σ-resonance and convergence of chiral perturbation theory

D. J. Cecile and S. Chandrasekharan
Duke University
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Motivation

- Region in quark mass where chiral perturbation theory is valid is an important question for QCD
- The role of resonances is particularly interesting
- The problem is non-perturbative
- Not many first principles studies exist
- We build a model of pions very similar to QCD and study the physics of the σ-resonance in it.

Model

Action

$$S = -\sum_x \sum_{\mu=1}^{d+1} \eta_{\mu,x} \left[\mathrm{e}^{i\phi_{\mu,x}} \overline{\psi}_x \psi_{x+\hat{\mu}} - \mathrm{e}^{-i\phi_{\mu,x}} \overline{\psi}_{x+\hat{\mu}} \psi_x \right] - \sum_x \left[m \overline{\psi}_x \psi_x + \frac{\tilde{c}}{2} \left(\overline{\psi}_x \psi_x \right)^2 \right]$$

strongly coupled U(1) gauge theory

mass

Anomaly

$$\psi_x = \begin{pmatrix} u_x \\ d_x \end{pmatrix} \qquad \overline{\psi}_x = \begin{pmatrix} u_x & d_x \end{pmatrix}$$

Two flavors

$$\left(\eta_{\mu,x}\right)^2 = 1,\ \mu = 1,2,3,4$$

$$\left(\eta_{5,x}\right)^2 = T$$

fictitious temperature

Model has symmetries of $N_f=2$ QCD

Observables

current-current susceptibility

$$Y_i = \frac{1}{dL^d}\left\langle \sum_{\mu=1}^{d}\left(\sum_x J_\mu^i(x)\right)^2 \right\rangle$$

Vector Current: $J_\mu^v(x) \longrightarrow Y_v$

Chiral Current: $J_\mu^c(x) \longrightarrow Y_c$

condensate susceptibility

$$\chi_\sigma = \frac{1}{L^d}\ \frac{1}{Z}\ \frac{\partial^2 Z}{\partial m^2}$$

ε-regime results

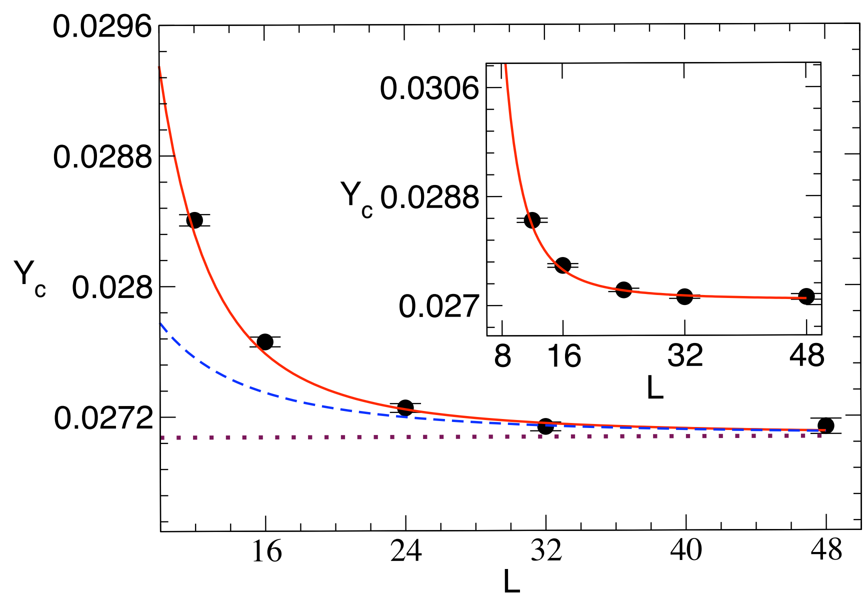
Parameters $\quad m = 0, \quad \tilde{c} = 0.3, \quad T = 1.7, \quad L_5 = 2$

$$Y_c = Y_v = \frac{F^2}{2}\left(1 + \frac{0.14046}{(FL)^2} + \frac{a}{(FL)^4}\right)$$

$F = 0.2327(1)$

$a = 1.91(9)$

$\chi^2/DOF = 1.2$



ε-regime results

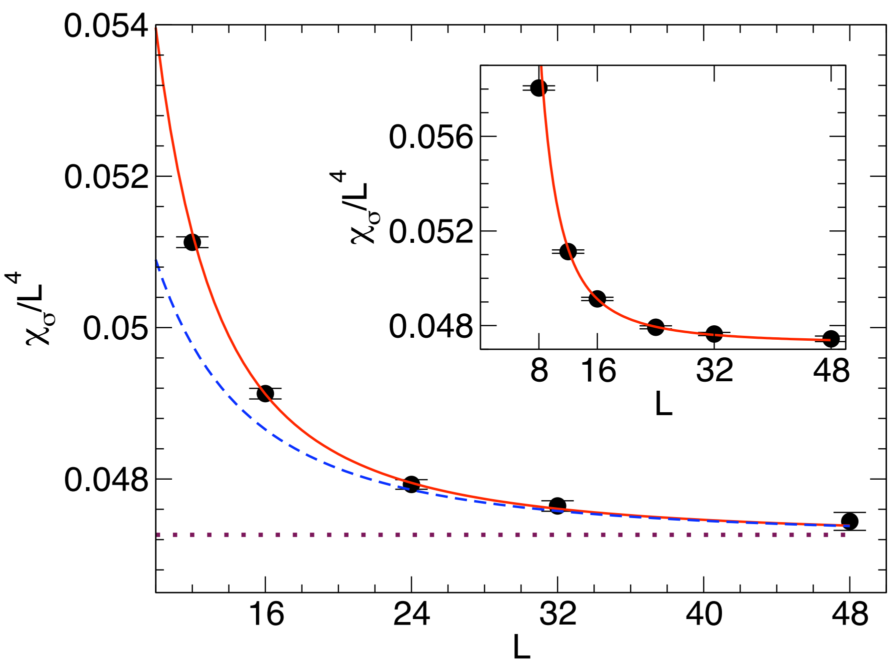
Parameters $m = 0, \quad \tilde{c} = 0.3, \quad T = 1.7, \quad L_5 = 2$

$$\chi_\sigma = \frac{\Sigma^2 L^4}{4}\left(1 + \frac{0.42138}{(FL)^2} + \frac{b}{(FL)^4}\right)$$

$\Sigma = 0.4346(2)$

$b = 1.72(11)$

$\chi^2/DOF = 0.2$



p-regime results

Finite size predictions at a fixed quark mass

$$Y_c = (F_\pi)^2[1 - 2\tilde{g}_1(LM_\pi)\xi + \mathcal{O}(\xi^2)],$$

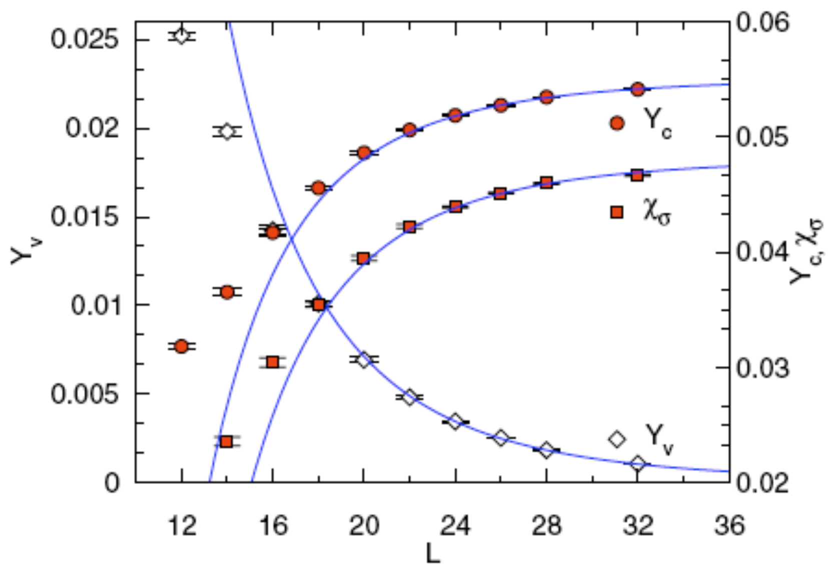
$$Y_v = (F_\pi)^2\left[-2L\frac{\partial \tilde{g}_1(LM_\pi)}{\partial L}\xi + \mathcal{O}(\xi^2)\right],$$

$$\chi_\sigma = (\langle\bar{q}q\rangle)^2 L^4[1 - 3\tilde{g}_1(LM_\pi)\xi + \mathcal{O}(\xi^2)],$$

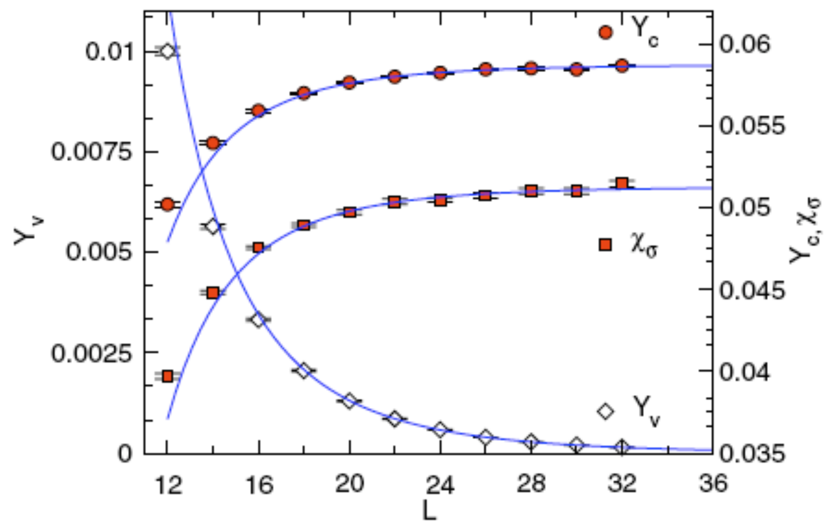
$$\tilde{g}_1(\lambda) = \sum_{n_1,n_2,n_3,n_4\neq 0}^{\infty} \frac{4}{\lambda\sqrt{n}} K_1(\lambda\sqrt{n}),$$

$$n = n_1^2 + n_2^2 + n_3^2 + n_4^2.$$

K_1 is a Bessel function of the second kind m=0.0002

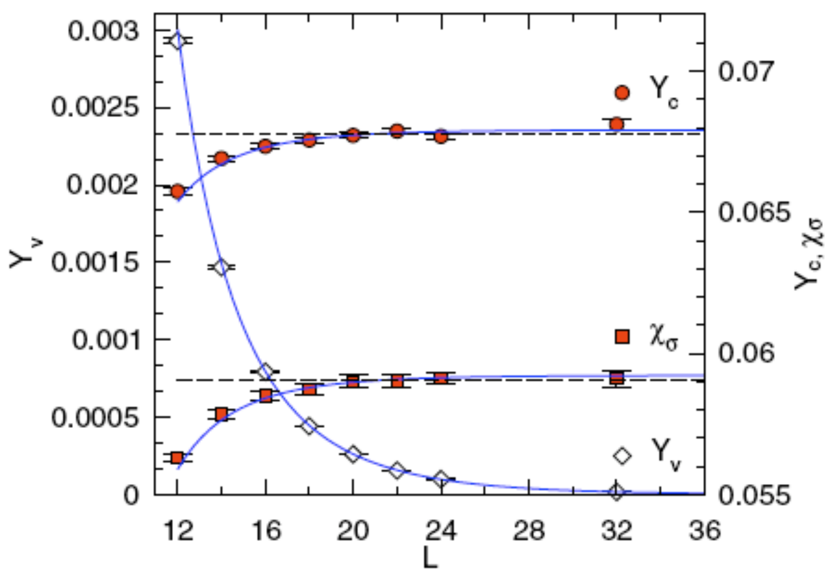


m=0.001

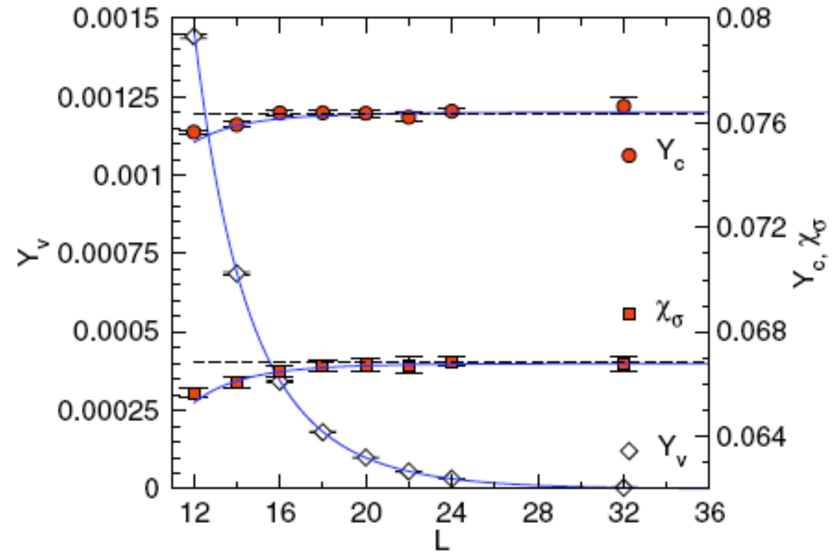


m	$\langle \bar{q}q \rangle$	F_π	M_π	χ^2	Fit range
0.0002	0.4392(2)	0.2348(1)	0.0400(2)	2.5	$24 \leq L \leq 32$
0.0005	0.4441(2)	0.2377(1)	0.0627(2)	1.1	$24 \leq L \leq 32$
0.0008	0.4499(2)	0.2406(1)	0.0789(1)	0.9	$22 \leq L \leq 32$
0.0010	0.4528(2)	0.2423(1)	0.0878(1)	0.8	$18 \leq L \leq 32$
0.0015	0.4606(2)	0.2467(1)	0.1070(2)	1.3	$18 \leq L \leq 32$
0.0020	0.4678(2)	0.2501(1)	0.1220(2)	1.8	$20 \leq L \leq 32$
0.0025	0.4740(2)	0.2538(1)	0.1356(2)	1.6	$16 \leq L \leq 32$
0.0035	0.4867(2)	0.2606(1)	0.1584(2)	0.9	$16 \leq L \leq 32$

m=0.0035



m=0.0065



m	$\langle \bar{q}q \rangle$	χ^2	F_π	χ^2	M_π	χ^2
0.0020	0.4668(3)	1.2	0.2498(1)	0.1	0.1226(2)	0.6
0.0025	0.4728(3)	0.7	0.2536(2)	0.9	0.1356(2)	1.6
0.0035	0.4861(3)	0.1	0.2603(1)	1.5	0.1584(2)	1.7
0.0050	0.5024(3)	0.2	0.2690(2)	1.1	0.1860(3)	0.7
0.0065	0.5170(3)	0.1	0.2764(2)	0.7	0.2083(4)	0.5
0.0075	0.5247(3)	0.2	0.2807(2)	1.6	0.2219(4)	0.9
0.0100	0.5433(2)	0.7	0.2912(2)	0.1	0.2521(5)	1.8

1-loop chiral peturbation theory

1-loop chiral perturbation theory predicts

$$F_\pi = F[1 - \xi' \log \xi' + 2\xi' c_F],$$
$$\langle \bar{q} q \rangle = \Sigma[1 - \tfrac{3}{2}\xi' \log \xi' + 3\xi' c_\Sigma],$$
$$M_\pi^2 = M^2[1 + \tfrac{1}{2}\xi' \log \xi' - \xi' c_M],$$

$$\xi' = M^2/(16\pi^2 F^2) \qquad M^2 = m\Sigma/F^2$$

Σ	F	c_Σ	c_F	c_M	χ^2
0.4354(3)	0.2329(2)	11.9(3)	19.3(5)	39(3)	1.1
0.4351(5)	0.2331(4)	12.3(5)	18.9(9)	37(3)	1.6

ε-region results

$$\Sigma = 0.4346(2) \qquad F = 0.2327(1)$$

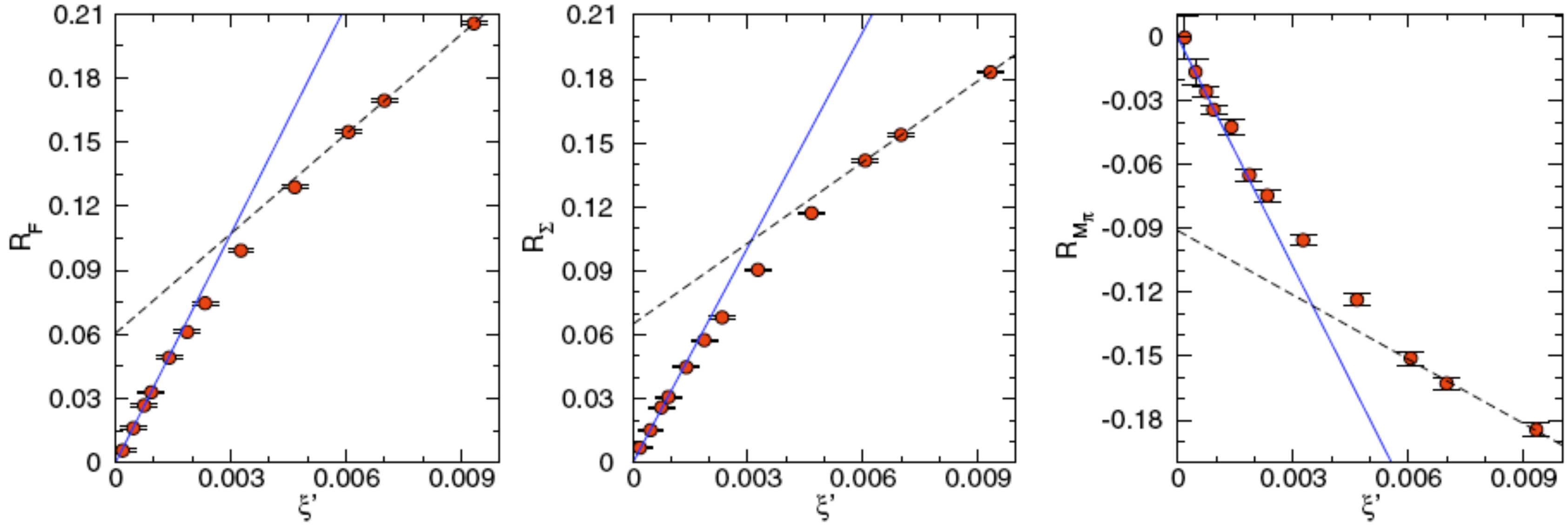
Region of 1-loop chiral perturbaton theory

$$F_\pi = F[1 - \xi' \log\xi' + 2\xi' c_F],$$
$$\langle \bar{q}q \rangle = \Sigma[1 - \tfrac{3}{2}\xi' \log\xi' + 3\xi' c_\Sigma],$$
$$M_\pi^2 = M^2[1 + \tfrac{1}{2}\xi' \log\xi' - \xi' c_M],$$

$$R_F \equiv F_\pi/F - 1 + \xi' \log\xi',$$
$$R_\Sigma \equiv \langle \bar{q}q \rangle/\Sigma - 1 + 3\xi' \log\xi'/2,$$
$$R_M \equiv M_\pi^2/M^2 - 1 - \xi' \log(\xi')/2.$$

R's linearly go to zero

in the region of 1-loop chiral perturbation theory



If 5% or less error is tolerated

$\xi' \leq 0.006$ is needed for 1-loop chiral perturbation to be valid!

$\xi' \geq 0.006$ another linear region!

A knee is present at $\xi' = 0.0035$. What is the reason?

σ-resonance and chiral pert. theory

In a weakly coupled linear sigma model one can show

$$c_\Sigma = \log(M_R/4\pi F) - \frac{7}{6} + \frac{8\pi^2}{3g_R},$$

$$c_M = \log(M_R/4\pi F) - \frac{7}{3} + \frac{8\pi^2}{g_R},$$

$$g_R = M_R^2/2F^2$$

where

$$M_\sigma^2 = M_R^2\left[1 + \frac{g_R}{16\pi^2}(3\pi\sqrt{3} - 13)\right].$$

Here M_σ is that mass of the σ particle

As M_σ becomes small

the region of validity of chiral Pert. theory shrinks

Using C_Σ= 12 and C_M=39 we get M_σ/F = 2

At the knee (i.e., m=0.0035) we find $M_\pi/F = 0.6$

Thus the knee occurs roughly when $M_\pi = M_\sigma/3$

In the current model

1-loop Chiral perturbation theory

breaks down when

$M_\pi \geq M_\sigma/3$

Take-home message

σ-resonance can also play an important role in determining the window of Ch.P.T

Important to match low energy constants from the ε-regime and the p-regime

Model calculations can teach us more about the role of resonances in Ch.P.T