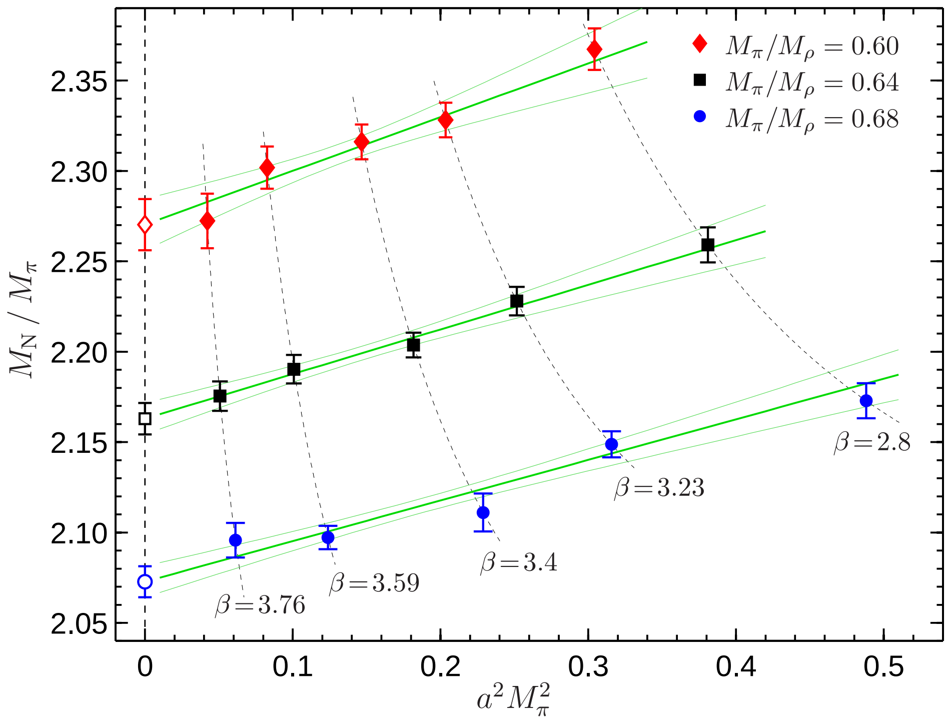
⟶ to monitor $1/N_{\mathrm{CG}}$ is cheaper than minimal eigenvalue of $|\gamma_5 D|$

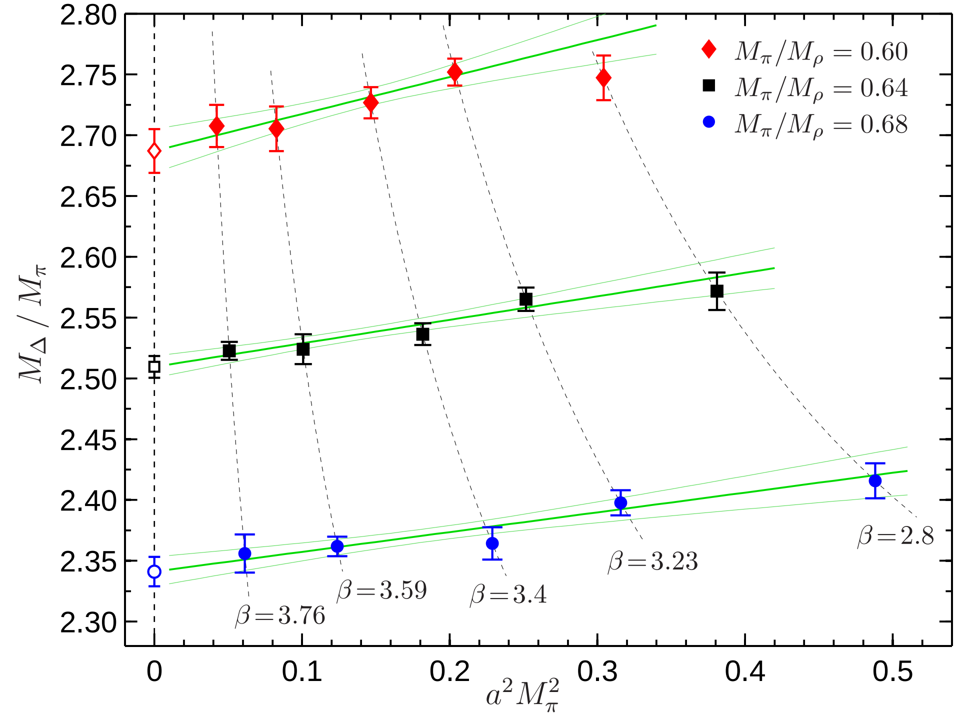
⟶ also R_{acc} and $e^{-\Delta H}$ are being monitored

⟶ see arxiv:0802.2706 [BMW Collab.] for details

Backup: scaling properties

Explicit scaling test for M_N and M_Δ in $N_f\!=\!3$ QCD:



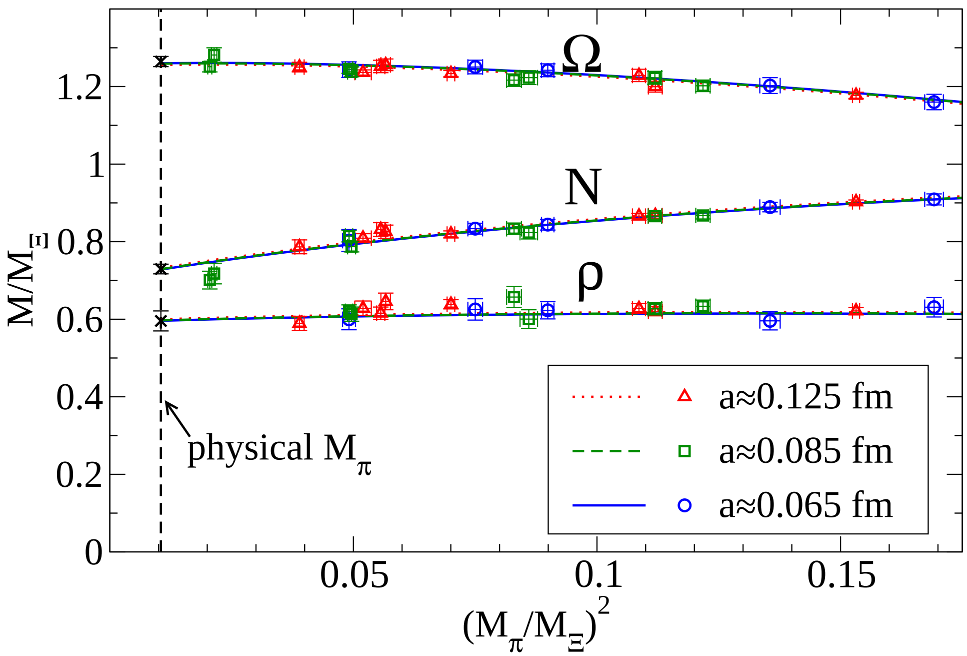


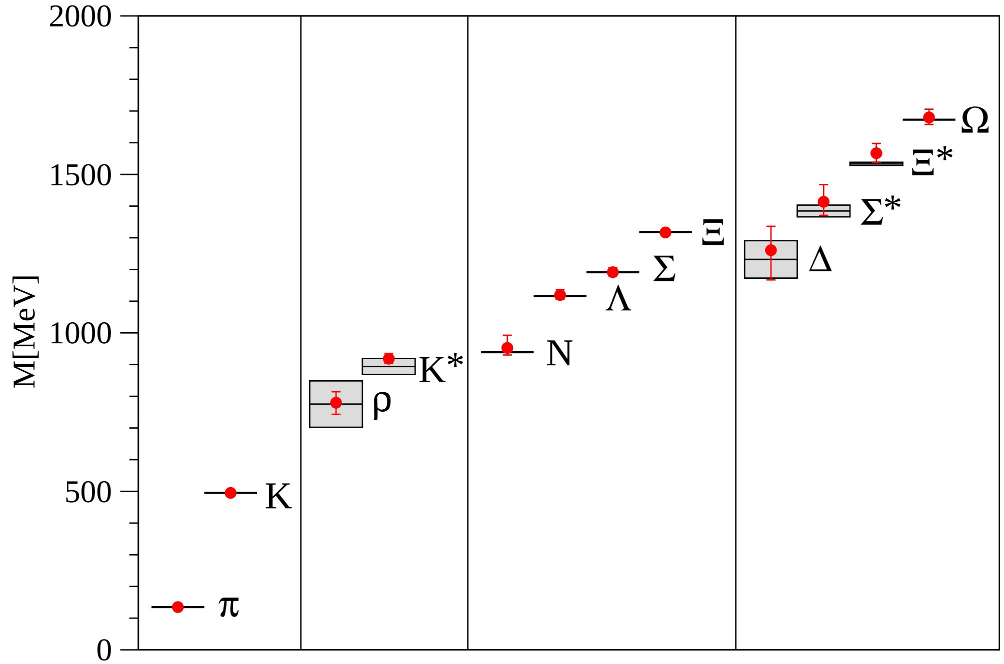
$\longrightarrow$ clean $O(a^2)$ scaling of 6-stout action out to $a\!\sim\!0.15\,\mathrm{fm}$

$\longrightarrow$ see arxiv:0802.2706 [BMW Collab.] for details

$\longrightarrow$ Th. Kurth: *Scaling study of dynamically smeared fermions*

Backup: octet/decuplet spectrum with extrapolation





- large volumes ($M_\pi L \geq 4$ maintained, larger/smaller volumes for check)
- light pions ($M_\pi \sim 190\,\mathrm{MeV}$ at two lattice spacings)
- three couplings ($a \sim 0.065, 0.085, 0.125\,\mathrm{fm}$)

$\longrightarrow$ S. Krieg: *The hadron spectrum in full QCD: setup and parameter selection*

$\longrightarrow$ Ch. Hoelbling: *The hadron spectrum in full QCD: analysis details and final result*