Wilson Chiral Perturbation Theory for twisted mass QCD at NLO

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Introduction

Lattice Simulation

- Machine power can simulate QCD at small mass region.
- ETMC's simulations reach $m_{\mathrm{q}} \sim a^2 \Lambda_{\mathrm{QCD}}^3$.

ChPT for twisted mass QCD

- regime : $m_{\mathrm{q}} \sim a \Lambda_{\mathrm{QCD}}^2$
- $O(a^2)$ effects are NLO.

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- $O(a^2)$ effects are NLO.

We need Wilson ChPT at small mass regime, $m_{\mathrm{q}} \sim a^2\Lambda_{\mathrm{QCD}}$.

Power Counting

Previous Study

- LO: $O(m, a)$
- NLO: LO^2

Power Counting

Our Study

- LO: $O(m, a)$, sub LO(SLO): $O(a^2, am)$
- NLO: LO^2 , $\mathrm{LO} \cdot \mathrm{SLO} \sim O(a^2 m, am^2)$, $\mathrm{SLO}^2 \sim O(a^4, a^3 m, a^2 m^2)$

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$O(a^2, am)$ effects:

- non-trivial phase structure appears at LO.
  cf. S.R.Sharpe & R.L.Singleton ('98), S.R.Sharpe & J.M.S.Wu ('04)

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- Vacuum expectation has divergence at 1-loop.

Talk Plan

1. Introduction
   - power counting
2. Leading Order
   - gap equation and phase structure
   - pion mass
3. NLO: 1-loop and renormalization
   - vertexes and 1-loop diagram
   - Local Counter Term
   - renormalization
4. Maximal Twist
5. Summary

LO Lagrangian

LO Lagrangian

$$\mathcal{L}_{\mathrm{LO}} = \frac{f^2}{4}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2}{4}\langle \Sigma M^\dagger + M\Sigma^\dagger \rangle - \frac{f^2}{4}\langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle$$
$$+ W_{45}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle \langle (\Sigma - \Sigma_0)\hat{a}^\dagger + \hat{a}(\Sigma - \Sigma_0)^\dagger \rangle$$
$$- W_{68}\langle \Sigma M^\dagger + M\Sigma^\dagger \rangle \langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle - W'_{68}\langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle^2$$

- $\Sigma = \Sigma_0^{1/2}\Sigma_{ph}\Sigma_0^{1/2}, \Sigma_0 = \exp[i\tau^3\phi], \Sigma_{ph} = \exp[i\tau^a\pi_a/f]$
- ϕ : vacuum expectation, π : pion fields, τ^a : Pauli Matrix
- $M = 2B_0(m + i\tau^3\mu), \hat{a} = 2W_0 a$

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- $2B_0\tilde{m} = 2B_0 m + 2W_0 a$
- $\tilde{W}'_{68} = W'_{68} - W_{68}$

Gap Equation

Gap equation, $dL^{(0)}_{\mathrm{LO}}/d\phi|_{\phi=\phi_0} = 0$

$$2B_0\tilde{m}\sin\phi_0 - (c_2a^2 - \tilde{c}_2 a 2B_0\tilde{m})\sin 2\phi_0 = 2B_0\mu\cos\phi_0 + \tilde{c}_2 a 2B_0\mu\cos 2\phi_0$$

The low energy constants,

- $c_2 = -64\tilde{W}'_{68}W_0^2/f^2, \tilde{c}_2 = 32W_{68}W_0/f^2$

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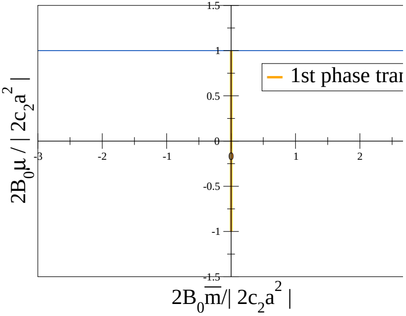
Non-trivial phase structure appeares.

Phase Structure



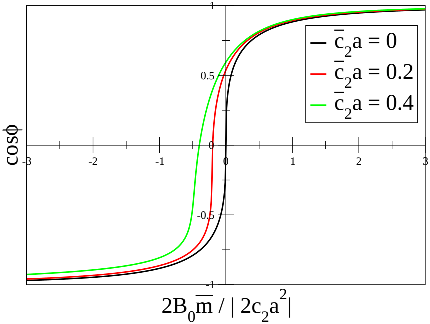


Figure: $c_2 < 0$, $2B_0\mu/|2c_2a^2| = 1$

- This phase structure is similar to previous study at NLO.
  cf. S.R.Sharpe & J.M.S.Wo('04)

Pion Mass

The $O(\pi^2)$ lagrangian

$$\mathcal{L}_{LO}^{(2)} = \frac{1}{2}\partial_\mu \pi_a \partial_\mu \pi_a + \frac{(m_\pi^a)^2}{2}\pi_a^2$$

The pion mass term

- $(m_\pi^{1,2})^2 = 2B_0 m' - 2c_2 a^2 \cos^2\phi_0 + 2\tilde{c}_2 a(2B_0 m')\cos\phi_0$
- $(m_\pi^3)^2 = (m_\pi^{1,2})^2 + (\Delta m_\pi^3)^2$
- $(\Delta m_\pi^3)^2 = 2c_2 a^2 \sin^2\phi_0 + 2\tilde{c}_2 a(2B_0 \mu')\sin\phi_0$
- The short-hand notation for quark mass term,

$$\begin{pmatrix} m' \\ \mu' \end{pmatrix} = \begin{pmatrix} \cos\phi_0 & \sin\phi_0 \\ -\sin\phi_0 & \cos\phi_0 \end{pmatrix} \begin{pmatrix} \tilde{m} \\ \mu \end{pmatrix}$$

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Pion mass splitting.

Vertexes

$O(\pi^3)$ and $O(\pi^4)$ Lagrangian

$$\mathcal{L}_{\rm LO}^{(3)} = \frac{\lambda_{3p}}{2f}\pi_3(\partial_\mu\pi_a)^2 + \frac{\lambda_3}{2f}\pi^2\pi_3$$

$$\mathcal{L}_{\rm LO}^{(4)} = \frac{\lambda_{4p}}{6f^2}(\pi_a\partial_\mu\pi_a)^2 + \frac{\lambda'_{4p}}{6f^2}\pi^2(\partial_\mu\pi_a)^2 + \frac{\lambda_4}{6f^2}(\pi^2)^2 + \frac{\lambda'_4}{6f^2}\pi^2(\pi_3)^2$$

These vertex factor λ is given by,

$$\lambda_{3p} = -c_0 a \sin\phi_0, \quad \lambda_{4p} = 1, \quad \lambda'_{4p} = -\left(1 + \frac{3}{2}c_0 a\cos\phi_0\right),$$

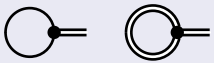
$$\lambda_3 = c_2 a^2 \sin 2\phi_0 - \tilde{c}_2 a(2B_0 m' \sin\phi_0 - 2B_0\mu'\cos\phi_0),$$

$$\lambda_4 = -\frac{1}{4}2B_0 m' + 2c_2 a^2\cos^2\phi_0 - 2\tilde{c}_2 a 2B_0 m'\cos\phi_0,$$

$$\lambda'_4 = -2c_2 a^2 \sin^2\phi_0 - 2\tilde{c}_2 a 2B_0\mu'\sin\phi_0$$

1-Loop Diagram for Vacuum Expectation

tadpole diagram



- ——— : $\pi_{1,2}$
- ═══ : π_3

1-Loop Diagram for Pion Mass

2-points function 1

2-points function 2

- ——— : $\pi_{1,2}$
- ═══ : π_3

1-Loop Diagram for Pion Mass

2-points function 1 $\longrightarrow$ LO^2

2-points function 2 $\longrightarrow$ SLO^2

- —— : $\pi_{1,2}$
- ═══ : π_3

Local Counter Term

$O(p^2 m), O(m^2)$: $\# = 2$

$\langle \Sigma M^\dagger + M\Sigma^\dagger \rangle \langle L_{\mu\mu} \rangle$, $\langle \Sigma M^\dagger + M\Sigma^\dagger \rangle^2$

$O(ap^2 m), O(am^2)$: $\# = 6$

$\langle L_{\mu\mu} \rangle \langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle \langle \Sigma M^\dagger + M\Sigma^\dagger \rangle$, $\langle L_{\mu\mu} \rangle \langle \hat{a} M^\dagger + M\hat{a}^\dagger \rangle$,
$\langle \partial_\mu \Sigma \hat{a}^\dagger + \hat{a}\partial_\mu \Sigma^\dagger \rangle \langle \partial_\mu \Sigma M^\dagger + M\partial_\mu \Sigma^\dagger \rangle$, $\langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle \langle \Sigma M^\dagger + M\Sigma^\dagger \rangle^2$,
$\langle M M^\dagger \rangle \langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle$, $\langle \hat{a} M^\dagger + M\hat{a}^\dagger \rangle \langle \Sigma M^\dagger + M\Sigma^\dagger \rangle$

$O(a^2 p^2), O(a^2 m)$: $\# = 6$

$\langle L_{\mu\mu} \rangle \langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle^2$, $\langle L_{\mu\mu} \rangle \langle \hat{a}\hat{a}^\dagger \rangle$, $\langle \partial_\mu \Sigma \hat{a}^\dagger + \hat{a}\partial_\mu \Sigma^\dagger \rangle^2$,
$\langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle^2 \langle \Sigma M^\dagger + M\Sigma^\dagger \rangle$, $\langle \hat{a}\hat{a}^\dagger \rangle \langle \Sigma M^\dagger + M\Sigma^\dagger \rangle$,
$\langle \hat{a} M^\dagger + M\hat{a}^\dagger \rangle \langle \Sigma \hat{a}^\dagger + \hat{a}\Sigma^\dagger \rangle$

where $L_{\mu\nu} = \partial_\mu \Sigma \partial_\nu \Sigma^\dagger$.

Renormalization

of divergence terms

- vacuum expectation : 6
- pion mass term : 13

- # of counter terms (14) < # of divergences terms (19)
- Can not renormalize

Renormalization

of indipendent divergence terms

- vacuum expectation : $6 \to 2$
- pion mass term : $13 \to 5$

- # of counter terms (14) $>$ # of divergences terms (7)
- mass term reducing by gap eq
  - $2B_0\mu' = -c_2a^2 \sin 2\phi_0 + \tilde{c}_2 a(2B_0 m' \sin\phi_0 - 2B_0\mu' \cos\phi_0)$
  - $(2B_0 m')(2B_0\mu') = -c_2a^2(2B_0 m') \sin 2\phi_0 + \tilde{c}_2 a(2B_0 m')^2 \sin\phi_0 + H.O.$
  - $(2B_0\mu')^2 = a(2B_0 m')(2B_0\mu') = a(2B_0\mu')^2 = a^2(2B_0\mu') = H.O.$
- Can renormalize

Maximal Twist Condition

(twisted) PCAC mass

$$m_{\mathrm{PCAC}} = \frac{\sum_{\vec{x}} \langle \partial_0 A_0^a(\vec{x}, t) P^a(0) \rangle}{2 \sum_{\vec{x}} \langle P^a(\vec{x}, t) P^a(0) \rangle} = 0, \quad a = 1, 2$$

- (twisted) PCAC mass at LO

$$m_{\mathrm{PCAC}} = \frac{(m_\pi^a)^2_{\mathrm{LO}} \cos \phi_0}{2 B_0 (1 + \tilde{c}_2 a \cos \phi_0)}$$

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$$\cos \phi_0 = 0$$

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- (twisted) PCAC mass at LO

$$m_{\mathrm{PCAC}} = \frac{(m_\pi^a)^2_{\mathrm{LO}} \cos\phi_0}{2B_0(1+\tilde{c}_2 a \cos\phi_0)}$$

- maximal twist condition

$$\cos\phi_0 = 0$$

- untwisted quark mass

$$2B_0\tilde{m} = -2B_0\mu\tilde{c}_2 a$$

Pion Mass at Maximal Twist

pion mass at maximal twist

- $(m_\pi^{1,2})^2_{\mathrm{LO}} = 2B_0\mu$
- $(m_\pi^{3})^2_{\mathrm{LO}} = 2B_0\mu + 2c_2a^2 - 2(\tilde{c}_2a)^2 2B_0\mu$
- $(m_\pi^{1,2})^2_{\mathrm{NLO}} = (m_\pi^{1,2})^2_{\mathrm{LO}}\{1 + C(\mu_{\mathrm{ChPT}}) + \mathcal{C}L_\pi^{1,2} + \mathcal{C}'L_\pi^{3}\}$

The coefficents is

- $C = -16(m_\pi^a)^2_{\mathrm{LO}}(2L_{68} + L_{45})/f^2 + a^2X_2/f^2$
- $\mathcal{C} = 0$
- $\mathcal{C}' = \frac{3}{4}$
- $L_\pi^a = \frac{(m_\pi^a)^2}{16\pi^2 f^2}\log\left(\frac{m_\pi^a}{\mu_{\mathrm{ChPT}}}\right)^2$

Summary and Working Problems

Summary

- Construct WChPT for twisted mass QCD at the regime, $m_{\mathrm{q}} \sim a^2$.
- It appears the non-trivial phase structure and pion mass splitting at LO.
- At NLO, a vacuum expectation has divergence, but it can be renormalized.
- Constract Pion mass at NLO.

Working Problems

- Decay constant at NLO
- Phase structure at NLO
- Maximal twist condition at NLO