Calculating the Light by Light Contribution to the Muon Anomalous Magnetic Moment Using Lattice QED

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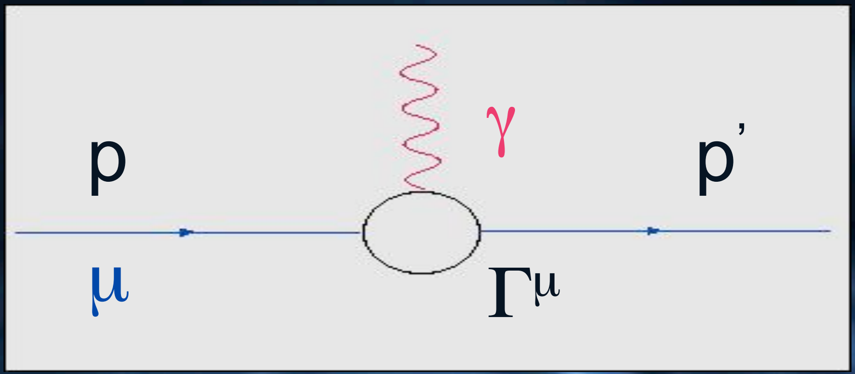
OUTLINE

- ✧ Definition of Anomalous Magnetic Moment
- ✧ Introduction to LBL diagram
- ✧ Approach to calculate the target diagram
- ✧ Preliminary results
- ✧ Summary & Future outlook

What is anomalous magnetic moment, F_2?



- To lowest order $\Gamma^\mu = \gamma^\mu$
- In general, after applying Lorentz Invariance, Ward and Gordon identities, we have

$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + (i\sigma^{\mu\nu}q_\nu/2m)F_2(q^2)$$

- In lowest order, $F_1 = 1$, and $F_2 = 0$
- Radiative corrections => $F_2 != 0$
- Expression for the magnetic moment of muon

$$\mu = g(e/2m)\mathbf{S}$$

where **S** is the muon spin, and

$$g = 2(F_1(0) + F_2(0)) = 2 + 2F_2(0)$$

$$\Rightarrow F_2 = (g - 2)/2$$

Introduction to LBL

- Muon anomalous magnetic dipole moment has been measured with a precision of 0.54 ppm at BNL

$$a_\mu \text{ (EXP)} = 11659208\ (6.3) \times 10^{-10}$$

- The measured quantity has reached a comparable level to the theoretical standard model prediction

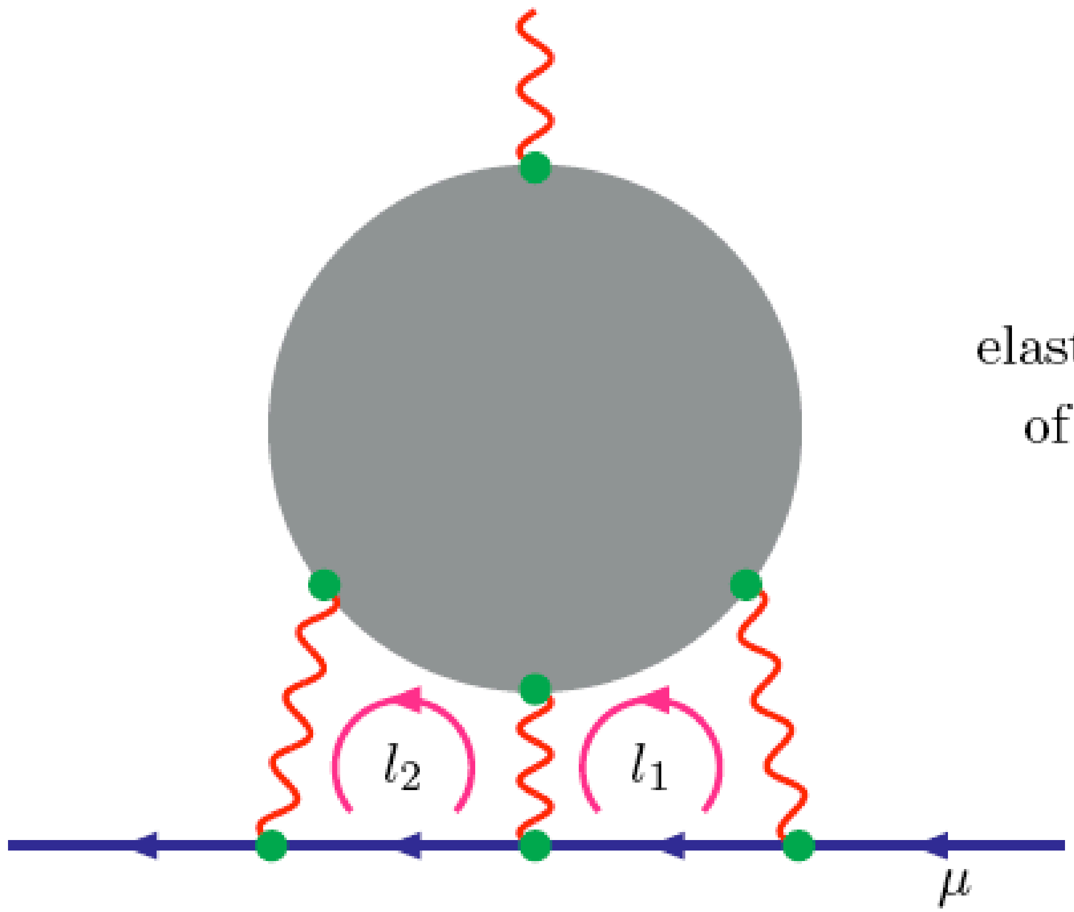
$$a_\mu \text{ (S M)} = 11659184.1\ (7.2)^{\text{Vac. Pol.}}\ (3.5)^{\text{LBL}}(0.3)^{\text{QED/Weak}} \times 10^{-10}$$

$$= 11659184.1\ (8.0) \times 10^{-10}$$

- The sensitivity between theoretical and experimental results can be attributed to some new physics such as super symmetry (SUSY)

$$\Delta a_\mu \text{ (EXP} - \text{S M)} = 23.9\ (9.9) \times 10^{-10}$$

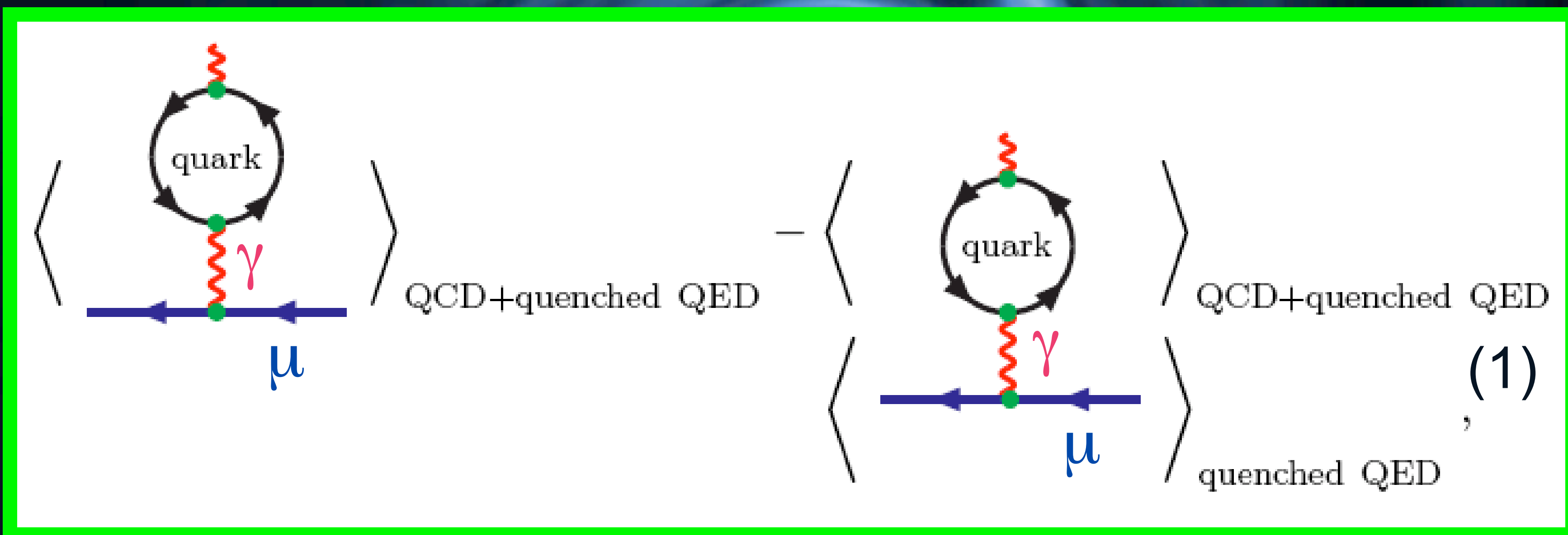
Target Diagram



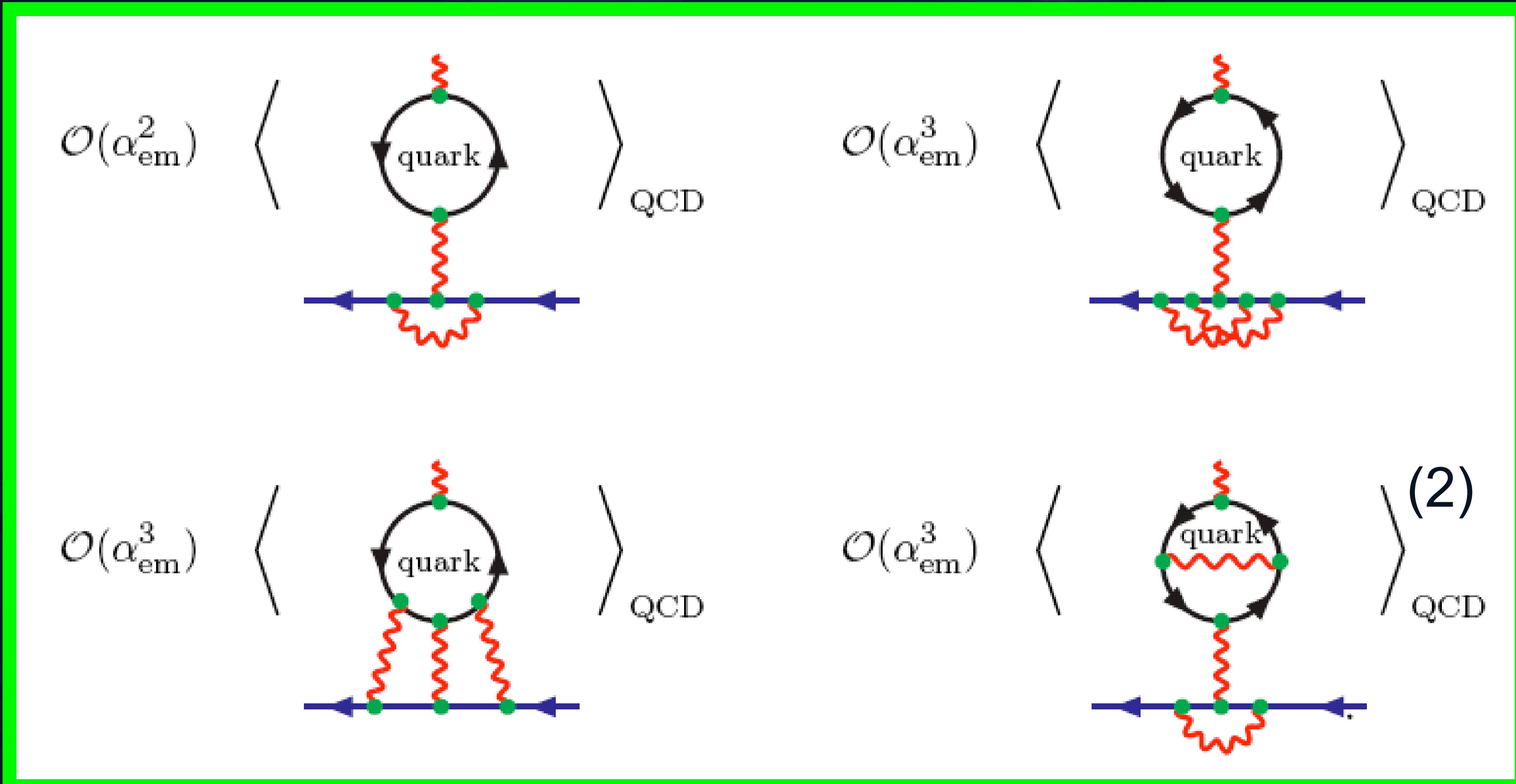
Note: Paul Rakow (QCDSF) gave a talk on Monday about calculating this diagram using a direct method.

Approach

Based on M. Hayakawa et al. (arXiv:hep-lat/0509016v2), the following method has been proposed to extract lbl (light-by-light) contribution:

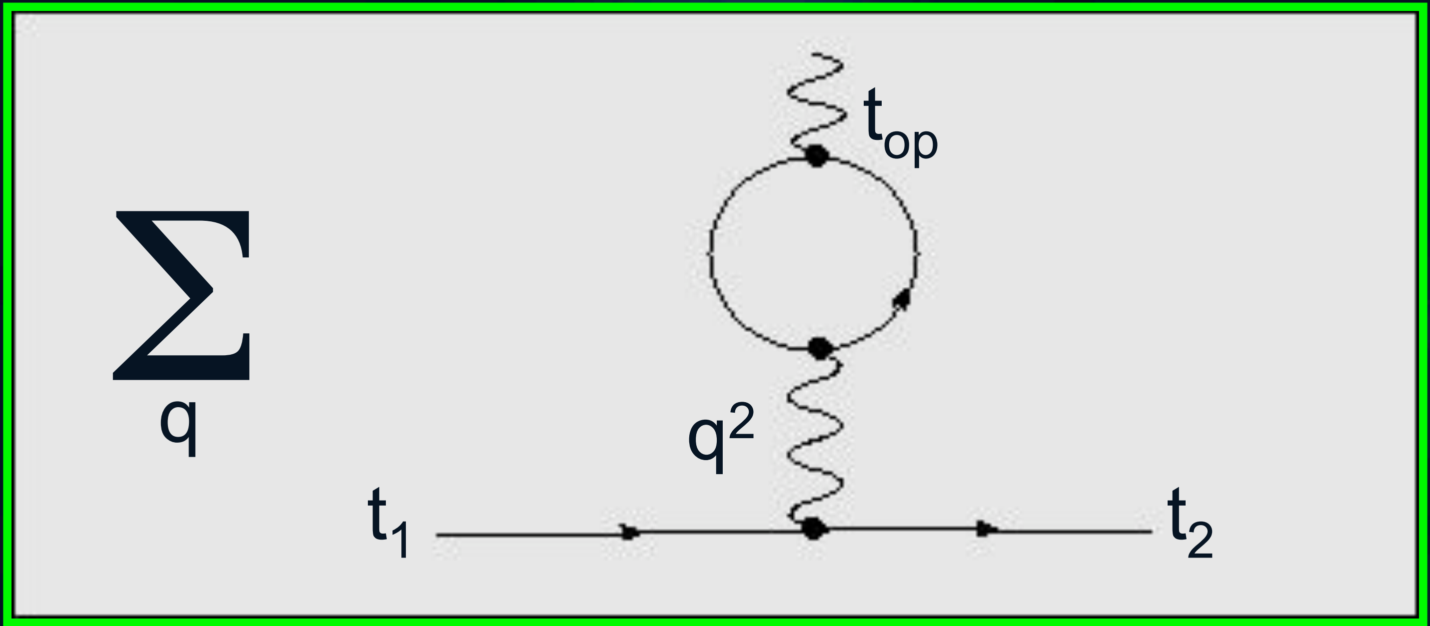


Analysis of the first term of eq. (1) yields:



Analysis of the second term of eq. (1) yields all the other diagrams in (2) except our target diagram (LBL)

Implementation of Lattice Calculation

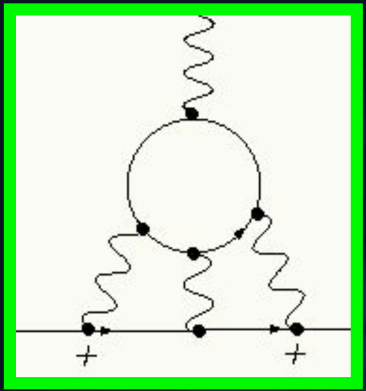


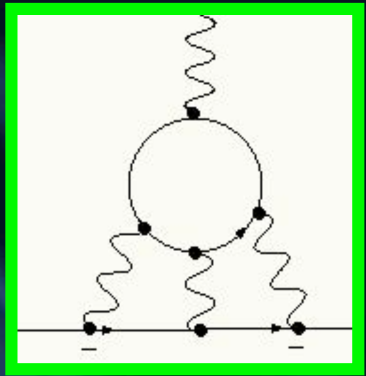
- Three-pt correlation functions are calculated for different projections at the external vertex
- The following two basic eqns are used to calculate F_2

$$F_1 - \frac{q^2}{4m^2} F_2 = \frac{G_3^t}{G_{PP}^2 G_{NP}^2} * \frac{V}{2} \quad (3)$$

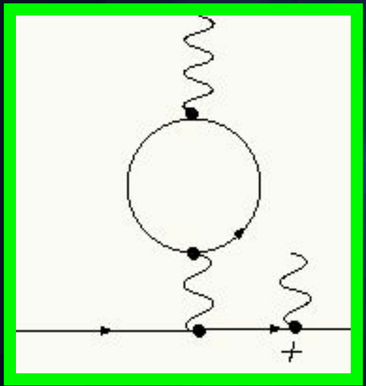
$$F_1 + F_2 = \frac{G_3^X}{G_{PP}^2 G_{NP}^2} * \frac{(E+M)}{p_y} * \frac{V}{2} \quad (4)$$

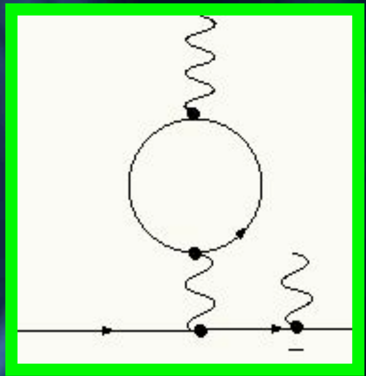
Analysis of the target diagram for positive & negative charge simulations :



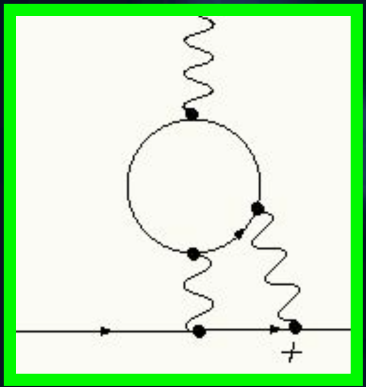


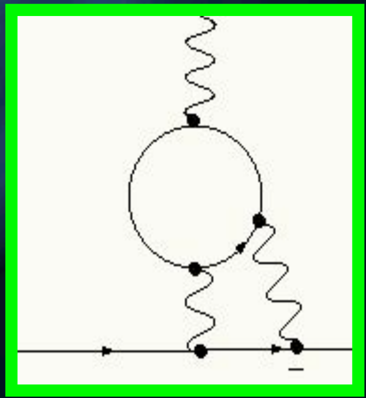
- Conserved current inserted at the internal vertices of loop & line





- cancelled out due to equal & opposite contributions





- cancelled out by itself due to Furry's theorem

Preliminary Results

- Ward Identity: WI is satisfied for both line & loop.
- It is rather trivial to show WI for Muon Loop, i.e.

$$q^{\nu}\Pi^{\mu\nu} = 0,$$

which is evident from the below results for the loop on 16^3x32x8 lattices.

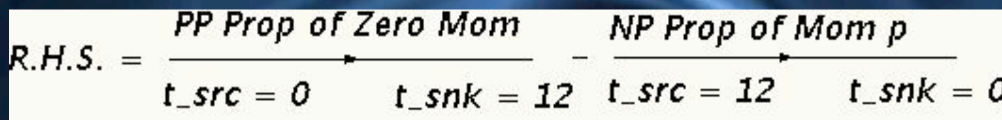
$\mu\ \nu$ = 0 0 MOM = 1 1 0 0 = -4.724959e-02 -1.128366e-03
$\mu\ \nu$ = 0 1 MOM = 1 1 0 0 = 4.724959e-02 1.128366e-03
.
.

WI for Muon Line

- WI for line is given by

$$-ik_\mu\left\langle q(\vec{p},t_2)J_\mu\vec{q}(\vec{q},t_1)\right\rangle = e^{ik_4t_2}\left\langle q(\vec{k}+\vec{p},t_2)\vec{q}(\vec{q},t_1)\right\rangle\delta(\vec{k}+\vec{p}-\vec{q}) - e^{ik_4t_1}\left\langle q(\vec{p},t_2)\vec{q}(\vec{k}-\vec{q},t_1)\right\rangle\delta(\vec{p}-\vec{k}-\vec{q})$$

- R.H.S. of the above eqn can simply be visualized for k_4=0 with the following two propagators:



- WI for Muon_Line with 16^3x32x8 Lattices:

ρ = 0 MOM = 1 0 0 0 = -4.686680e-01 4.115704e+02

ρ = 1 MOM = 1 0 0 0 = 3.990371e+00 1.452571e+00

ρ = 2 MOM = 1 0 0 0 = -5.496471e+00 -6.494709e+01

ρ = 3 MOM = 1 0 0 0 = 9.631317e+02 -1.213289e+01

muon 2pt PP FUNCTION MOM = 0 0 0 12 = 4.811456e+01 -1.044212e-01

muon 2pt NP FUNCTION MOM = 1 0 0 0 = 8.067967e+00 1.500217e-01

l.h.s = (4.115704e+02)*2*sin(PI/16)/(5-0.99) = 40.04658446966

r.h.s. = 48.11456 - 8.067967 = 40.046593

LBL Preliminary Results

- Simulation Parameters:

  Lattices: 16^3x32x8 (Coulomb Gauge Fixed)

  Mass = 0.4;

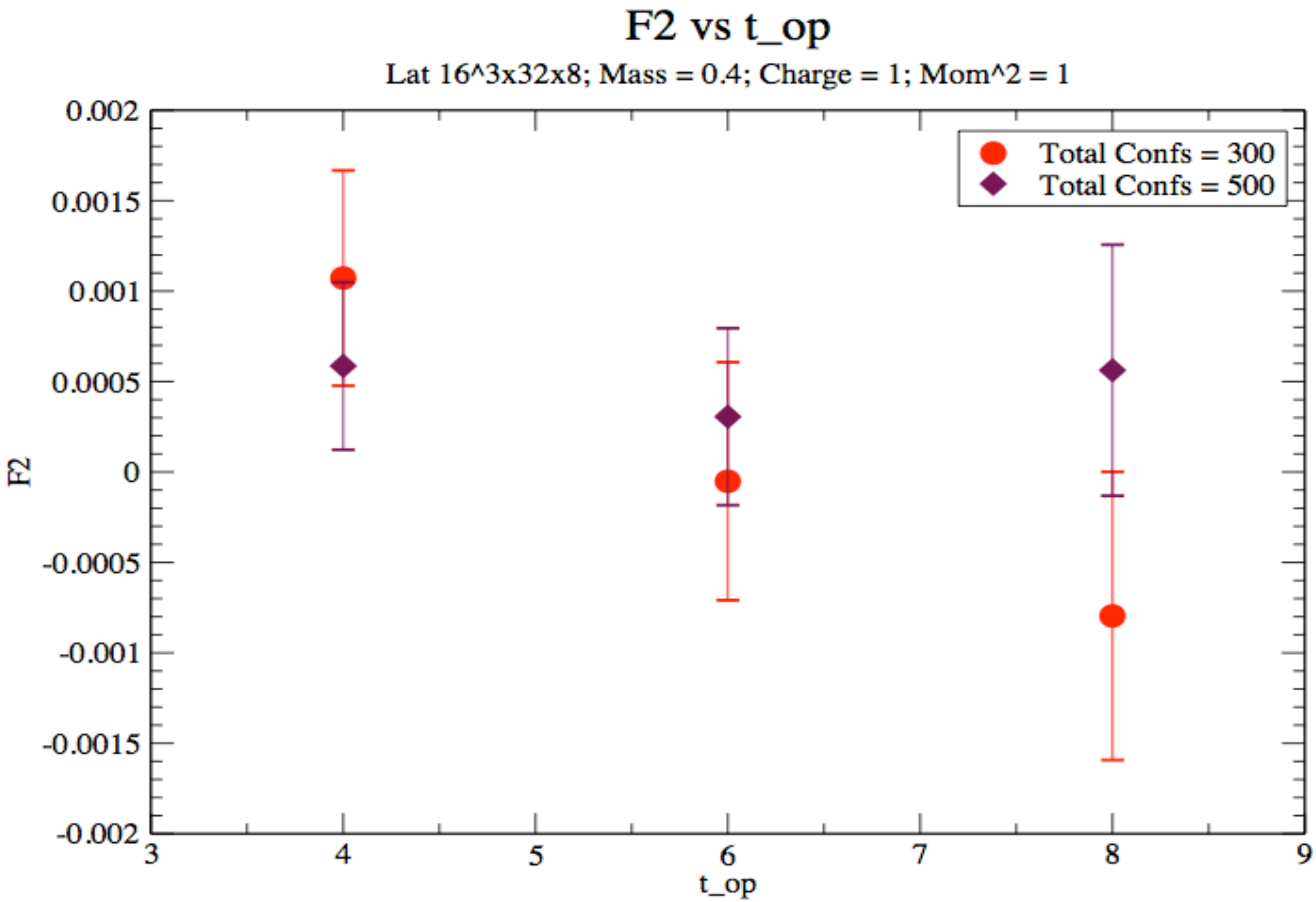
  Charge = 1;

  $t_{src} = 0$

  $t_{snk} = 12$

  Mom^2 (in-coming prop) = 0

  Mom^2 (out-going prop) = 1


F2 vs t_op
Lat 16^3x32x8; Mass = 0.4; Charge = 1; Mom^2 = 1
Total Confs = 300
Total Confs = 500
F2
t_op
0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
-0.0015
-0.002
3
4
5
6
7
8
9

Preliminary Result

- After averaging over t_{op} = 4, 6, and 8 under Jack Knife Blocks, We have

  F_2 = 0.00048 +/- 0.00036

- Error is roughly one order of magnitude more than the Expected signal

- F_2 in perturbation theory has been calculated to be in the order of α^3/π^3, which translates into ~1.6e-05 (for e = 1)

Summary & Future Outlook

- We have all the machinery to calculate the LBL term.
- No signal yet.
- Need to improve the statistics.
  - use random source for the loop
  - simulate at the larger volume
- If we can achieve the signal for LBL term in pure QED, it is straight forward to include QCD by replacing the fermionic loop with a hadronic blob.