The hadron spectrum in full QCD: Setup and parameter selection

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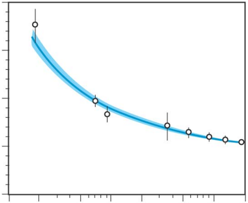
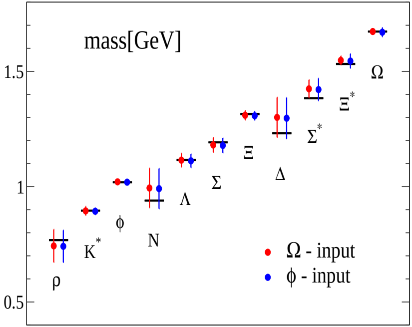
The XXVI International Symposium on Lattice Field Theory
College of William & Mary
July 2008

Outline

Status

- Asymptotic freedom: good agreement between theory and experiment





- Good evidence that QCD describes the strong interaction in the non-perturbative domain (e.g. CP-PACS '07, $N_f = 2 + 1$, $210\text{MeV} \leq M_\pi \leq 730\text{MeV}$, $a \simeq 0.087\,\text{fm}$, $L \lesssim 2.8\,\text{fm}$, $M_\pi L \simeq 2.9$)
- However, systematic errors not yet under control

Controlling systematic errors

- Include u, d and s quarks int the simulation with an action, whose universality class is QCD.
- Use large volumes ($M_\pi L \gtrsim 4$) to guarantee small finite-size effects and at least one simulation at a significantly larger volume to confirm the smallness of these effects.
- Use controlled interpolations and extrapolations of the results to physical m_{ud} and m_s
- Use controlled extrapolations to the continuum limit, requiring that the calculations be performed at no less than three values of the lattice spacing.

Choice of action

- Explicit form of our action (thin links $U_{n,\mu}$, smeared links $V_{n,\mu}$)

$$
\begin{aligned}
S &= S_G^{\mathrm{Sym}} + S_F^{\mathrm{SW}} \\
S_G^{\mathrm{Sym}} &= \beta \left[\frac{c_0}{3} \sum_{\mathrm{plaq}} \mathrm{Re\,Tr}\,(1 - U_{\mathrm{plaq}}) + \frac{c_1}{3} \sum_{\mathrm{rect}} \mathrm{Re\,Tr}\,(1 - U_{\mathrm{rect}}) \right] \\
S_F^{\mathrm{SW}} &= S_F^{\mathrm{W}}[V] - \frac{c_{\mathrm{SW}}}{4} \sum_n \sum_{\mu,\nu} \overline{\psi}_n \, \sigma_{\mu\nu} F_{\mu\nu,n}[V] \, \psi_n \,,
\end{aligned}
$$

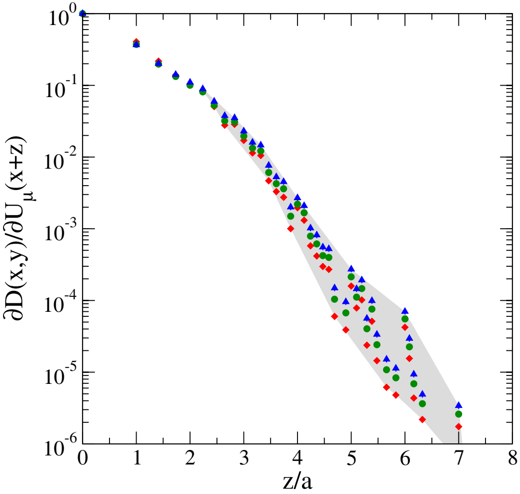
- with $c_{\mathrm{SW}} = 1, \quad c_1 = -1/12, \quad c_0 = 1 - 8c_1 = 5/3\,$. Smearing:

$$
\begin{aligned}
V^{(n+1)} &= e^{\rho S^{(n)}} U^{(n)}, \\
S^{(n)} &= \frac{1}{2}(\Gamma^{(n)} V^{(n)\dagger} - V^{(n)} \Gamma^{(n)\dagger}) - \frac{1}{6} \mathrm{Re\,Tr}(\Gamma^{(n)} V^{(n)\dagger} - V^{(n)} \Gamma^{(n)\dagger}) \\
\Gamma^{(n)}_{n,\mu} &= \sum_{\nu \neq \mu} V^{(n)}_{n,\nu} V^{(n)}_{n+\nu,\mu} V^{(n)\dagger}_{n+\mu,\nu}
\end{aligned}
$$

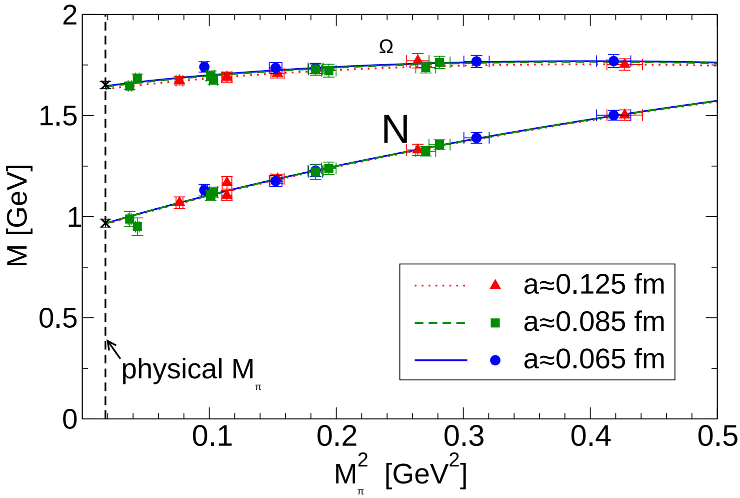
Locality properties of the action

- locality in position space:
  $|D(x,y)| < \text{const}\, e^{-\lambda|x-y|}$ with $\lambda = O(a^{-1})$ for all couplings.
  Our case: $D(x,y) = 0$ as soon as $|x-y| > 1$
  (despite 6 smearings).
- locality in space of gauge fields:
  $|\delta D(x,y)/\delta A(z)| < \text{const}\, e^{-\lambda|(x+y)/2-z|}$ with $\lambda = O(a^{-1})$ for all couplings.

Locality properties of the action



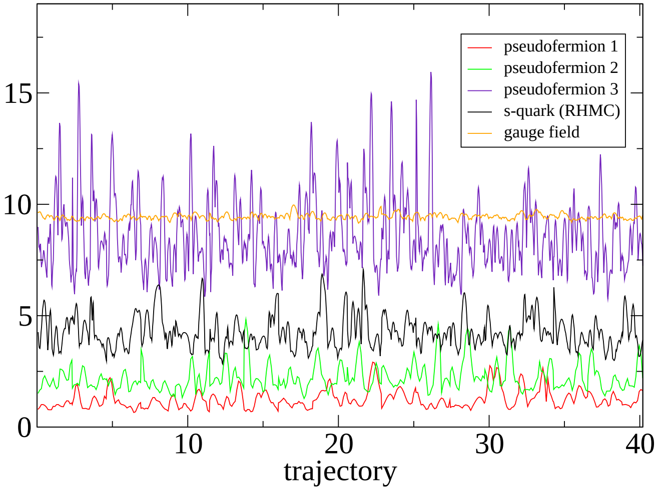
Scaling of this action



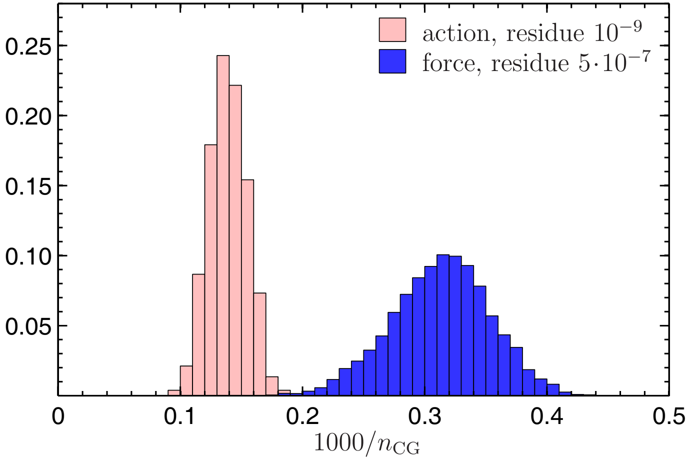
Dynamical fermion Algorithm

- Action
  - Clover tree level improved Wilson fermions
  - Symanzik improved gauge action
  - Stout links
- Algorithm
  - Rational HMC for strange quark
  - Mass preconditioning (“Hasenbusch trick”)
  - Multi scale integration scheme (“Sexton-Weingarten”)
  - Omelyan integrator (“non equidistant leap frog”), increasing integration precision
  - Mixed precision inverters

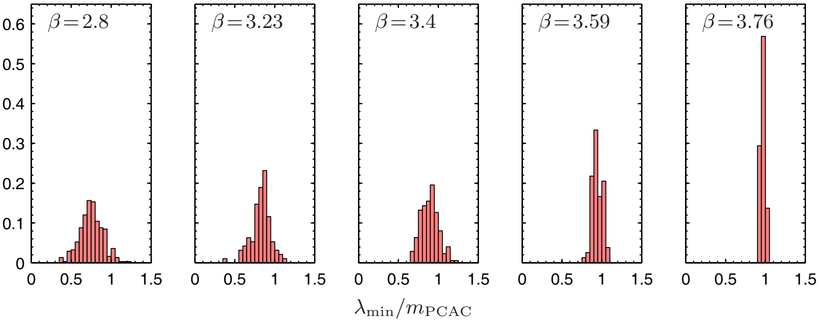
Fermionic force history



Inverse iteration count distribution

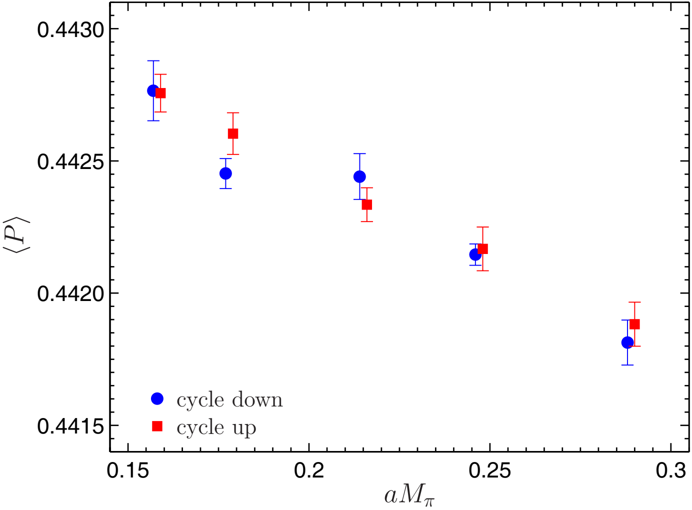


λ_{min}^{-1} distribution



$\mapsto$ Simulations

"Thermal cycle"

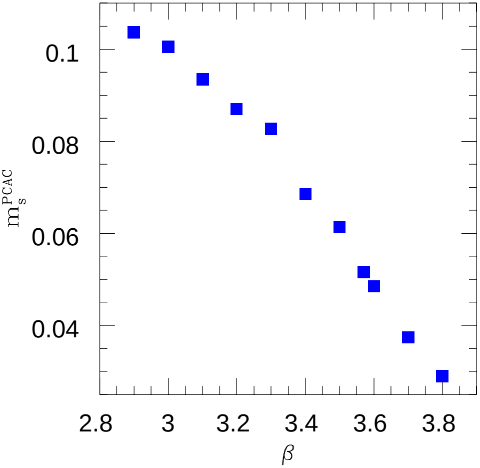


Setting the strange quark mass

- We use a $N_f = 3$ simulation to set the strange quark mass
- For each beta, search for the κ where

$$\frac{m_{ps}}{m_v} = \frac{\sqrt{2m_K^2 - m_\pi^2}}{m_\Phi}$$

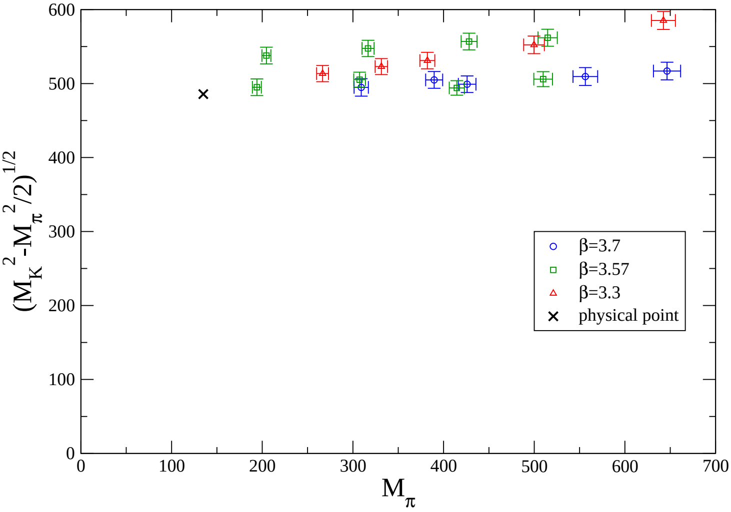
- We determined the β dependency in the range ($\beta = 2.9...3.8$)



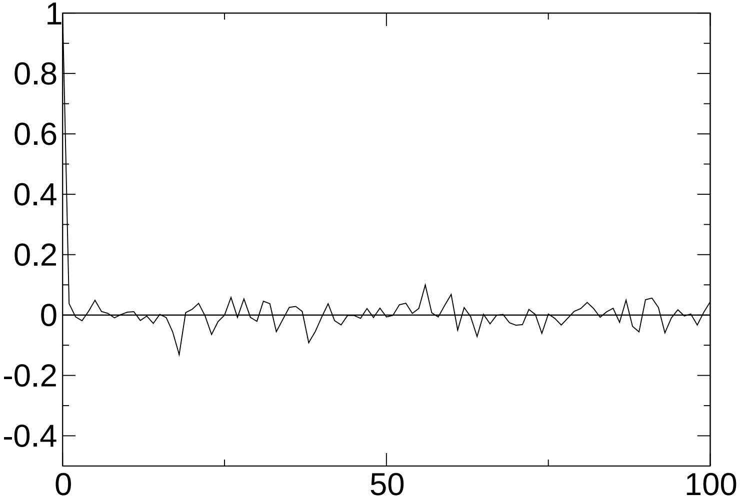
Simulation points

β	am_{ud}	M_π [GeV]	am_s	$L^3 \times T$	# traj.
3.3	-0.0960	.55	-0.057	$16^3 \times 32$	10000
	-0.1100	.45	-0.057	$16^3, 32^3 \times 32$	1450,1800
	-0.1200	.36	-0.057	$16^3 \times 64$	4500
	-0.1233	.32	-0.057	$16^3, 24^3, 32^3 \times 64$	5000,2000,1300
	-0.1265	.26	-0.057	$24^3 \times 64$	2100
3.57	-0.0318	.46,.48	0.0, -0.01	$24^3 \times 64$	3300
	-0.0380	.39,.40	0.0, -0.01	$24^3 \times 64$	2900
	-0.0440	.31,.32	0.0, -0.007	$32^3 \times 64$	3000
	-0.0483	.19,.21	0.0, -0.007	$48^3 \times 64$	1500
3.7	-0.007	.58	0.0	$32^3 \times 96$	1100
	-0.013	.50	0.0	$32^3 \times 96$	1450
	-0.020	.40	0.0	$32^3 \times 96$	2050
	-0.022	.36	0.0	$32^3 \times 96$	1350
	-0.025	.29	0.0	$40^3 \times 96$	1450

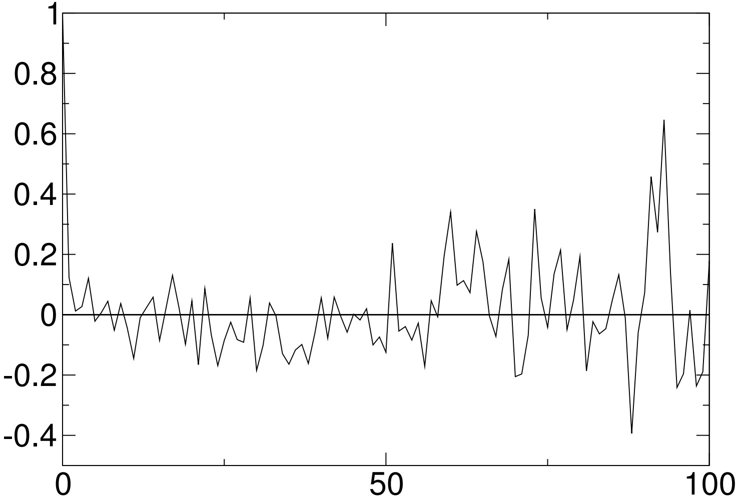
“Landscape” plot



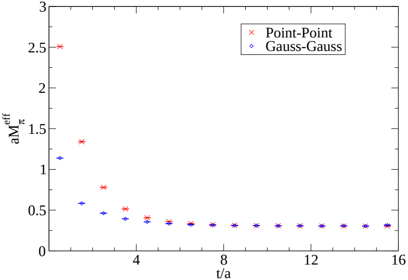
Nucleon autocorrelation ($M_\pi = 550\,\text{MeV}$, $\beta = 3.3$)

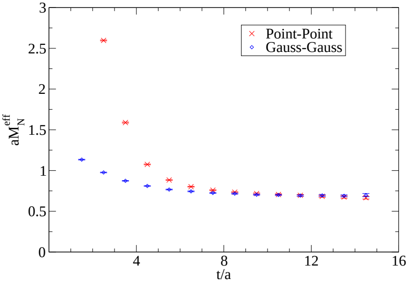


Pion autocorrelation ($M_\pi = 190\,\text{MeV}$, $\beta = 3.57$)



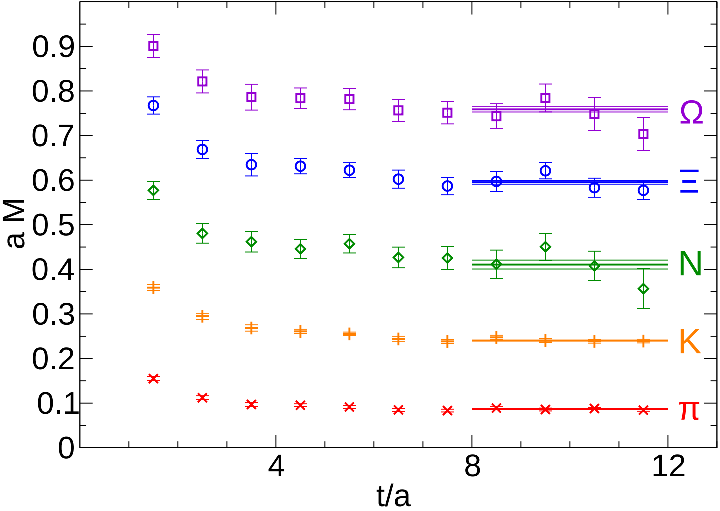
Sources





- Gaussian sources $r = 0.32$ fm
- Coulomb gauge
- Gauss-Gauss less contaminated by excited states

Effective masses and correlated fits



Setting the lattice spacing via hadron mass

The particle selected should have a mass

1. that is experimentally well known
2. that is independent of light quark mass $\rightarrow$ large strange content
3. that can be simulated with small statistical errors $\rightarrow$ octet better suited than decuplet

All points cannot be fulfilled simultaneously, but

- Ξ: largest strange content of the octet, but still dependent on ud mass
- Ω: member of the decuplet, but largest strange content of particles included in analysis

Conclusions

- To control systematic errors in spectrum calculations, we
  1. include dynamical $N_f = 2 + 1$ degrees of freedom,
  2. use an action in the universality class of QCD,
  3. use lattices with $M_\pi L \gtrsim 4$,
  4. use controled chiral extrapolations,
  5. and include ensembles at three different β values.
- Our simulation algorithm is save even at $M_\pi < 200$ MeV.
- Our action has a very good scaling behavior.

Result