Nucleon structure functions with dynamical (2+1)-flavor domain wall fermions

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Talk at Lattice 2008, Williamsburg, July 18, 2008

RBC and UKQCD-DWF collaborations produced some dynamical DWF ensembles using QCDOC's:

- $24^3 \times 64 \times 16$ (2.7fm across), $16^3 \times 32 \times 16$ (1.8 fm), ...
- Iwasaki gauge action, $\beta = 2.13$, and Domain-Wall Fermions (DWF) quarks, $M_5 = 1.8$,
- $m_{\rm strange}a = 0.04$, $m_{\rm ud}a = 0.03$, 0.02, 0.01 and 0.005, with $a^{-1} \sim 1.7$ GeV.

Best ever hadron structure calculations: flavor and chiral symmetries and lattice volume.

- Very accurate determination of kaon bag parameter, $B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.524(10)(28)$,
- beginning to see SU(3) chiral perturbation failure, e.g. NLO corrections $\sim 0.5\times$LO.

Here we report some low moments of isovector nucleon structure functions calculated by Takeshi Yamazaki, Huey-Wen Lin, Shoichi Sasaki, Tom Blum, James Zanotti, Robert Tweedie, ... and are now nearly final.

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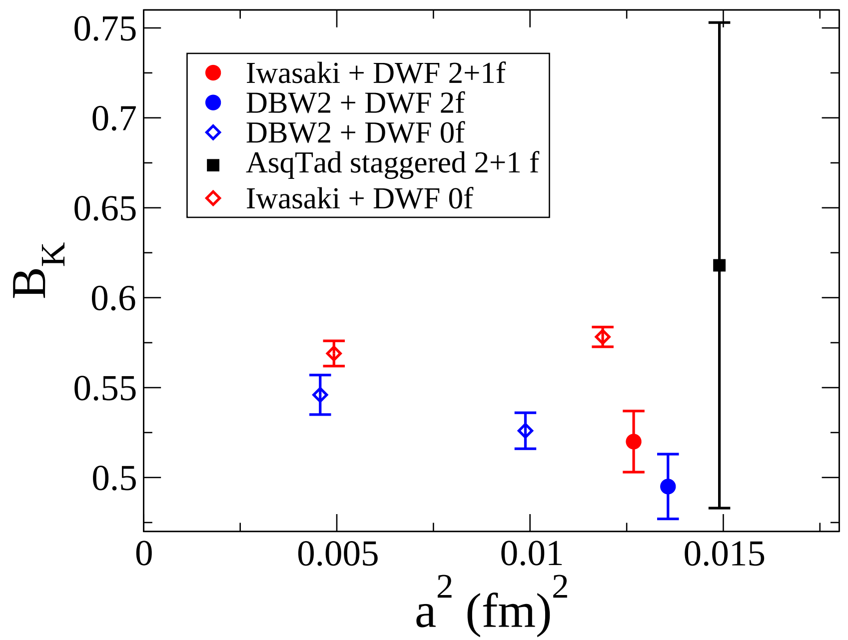
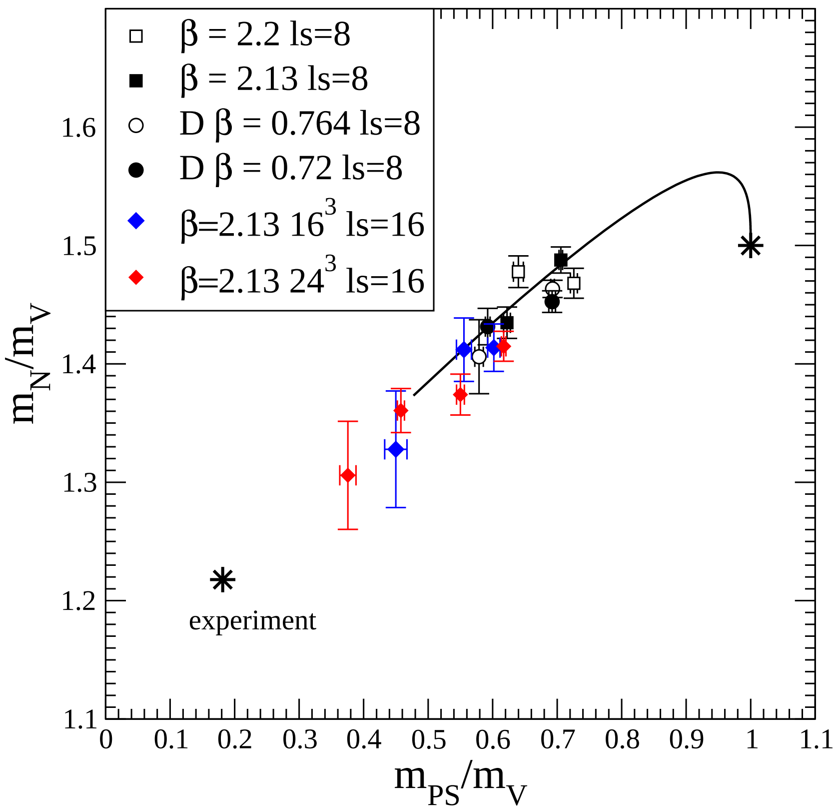
RBC/UKQCD $N_f = 2+1$ dynamical DWF ensemble: from Ω^- and K masses we estimate

- Lattice cutoff $a^{-1} = 1.73(2)$ GeV, or physical volume is $(2.74(3)\text{fm})^3$,
- physical strange mass is 0.035(1)+0.003 in lattice units.





The best ever kaon and nucleon calculations in regard of good flavor and chiral symmetries.

- $m_\pi = 0.67, 0.56, 0.42$ and 0.33 GeV, $m_N = 1.55, 1.39, 1.22$ and 1.15 GeV.
- Very accurate constraints on CKM matrix: $B_K^{\overline{\rm MS}}(2\text{GeV}) = 0.524(10)(28)$, $K_{l3}\ f_+(0) = 0.964(5)$, ...
- Beginning to tell where SU(3) chiral perturbation fails, e.g. NLO corrections $\sim 0.5\times$LO.

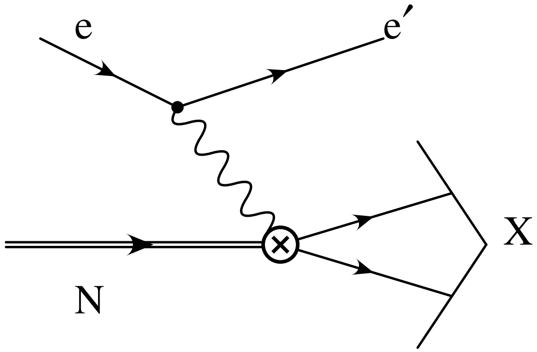
The isovector form factors are defined in the following:

$$\begin{aligned}\langle p|V_\mu(0)|p\rangle &= \overline{u}_p\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}q_\nu F_2(q^2)/2m_N\right]u_p,\\ \langle p|A_\mu(0)|p\rangle &= \overline{u}_p\left[\gamma_\mu\gamma_5 G_A(q^2) + iq_\mu\gamma_5 G_P(q^2)\right]u_p,\end{aligned}$$

where $V_\mu = \overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d$ and $A_\mu = \overline{u}\gamma_\mu\gamma_5 u - \overline{d}\gamma_\mu\gamma_5 d$ are isovector vector and axial vector currents.

Nucleon structure functions are measured in deep inelastic scatterings (and RHIC/Spin):

- DIS



$$\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\; W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

  - unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$,
  - polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma}q_\rho\left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot SP_\sigma}{\nu^2}g_2(x,Q^2)\right)$,

  with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

- The same structure functions appear in RHIC/Spin and Drell-Yang, which may also provide $\langle 1\rangle_{\delta q}$ or $\langle P,S|\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$.

Moments of the structure functions are accessible on the lattice:

$$2\int_0^1 dx x^{n-1} F_1(x,Q^2) = \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),$$

$$\int_0^1 dx x^{n-2} F_2(x,Q^2) = \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2),$$

$$2\int_0^1 dx x^{n} g_1(x,Q^2) = \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),$$

$$2\int_0^1 dx x^{n} g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1} \sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n \rangle_q(\mu)$, $\langle x^n \rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle x^n \rangle_{\delta q}(\mu)$ which may be provided by polarized Drell-Yang and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P} = 0$.

Our formulation follows the standard one:

- Two-point function: $G_N(t) = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}}\langle T B_1(x) B_1(0)\rangle]$, using $B_1 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ for proton.
- Three-point functions: appropriate operator inserted between source and sink.
- Source and sink are separated by 12 lattice spacings. 4 source positions.
- Gaussian smearing is employed to enhance ground-state signals.
- All the matrix elements except d_1 are iso-vector.

Number of configurations and pion mass

$m_f a$	# of config.'s	meas. interval	N_{sources}	m_π (GeV)	m_N (GeV)
0.005	932	10	4	0.33	1.15
0.01	356	10	4	0.42	1.22
0.02	98	20	4	0.56	1.39
0.03	106	20	4	0.67	1.55

Good chiral and flavor symmetries of DWF help:

- negligible unwanted mixings,
- straight-forward non-perturbative renormalization: RBC-standard RI-MOM (Rome-Southampton).

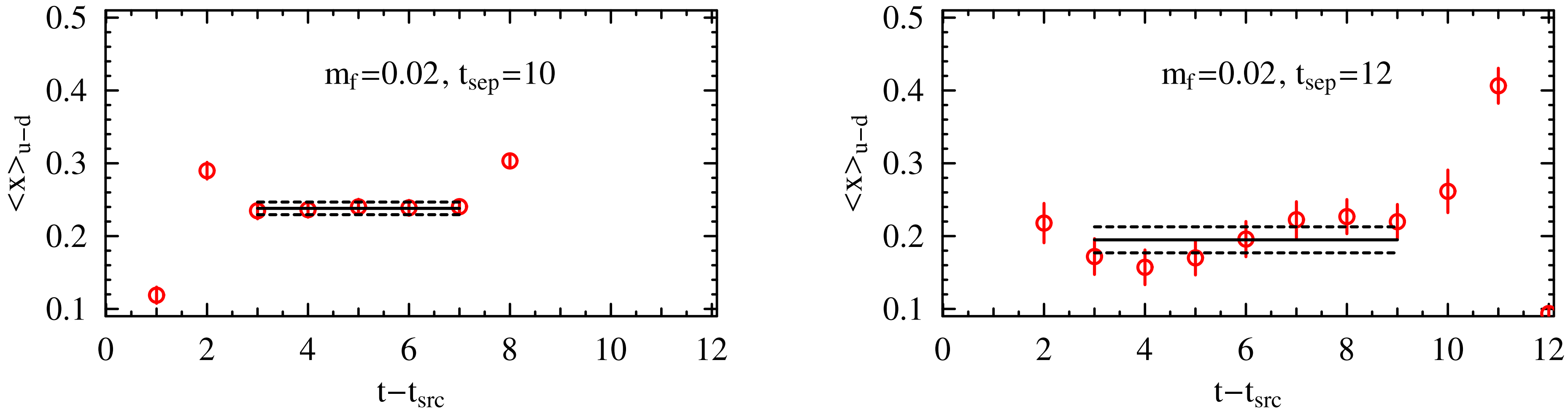
In particular, ratios such as g_A/g_V or $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$ are naturally renormalized.

Two possibly important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.

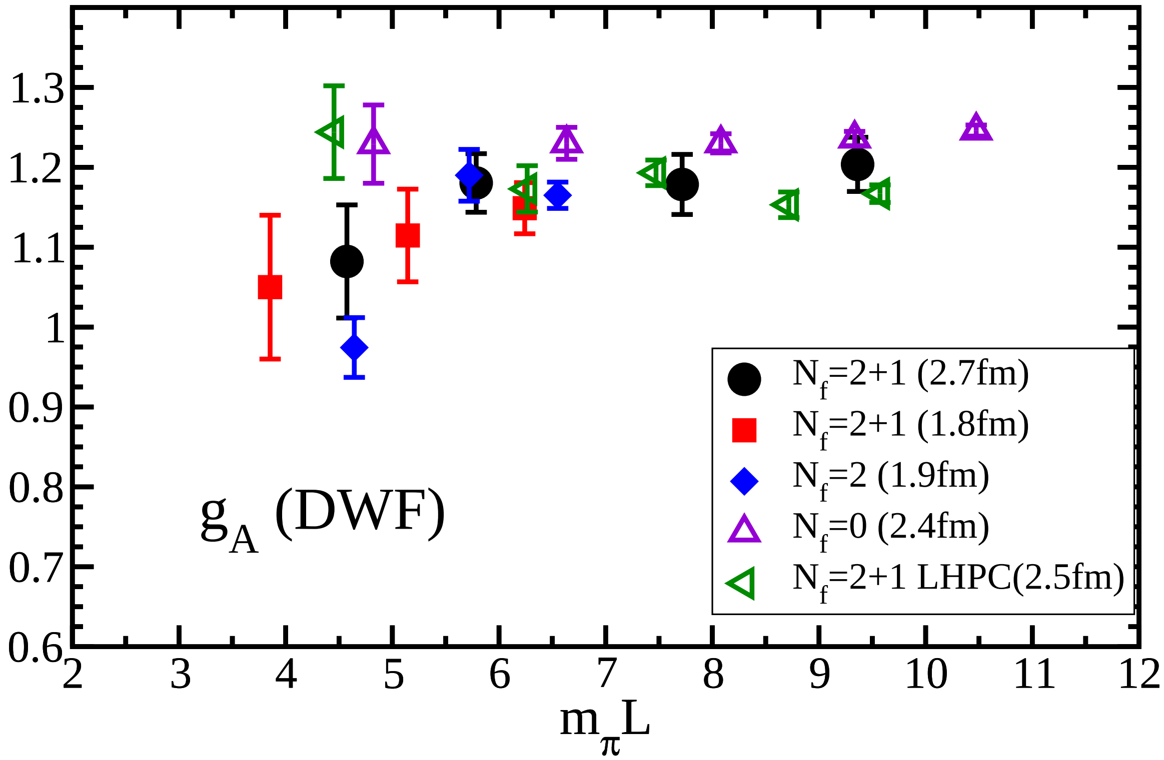


- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

In this study we choose separation 12 or 1.37 fm: lighter quarks should help maintaining the signal .

Spatial volume: in Lattice 2007 Takeshi Yamazaki reported significant finite-size effect in axial charge:

- measured in neutron β decay, $g_A/g_V = 1.2695(29)$, decides neutron life.

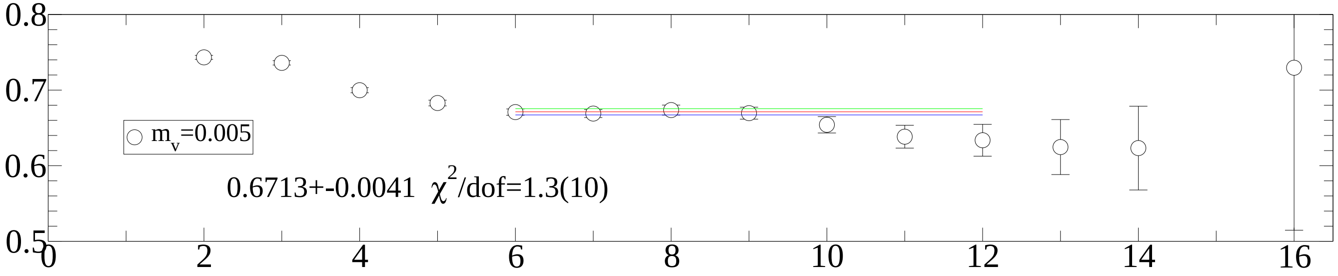


Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

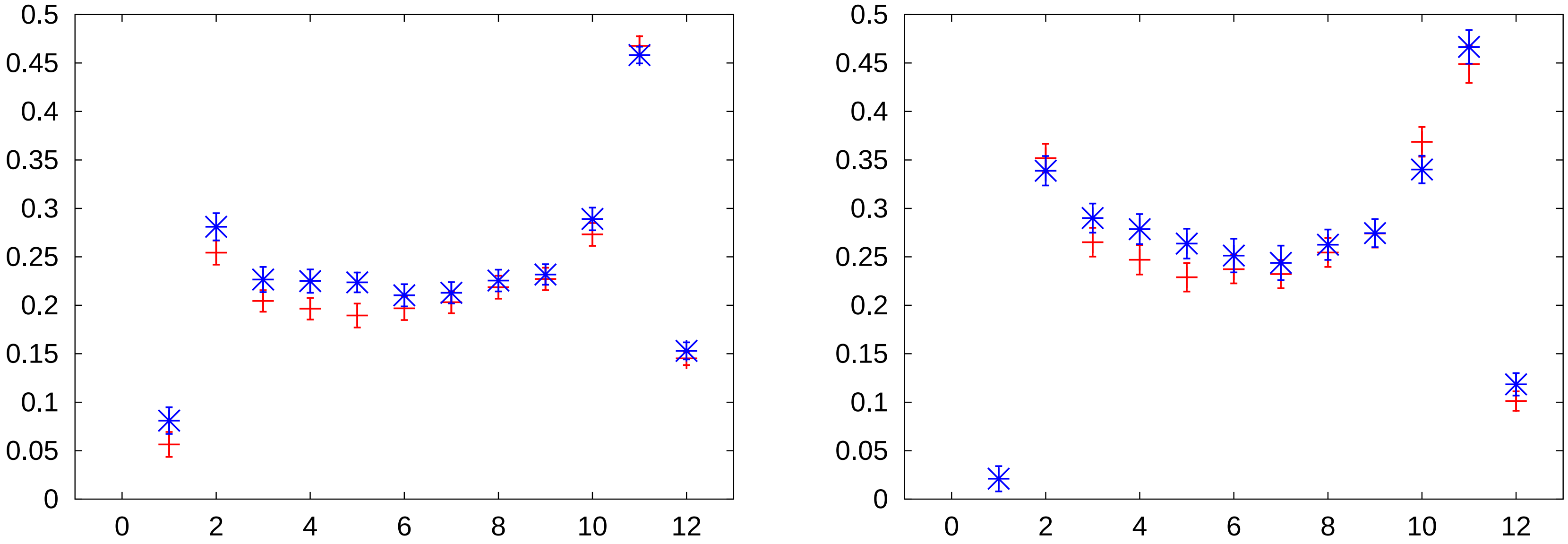
- Heavier quarks: consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
  - elastic form factors demand big volumes.
- Does not necessarily mean inelastic structure functions do: need further investigation.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

Mass signal: $m_f = 0.005$



Bare three-point functions: $\langle x \rangle_{u-d}$ (left) and $\langle x \rangle_{\Delta u - \Delta d}$ (right), for $m_f = 0.005$ (red +) and 0.01 (blue ×):

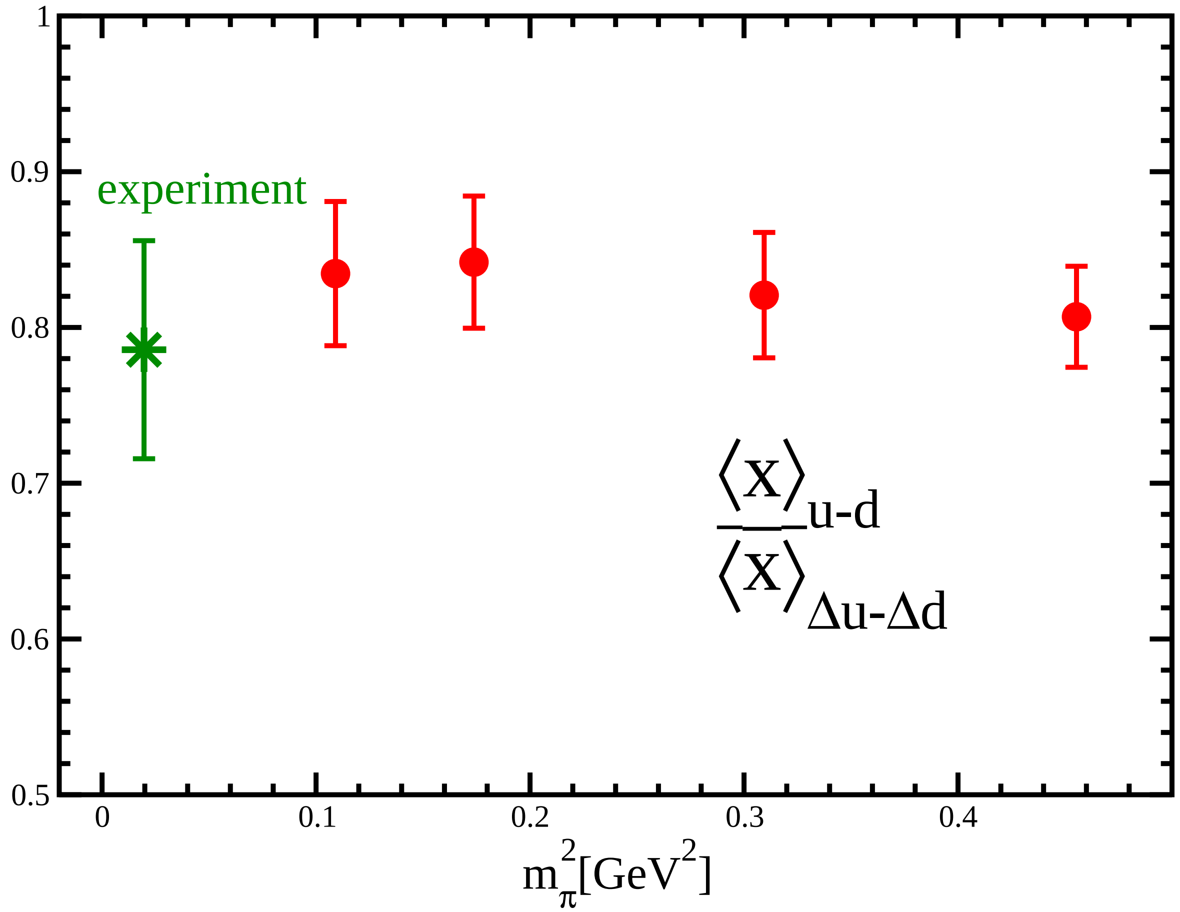


Similar in quality with 2-flavor, time separation 12.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Ratio, $\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice),

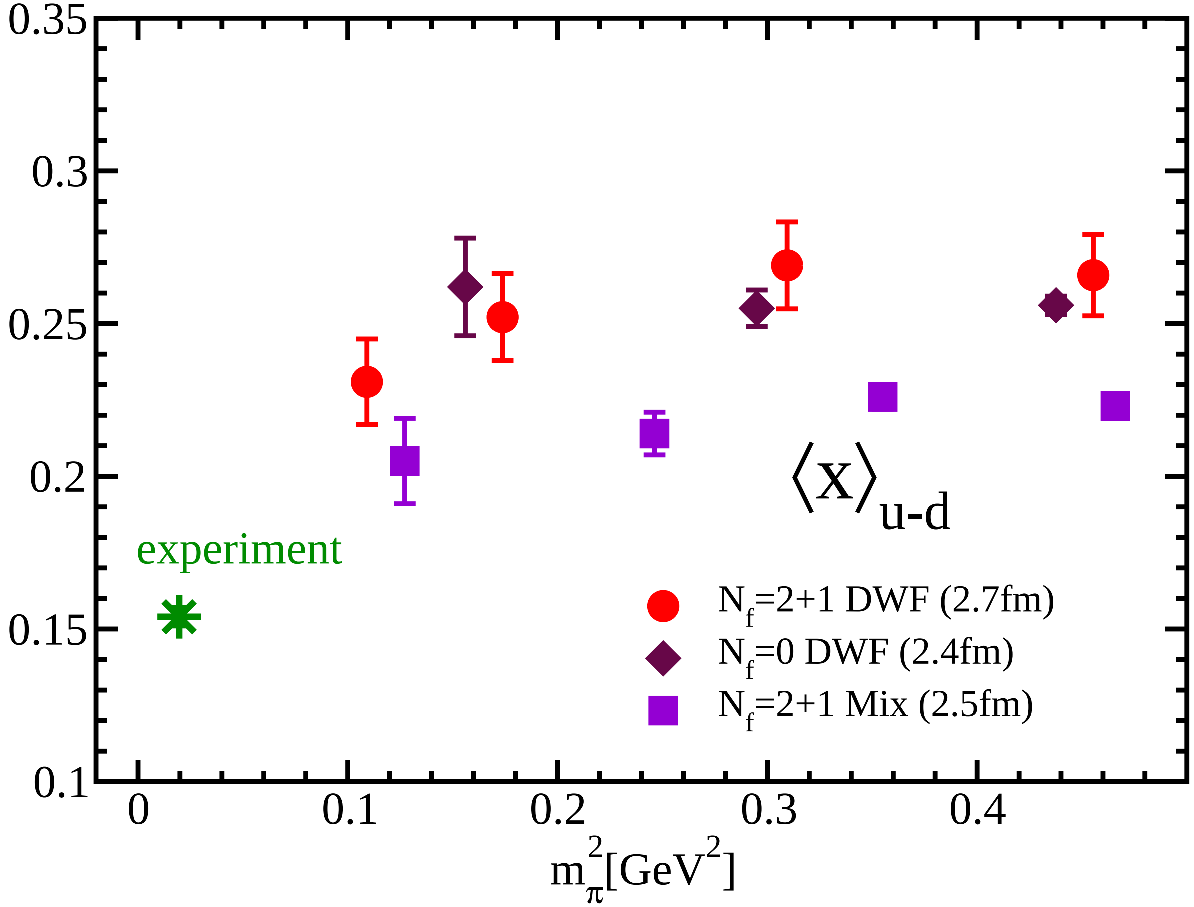


consistent with experiment, no discernible quark-mass dependence.
No finite-size effect seen, in contrast to g_A/g_V which is also naturally renormalized on the lattice.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$.

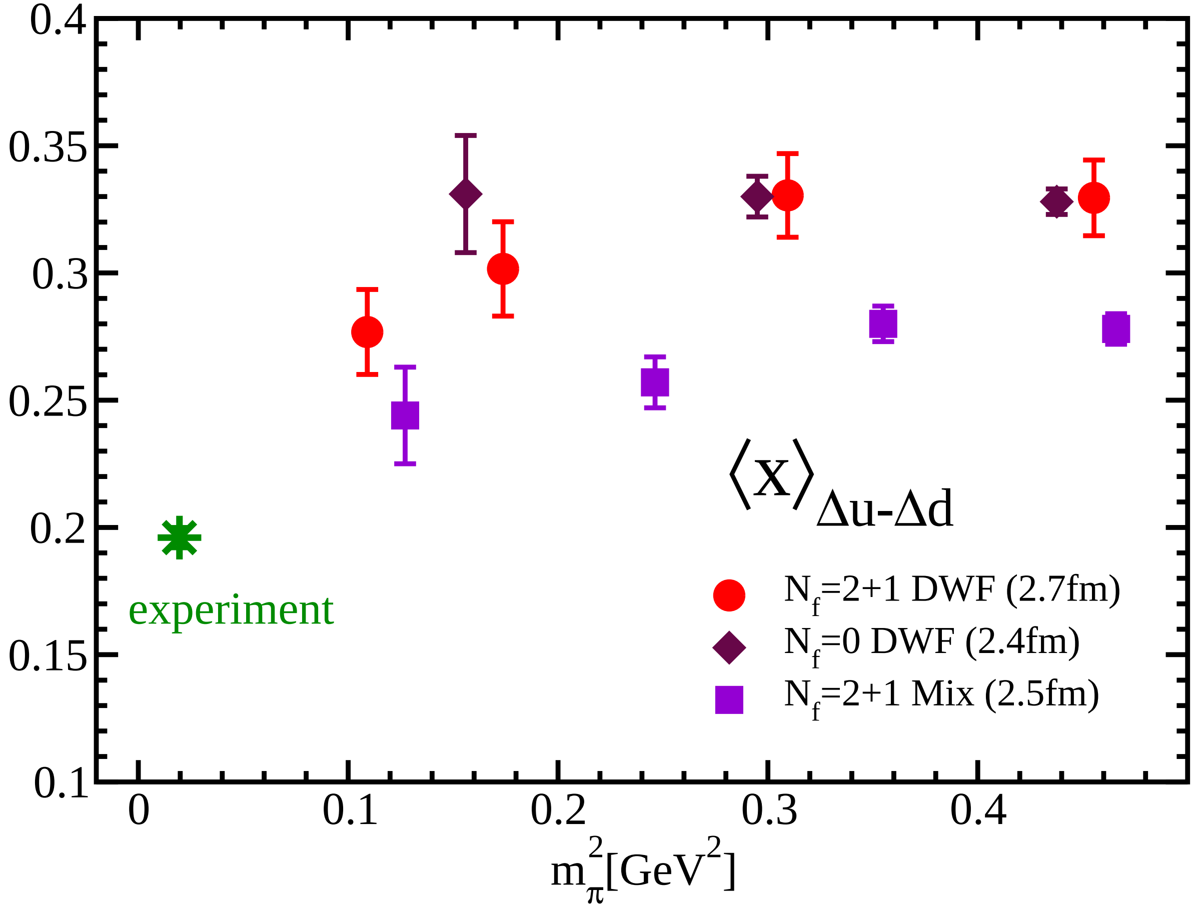
Light quarks or finite volume? Need further investigation.

Why differ from LHPC/MILC? NPR? Unitarity? Source/sink?

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67, 0.56, 0.42$ and 0.33 GeV; $m_N = 1.55, 1.39, 1.22$ and 1.15 GeV,

Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 1.15(3)$,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(3)$.

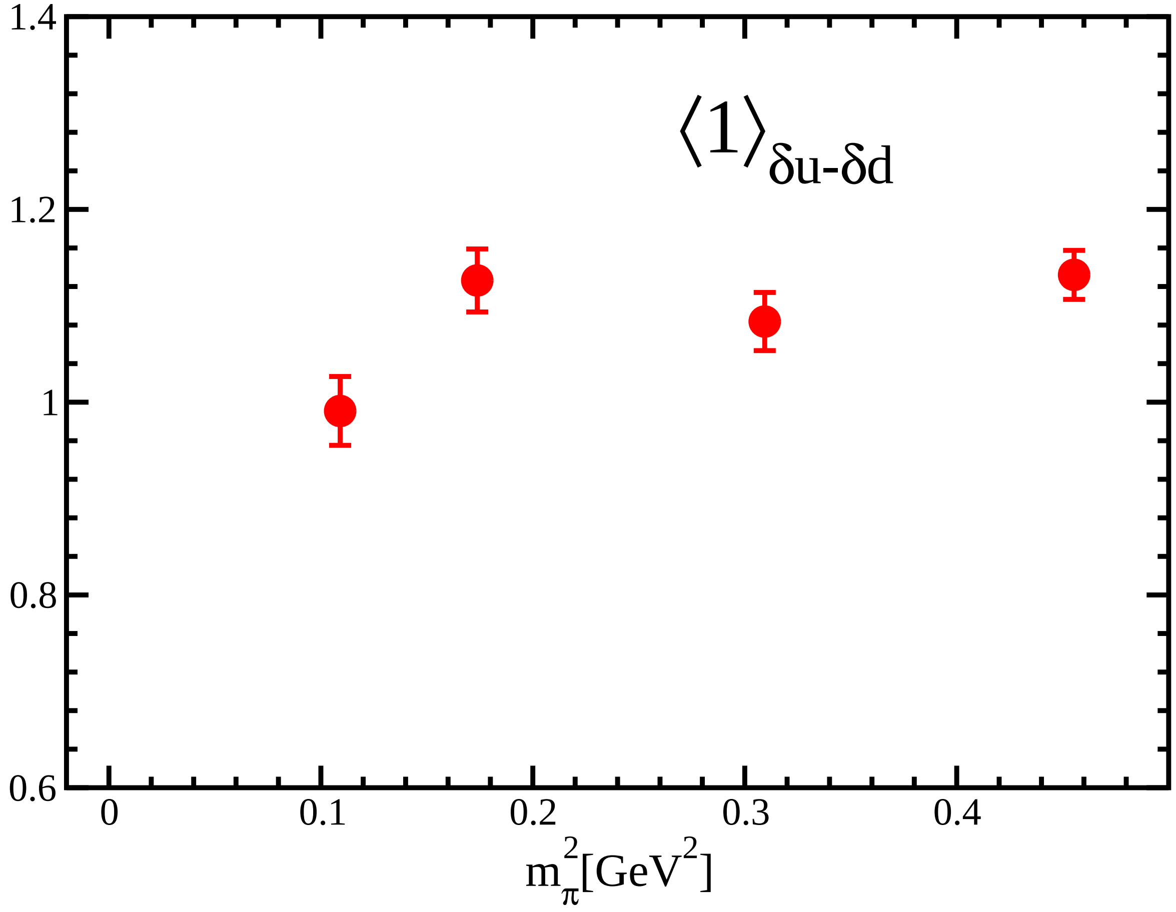
Light quarks or finite volume? Need further investigation.

Why differ from LHPC/MILC? NPR? Unitarity? Source/sink?

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Transversity fraction, $\langle 1 \rangle_{\delta u - \delta d}$, with preliminary NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 0.783(3)$,

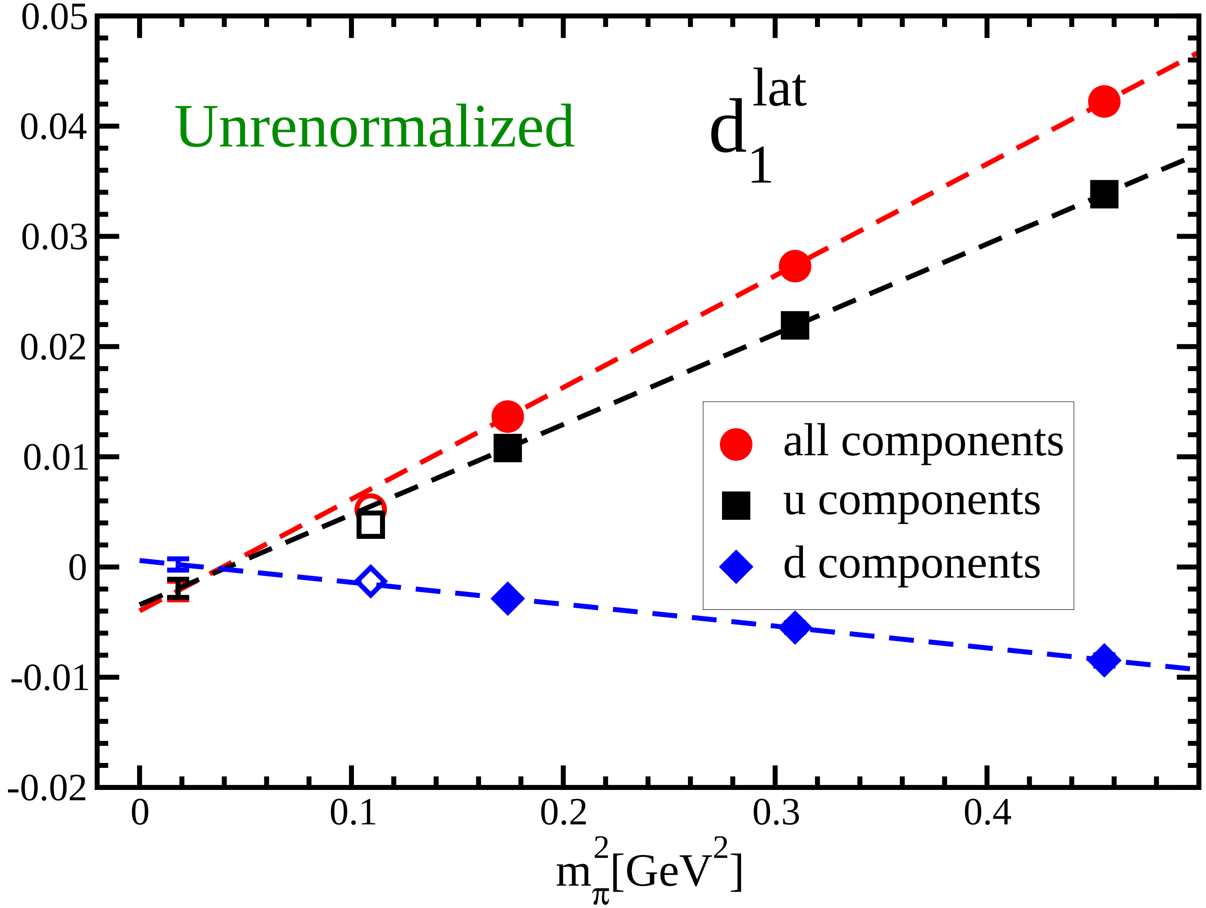


Preliminary NPR is $Z^{\overline{\rm MS}}(2{\rm GeV}) = 0.783(3)$.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Twist-3, d_1, yet to be renormalized



Chirally well-behaved, small, and in consistency with Wandzura-Wilczek relation.

Conclusions

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations with, $a^{-1} = 1.73(2)$ GeV, $(2.74(3)\text{fm})^3$ box, $m_{\text{res}} = 0.00315(2)$, $m_{\text{strange}} = 0.04$, $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV:

While elastic form factors such as the axial charge, g_A, suffer large finite-volume effect,

- a similarly naturally renormalized ratio, $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$, of momentum and helicity fractions does not show such effect.
  - It is consistent with experiment, and
  - does not show any discernible quark-mass dependence.
- Lightest points show an encouraging trend toward experiments in both momentum and helicity fractions.
  - Light quark or finite volume? Plan to check the smaller 1.8-fm box.
  - But they are different from corresponding LHPC/MILC results. NPR? Unitarity? Source/sink?
- Transversity moment is obtained.
- Twist-3 moment, d_1, is chirally well-behaved, small, and consistent with Wandzura-Wilczek relation.

Structure function renormalizations are now complete: typically 15-20 % effect.
Exploring auxiliary determinant for much larger volume.