Stochastic quantization of a finite temperature lattice field theory in the real time formula

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Time evolution of the system,
Relaxation, and
Transport problem,

……one of the final goads of theoretical Physics.

But…

It is very very difficult to solve dynamics,

especially for many-body system.

Lattice field theory is
quite powerful tool !! But.

Well defined only for Euclid time.

Euclid time is really a “time”?

Minkowski space		Euclidian space
Cauchy problem Initial condition	⟷	Poisson problem Boundary condition

Analytic continuation is indispensable to discuss time dependence.

Finite temperature field theory in real time formula was well established in '80s.

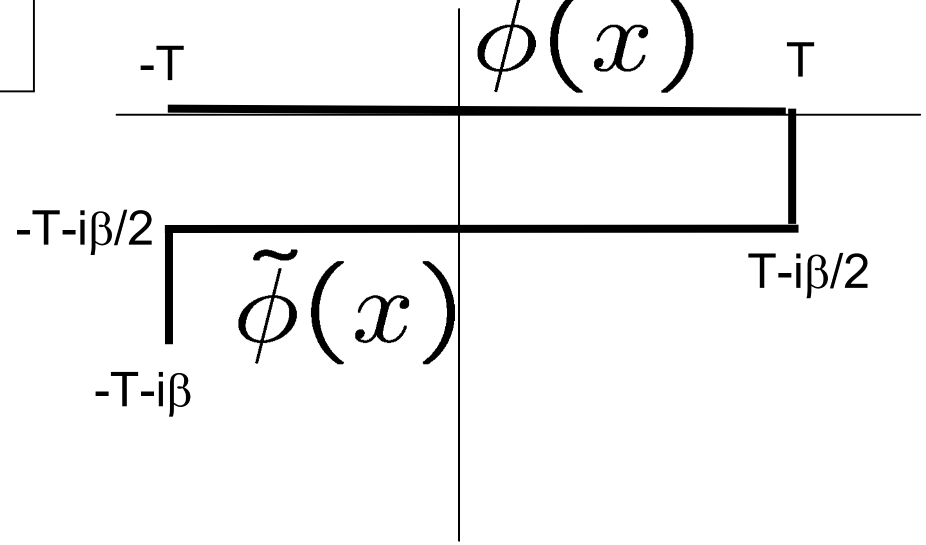
Takahashi –Umezawa
Thermo-field Dynamics

- Operator formula
- Doubled field by tilder operation.

$$H_T = H - \tilde{H}$$

Niemi-Semenoff
Complex time path

- Path-integral
- Anti-chronological field



Simulation of Nonequilibrium system !!

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Simulating Nonequilibrium Quantum Fields with Stochastic Quantization Techniques

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Lattice simulations of real-time quantum fields

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Schwinger-Keldysh type closed time path + Stochastic Quantization Method

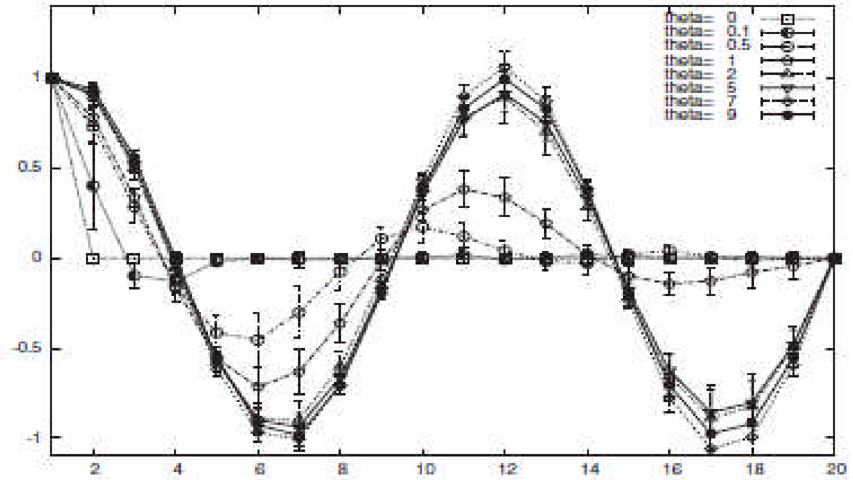


FIG. 1. Re$C(\hat{t})$ vs $\hat{t}$ for a free-field theory with mass $\hat{m} = 2.315$. The Langevin evolution, shown for $\vartheta = 0$–9 in units of a^2, converges to the correct result with period $2\pi\gamma/\hat{m}$.

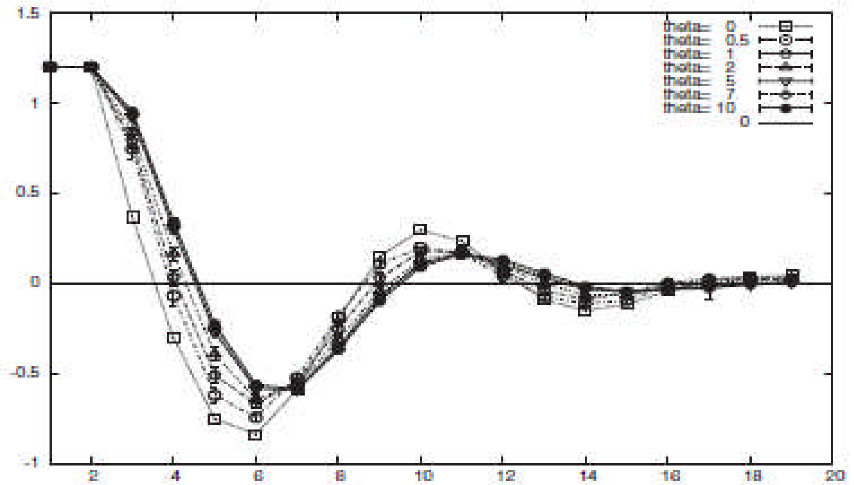


FIG. 2. Re$G(\hat{t})$ vs $\hat{t}$ for the interacting theory with $\lambda = 1$. As starting configuration ($\vartheta = 0$) the classical result is taken, and the Langevin updating incorporates quantum corrections.

large-$\hat{t}$ boundary conditions (no coupling to $\hat{t} = N_t$). In this case, we consider $\hat{\phi}(\hat{t} = 1, \hat{\mathbf{x}}) = \hat{\phi}(\hat{t} = 2, \hat{\mathbf{x}}) = \hat{\phi}_{\text{class}}(\hat{t} = 1, \hat{\mathbf{x}})$ to set the *initial conditions*. Below, we will also use $c_{N_t - 1} = 2$ for fixed large-$\hat{t}$ boundary conditions in the case of a noninteracting field for comparison, and we set $\hat{\phi}(\hat{t} = 1, \hat{\mathbf{x}}) = 1$ and $\hat{\phi}(\hat{t} = N_t, \hat{\mathbf{x}}) = 0$. The classical field configurations $\hat{\phi}_{\text{class}}(\hat{t}, \hat{\mathbf{x}})$ have been obtained by numerically

$$G(\hat{t}) = C(\hat{t}) - \left\langle \frac{1}{N_s^3} \sum_{\hat{\mathbf{x}}} \hat{\phi}(1, \hat{\mathbf{x}}) \right\rangle \left\langle \frac{1}{N_s^3} \sum_{\hat{\mathbf{x}}'} \hat{\phi}(\hat{t}, \hat{\mathbf{x}}') \right\rangle \quad (11)$$

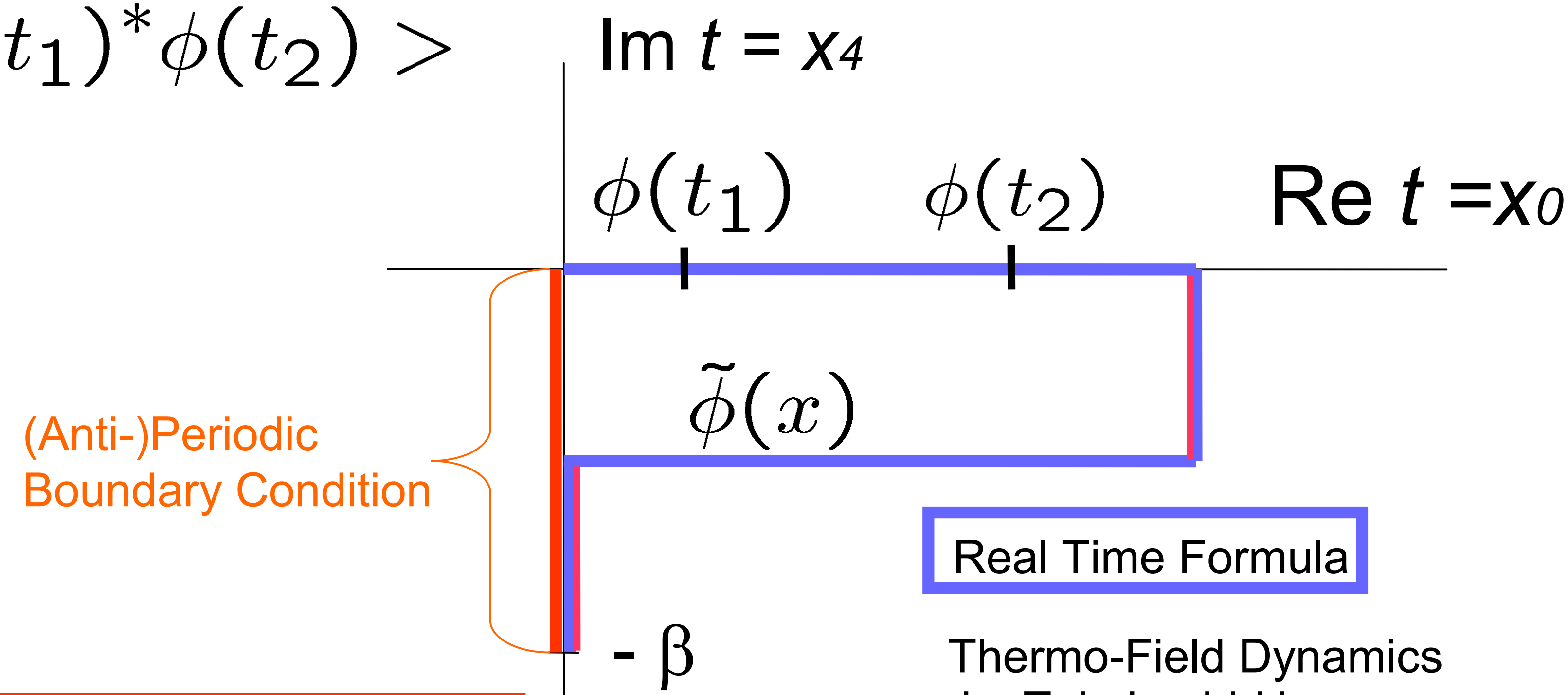
for $\lambda = 1$ and $\hat{m} = 0$. In Fig. 3 a different starting configuration is considered for the same $\hat{\phi}_{\text{class}}(1, \hat{\mathbf{x}})$ initial condition as in Fig. 2. The same data is presented as a function of the Langevin time $\hat{\vartheta}$ in Fig. 4 to see the

“Nonequilibrium system” seems difficult to control → Finete temperature equilibrium system with “**real**” time is enough for us

Real time correlation

$$< \phi(t_1)^* \phi(t_2) >$$

Complex *t*



Real Time Formula

Imaginary Time Formula

$$e^{iS_M} \to e^{-S_E}$$

Wick rotation

Probability distribution

⇩

MonteCarlo Numerical simulation of Lattice quantum field theory

Thermo-Field Dynamics by Takahashi-Umezawa

Matsumoto et al;

Path-Integral by Niemi-Semenoff; Niegawa

$$e^{iS_M}$$

Sigh Problem

Parisi-Wu type Stochastic Quantization

Parisi-Wu Stochastic Quantization

- Euclidian Action S_E
- Suppose stochastic process with an additional fictitious time τ

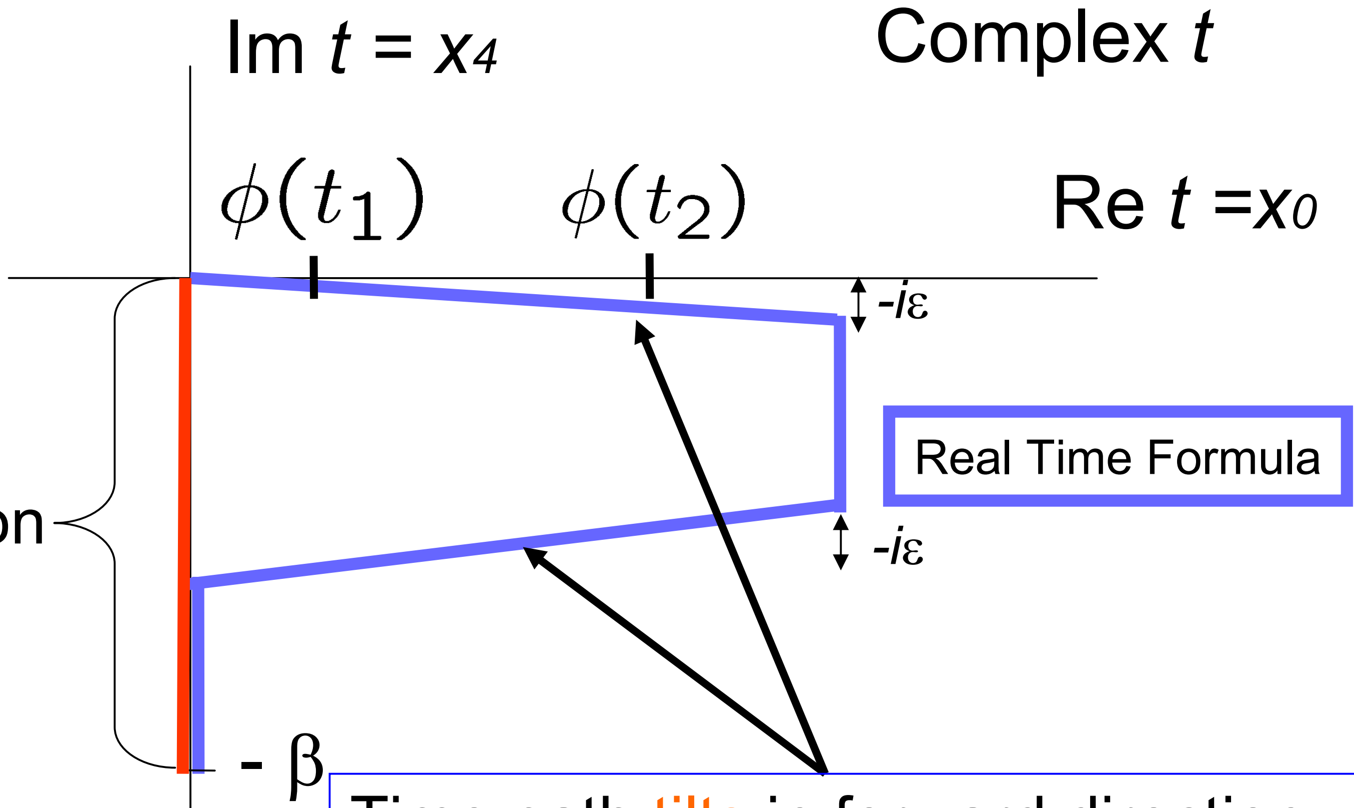
$$\frac{d\phi(x,\tau)}{d\tau} = -\frac{\delta S_E}{\delta\phi(x,\tau)} + \eta(\tau)$$

$$< \eta(\tau) >= 0, < \eta(\tau)\eta(\tau') >= 2\delta(\tau - \tau')$$

- Take the equilibrium limit in $(\tau \rightarrow \infty)$

Then we can obtain quantum expectation of Euclidian theory.

Our plan



(Anti-)Periodic Boundary Condition

⇕

KMS condition
Thermal equilibrium
(by Niegawa)

Time path tilts in forward direction
(Feynman causality,
Hamiltonian has spectra bounded from below)

Imaginary Time Formula

$$e^{iS_M} \rightarrow e^{-S_E}$$

Wick rotation

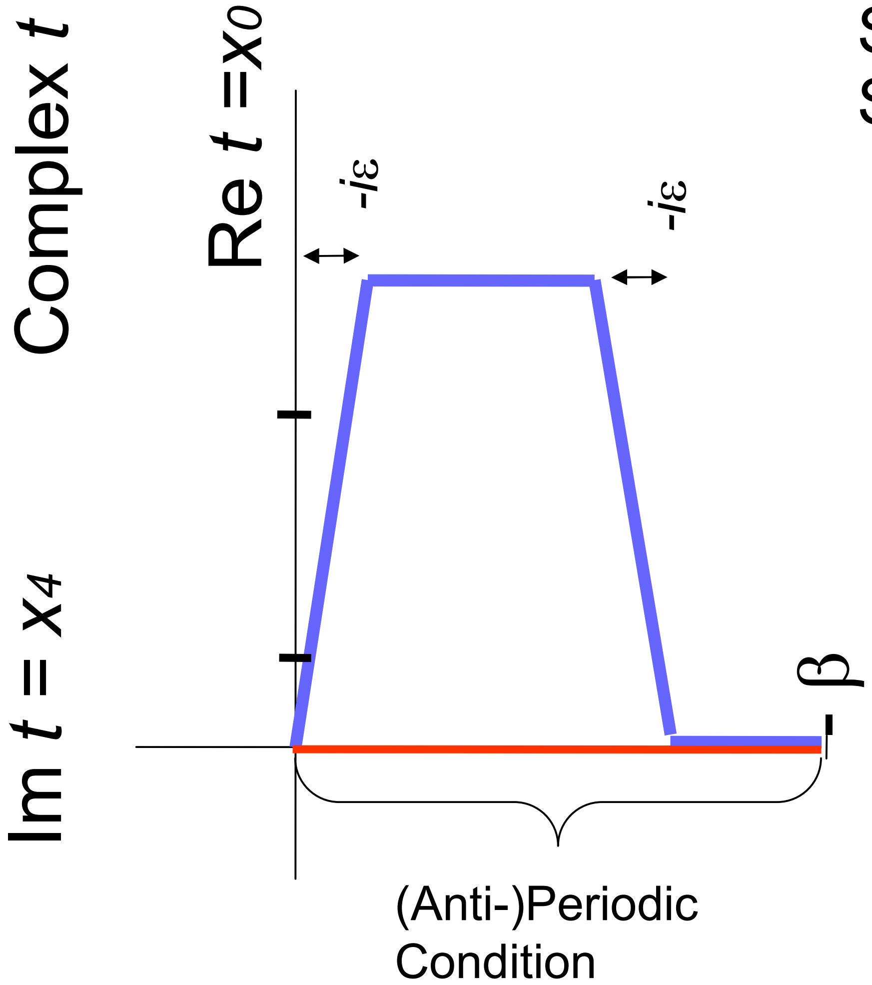
Small $-i\varepsilon$ in *real time path*.

⇕ Nakazato and Yamanaka

Converge stochastic process in SQM

⇐

$$e^{iS_M}$$



Stochastic Quantization
Starts from Euclidian Action

$$S_E$$

We extend
Euclidian action to
complex plane

$$\frac{d\phi}{d\tau} = -\frac{\delta S_E}{\delta \phi(\tau)} + \eta(\tau)$$

$$\frac{\delta S_E}{\delta \phi(t_c, \boldsymbol{x}, \tau)} = \Delta t_c \; [\partial_{t_c}\partial_{t_c}\phi(t_c, \boldsymbol{x}, \tau) \cdots$$

Contour dependent phase

Numerical simulation

- Scalar field $\lambda|\phi|^4$

  $ma = 0.2, \quad \lambda = 0.01, 0.05, 0.1$

- Lattice size

  16X16X16X40, tilt = 0.05

  40= 15 + 5 + 15 + 5,

  real imaginary real imaginary

- Stochastic process $\Delta\tau$=0.00002

  Take average for each 5000 steps X50times

- Annisotropic

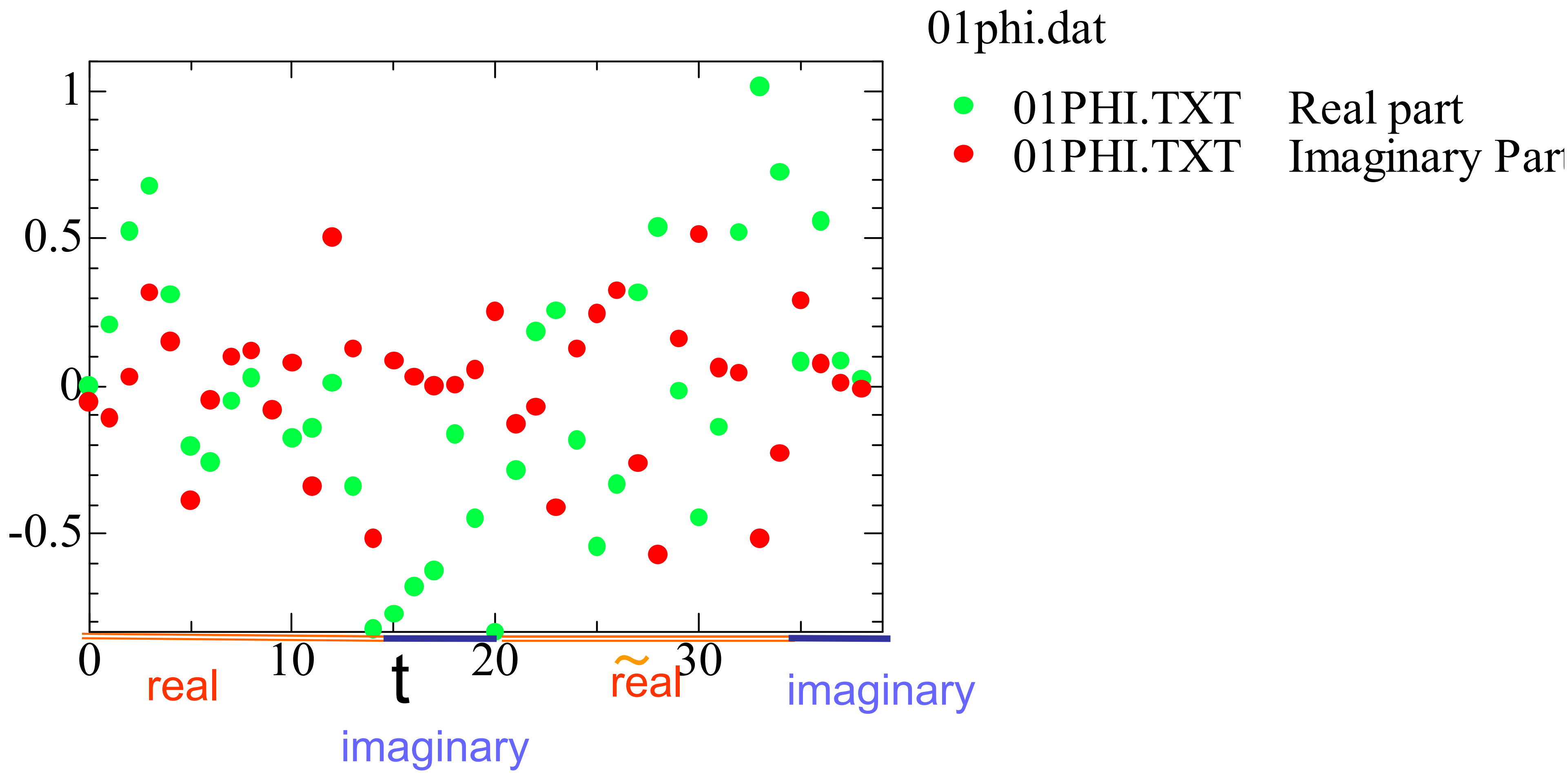
  spatial lattice size

  Courant condition

  = time-like lattice size × γ, γ=4

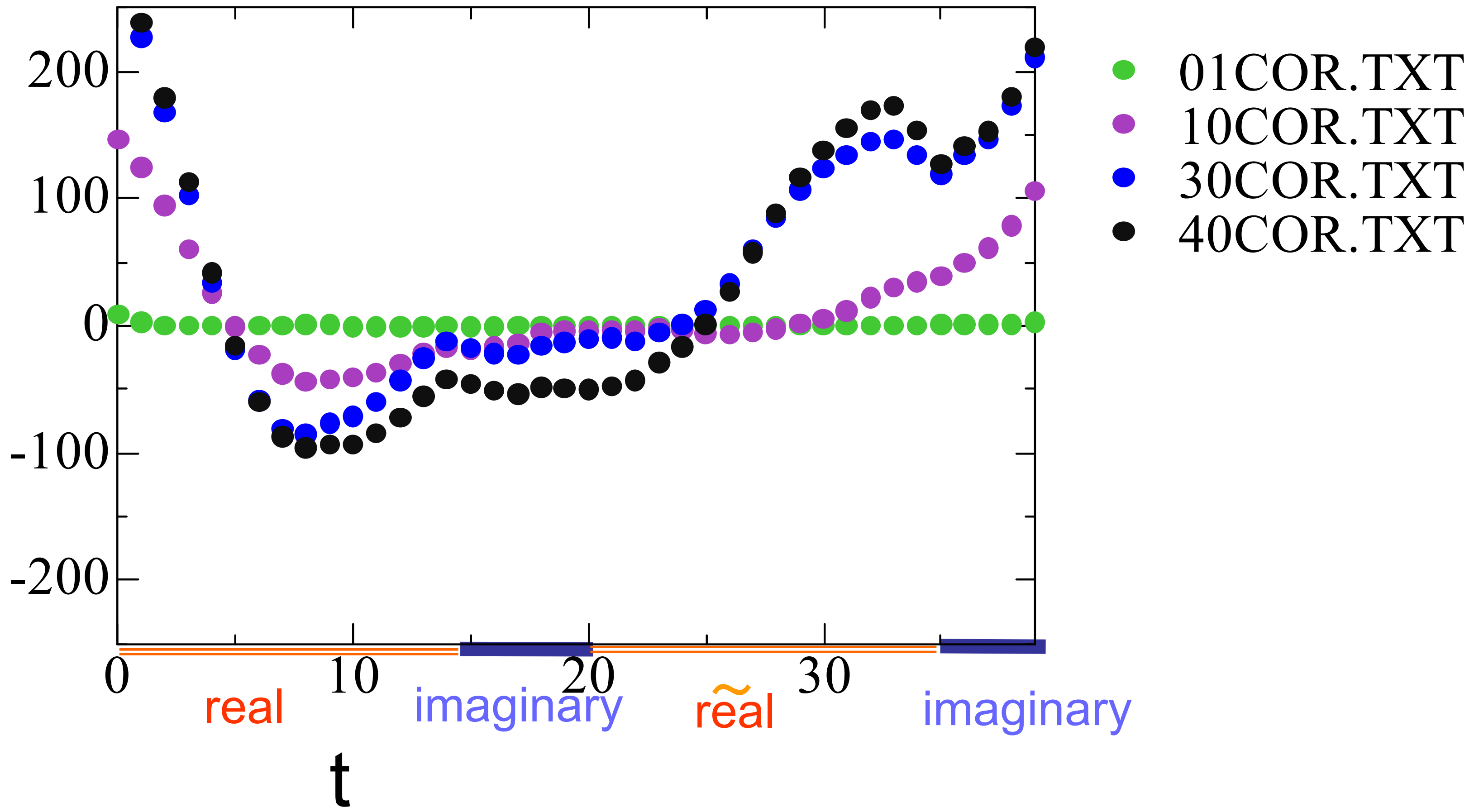
$$< \sum_{x} \phi(x,t) >$$

Hotstart first 5000 average



ma = 0.2, λ = 0.05

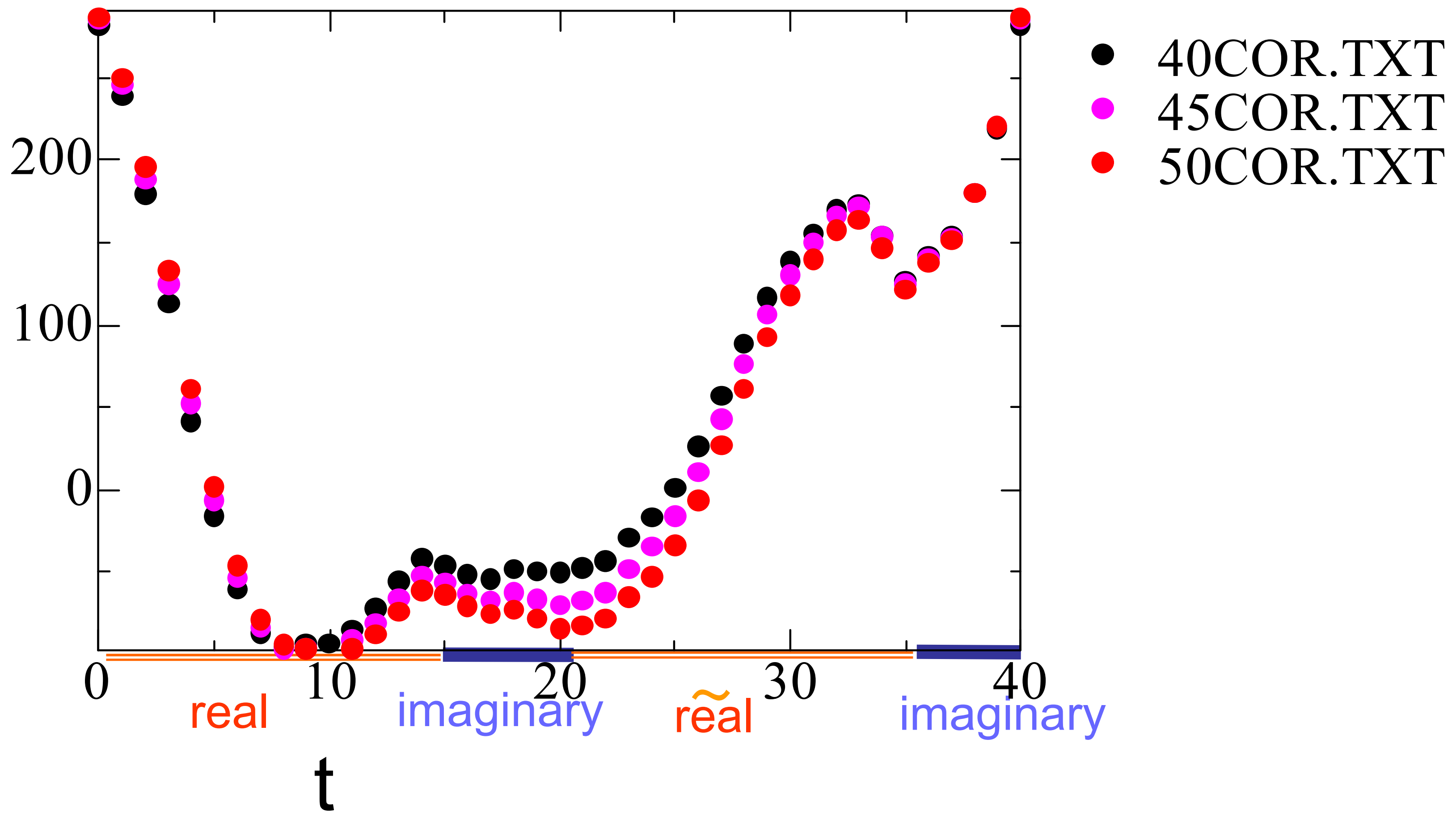
$$< \sum_{\boldsymbol{x}} \phi^*(\boldsymbol{x}, t=0)\phi(\boldsymbol{x}, t) >$$



ma = 0.2, λ = 0.05

Average of 5000 steps

$$< \sum_{\boldsymbol{x}} \phi^*(\boldsymbol{x}, t = 0)\phi(\boldsymbol{x}, t) >$$

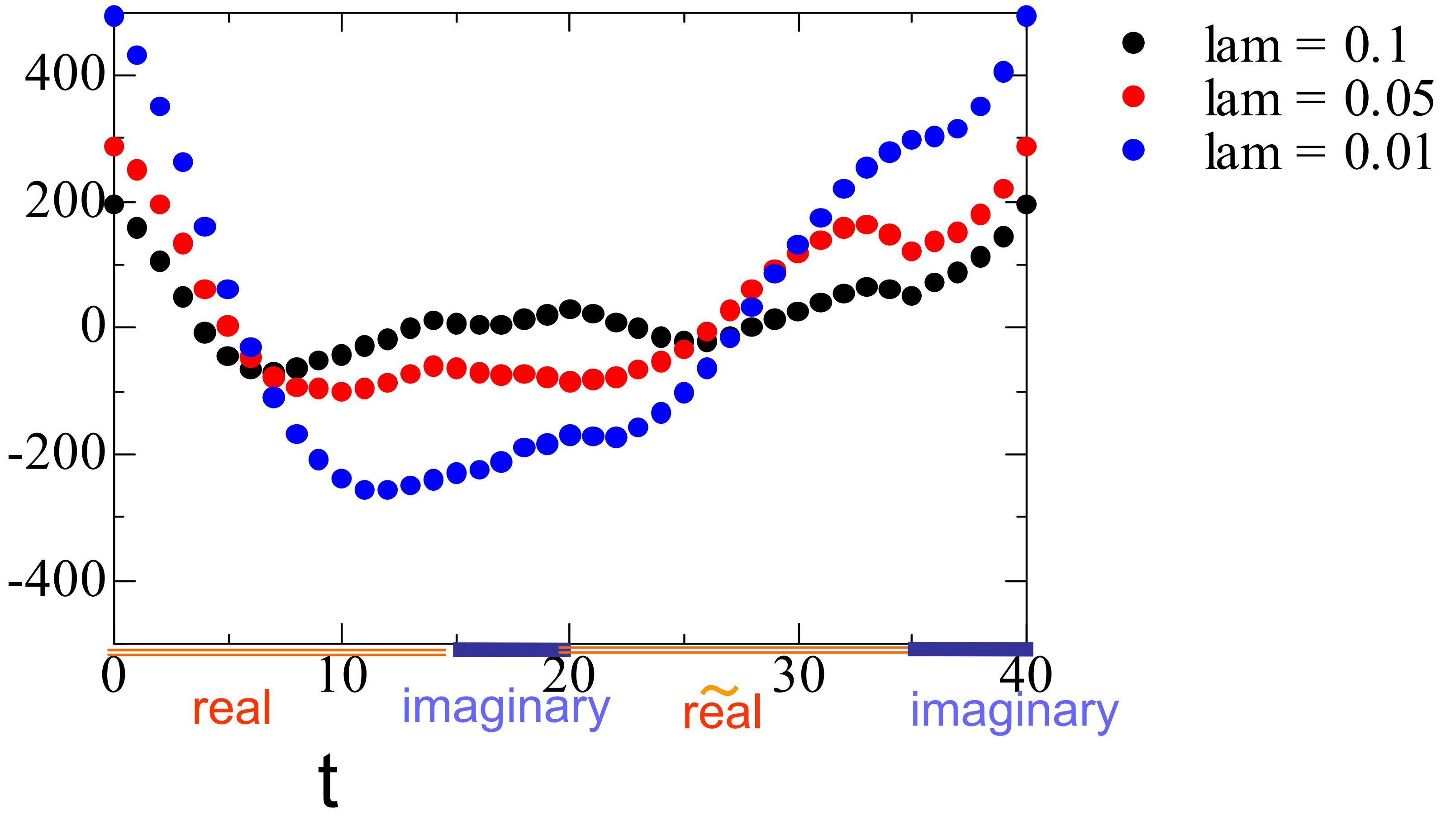


ma = 0.2, λ = 0.05

Average of 5000 steps

$$< \sum_{x} \phi^*(x, t=0)\phi(x,t) >$$

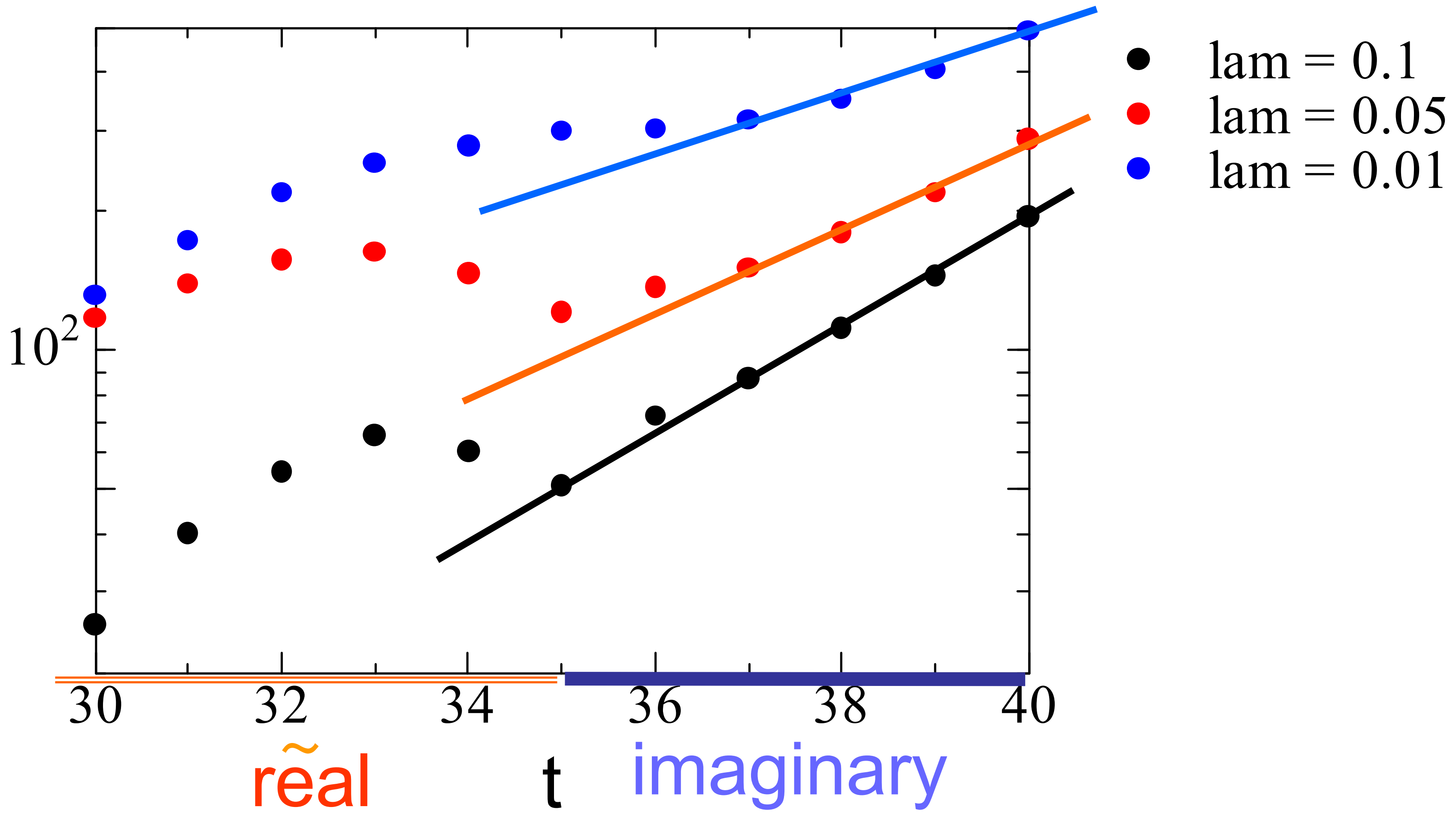
coupling λ dependences



ma = 0.2

50-th average of 5000 steps

$$\log < \sum_{\boldsymbol{x}} \phi^*(\boldsymbol{x}, t=0)\phi(\boldsymbol{x}, t) >$$



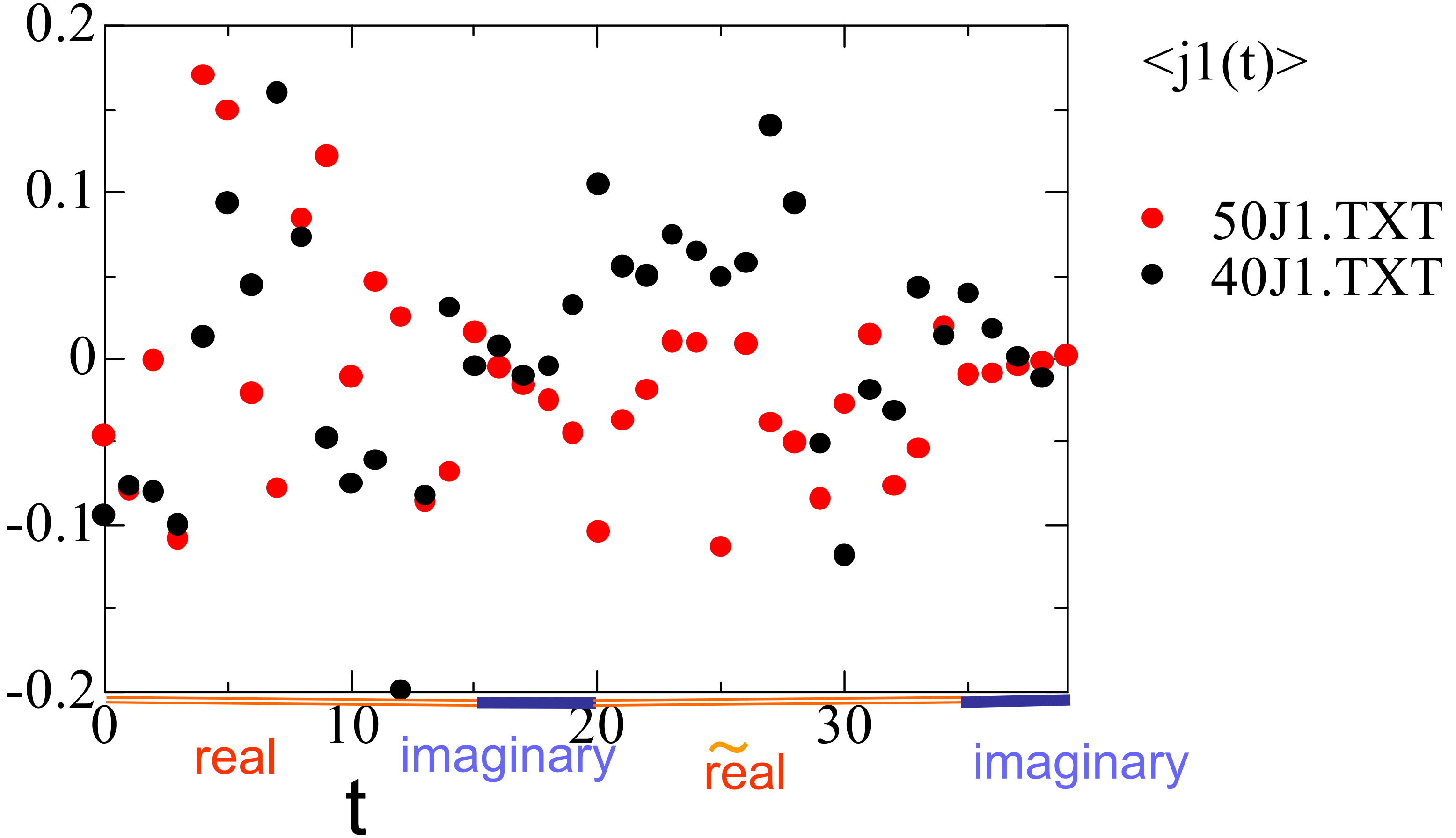
ma = 0.2

50-th average of 5000 steps

$$< \sum_{\boldsymbol{x}} J_x(\boldsymbol{x}, t) >$$

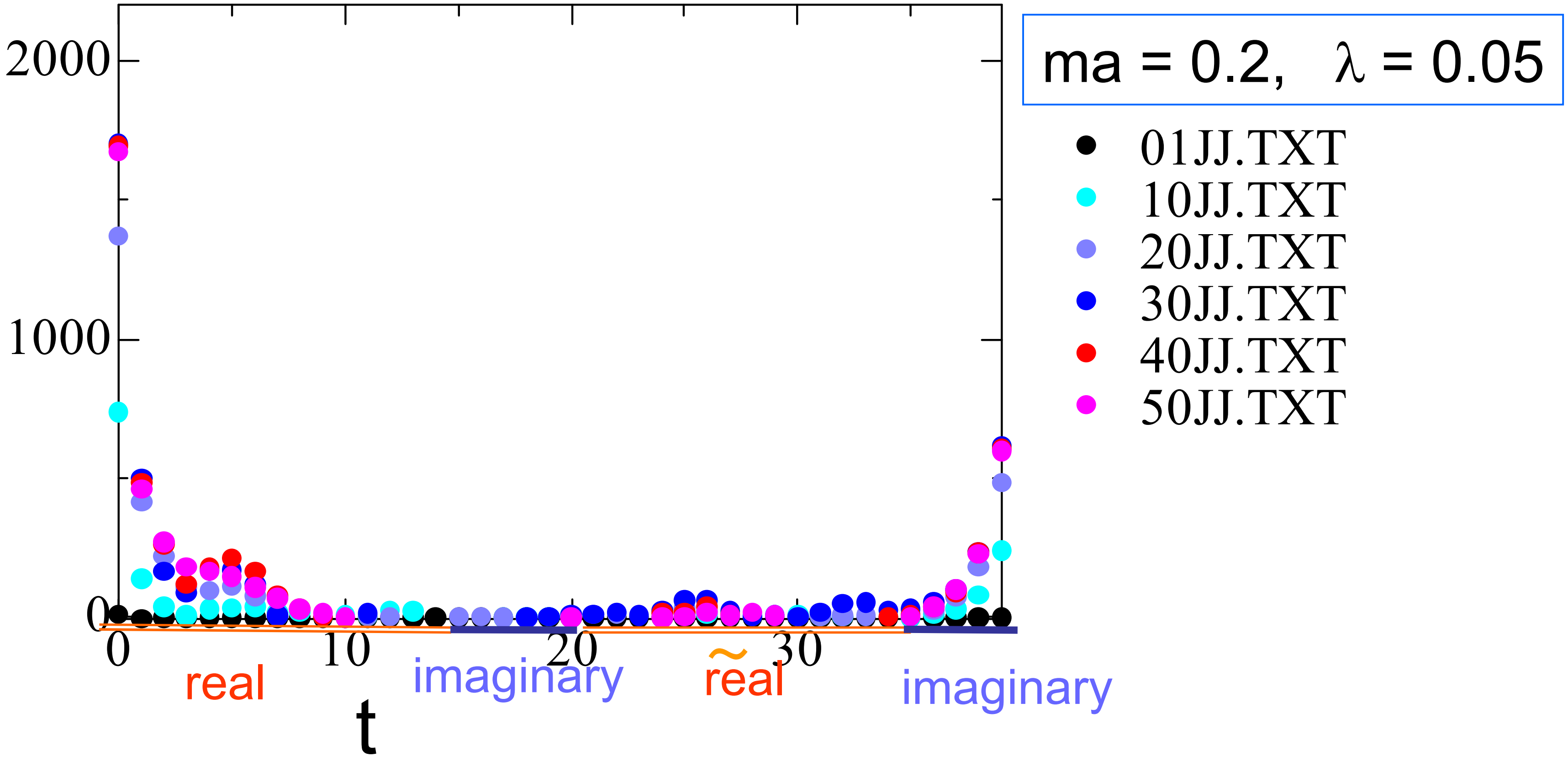
ma = 0.2, λ = 0.05

Average of 5000 steps.
(40th and 50th)



$$\boldsymbol{J} = \text{Im}\left(\phi^*(\boldsymbol{x}, t)\nabla\phi(\boldsymbol{x}, t)\right)$$

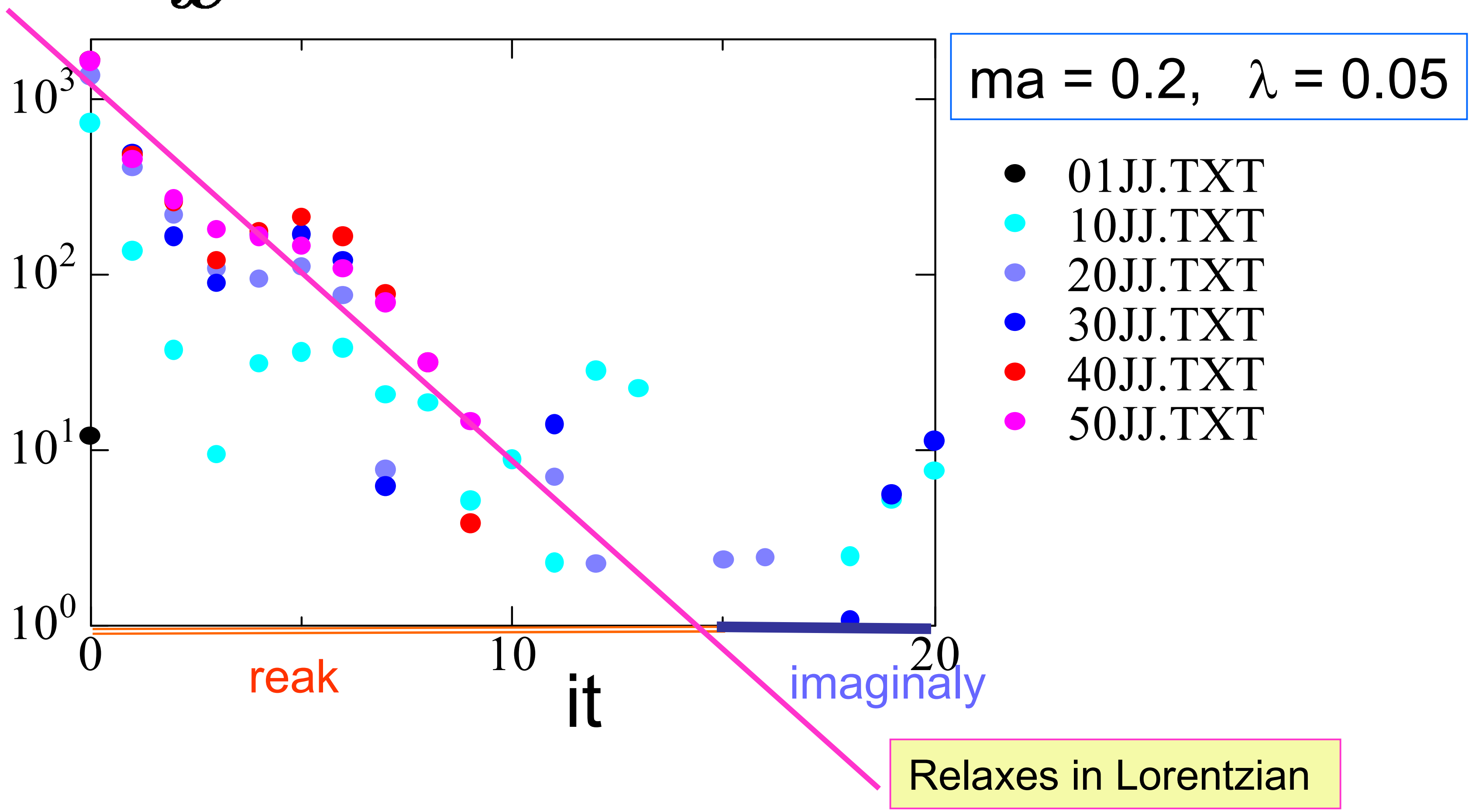
$$< \sum_{\boldsymbol{x}} J_x(\boldsymbol{x}, t=0) J_x(\boldsymbol{x}, t) >$$



$$\boldsymbol{J} = \mathrm{Im}\left(\phi^*(\boldsymbol{x}, t) \nabla \phi(\boldsymbol{x}, t)\right)$$

Average of 5000 steps

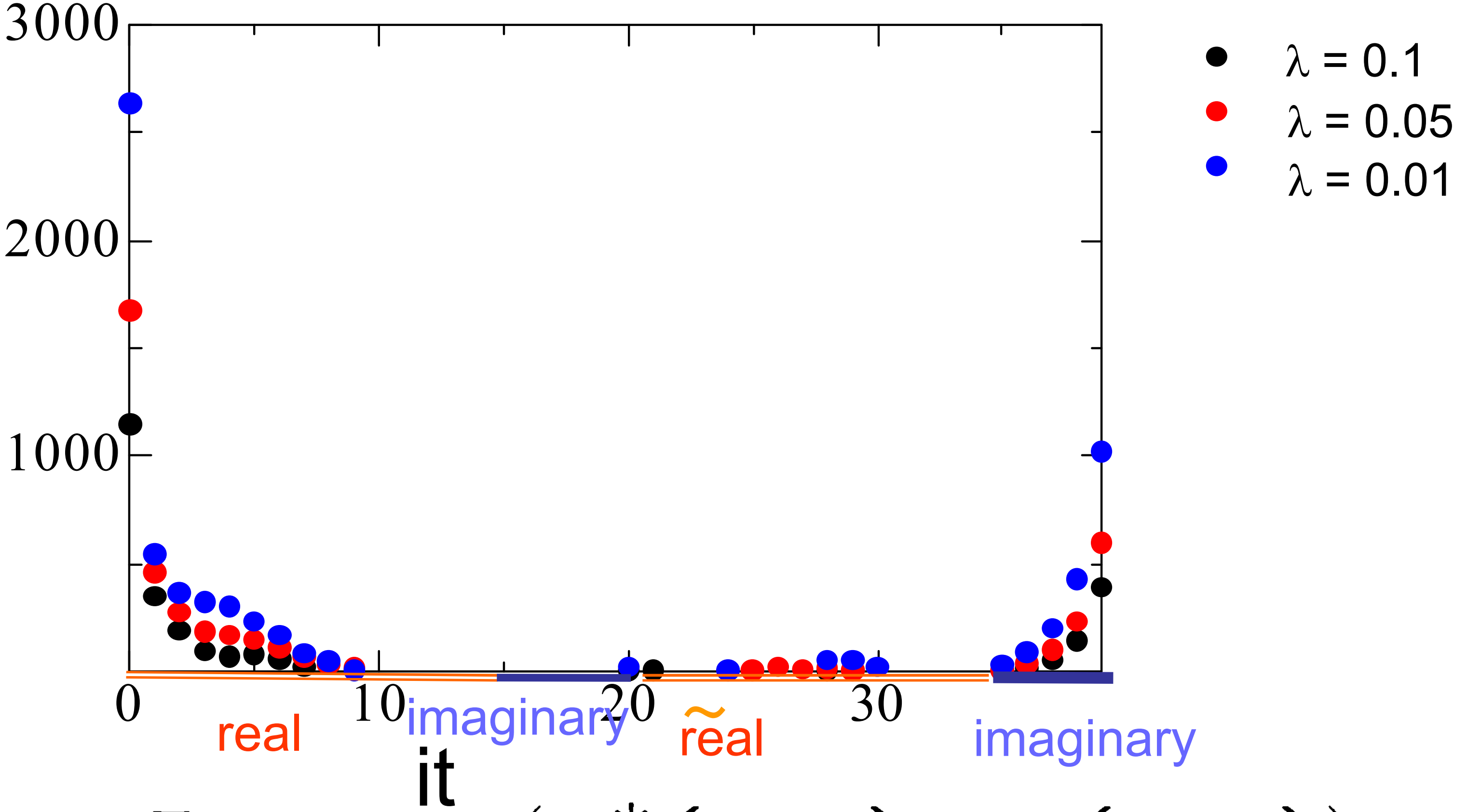
$$< \sum_{\boldsymbol{x}} J_x(\boldsymbol{x}, t=0) J_x(\boldsymbol{x}, t) >$$



$$\boldsymbol{J} = \mathrm{Im}\left(\phi^*(\boldsymbol{x}, t) \nabla \phi(\boldsymbol{x}, t)\right)$$

Average of 5000 steps

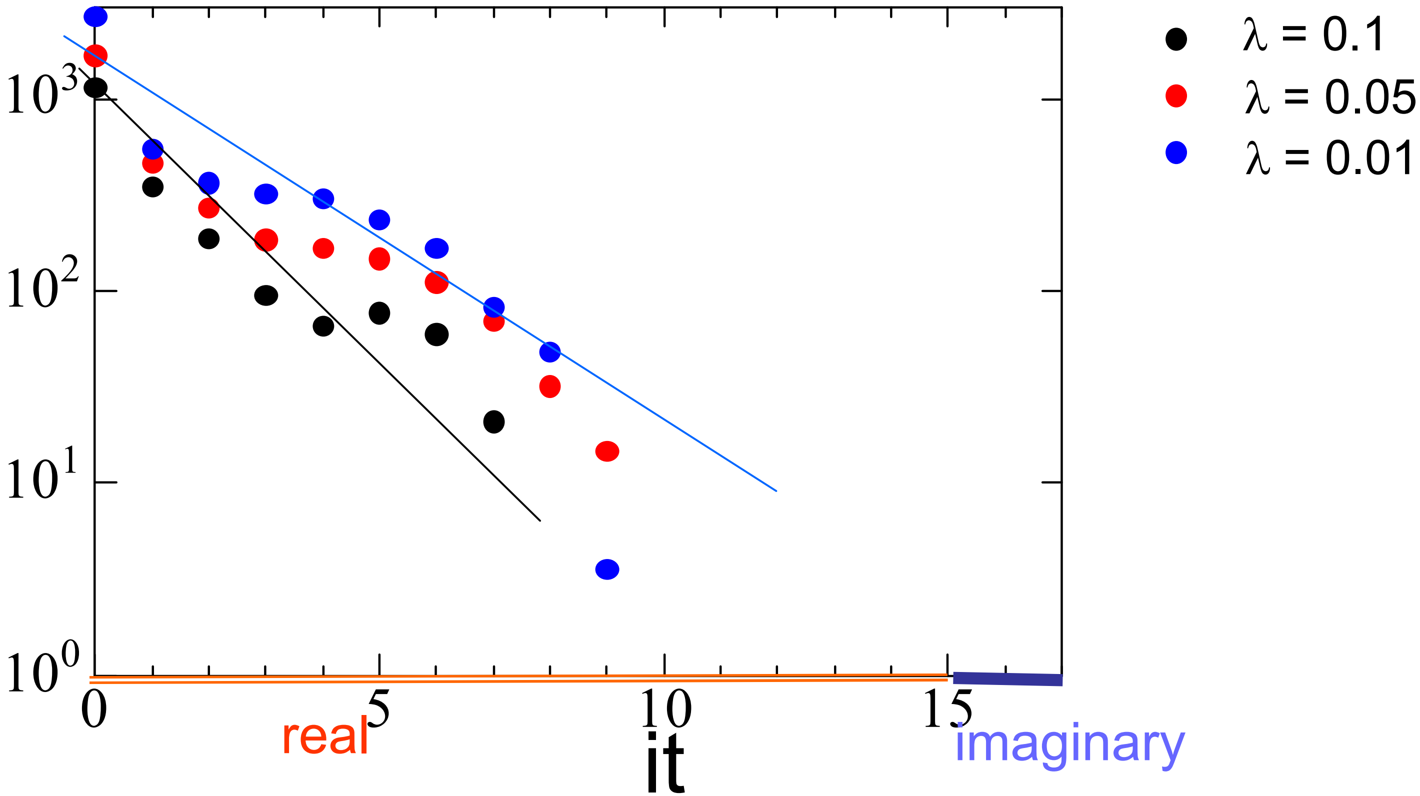
$$< \sum_{x} J_x(x, t=0) J_x(x, t) >$$



$$J = \mathrm{Im}\left(\phi^*(x,t)\nabla\phi(x,t)\right)$$

coupling λ dependences

$$< \sum_{\boldsymbol{x}} J_x(\boldsymbol{x}, t = 0) J_x(\boldsymbol{x}, t) >$$



$$J = \text{Im}\left(\phi^*(\boldsymbol{x}, t) \nabla \phi(\boldsymbol{x}, t)\right)$$

coupling λ dependences

50-th average of 5000 steps

We want to simulate numerically finite temperature system with real time.

- Our results seem to converge even with Minkowski time.
- Current correlation ⇒relaxation-like behavior appears

conductivity ?

- Coupling dependence
- Need to check
  - Contour dependence
  - Tilt dependence
  - Consistency with the results of imaginary time method