# Energy dependence of nucleon-nucleon potentials

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in collaboration with

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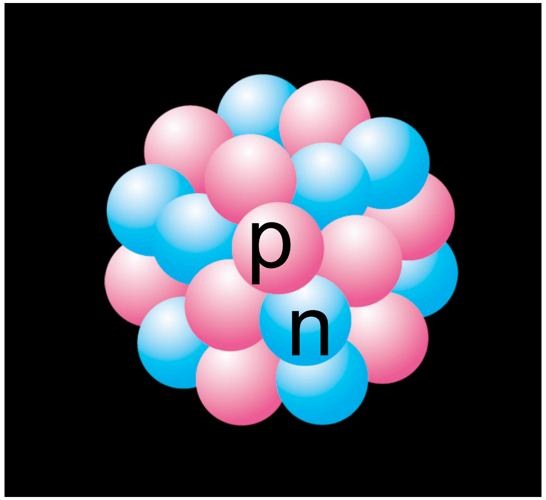
# Introduction

# What binds protons and neutrons inside a nuclei ?



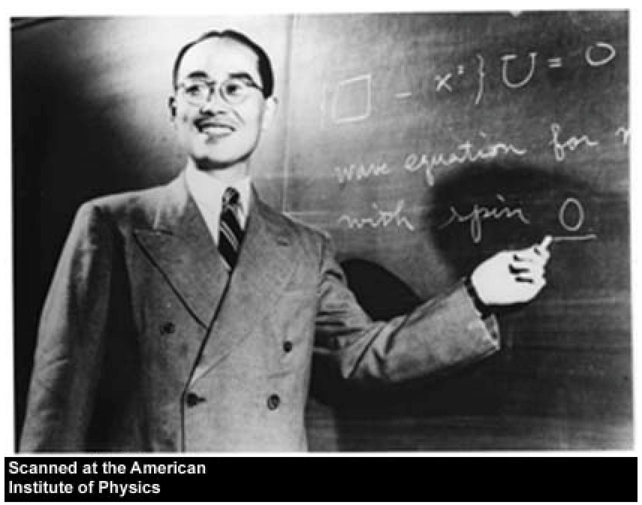

gravity: too weak

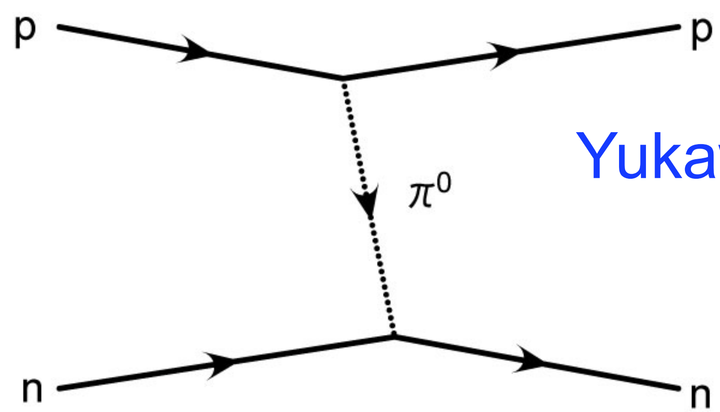
Coulomb: repulsive between pp
no force between nn, np

New force (nuclear force) ?

1935 H. Yukawa

introduced virtual particles (mesons) to explain the nuclear force



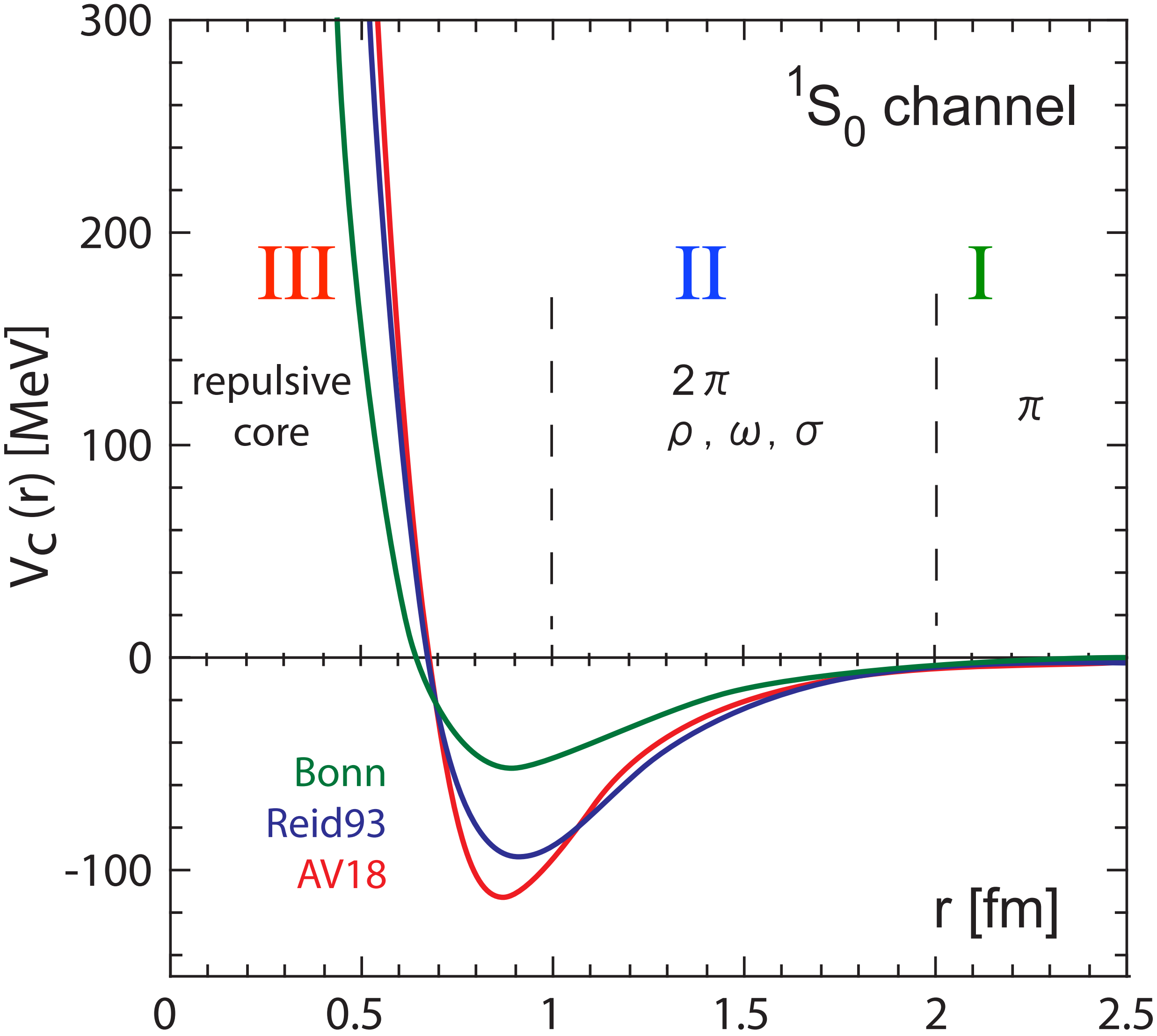




Yukawa potential

$$V(r) = \frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$$

1949 Nobel prize

# A current understanding of the nuclear potential





I Long range part

one pion exchange potential (OPEP)

II Medium range part

$\sigma, \rho, \omega$ exchange

$2\pi$ exchange

III Short range part

repulsive core (RC)

R. Jastrow(1951)

quark ?

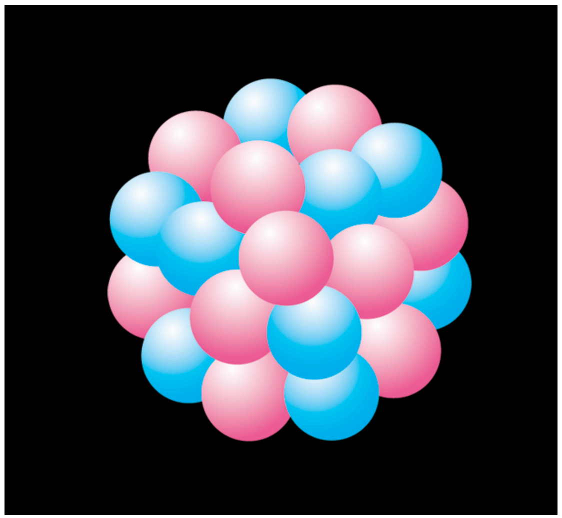
Bonn: Machleidt, Phys.Rev. C63('01)024001

Reid93: Stoks et al., Phys. Rev. C49('94)2950.

AV18: Wiringa et al., Phys.Rev. C51('95) 38.

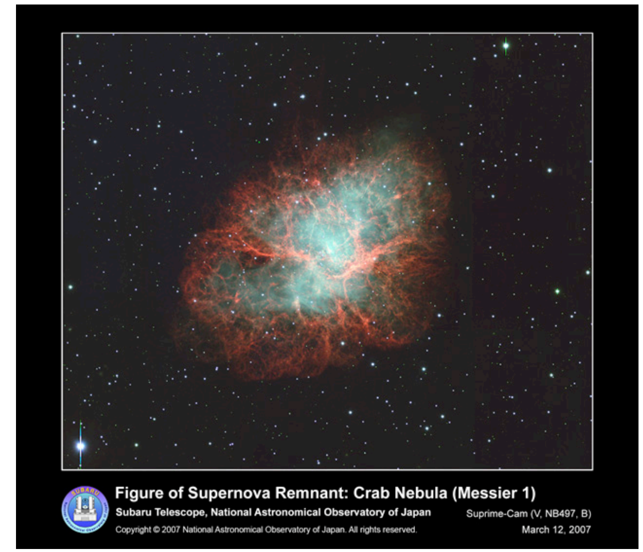
# Importance of repulsive core

stability of nuclei



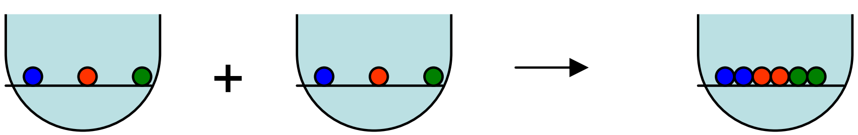
maximum mass of neutron star

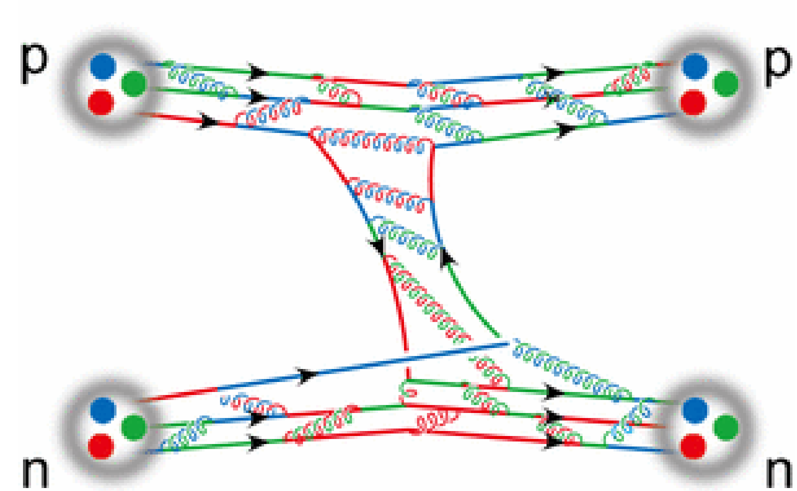

explosion of type II supernova




Origin of RC: “The most fundamental problem in Nuclear physics.”

Note: Pauli principle is not essential for the “RC”.



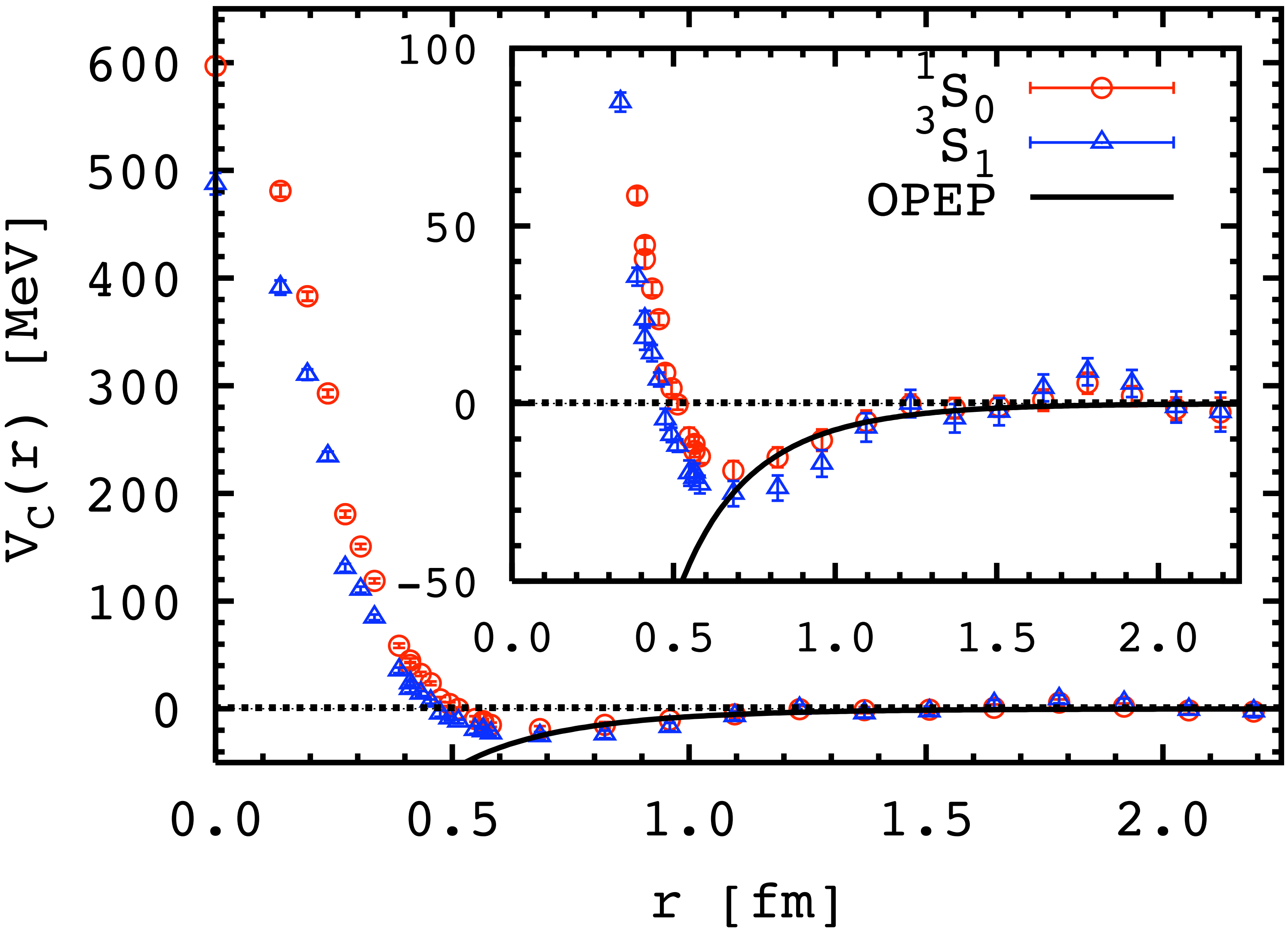



QCD based explanation is needed.

# An “answer” by lattice QCD simulations

N. Ishii S. Aoki and T. Hatsuda, Phys.Rev.Lett. 90(2007)0022001

## NN (effective) central potentials




## Quenched QCD

$m_\pi \simeq 0.53$ GeV

$E \simeq 0$

“The achievement is both a computational *tour de force* and a triumph for theory.”

Nature Research Highlights 2007

## Our strategy

Wave function $\varphi_E(\mathbf{x}) = \langle 0|N(\mathbf{x},0)N(\mathbf{0},0)|NN;E\rangle$

(equal-time BS amplitude) 2N state with energy E

↓ Schrödinger equation

Potential $V(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E + \frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$

## Some questions

1. $V(\mathbf{x})$ may depend on energy $E$.
2. $V(\mathbf{x})$ may depend on nucleon fields $N(x)$.

**This talk: focus on energy dependences.**

A. $V(x)$ from an integrable model in 2 dimensions.

B. NN potential at $E \neq 0$ in quenched QCD.

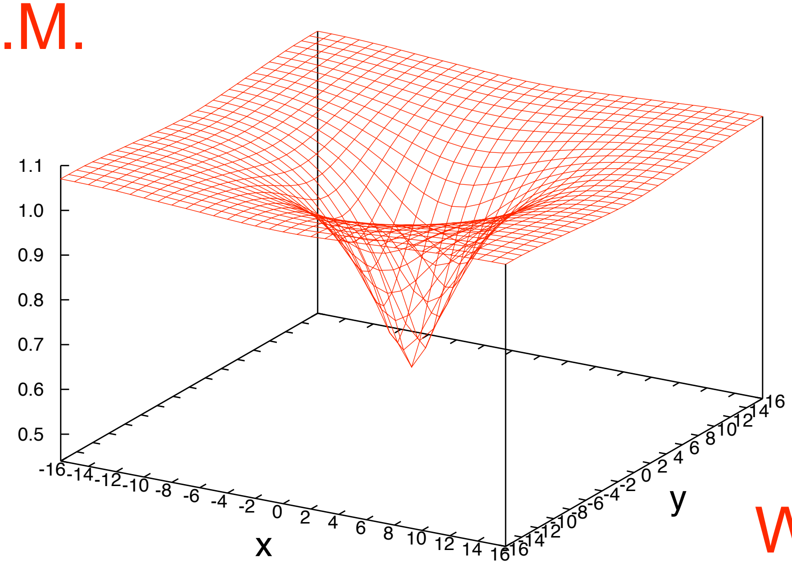
# Example: $\pi\pi$

Sasaki-Ishizuka, arXiv:0804.2941[hep-lat]

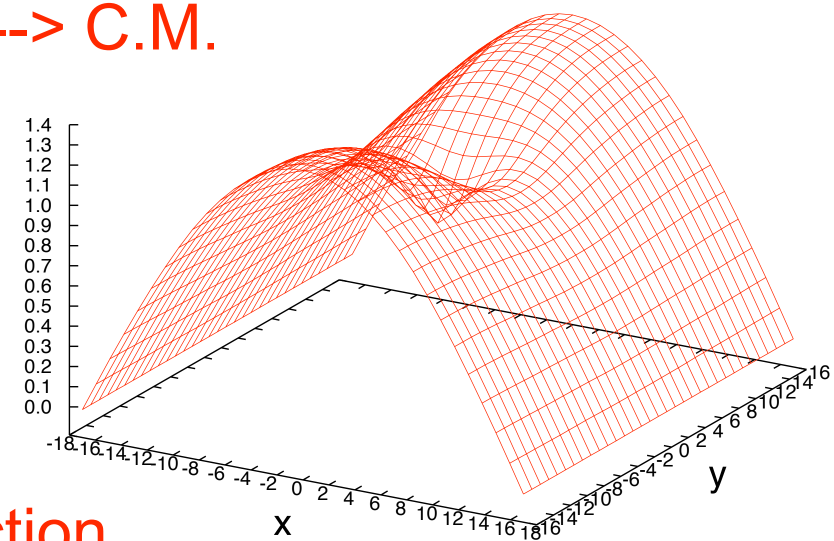
$$p = 0,\ t = 0$$

$$p_x = 2\pi/L,\ t = v\gamma x$$

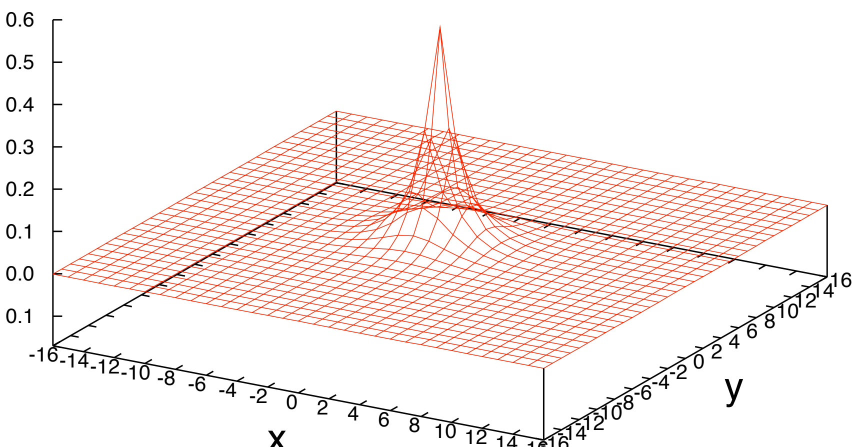
C.M.

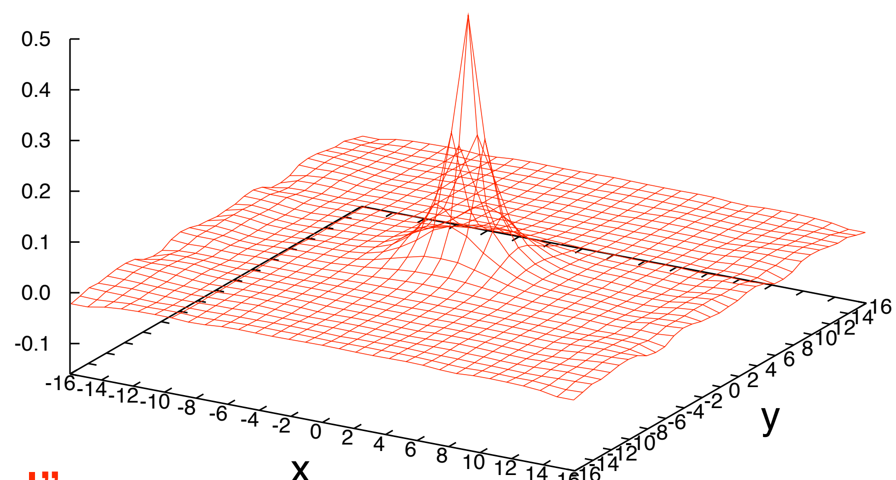



Lab.--> C.M.




Wave function




"potential"

# Potentials from an integrable model

Ref. S. Aoki, J. Balog and P. Weisz, arXiv:0805.3098[hep-th]

# Ising Field Theory in 2 dimensions

**Bethe-Salpeter wave function**

$M$: mass, $\theta$: rapidity

$$\Psi(r,\theta) = i\langle 0|\sigma(x,0)\sigma(0,0)|\theta,-\theta\rangle^{\text{in}}$$

$$p = M(\cosh(\theta), \sinh(\theta))$$
$$r = Mx$$

spin fields

Result by P. Fonseca and A. Zamolodchikov, hep-th/0309228.

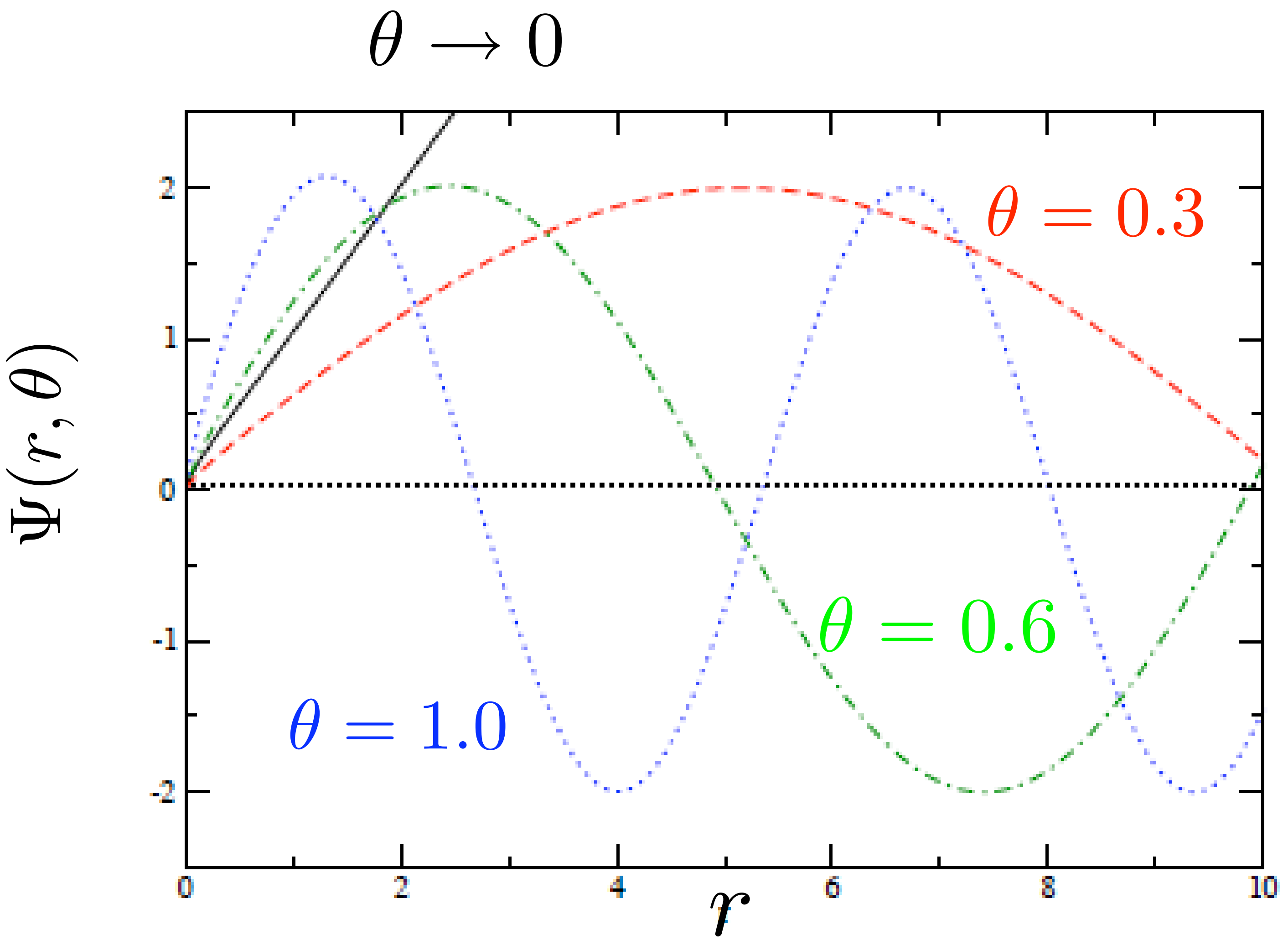
$$\Psi(r,\theta) = \frac{e^{\chi(r)/2}}{\text{ch}\theta}\left[\Phi_+(r,\theta)^2\cosh\left(\frac{\varphi(r)}{2}-\theta\right) - \Phi_-(r,\theta)^2\cosh\left(\frac{\varphi(r)}{2}+\theta\right)\right]$$

$$\Phi'_\pm(r,\theta) = \frac{1}{2}\text{sh}(\varphi(r)\pm\theta)\Phi_\mp(r,\theta)$$

$$\frac{1}{r}\left[r\varphi'(r)\right]' = \frac{1}{2}\text{sh}(2\varphi(r))$$
$$\frac{1}{r}\left[r\chi'(r)\right]' = \frac{1}{2}\left[1-\text{ch}(2\varphi(r))\right]$$

Solve these equations numerically with appropriate boundary conditions by *Mathematica.*

# Bethe-Salpeter wave function



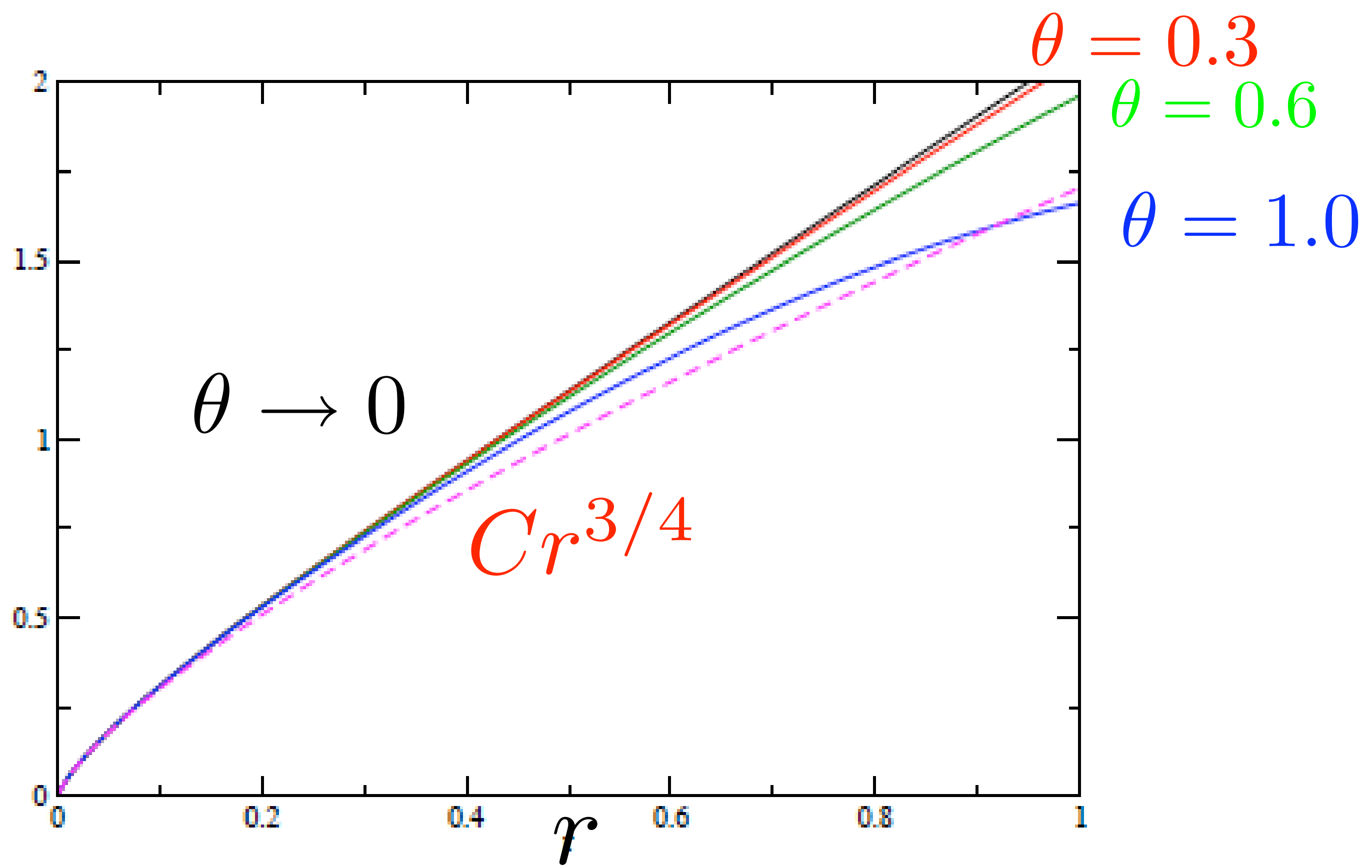
$r \to 0$ $\Psi(r,\theta) \sim C r^{3/4} \sinh(\theta) + O(r^{7/4})$

OPE $\sigma(x,0)\sigma(0,0) \sim G(r)\mathbf{1} + c r^{3/4}\mathcal{E}(0) + \cdots$

(Operator Product Expansion)

mass operator of dim=1

normalized BS wave function $\Psi(r,\theta)/\sinh(\theta)$



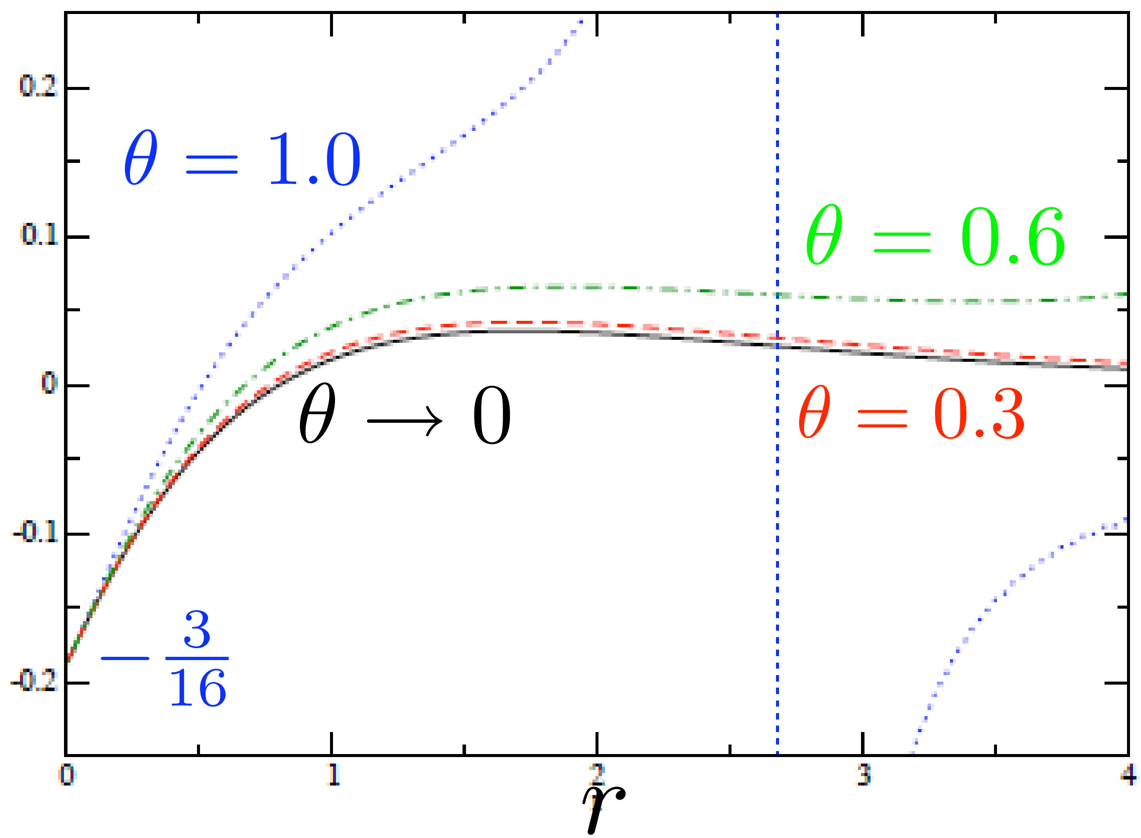
# BS Potentials

$$V_\theta(r) = \frac{\Psi''(r,\theta) + \sinh^2\theta\,\Psi(r,\theta)}{\Psi(r,\theta)} \quad \xrightarrow{r \to 0} \; \sim -\frac{3}{16}\frac{1}{r^2}$$

OPE

Universal($\theta$-independent) at small $r$

$r^2 V_\theta(\mathrm{r})$




Energy dependence is small up to $\theta \simeq 0.6$

potentials are almost identical betweeen $\theta = 0$ and 0.3

Energy dependence is weak at low energy !

# Nucleon-Nucleon Potential at non-zero Energy in Quenched QCD

K. Murano, S. Aoki, T. Hatsuda, N. Ishii, H. Nemura

Work in progress

# Set-up of numerical simulations

quenched QCD on $32^3 \times 48$ lattice

plaquette gauge at $\beta = 5.7$: $a \simeq 0.137$ fm

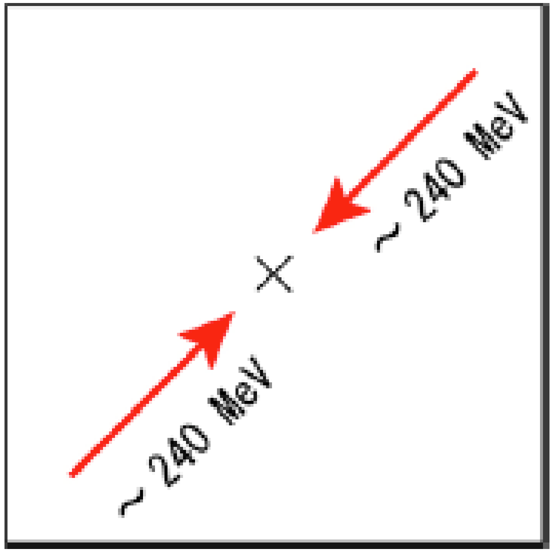
Wilson quark with anti-periodic BC

$m_\pi \simeq 530$ MeV, $m_N \simeq 1334$ MeV

$\mathbf{p}_{\min} = \frac{\pi}{L}(1,1,1)$ $\quad |\mathbf{p}_{\min}| \simeq 240$ MeV

$$E = \frac{k^2}{m_N} \simeq 50 \text{ MeV}$$

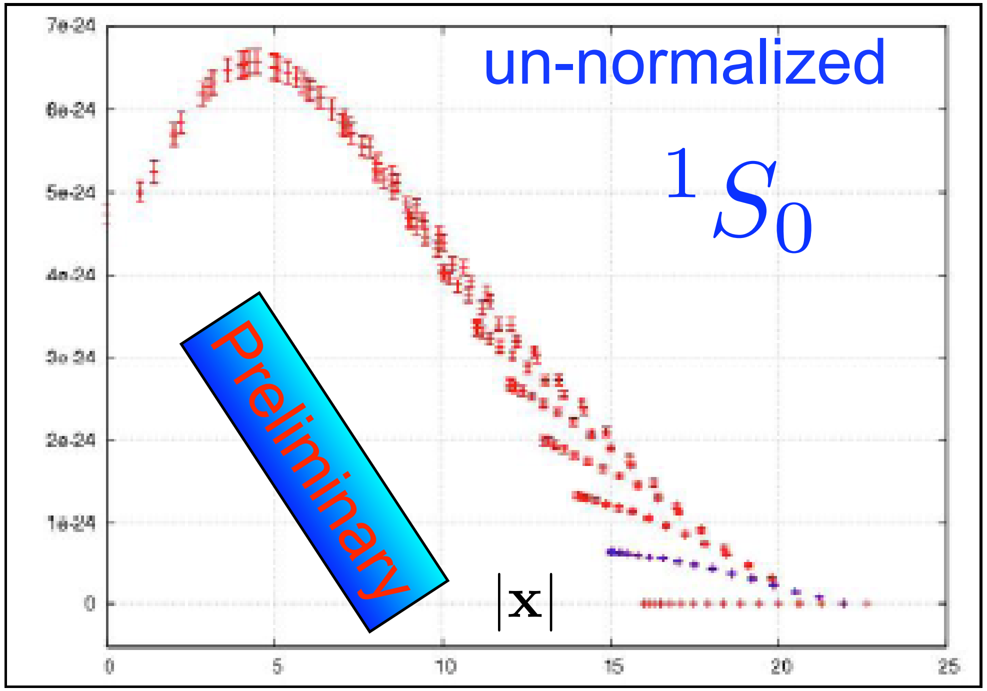
$N_{\text{conf}} = 439 \Rightarrow N_{\text{conf}} = 700$

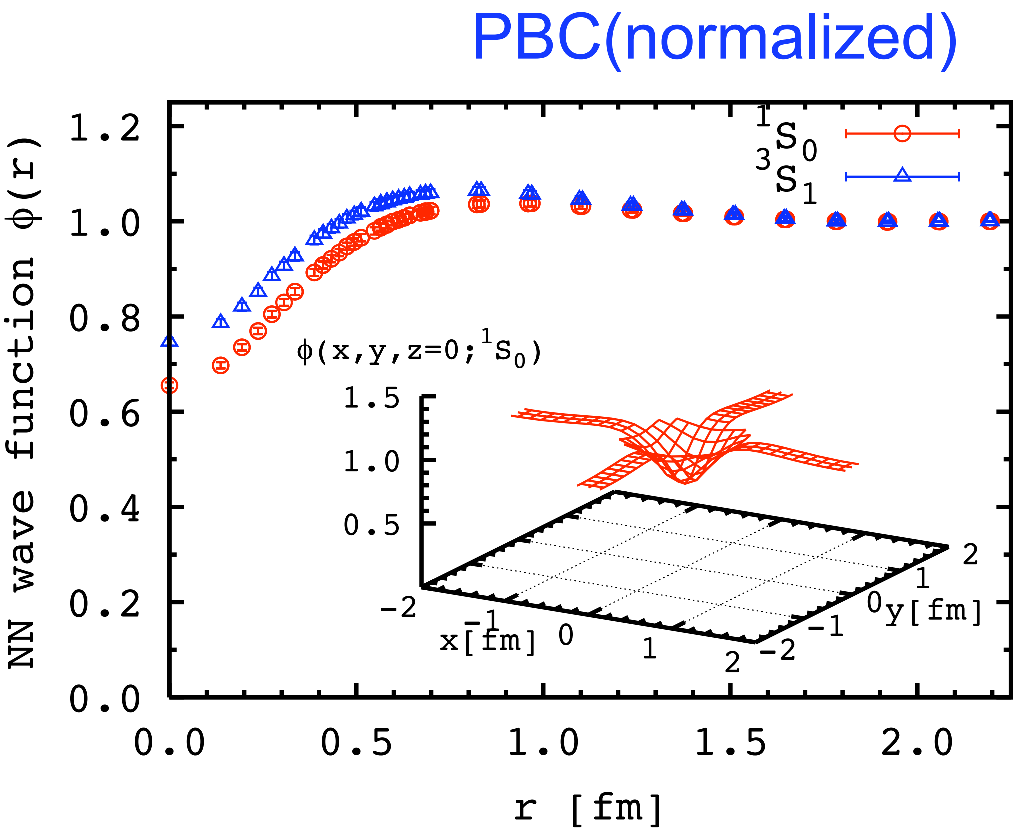


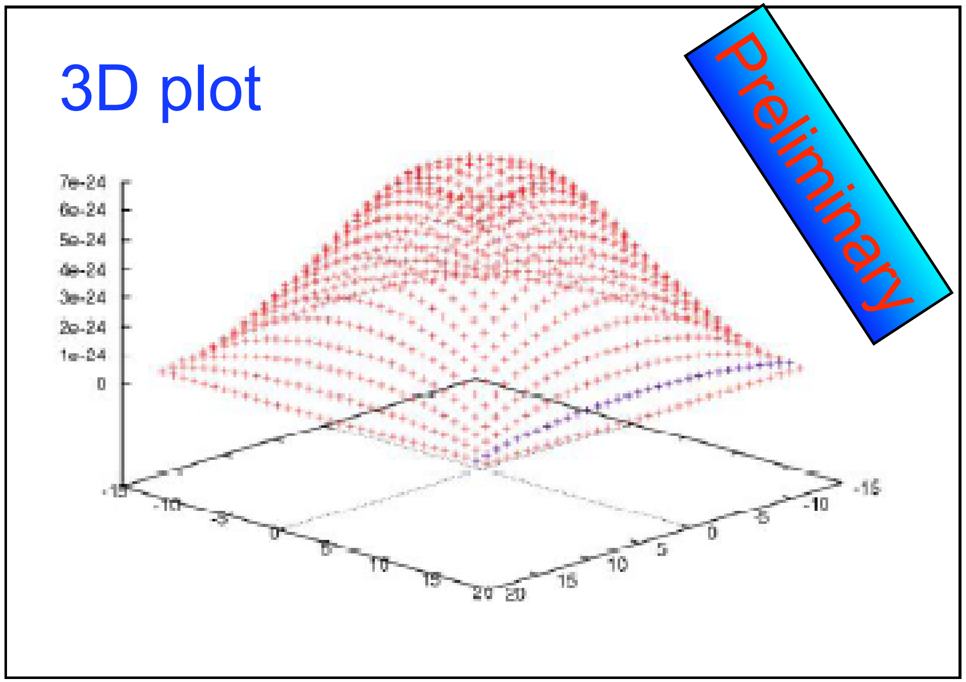
# Wave function with APBC

$t = 8$

$N_{\rm conf} = 439$



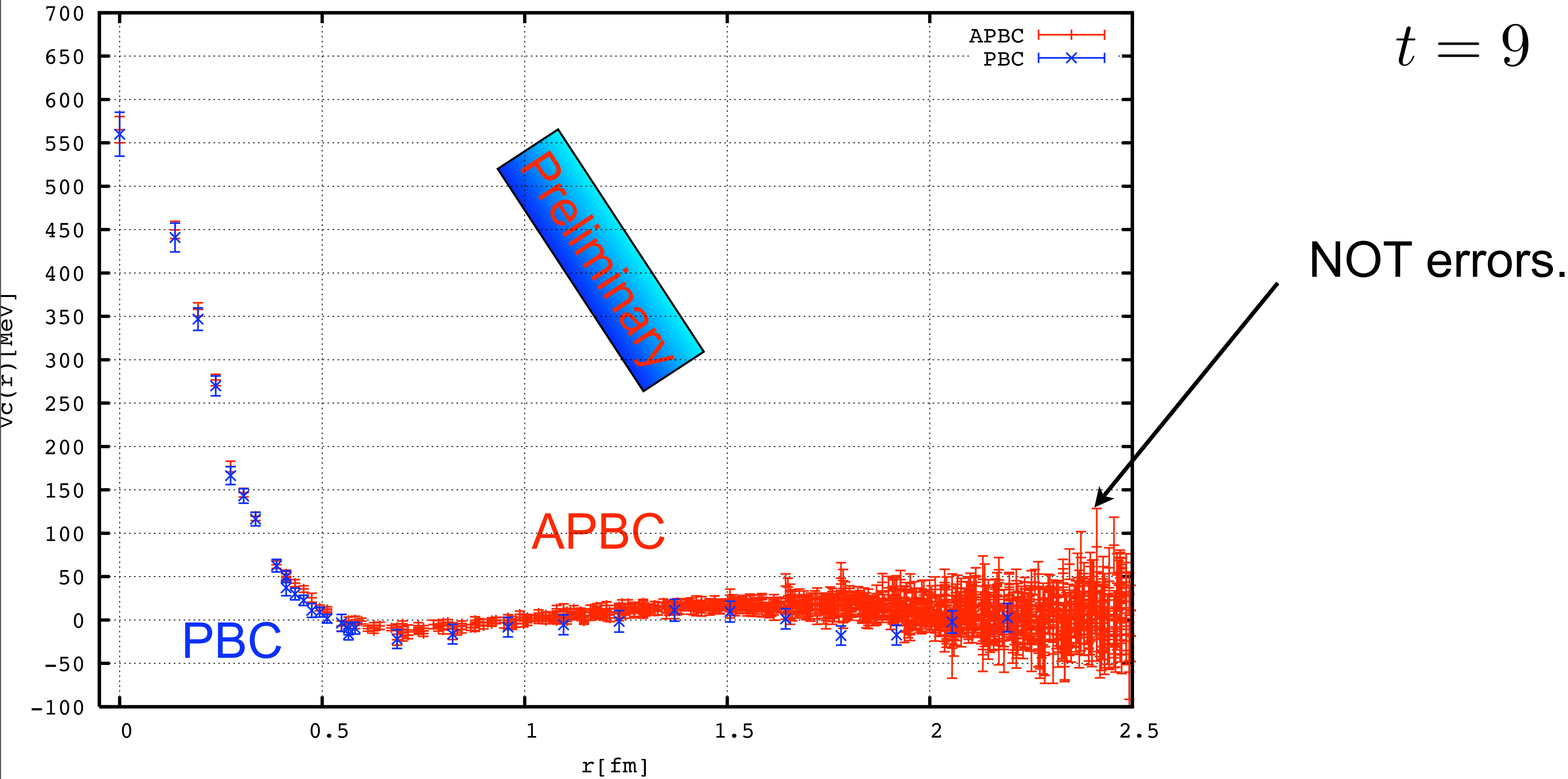




Wave functions are different between APBC and PBC.

# Potentials with APBC

$N_{\rm conf} = 700$

$t = 9$




Free theory on the finite box

$$(\nabla^2 + k^2)G(\mathbf{x}; k^2) = 0$$

$$\Rightarrow \quad V(\mathbf{x}) = 0$$

Correct

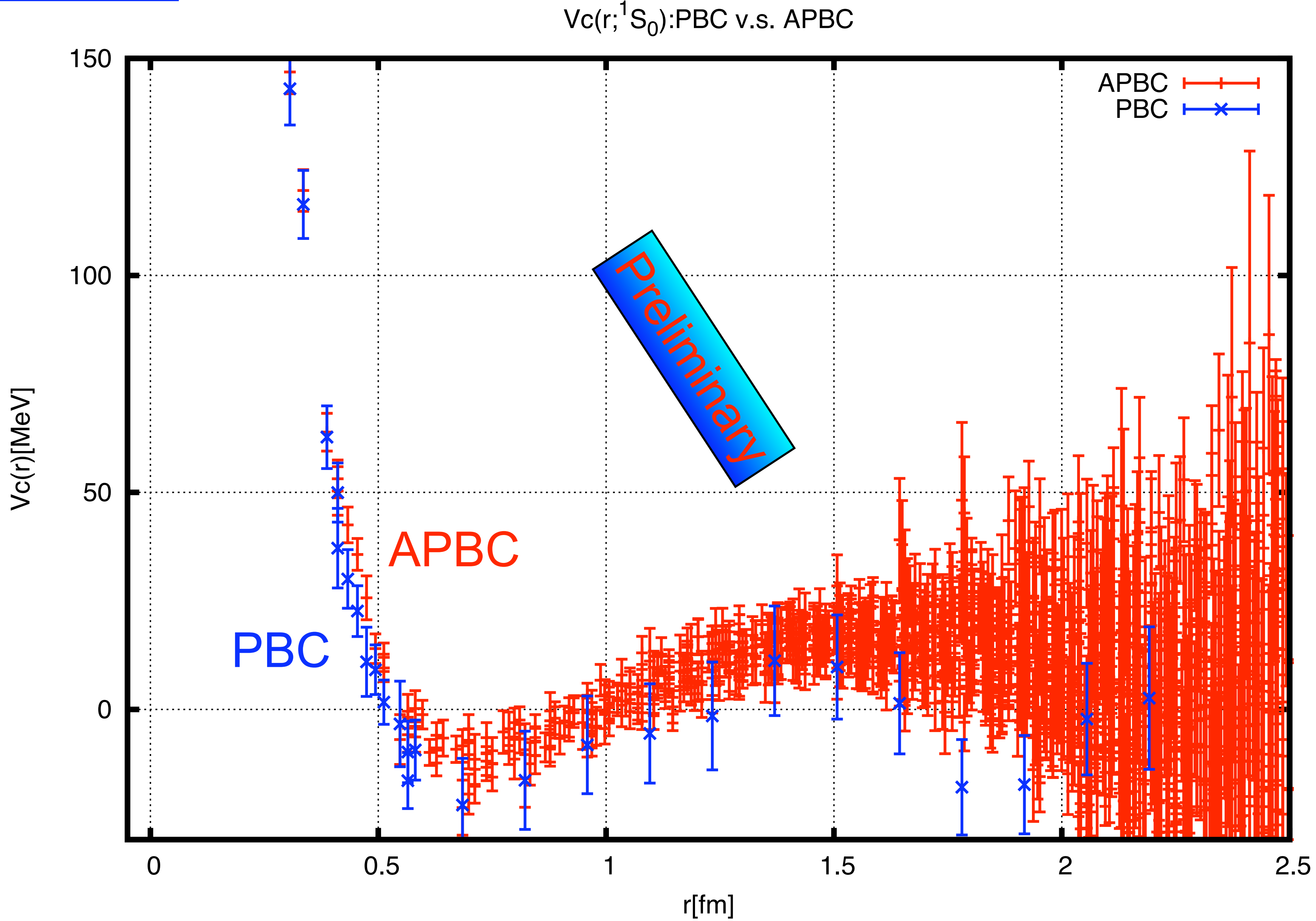
With difference operator $\nabla_L^2$

$$(\nabla_L^2 + k^2)G(\mathbf{x}; k^2) \neq 0$$

$$\Rightarrow \quad V_L(\mathbf{x}) \neq 0$$

(Lattice) Artifact

Zoom-In




Potentials are almost identical between APBC and PBC !

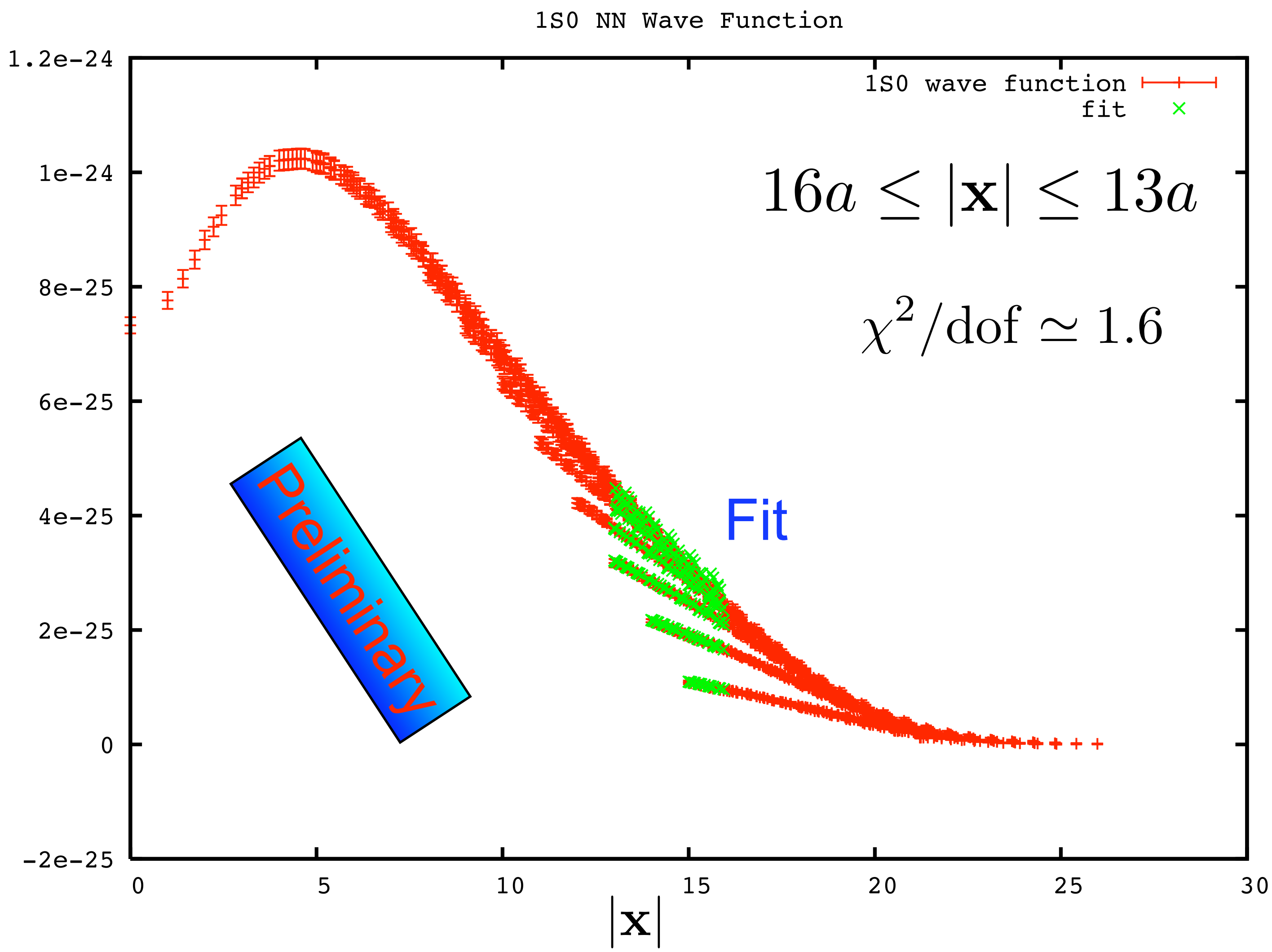
# Fit of the wave function at large distance

$$(\nabla^2 + k^2)G(\mathbf{x}; k^2) = -\delta_L(\mathbf{x})$$

Green's function

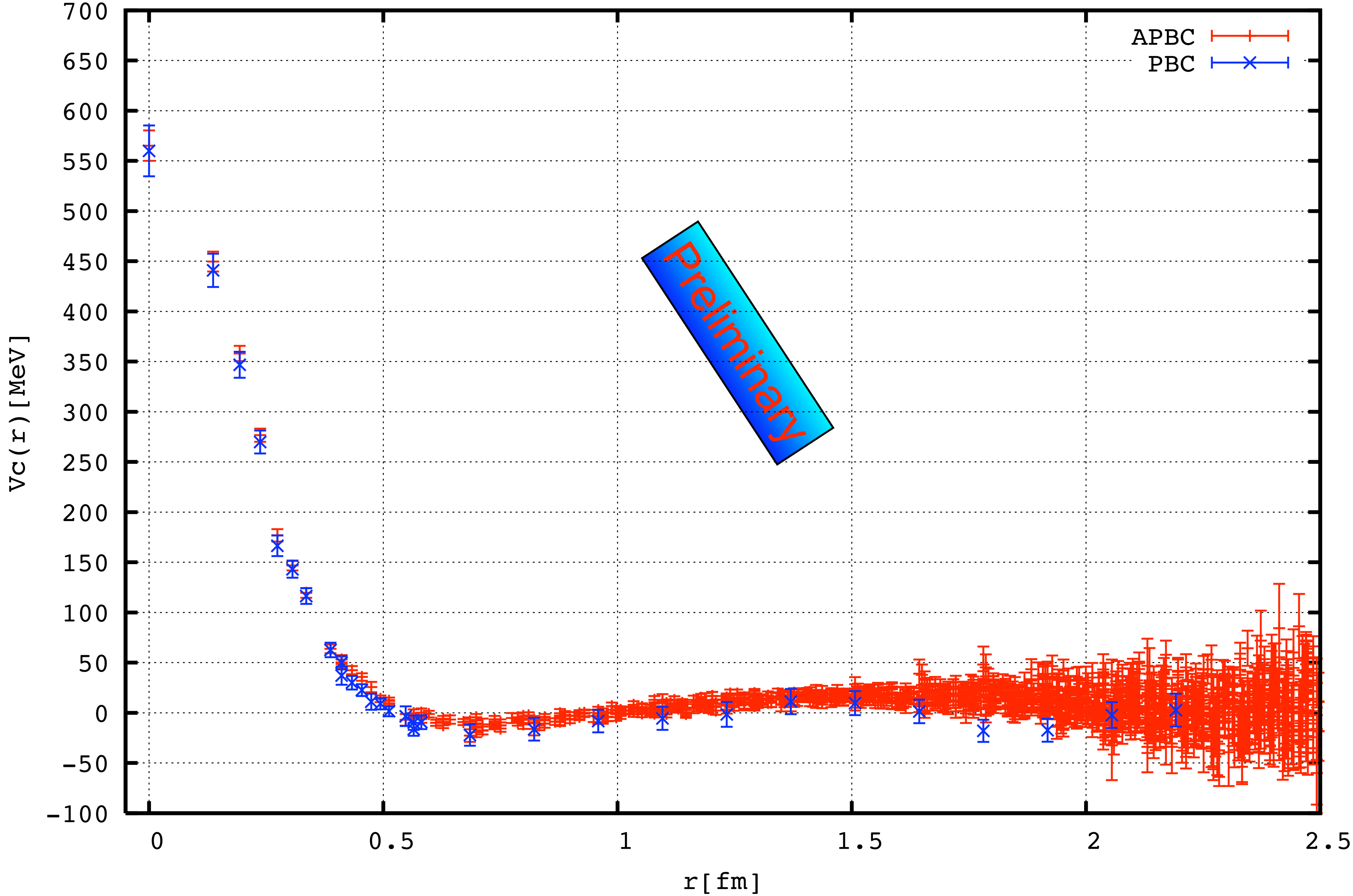
$$G(\mathbf{x}; k^2) = \frac{1}{L^3} \sum_{\mathbf{n} \in \Gamma} \frac{e^{i(2\pi/L)\mathbf{n}\cdot\mathbf{x}}}{(2\pi/L)^2 \mathbf{n}^2 - k^2}$$

$$\Gamma = \{(n_x + 1/2, n_y + 1/2, n_z + 1/2) | n_x, n_y, n_z \in \mathbf{Z}\}$$




$$k^2 = 0.0340(22)$$

# Discussions




Energy dependence of NN potentials seems small at $E \leq 50$ MeV

Non-local potential ⟷ Energy dependent potential

$$\left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y}) \qquad V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E+\frac{\nabla^2}{2m}\right)\varphi_E(\mathbf{x})$$

$U(\mathbf{x},\mathbf{y})$ contains "off-shell" informations

**Derivative expansion**

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta(\mathbf{x}-\mathbf{y})$$

spins

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + V_T(r)S_{12} + O(\nabla)$$

tensor operator

$$S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) \qquad r=|\mathbf{x}|$$

Our result → Non-locality is very weak. Why ?

Universality of potentials at short distance might be understood by OPE.

→ Repulsive core is energy/operator independent ?

# Alternative: Construct energy-independent local potential

The inverse scattering theory suggests that there exist an unique energy independent local potential, which gives the correct phase shift at all energies.

"only on-shell information"

Ex. 1-dimension

$$\left(-\frac{d^2}{dx^2} + V(x)\right)(\Lambda_E(x)\varphi_E(x)) = E(\Lambda_E(x)\varphi_E(x))$$

"correct wave function"

local potential

$$V_E(x)\varphi_E(x) = \left(E + \frac{d^2}{dx^2}\right)\varphi_E(x)$$

$$V(x)\Lambda_E(x) = V_E(x)\Lambda_E(x) + \Lambda''_E(x) + 2\Lambda'_E(x)(\log\varphi_E(x))'$$

This equation can easily be solved for $\Lambda_E(x)$, if $V(x)$ is given.

How can we obtain $V(x)$ ?

# A proposal

Consider a finite box with size $L$. $\quad k_n \simeq \frac{2\pi n}{L}, \ n = 0, 1, 2, \cdots$

$E = k_n^2$ is given, $\varphi_E(x) = 0$ at $\Omega_n = \{x_0, x_1, \cdots, x_n\}$

$$V(x)\Lambda_E(x)\varphi_E(x) = V_E(x)\varphi_E(x)\Lambda_E(x) + \Lambda''_E(x)\varphi_E(x) + 2\Lambda'_E(x)\varphi_E(x)'$$

$$V_E(x)\varphi_E(x) = \left(E + \frac{d^2}{dx^2}\right)\varphi_E(x) \equiv K_E(x)$$

$$0 = K_E(x_i)\Lambda_E(x_i) + 2\Lambda'_E(x_i)\varphi_E(x_i)' \qquad x_i \in \Omega_n$$

$n \to \infty$, $\Omega_n$ becomes dense in $[0, L]$

$$V(x) = \lim_{E\to\infty} \{V_E(x) - 2X_E(x)(\log(\varphi_E(x)))' - X_E(x)' + X_E(x)^2\}$$

$$\Lambda_E(x) = \lim_{E\to\infty} \exp[-\int_0^x d\,y\,X_E(y)]$$

$$X_E(x) = \frac{K_E(x)}{2\varphi_E(x)'}, \text{ or a interpolation of } \frac{K_E(x_i)}{2\varphi_E(x_i)'}$$

If this limit exists, the energy-independent local potential is obtained.

# Works in progress

1. full QCD with lighter quark mass
2. tensor force

→ Talk by N. Ishii
This session at 3:50

3. hyperon-nucleon potential

→ Talk by H. Nemura
This session at 4:10

4. OPE and universality of repulsive core