Nucleon Structure with Dynamical Domain Wall Fermions at a=0.084 fm

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Motivation & outline

- Goal: to understand the full quark and gluon structure of nucleons
- Previously with mixed action, Asqtad sea & DW valence quarks
  [M. Lin (Monday); J. Negele (Friday); Phys. Rev. D77:094502,2008]
- New phase: dynamical DW fermions
  (in collaboration with RBC/UKQCD)
  - chiral invariance is crucial to preserve operators from mixing
- Early results obtained on
  - JLab cluster
  - ANL BlueGene/P, early usage time
  - Encanto, New Mexico Computing Applications Center (NMCAC)

Motivation & outline

1. Simulation details
2. Renormalization and scale
3. Techniques
   - correlator plateaus
   - formfactor extraction: overdetermined analysis
   - selecting momentum combinations
4. Formfactor results

Simulation details

RHMC generated g.f. trajectories [RBC and UKQCD collab, arXiv:0804.0473]

- Iwasaki gauge action with $\beta = 2.13$ (coarse) and $\beta = 2.25$ (fine)
- $N_f = 2 + 1$ dynamical & valence quarks with Domain Wall fermion action with $L_s = 16$
- Spatial volume $(2.74\,\mathrm{fm})^3$ and $(2.69\,\mathrm{fm})^3$, respectively

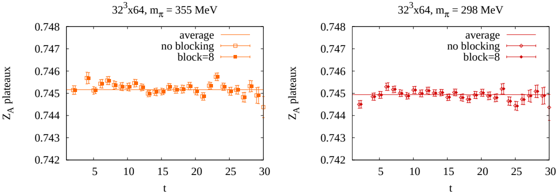
	$a\,[fm]$	#	am_l/am_h	$am_{res} \times 10^3$	$m_\pi [MeV]$
$24^3 \times 64$	0.114	3208	$0.005/0.040$	$3.15(1)$	$329(5)$
$32^3 \times 64$	0.084	840	$0.006/0.030$	$0.663(4)$	$355(6)$
		1592	$0.004/0.030$	$0.665(3)$	$298(5)$

- Increase statistics by placing 4 sources and computing nucleon & antinucleon on each trajectory
- Use two sink momenta $\vec{P'} = \vec{0}$ and $\vec{P'} = (-1, 0, 0)$

Essential quantities from pseudoscalar meson

$$Z_A = \frac{\langle \pi | \mathcal{A}_0 | 0 \rangle}{\langle \pi | A_0 | 0 \rangle}$$

$$Z_A \approx \frac{\langle \mathcal{A}_0(t) \tilde{J}_5(0) \rangle}{\langle A_0(t) \tilde{J}_5(0) \rangle}$$



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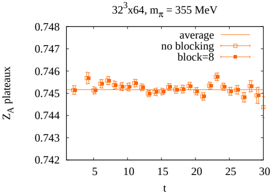
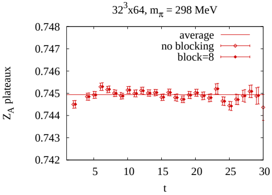




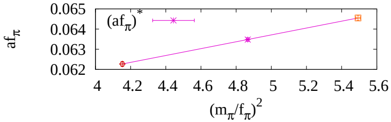
m_π, f_π, m_{res} from simultaneous fit, $t \in [10; 54]$:

$$\langle A_0(t) \tilde{J}_5(0) \rangle = c_{smear} A_5 (e^{-m_\pi t} - e^{-m_\pi (L_t - t)}),$$
$$\langle J_5(t) \tilde{J}_5(0) \rangle = c_{smear} B_5 (e^{-m_\pi t} + e^{-m_\pi (L_t - t)}),$$
$$\langle J_{5q}(t) \tilde{J}_5(0) \rangle = c_{smear} m_{res} B_5 (e^{-m_\pi t} + e^{-m_\pi (L_t - t)})$$

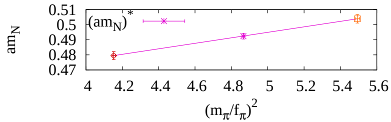
am_π	$m_\pi L$	$am_{res} \cdot 10^3$	af_π	Z_A	$Z_A g_V^B$
0.19004(11)	4.56	3.151(12)	0.08615(13)	0.71722(4)	0.991(4)
0.15129(15)	4.84	0.663(4)	0.06455(13)	0.74516(4)	0.996(8)
0.12690(13)	4.06	0.665(3)	0.06227(12)	0.74494(4)	0.994(11)

Setting scale on fine lattices

- $24^3 \times 64$, $m_l = 0.004$: $a = 0.1141(18)\,\text{fm}$ (χPT extrapolated m_Ω)
  [C. Allton et al, RBC and UKQCD collab., arXiv:0804.0473]
- $32^3 \times 64$: only two points $m_\pi = 355\,\text{MeV}$ and $m_\pi = 298\,\text{MeV}$; compare (af_π) and (am_N) on coarse and fine lattices



$$\frac{(af_\pi)^*}{(af_\pi)^{coarse}} = 0.7369(15)$$



$$\frac{(am_N)^*}{(am_N)^{coarse}} = 0.7530(54)$$

$$\boxed{a^{fine} = 0.0841(14)\,\text{fm}}$$

Polarized nucleon ME of Twist-two operators

- Vector formfactors

$$\langle P' \left| \bar{q}\gamma^\mu q \right| P\rangle = \bar{U}(P')\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_N}\right]U(P),$$

$$F_1(Q^2) \rightarrow g_V\,,\ \langle r_1^2\rangle \qquad F_2(Q^2) \rightarrow \kappa_V\,,\ \langle r_2^2\rangle$$

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- Axial vector formfactors

$$\langle P' \left| \bar{q}\gamma^\mu\gamma^5 q \right| P\rangle = \bar{U}(P')\left[G_A(Q^2)\gamma^\mu\gamma^5 + G_P(Q^2)\frac{q^\mu}{2m_N}\gamma^5\right]U(P),$$

$$G_A(Q^2) \to g_A = \langle 1\rangle_{\Delta q} = \frac{1}{2}\Sigma_q,\ m_D \qquad G_P(Q^2) \to \text{soft pion pole}$$

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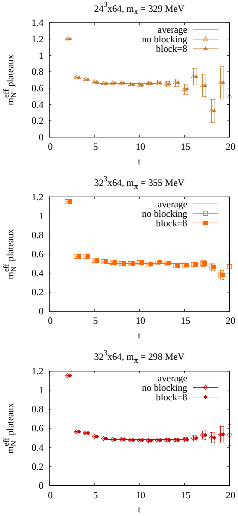
- Generalized formfactors ($n = 2, 3$)

$$\mathcal{O}^{\mu_1\cdots\mu_n}_{[\gamma^5]} = \bar{q}\gamma^{\{\mu_1}\left[\gamma^5\right] i\overleftrightarrow{D}^{\mu_2}\cdots i\overleftrightarrow{D}^{\mu_n\}}q,$$

$$\langle P' \left|\mathcal{O}^{\mu_1\cdots\mu_n}_{[\gamma^5]}\right| P\rangle = \bar{U}(P')\left[\overset{(\sim)}{A}_{n0}(Q^2)\gamma^{\{\mu_1}[\gamma^5]\bar{P}^{\mu_2}\cdots\bar{P}^{\mu_n\}} + \cdots\right]U(P),$$

$$A_{n0}(0) = \langle x^{n-1}\rangle_q \qquad \tilde{A}_{n0}(0) = \langle x^{n-1}\rangle_{\Delta q}$$

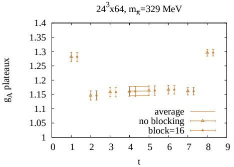
Two-point correlators

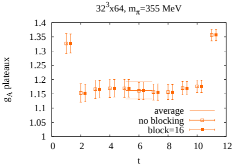


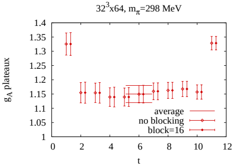
$$C_{2pt}(\tau, P) = \sum_{\vec{x}} e^{-i\vec{P}'\vec{x}} \langle N(\tau, x)\bar{N}(0,0)\rangle$$

- Gaussian smeared quark source
  - coarse $\sqrt{\langle r^2 \rangle} \approx 4.0a$
  - fine $\sqrt{\langle r^2 \rangle} \approx 6.0a$
- Small oscillations from DWF action reflection positivity breaking
- Nucleon mass from $m_N^{eff}(t)$ plateaux fits
  - coarse $t \in [5; 12]$
  - fine $t \in [6; 15]$

Three-point correlators







$$C^{\mathcal{O}}_{3pt}(T,\tau;P',q) = \sum_{\vec{x},\vec{y}} e^{-i\vec{P}'\vec{y}+i\vec{q}\vec{x}} \langle N(T,y)\mathcal{O}(\tau,x)\bar{N}(0,0)\rangle,$$

$$R^{\mathcal{O}}_{3pt}(T,\tau;P',P) = C^{\mathcal{O}}_{3pt}(T,\tau;P',P'-P) \times \left(\begin{array}{c} C_{2pt} \text{ combination} \\ \text{at } T,\ \tau,\ T-\tau \end{array} \right)$$

$$\to \langle P' \,|\mathcal{O}|\, P\rangle, \quad T \to \infty$$

- Sufficient source-sink separation corresponding to $T \approx 1.0 fm$
  - coarse $T/a = 9$
  - fine $T/a = 12$
- Operator plateaux
  - coarse $t \in [4;\ 5]$
  - fine $t \in [5;\ 7]$

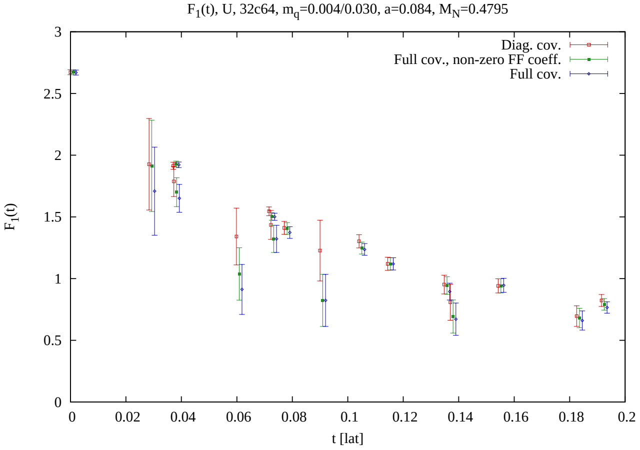
Overdetermined analysis

$$\langle R_\alpha(t)\rangle = A_{\alpha i}(t)\mathcal{F}_i(t), \qquad \alpha = \{P, P', \mu_1 \cdots \mu_n\}$$

$$\mathcal{F}_i = \left[A^T C^{-1} A\right]^{-1}_{ij} \left[A^T C^{-1}\right]_{j\alpha} \langle R_\alpha\rangle, \qquad \langle \delta R_\alpha \delta R_\beta\rangle = C_{\alpha\beta}$$

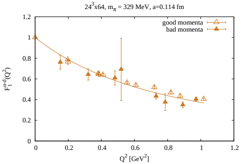
Variation weighted average:

- uncorrelated (diag. cov.)
- correlated (full. cov.)
- correlated, all components

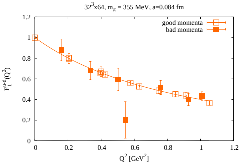


Selecting momentum combinations

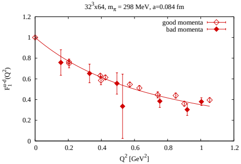
Remove noisy momenta combinations: $\vec{P}' \neq 0$ & $\vec{P}^2 \geq 4$



$$(m_D^{all})^2 = 1.567(38)GeV^2$$
$$(m_D^{good})^2 = 1.561(38)GeV^2$$

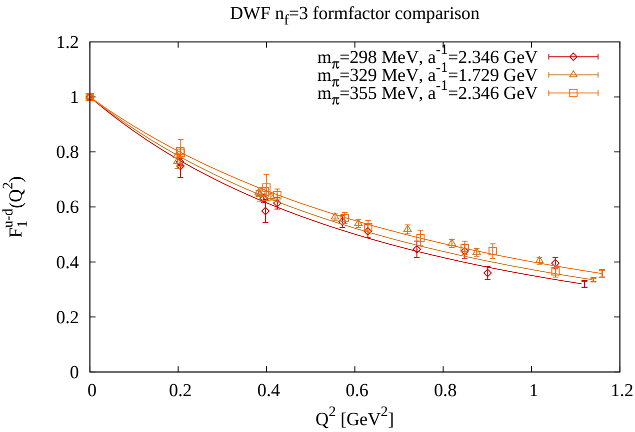


$$(m_D^{all})^2 = 1.726(78)GeV^2$$
$$(m_D^{good})^2 = 1.726(77)GeV^2$$



$$(m_D^{all})^2 = 1.453(62)GeV^2$$
$$(m_D^{good})^2 = 1.451(61)GeV^2$$

Dirac isovector formfactor $F_1^v(Q^2)$ at three m_π values



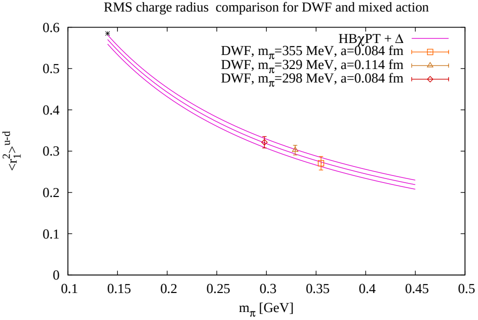
Dipole fit in range $0 <= Q^2 <= 0.4\,\mathrm{GeV}^2$

Dirac isovector RMS radius $\langle r_1^2 \rangle^{u-d}$

One-parameter fit from one-loop HBχPT + Δ-resonance

$$\langle r_1^2 \rangle^v = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7g_A^2 + \left(2 + 10g_A^2\right)\log\left[\frac{m_\pi}{\lambda}\right]\right\} - \frac{12B_{10}^{(r)}(\lambda)}{(4\pi F_\pi)^2}$$
$$+ \frac{c_A^2}{54\pi^2 F_\pi^2}\left\{26 + 30\log\left[\frac{m_\pi}{\lambda}\right] + 30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right]\right\}$$

Ph. value
$\langle r_1^2 \rangle^{u-d} = 0.585\,\mathrm{fm}^2$
[Mergell, Meissner and Drechsel, hep-ph/9506375]



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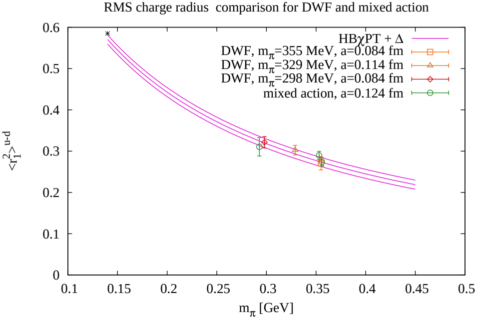

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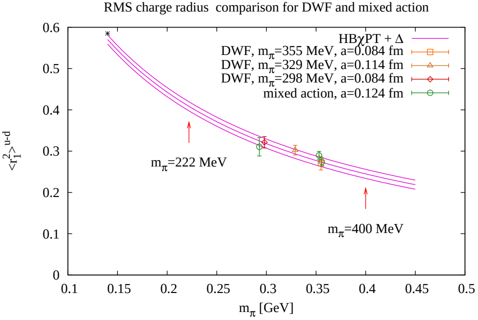
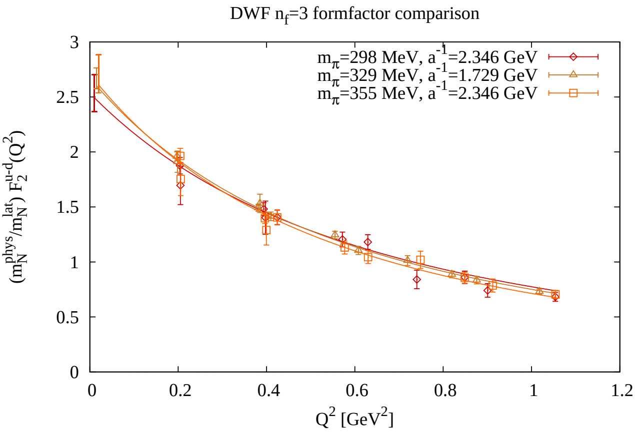


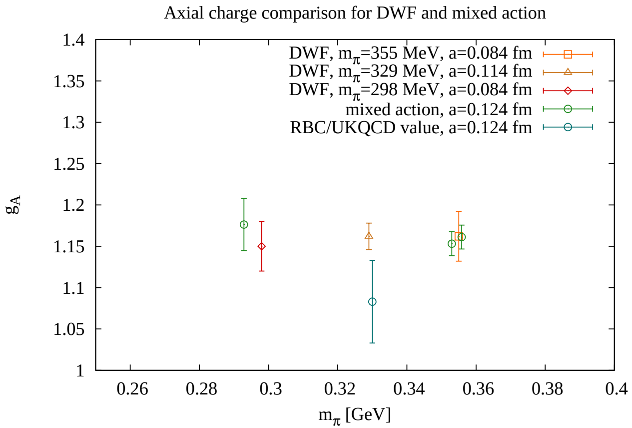
Pauli isovector formfactor $F_2^v(Q^2)$ at three m_π values



Dipole fit in range $0.2 <= Q^2 <= 0.8\,\mathrm{GeV}^2$

$(\kappa_v)^{\text{exp}} = 3.706\ldots$

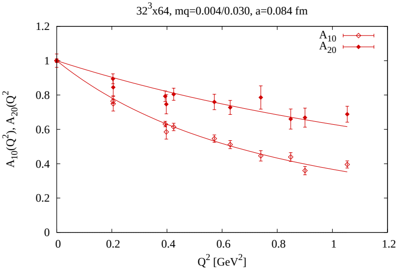
Axial charge comparison

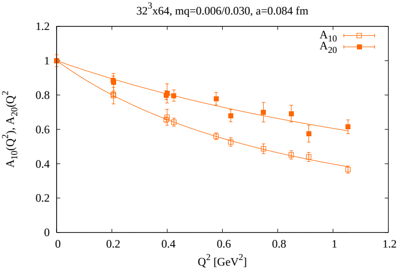


Earlier obtained value for g_A from

[T. Yamazaki et al, Phys. Rev. Lett. 100:171602, 2008]

Generalized spin-independent formfactors: A_{10}^{u-d} and A_{20}^{u-d}

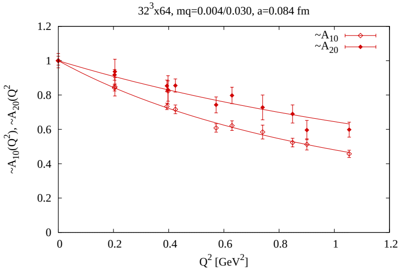


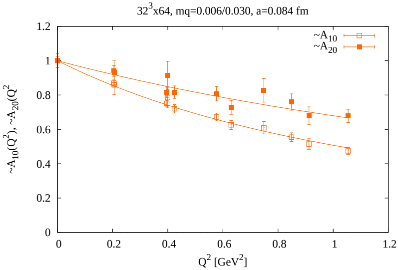


$$\delta A_{10}^{u-d}(0)/A_{10}^{u-d}(0) = 0.011$$
$$\delta A_{20}^{u-d}(0)/A_{20}^{u-d}(0) = 0.039$$

$$\delta A_{10}^{u-d}(0)/A_{10}^{u-d}(0) = 0.008$$
$$\delta A_{20}^{u-d}(0)/A_{20}^{u-d}(0) = 0.035$$

Generalized spin-dependent formfactors: $\tilde{A}_{10}^{u-d}$ and $\tilde{A}_{20}^{u-d}$





$$\delta A_{10}^{u-d}(0)/A_{10}^{u-d}(0) = 0.026$$
$$\delta A_{20}^{u-d}(0)/A_{20}^{u-d}(0) = 0.042$$

$$\delta A_{10}^{u-d}(0)/A_{10}^{u-d}(0) = 0.025$$
$$\delta A_{20}^{u-d}(0)/A_{20}^{u-d}(0) = 0.040$$

Summary and Outlook

- Used techniques are proven to give good results for affordable number of configurations
- Results are consistent for two quark action types and two lattice spacing values
- Accumulating more statistics for fine lattices is necessary for higher GPD moments
- Adding more m_π points for χPT extrapolation to physical point is essential
- Renormalization factors are required to get $\langle x \rangle_{(\Delta)q}$, J_q