The $B\to\pi\ell\nu$ form factor and $|V_{ub}|$ from unquenched lattice QCD

Ruth Van de Water
for the Fermilab Lattice and MILC Collaborations

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Outline

I. Introduction: $B \to \pi \ell \nu$, $|V_{ub}|$, & new physics

II. Lattice simulation details: actions and parameters

III. Lattice calculation of the $B \to \pi \ell \nu$ form factor

- ✦ Chiral-continuum extrapolation with staggered chiral perturbation theory
- ✦ Systematic error budget

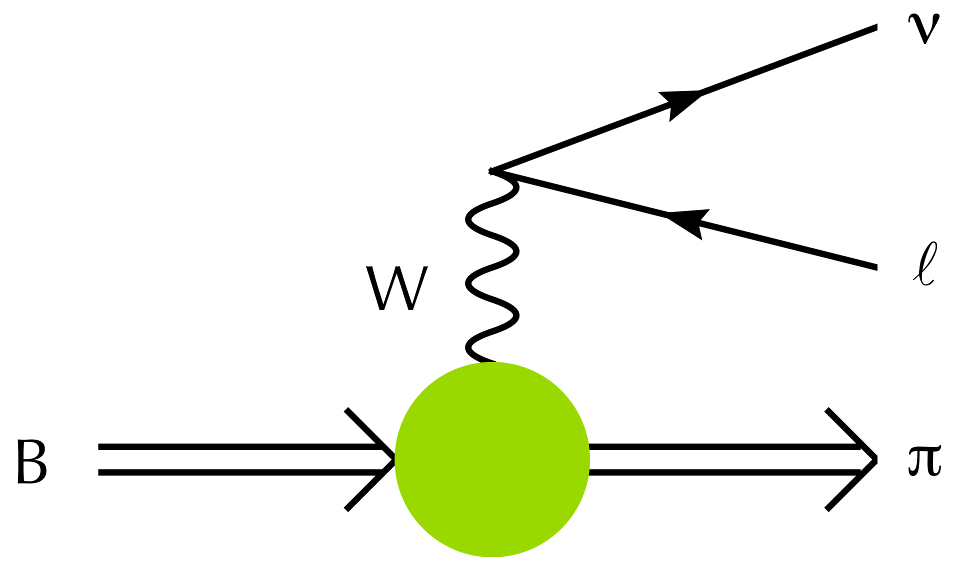
IV. Determination of $|V_{ub}|$

- ✦ Model-independent z-parameterization of form factors
- ✦ Combined z-fit of lattice and 12-bin BABAR data

V. Conclusions

- ✦ Comparison of $|V_{ub}|$ with other determinations
- ✦ Future plans

Why study B→πℓν?



$$q^2 = m_B^2 + m_\pi^2 - 2m_B E_\pi$$

- Decay parameterized by two form factors, $f_+(q^2)$ and $f_0(q^2)$:

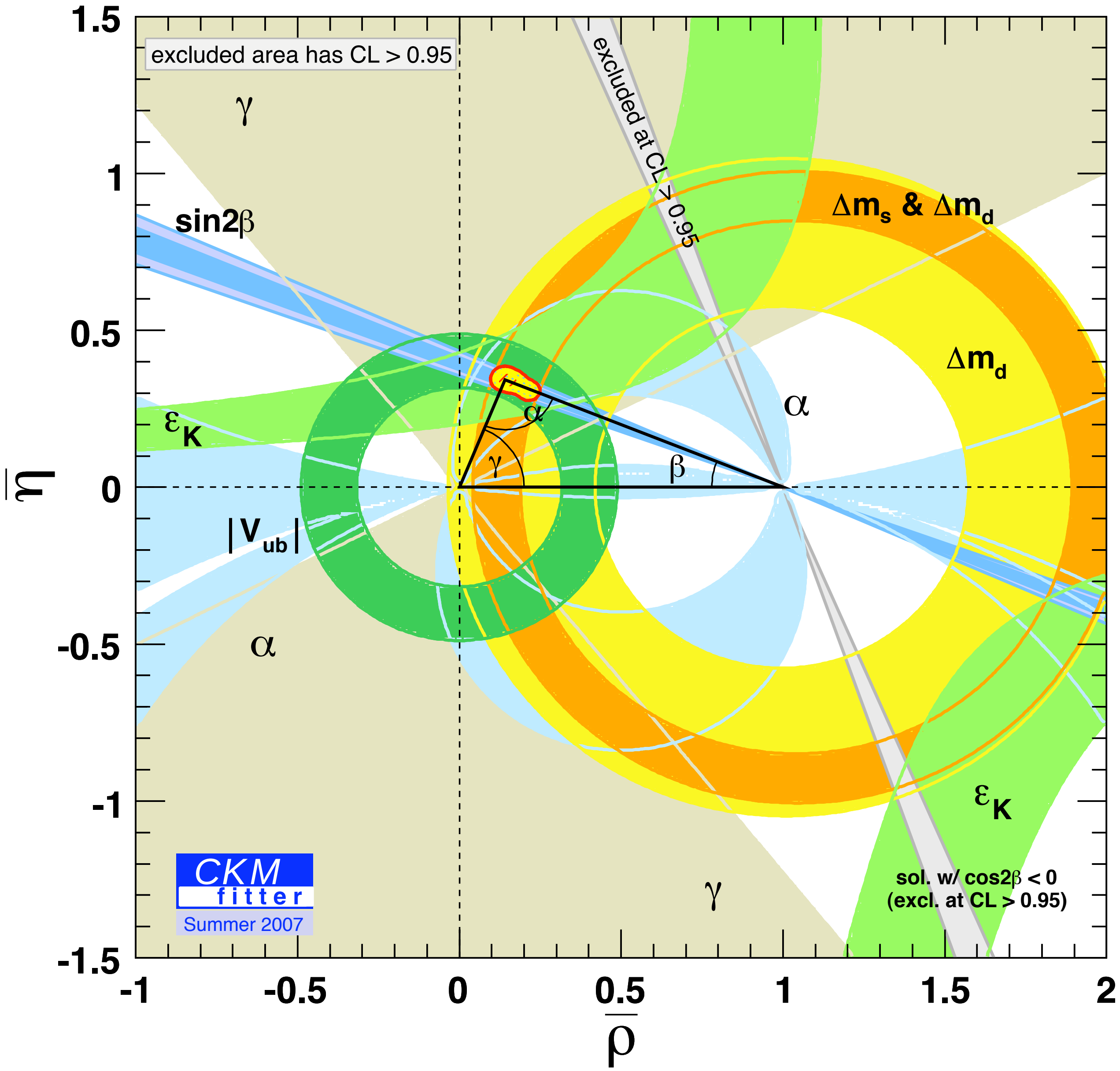
$$\langle \pi(p_\pi)|\mathcal{V}^\mu|B(p_B)\rangle = f_+(E_\pi)\left[p_B + p_\pi - \frac{m_B^2 - m_\pi^2}{q^2} q\right]^\mu + f_0(E_\pi)\frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

- Experiments measure the differential decay rate -- can use to extract form factor **shapes**:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3}\left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} |f_+(q^2)|^2$$

- Lattice QCD calculations needed to determine **normalization** and extract the CKM matrix element $|V_{ub}|$

$|V_{ub}|$ and the CKM unitarity triangle



- $|V_{ub}|$ constrains the apex $(\bar{\rho}, \bar{\eta})$ of the unitarity triangle:

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1 - \frac{\lambda^2}{2}} \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

  - $\lambda = |V_{us}|$ known to ~1%
  - $|V_{cb}|$ known to ~2%

- Width of green error ring currently dominated by ~10-15% uncertainty in $|V_{ub}|$

- sin(2β) currently constrains the height to better than ???% and is still improving

- ∴ A precise determination of $|V_{ub}|$ will allow a **strong test of CKM unitarity and sensitive probe of new physics**

Lattice calculation of the $B \to \pi \ell \nu$ semileptonic form factor

Light quark and heavy quark actions

- MILC 2+1 flavor gauge configurations [Phys.Rev.D70:114501,2004]
  - Symanzik improved gluon action
  - **Asqtad-improved staggered light (up,down) quark action**
  - Multiple light quark masses down to $m_s/10$ and two lattice spacings

a(fm)	L	am_l/am_s	am_π	# configs.
0.09	28	0.0062/0.031	$0.14856(^{+49}_{-46})$	557
0.09	28	0.0124/0.031	$0.20655(^{+45}_{-46})$	534
0.12	24	0.005/0.05	$0.15514(^{+28}_{-27})$	529
0.12	20	0.007/0.05	$0.18341(^{+25}_{-29})$	836
0.12	20	0.01/0.05	$0.21731(^{+29}_{31})$	592
0.12	20	0.02/0.05	$0.30198(^{+29}_{-29})$	460

- **Clover-improved action with Fermilab interpretation for b-quark**
  - Adjust hopping parameter to reproduce experimental B_s meson mass
  - Adjust bare quark mass and clover coefficient to mach lattice action onto continuum HQET action through $O(1/m_Q)$ such that leading errors scale as $O(\alpha_S \Lambda/m_Q)$

Heavy-light currents

- To calculate the B→$\pi\ell\nu$ form factor we compute 3-point functions of the temporal and spatial vector currents: $\langle \pi | \widehat{V}_\mu | B \rangle$
  - We remove the leading heavy quark discretization effect by rotating the heavy quark field
- We multiply the lattice amplitude by the appropriate renormalization factor to get the continuum one:

$$\langle \pi | V_\mu | B \rangle = Z^{hl}_{V_\mu} \times \langle \pi | \widehat{V}_\mu | B \rangle \qquad Z^{bl}_{V_\mu} = \rho_{V_\mu} (Z^{bb}_V Z^{ll}_V)^{1/2}$$

  - The Z-factors are computed **nonperturbatively**:

$$Z^{qq}_V \times \langle B(0) | \overline{\psi}_q \gamma_0 \psi_q | B(0) \rangle = 1 \qquad (q = b, l)$$

  - The correction factor, ρ, is a ratio expected to be close to 1 and is calculated to 1-loop in lattice perturbation theory:

$$\rho_{V_\mu} \equiv Z^{bl}_{V_\mu} / (Z^{bb}_V Z^{ll}_V)^{1/2}$$

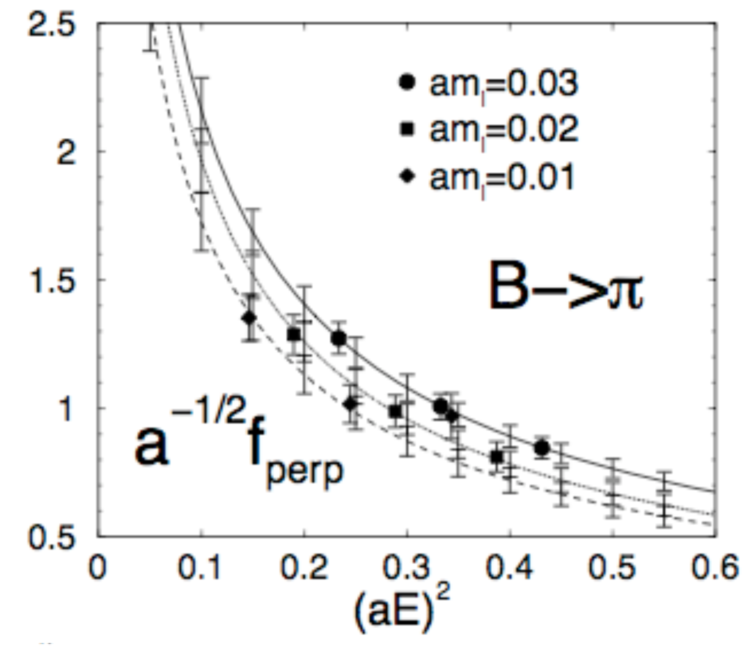
★ This analysis was **BLINDED**: an unknown offset was put into the ρ−factors and not revealed until the central value and error budget for $|V_{ub}|$ were finalized ★

Current status of lattice calculations

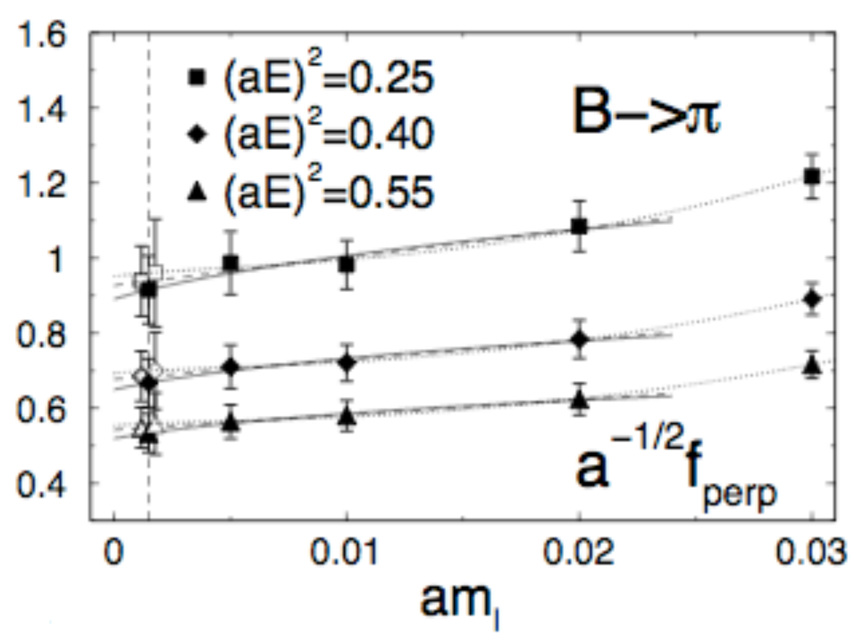
- Only two unquenched 2+1 flavor calculations of the $B\to\pi\ell\nu$ form factor:
  - Both use the MILC gauge configurations
  - **Fermilab-MILC** uses Fermilab quarks [arXiv:hep-lat/0510113]
  - **HPQCD** uses nonrelativistic (NRQCD) heavy quarks [Phys.Rev.D73:074502,2006, Erratum-ibid.D75:119906,2007]
- The analysis procedure is similar for both groups
  - In both cases, it is a four-step procdure
  - We improve upon the the analysis method in all four steps . . .

Improvements over previous calculations: I

(1) Previous unquenched analyses first interpolate and extrapolate lattice data using the BK parameterization which builds in the B^* pole to get the form factors $f_{||}$ and $f_\perp$ at fiducial values of E_π^2



- This **builds in model-dependence** at the very first step of the analysis



(2) Next they extrapolate lattice data separately at each value of E_π^2 to physical light quark mass using staggered χPT [Lee & Sharpe, Aubin & Bernard, Sharpe & RV]

- This does not enforce the condition that the low-energy constants must be the same at all pion energies

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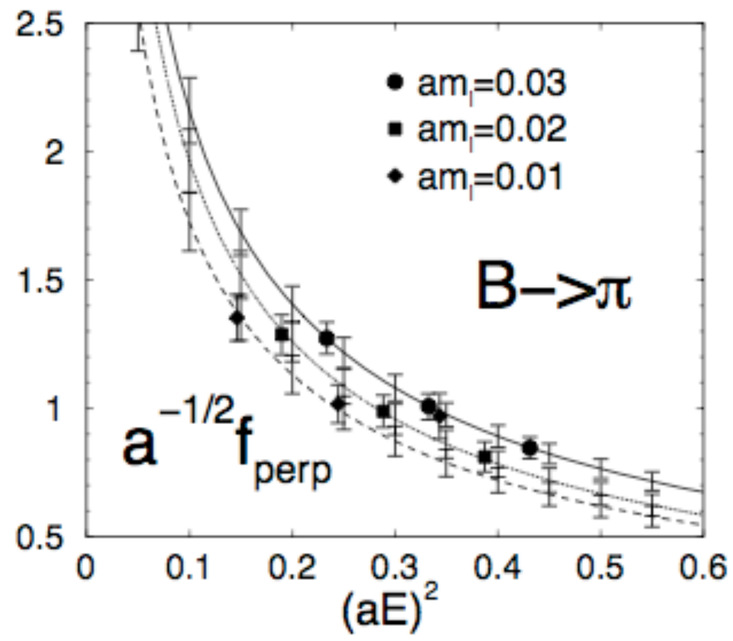

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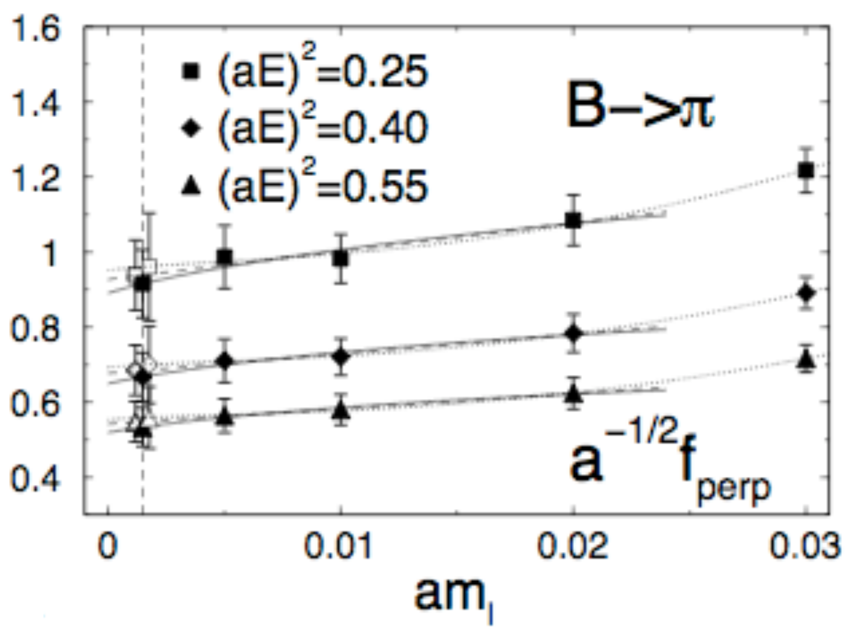

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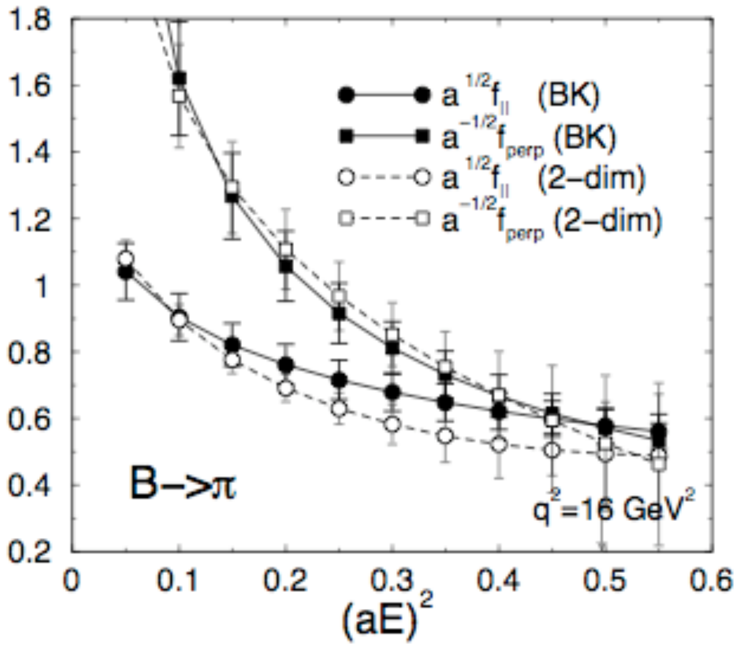
★ We perform a **SIMULTANEOUS FIT TO ALL DATA** (m_q, E_π) using rSχPT ★

Improvements to previous calculations: II



(3) Then previous analyses interpolate and extrapolate in q^2 again using the BK parameterization to determine the continuous function $f_+(q^2)$

(4) Last they combine $f_+(q^2)$ with the experimentally-measured $B\to\pi\ell\nu$ branching fraction and B-meson lifetime and integrate over q^2 to get $|V_{ub}|$:

$$\Gamma(q_{\rm min})/|V_{ub}|^2 = \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 (d\Gamma/dq^2)/|V_{ub}|^2$$

- This again introduces model dependence through the choice of fit function for the q^2 extrapolation
- It also increases the total uncertainty in $|V_{ub}|$ by combining the lattice and experimental results in a non-optimal way

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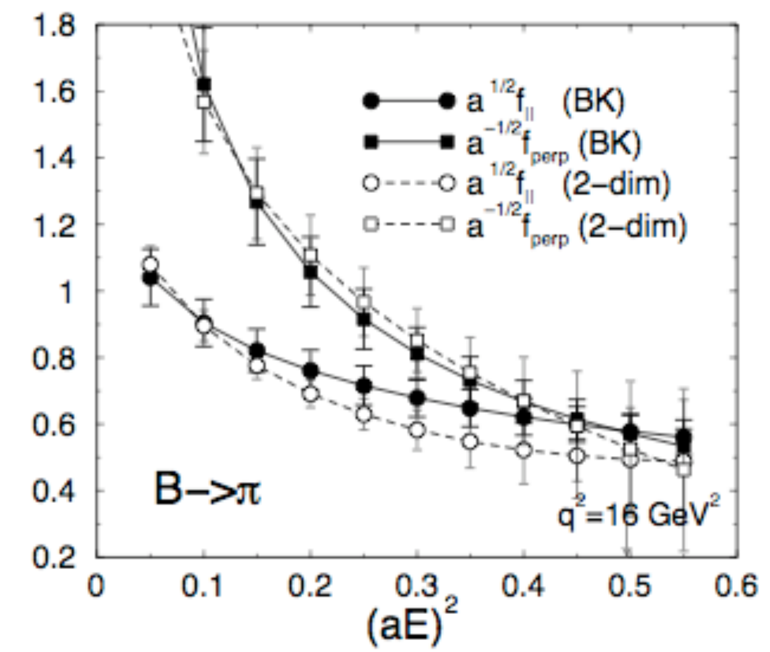

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★ We perform a **SIMULTANEOUS FIT TO LATTICE AND EXPERIMENTAL DATA** using the **MODEL-INDEPENDENT Z-PARAMETERIZATION** for the form factor ★

Staggered χPT for heavy-light form factors

- In the B-meson rest frame it is easiest to calculate the form factors $f_{||}$ and $f_\perp$:

$$f_{||}(E_\pi) = \langle \pi | V^0 | B \rangle / \sqrt{2m_B}$$
$$f_\perp(E_\pi) p_i = \langle \pi | V^i | B \rangle / \sqrt{2m_B}$$

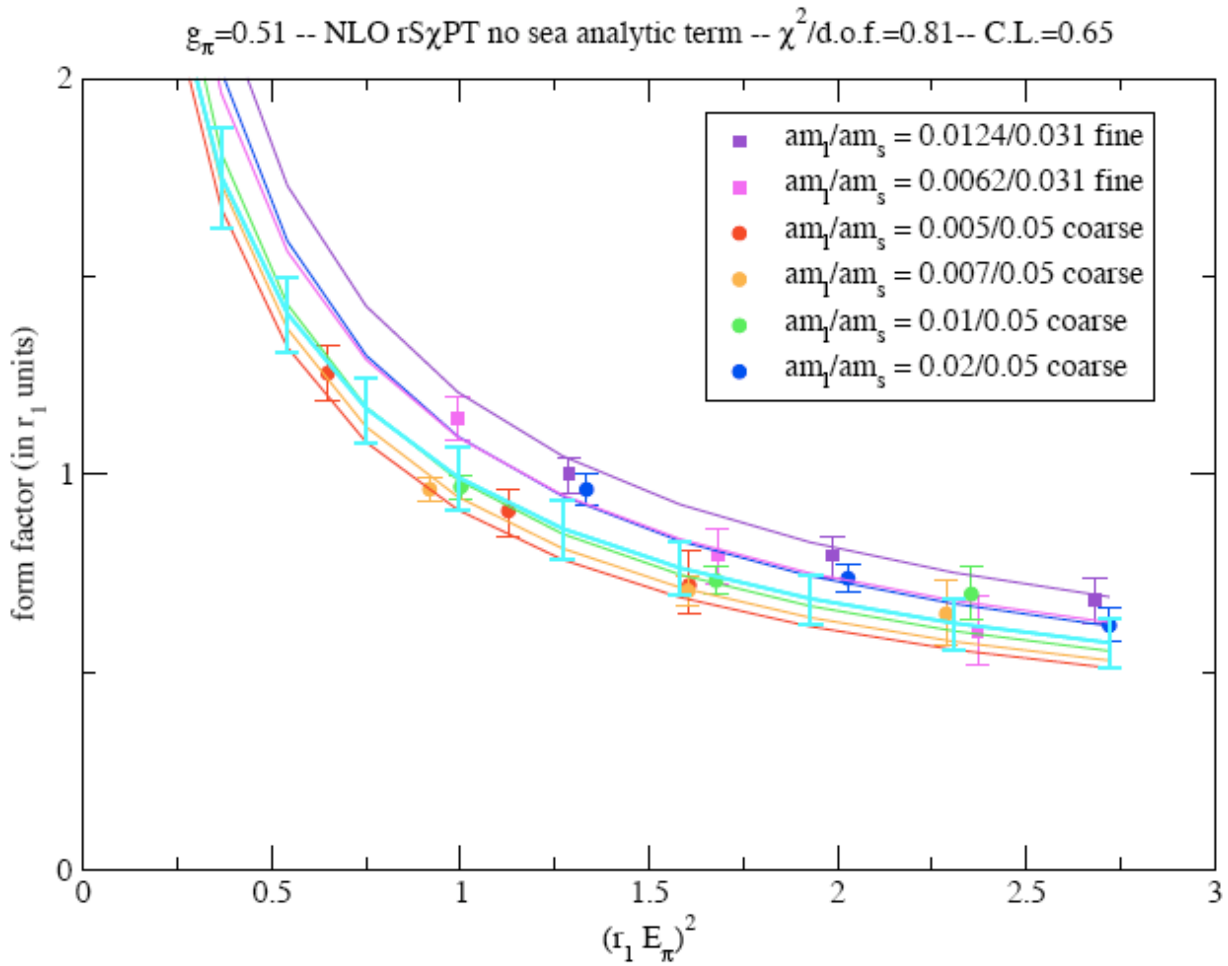
- The NLO staggered χPT expressions for $f_{||}$ and $f_\perp$ are [Aubin & Bernard]:

$$\boxed{f_{\|,\perp} = c_0 \left[1 + \text{chiral logs} + c_1 m_l + c_2(2m_u + m_s) + c_3 E_\pi + c_4 E_\pi^2 + c_5 a^2 + \right] + \mathcal{O}(m_q^2, E_\pi^3)}$$

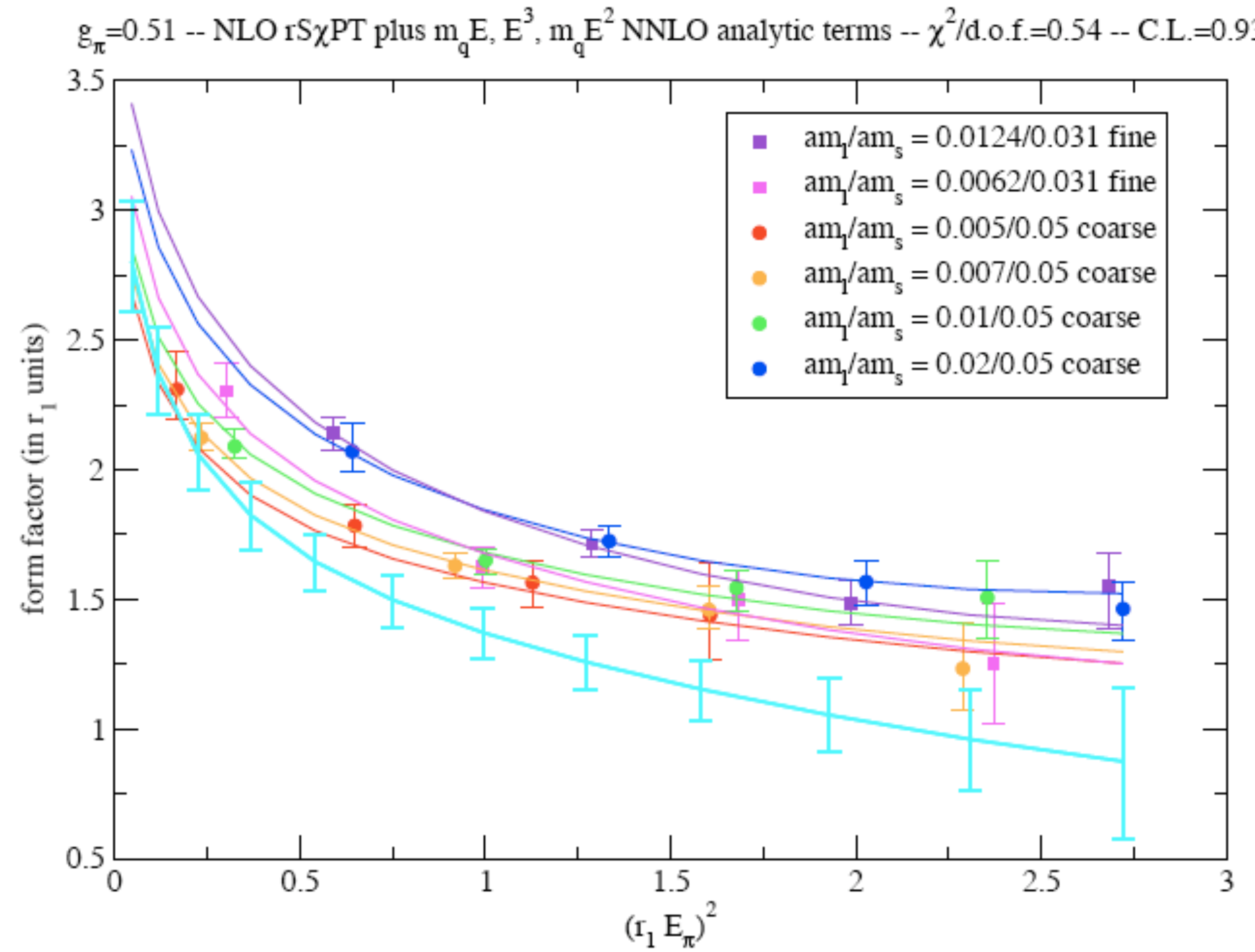
- These expressions come from an expansion in E_π, which is ≈1GeV for our highest-momentum data points that have lattice momentum ap=(1,1,1)
  - Although this is clearly beyond the range of χPT, rSχPT provides the **only theoretically-motivated extrapolation formulae that we have**
  - We therefore add NNLO analytic terms to allow us to fit our entire data set
- Because our statistical errors are large, however, we cannot rule out a simple polynomial fit, and we use the difference from the preferred rSχPT fit to estimate the fitting systematic uncertainty

Chiral-continuum extrapolations of $f_{||}$ and $f_{\perp}$

$f_{\perp}$



$f_{||}$



- Correlated simultaneous fit to all $f_{\perp}$ data using NLO rSχPT
- Separate correlated fit to all $f_{||}$ data using NLO rSχPT plus NNLO analytic terms
- **Cyan** curves show continuum-chiral extrapolated form factors with stat. errors

Systematic errors in f_+

Systematic errors in f_+

Errors shown for a representative value of q^2

source	$f_+(25\ \mathrm{GeV}^2)$
statistics	8%
chiral-cont. extrapolation	7%
discretization effects	4%
matching heavy-light currents	3%
m_b uncertainty	2%
$\hat{m}, m_s$ uncertainty	1%
g_π uncertainty	1%
total	12%

Note that the form factor

$f_\perp$ contributes > 80% of f_+

for all values of q^2

Systematic errors in f_+

- Compare preferred rSχPT fit result with alternative polynomial fit

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- Statistical errors in Z_V^{bb} and Z_V^{ll} plus estimate of higher-order corrections to ρ^{bl}

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- Vary continuum light quark masses over range determined by MILC light pseudoscalar meson fits
- Vary g_π, the B-B*-π coupling, over range determined by fits to experimental D-decay data

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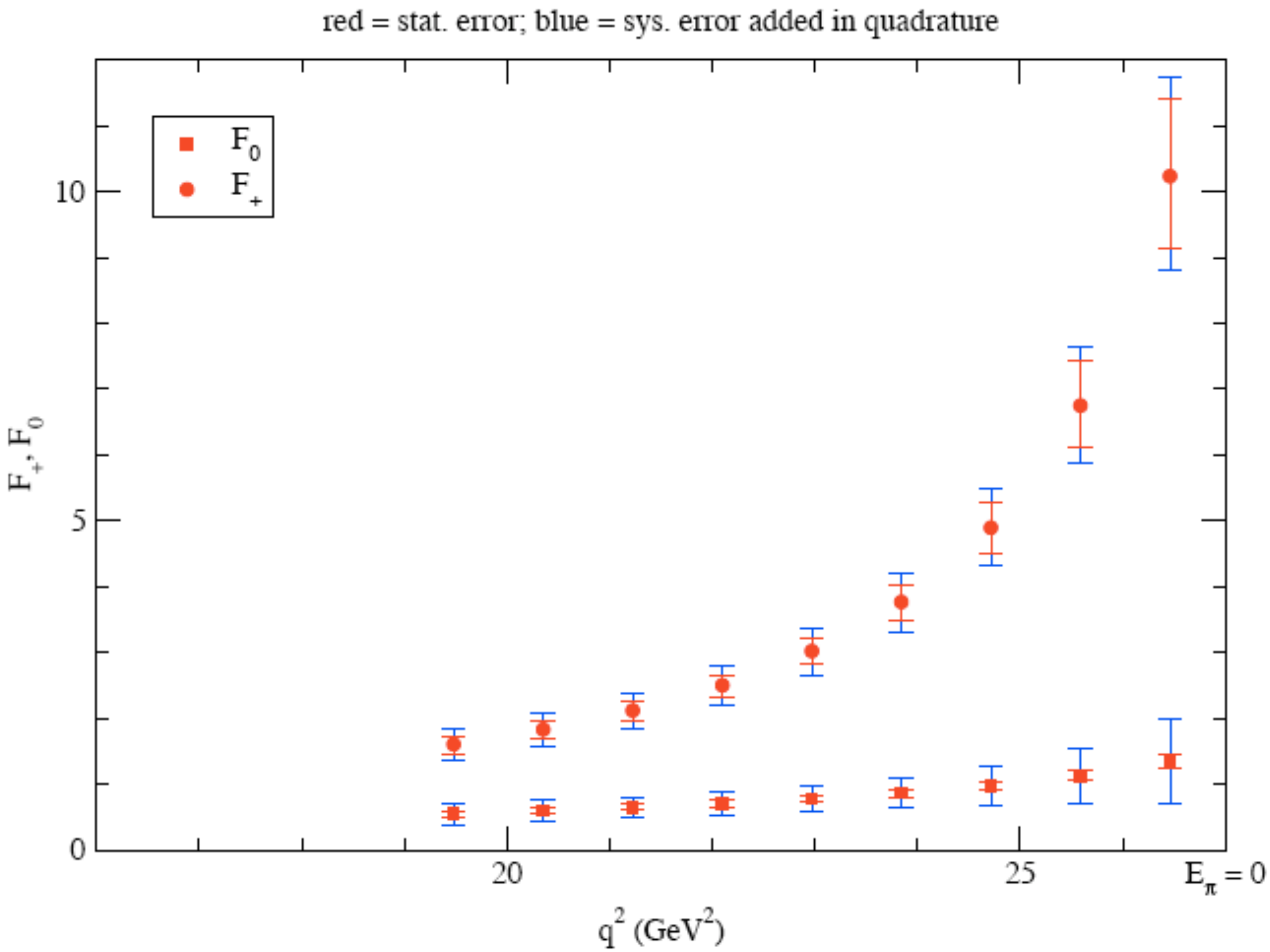
Full error budget for f_+

- Preserve correlations between q^2 bins for later determination of $|V_{ub}|$

q^2 (GeV^2)	26.46	25.59	24.72	23.84	22.97	22.1	21.23	2 0.35	19.48
statistics	11.74	9.58	7.93	6.97	6.74	7.04	7.57	8.13	8.66
χPT fit ansatz	6.58	6.06	6.59	7.26	7.81	8.23	8.6	9.03	9.6
g_π uncertainty	0.44	0.72	1.42	2.0	2.32	2.38	2.25	2.01	1.75
finite volume errors	0.08	0.1	0.03	0.14	0.4	0.68	0.86	0.76	0.13
scale uncertainty	0.0	0.28	0.48	0.62	0.74	0.84	0.95	1. 06	1.19
$\hat{m}$ uncertainty	0.18	0.18	0.18	0.19	0.21	0.22	0.24	0.27	0.29
$m_{strange}$ uncertainty	0.61	0.6	0.61	0.64	0.69	0.75	0. 83	0.92	1.03
m_b uncertainty	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
light quark and gluon discretization errors	2.4	2.4	2.4	2.4	2.4	2. 4	2.4	2.4	2.4
heavy quark discretization errors	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
p-dependent discretization errors	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
nonperturbative Z_V determinations	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4	1.4
perturbative ρ determination	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
total systematic	8.79	8.43	8.91	9.54	10.05	10.42	10.71	11.02	11.45

The $B \to \pi \ell \nu$ form factors f_+ and f_0



✦ For f_+ (the form factor measured by experiments), about 12% errors for all q^2

Determination of $|V_{ub}|$

Exclusive determination of $|V_{ub}|$ from $B \to \pi \ell \nu$

The Problem:

- Traditional lattice QCD methods can only accurately calculate form factors at **high q^2** (low E_π^2)

Current "Solutions":

1. Accept lattice limitations and only compare lattice and experiment in the region where lattice data exists -- extremely conservative

2. Use a model which contains shape information (e.g. BK parameterization) to extrapolate lattice data to zero q^2 -- difficult to quantify systematic errors due to choice of model

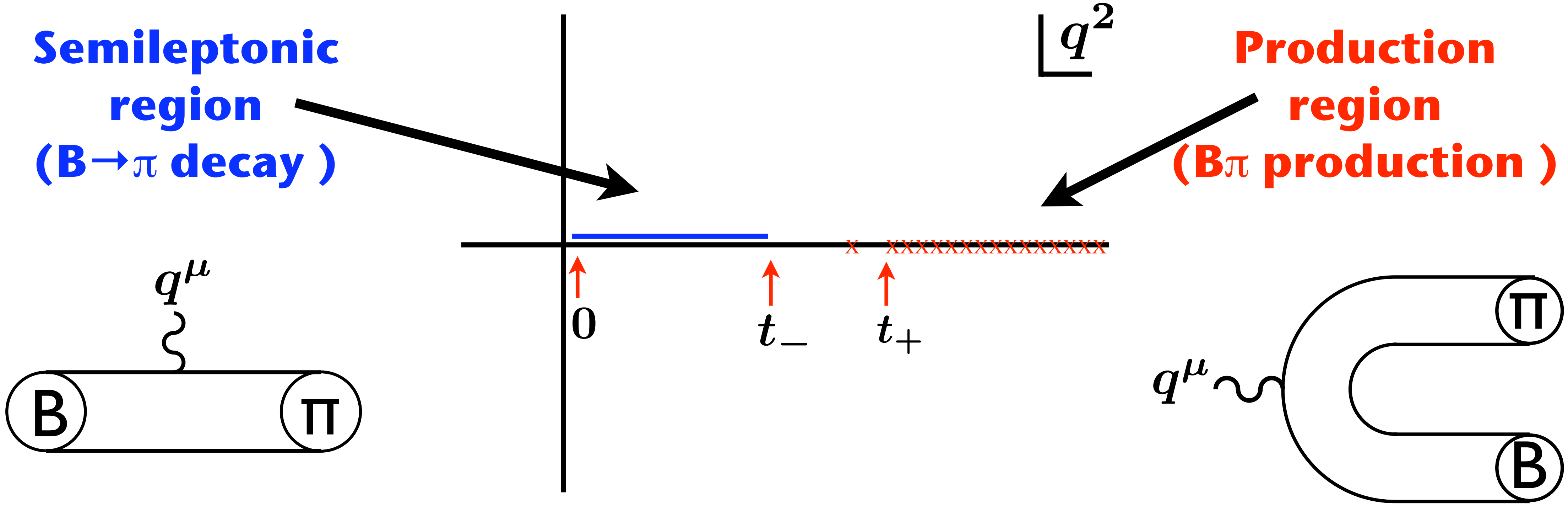
3. Generate lattice data at lower using an alternative method such as Moving NRQCD [**Foley *et. al.***] -- requires additional work

... is this the best we can do?

... or can we get more information about the form factors from the lattice data we already have without introducing model dependence?

Analytic structure of semileptonic form factors

- $f(q^2)$ analytic except when $q^2=m^2$ of a physical state:



$0 < q^2 < t_- \equiv (m_B - m_\pi)^2$ $\qquad$ $q^2 > t_+ \equiv (m_B + m_\pi)^2$

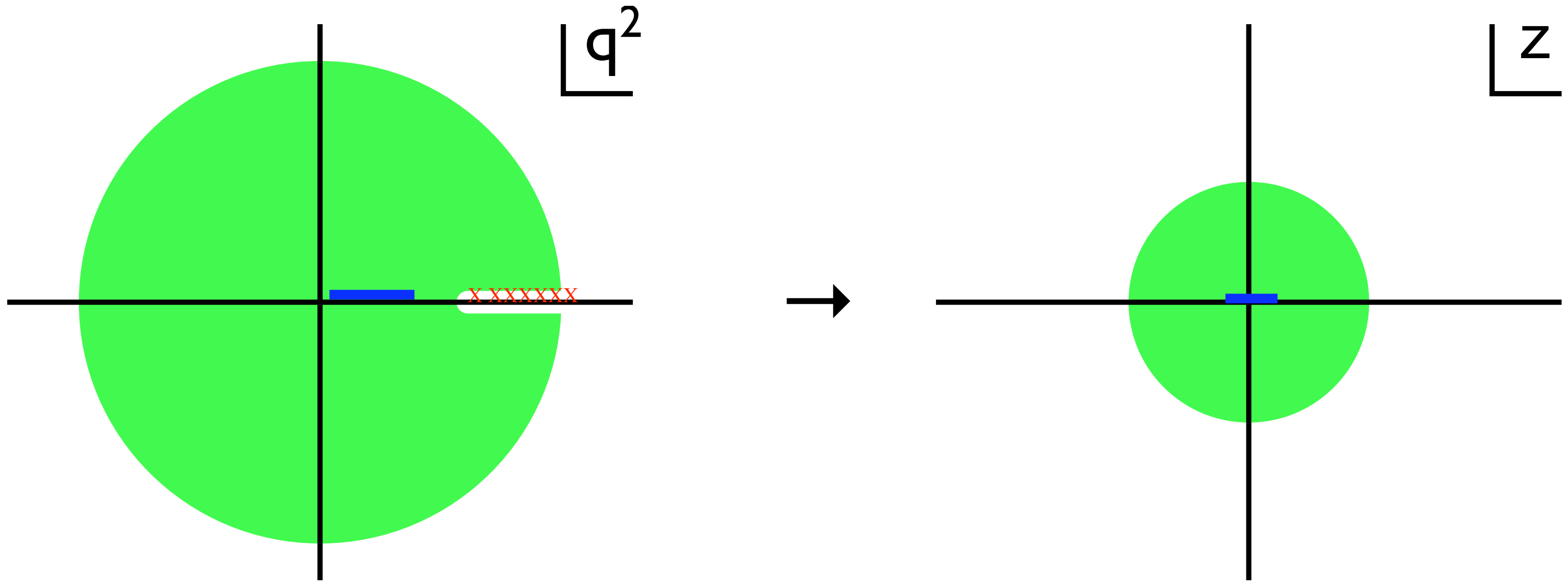
- Therefore $f(q^2)$ analytic below the production region except at the B* pole
- Analytic functions can always be written as convergent power series -- **how can we use this?**

[images courtesy of R. Hill]

A better variable for semileptonic form factors

- Consider mapping the variable q^2 onto a new variable, z, in the following way: $$z = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$$

    - maps **$q^2 > t_+$** (the production region) onto **$|z|=1$**
    - maps **$q^2 < t_+$** (includes the semileptonic region) onto **$z=[-1,1]$**



- Can choose the free parameter t_0 to make the maximium $|z|$ in the semileptonic region as small as possible -- we choose 0.65 t_-

- For semileptonic decays this maps the physical region, **$0 < t < t_-$**, onto:

B→πℓν: −0.34 < z < 0.22
D→πℓν: −0.17 < z < 0.16
D→Kℓν: −0.04 < z < 0.06
B→Dℓν: −0.02 < z < 0.04

z-expansion of semileptonic form factors

- In terms of z, form factors have simple form:

$$P(t)\,\phi(t,t_0)\,f(t) = \sum_{k=0}^{\infty} a_k(t_0) z(t,t_0)^k$$

Vanishes at subthreshold (e.g. B*) poles

"Arbitrary" analytic function -- choice only affects particular values of coefficients (a's)

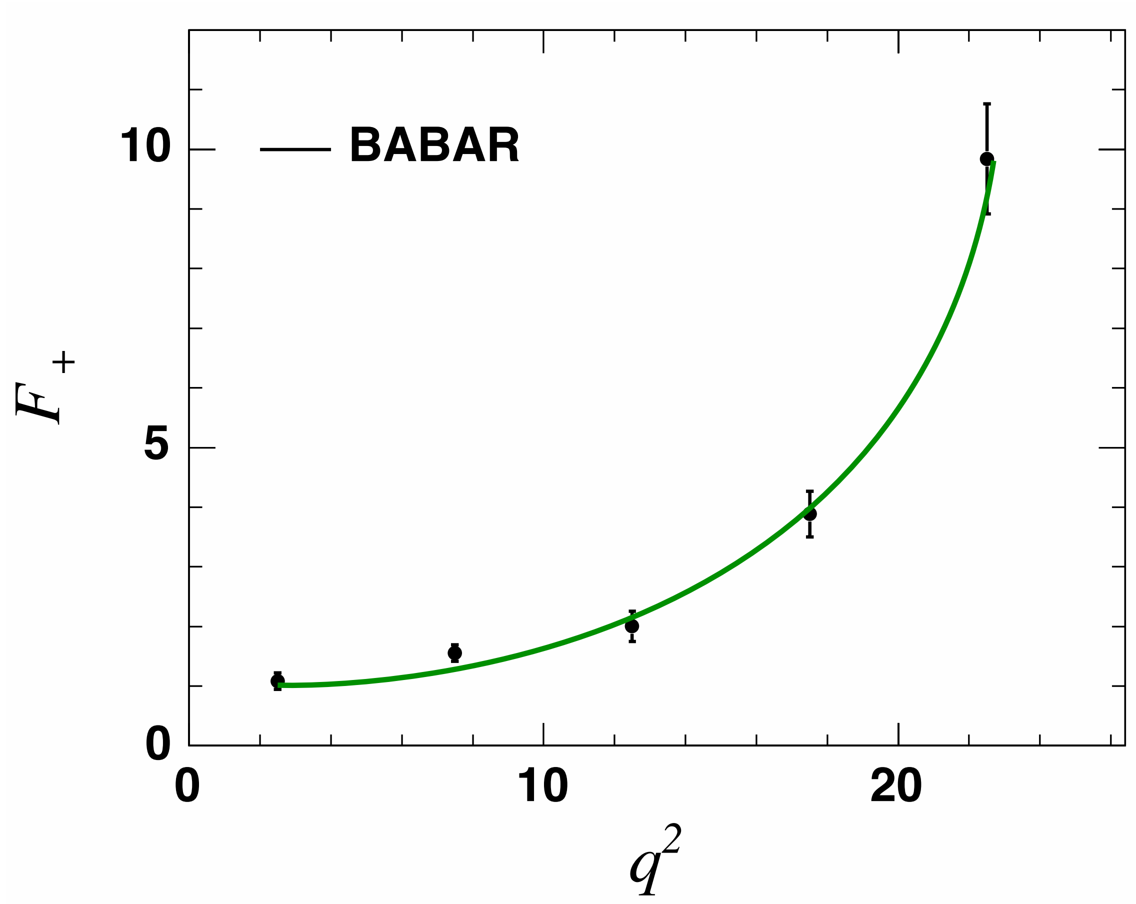
- Unitarity constrains the size of the coefficients:

Constraint holds for any value of N

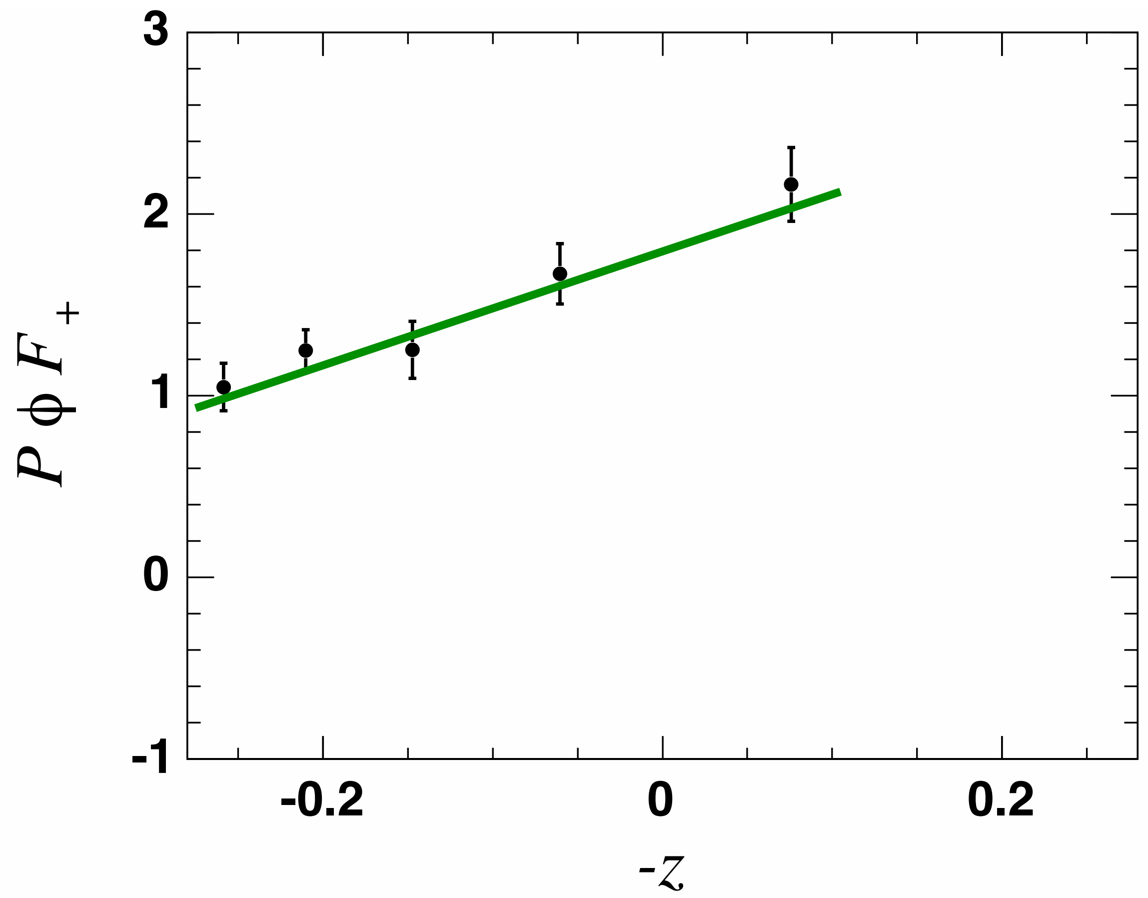
$$\sum_{k=0}^{N} a_K^2 \leq 1$$

- Thus, in combination with the small range of |z|, **one needs only a small number of parameters** to obtain the form factors to a high degree of accuracy

Effect of z-remapping on $B\to\pi\ell\nu$ form factor



Striking curvature in $B\to\pi\ell\nu$ form factor data versus q^2



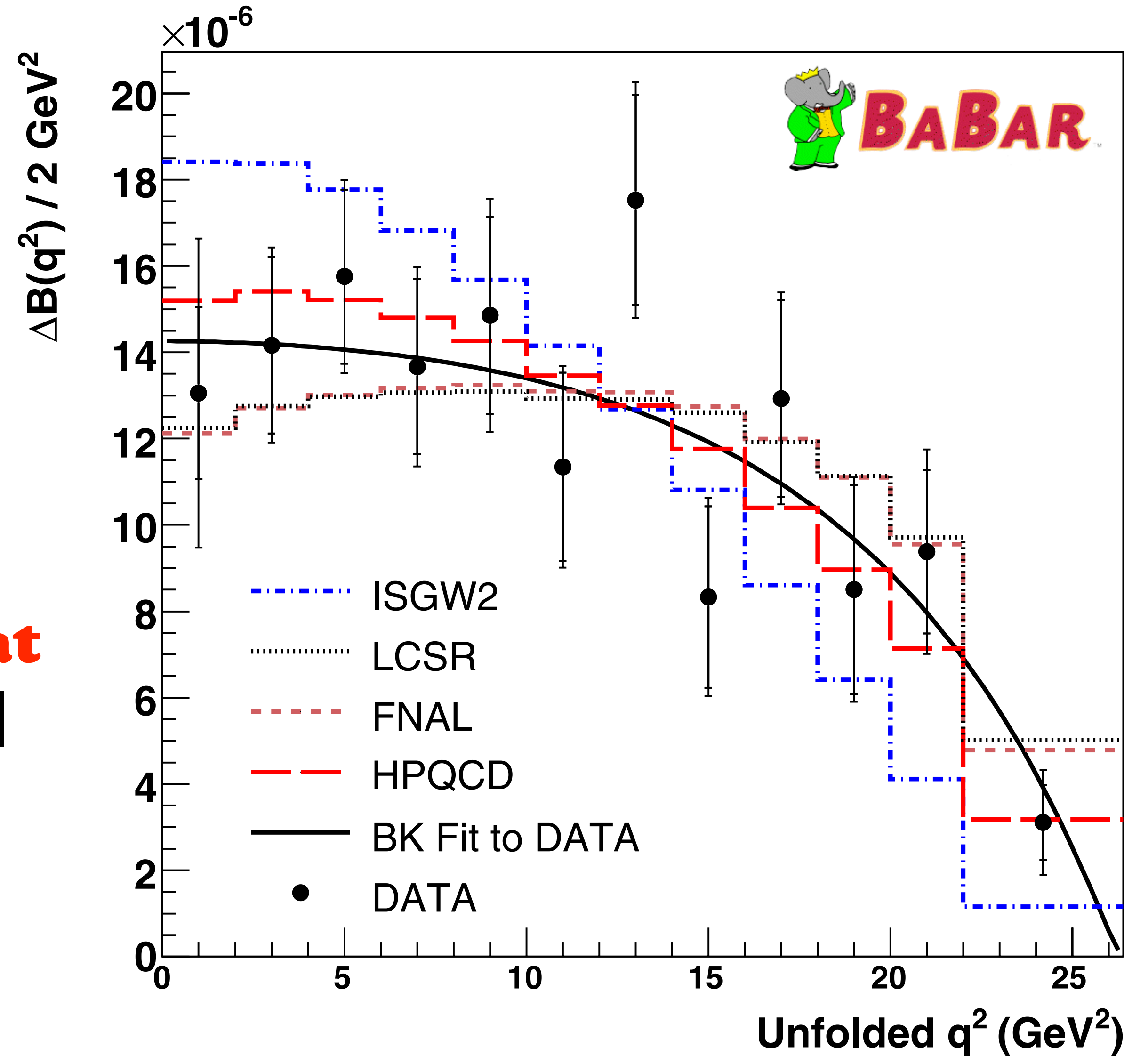
No visible curvature after remapping

- Curvature in data due to well-understood perturbative QCD effects
- Data completely described by a **normalization** and a **slope**, and constrains the size of possible curvature

12-bin BABAR determination of $B \to \pi \ell \nu$

- In 2007 BABAR published **the best determination of the shape of the $B \to \pi l \nu$ form factor yet** with measurements in 12 separate q^2 bins [cite]

- This suggests that **lattice QCD need only provide a precise normalization at one q^2 value** in order to determine $|V_{ub}|$



- In order to minimize the error in $|V_{ub}|$, however, we propose a slightly more sophisticated approach for combining lattice and experimental data . . .

The program for lattice and experiment

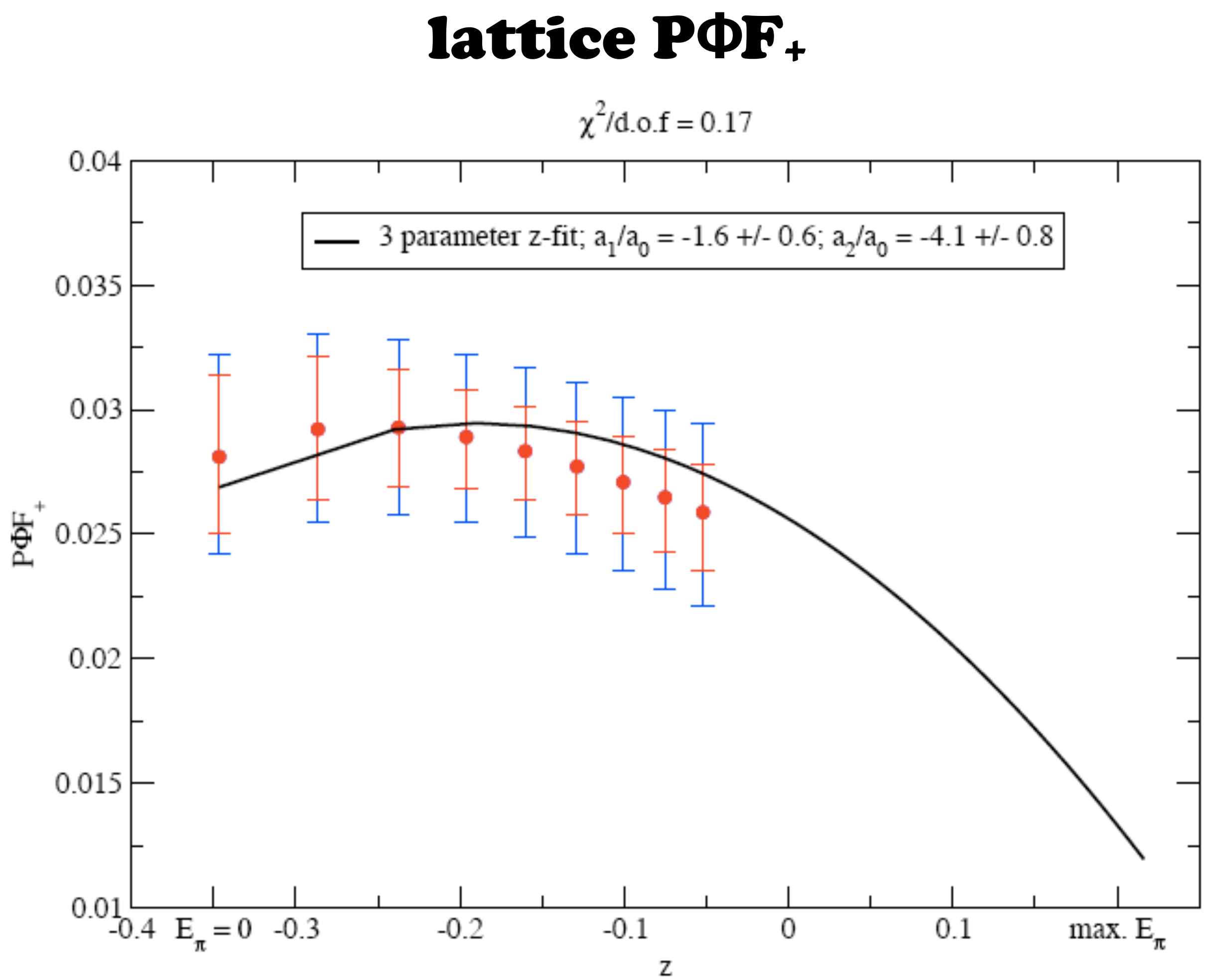
1. Fit experimental and lattice data in terms of z expansion
2. Determine and compare the slopes (and curvature) in z
3. If consistent, fit lattice and experimental data simultaneously with an unknown relative offset to determine $|V_{ub}|$

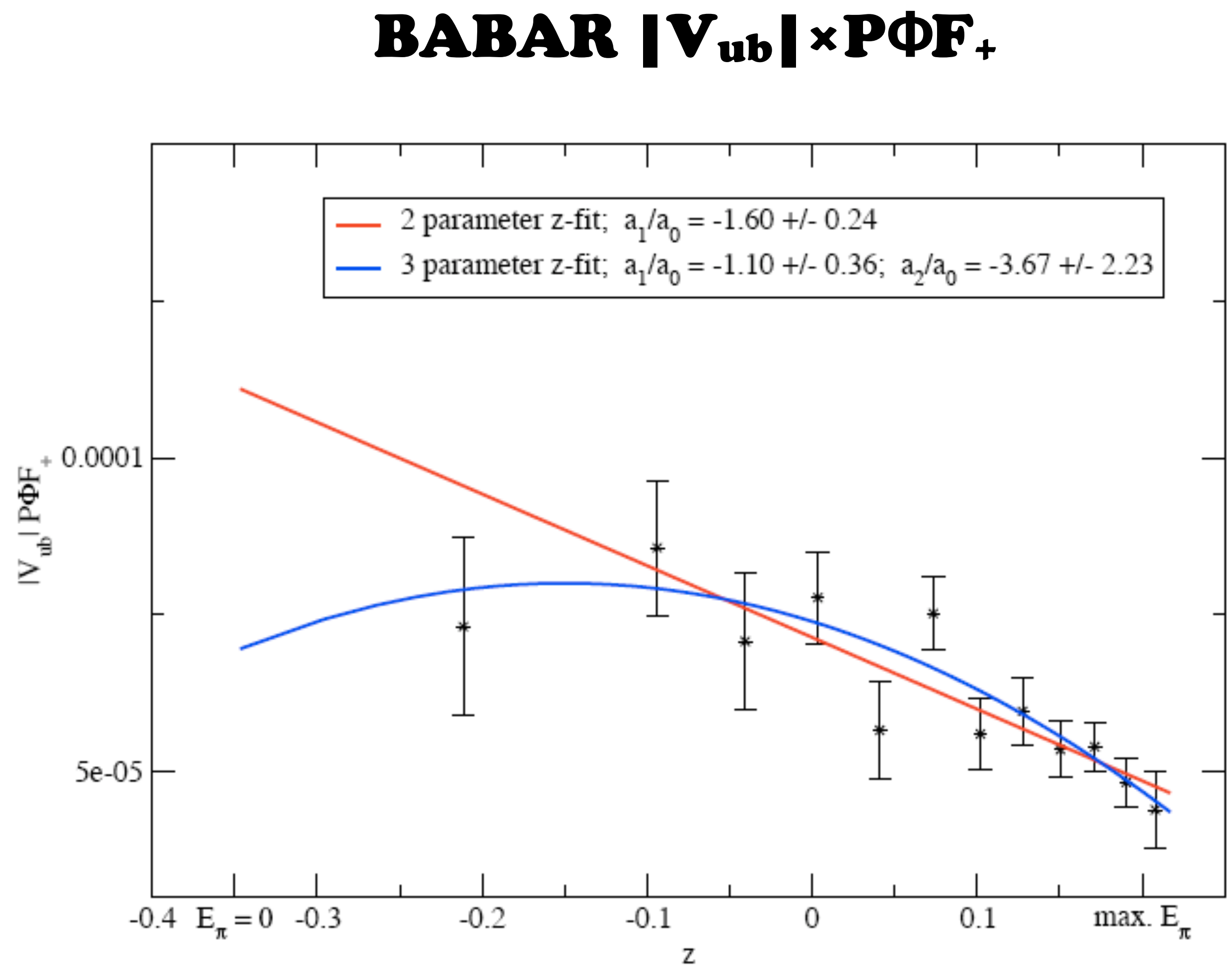
Advantages to this approach:

- **Model-independent**
- Can quantify the agreement between lattice and experiment using slope measurements
- **Systematically improvable** -- as data gets more precise can add more terms in z
- **Minimizes error in |Vub|** by using all of the lattice and experimental data in a single fit

. . . but does it work???

Consistency check: separate z-fits



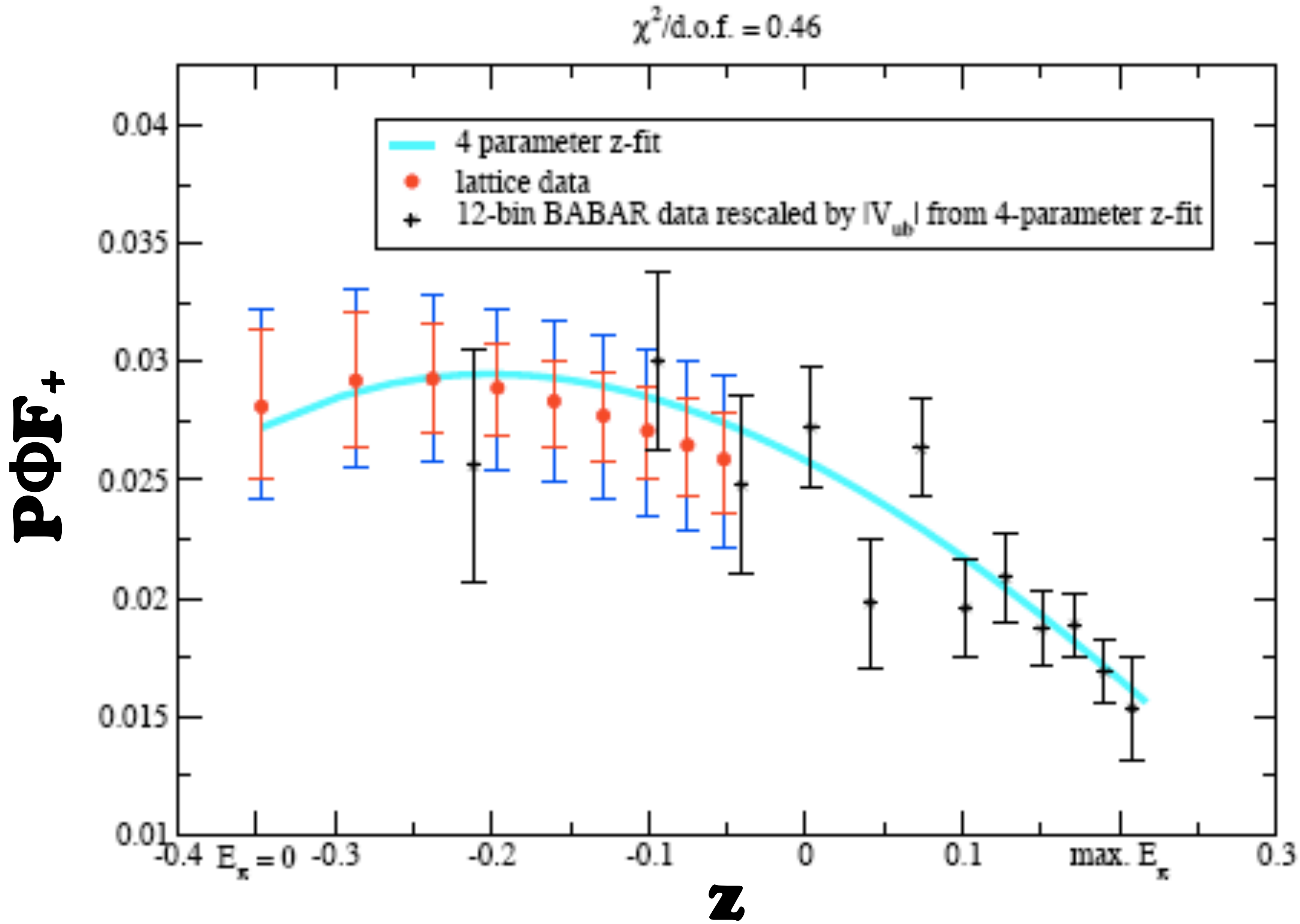


- Lattice data determines both the slope and curvature
- Experimental data consistent with zero curvature
- Lattice and experimental slope and curvature **agree within uncertainties**

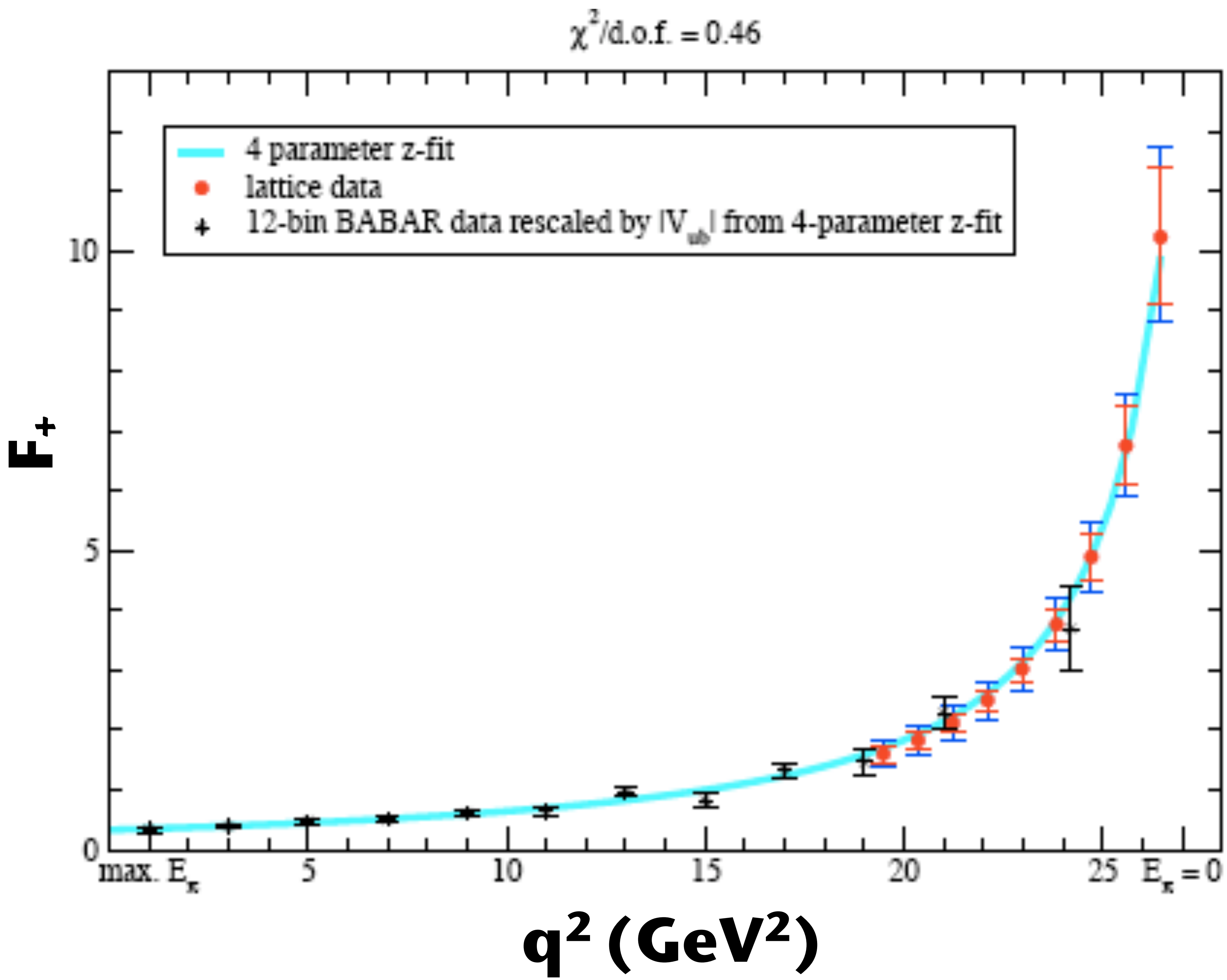
⇒ Proceed to simultaneous fit of lattice and experimental data

Simultaneous z-fit to determine $|V_{ub}|$

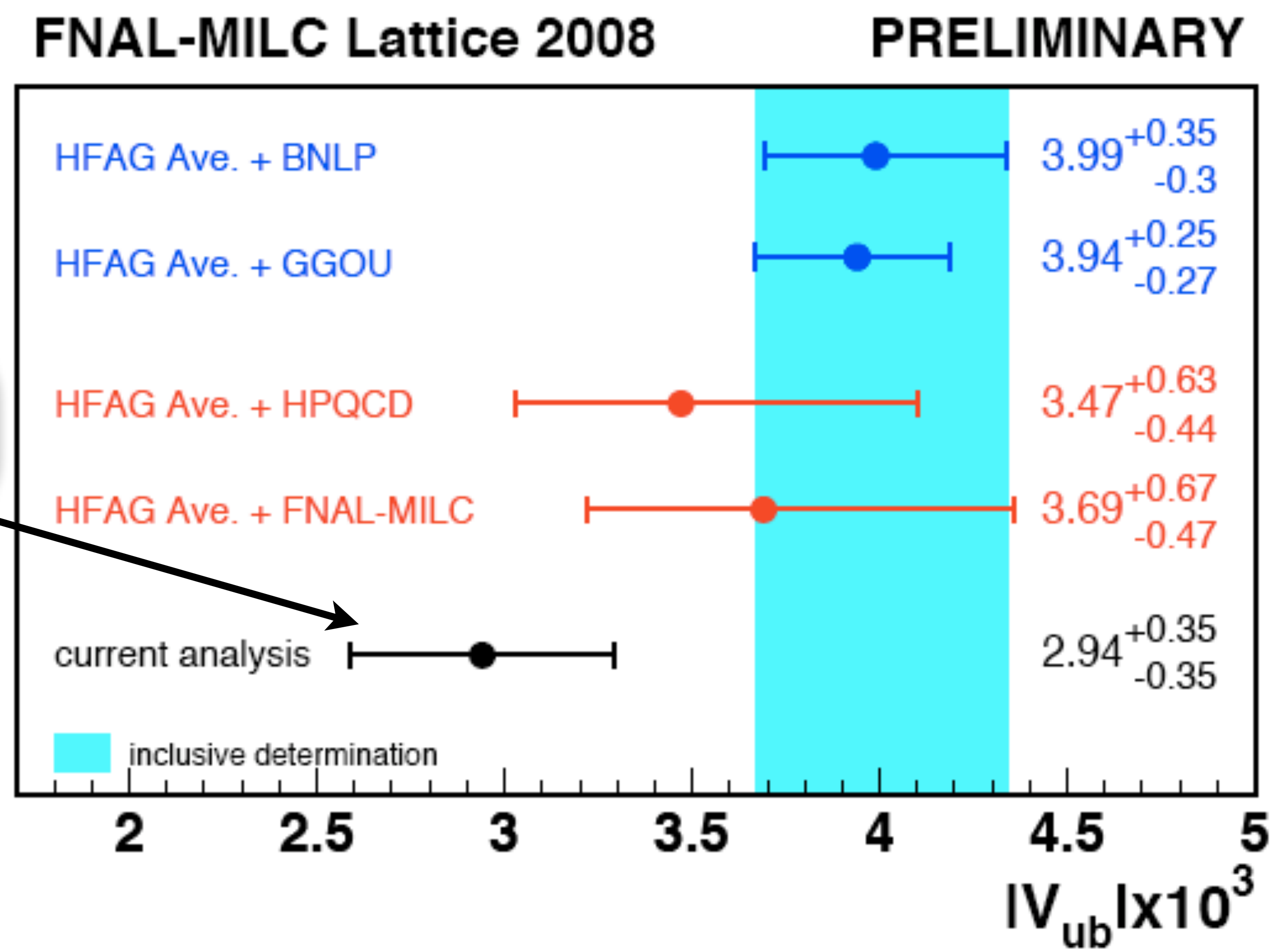
- Fit lattice and 12-bin BABAR experimental data together to z-expansion leaving relative normalization factor ($|V_{ub}|$) as a free parameter



Simultaneous fit in standard variables



Preliminary result for $|V_{ub}|$



$$|V_{ub}| \times 10^3 = 2.94 \pm 0.35$$

(~12% total error)

(~15% errors)

- Consistent with exclusive determinations from 2+1 flavor lattice QCD
  - **Error in $|V_{ub}|$ reduced primarily by combined z-fit** to lattice and experimental data
- Approximately 2-σ below inclusive determinations

Summary and outlook

- We have calculated the $B\to\pi\ell\nu$ form factor $f_+(q^2)$ in 2+1 flavor lattice QCD
  - Extrapolated data to the continuum limit and physical quark masses with **simultaneous fit to all $f_{||}$ ($f_\perp$) data using rSχPT** expressions
- Exclusive determination of $|V_{ub}|$ limited by the ability of lattice QCD to accurately calculate form factors at low q^2
  - Typically dealt with by using a model to input information about the shape of $f_+(q^2)$ versus. q^2 -- this introduces unknown systematic error
  - **Analyticity**, **unitarity**, and **heavy quark physics** can be combined to constrain the shape of semileptonic form factors in a **model-independent** way using only a small number of fit parameters
- We have **determined $|V_{ub}|$ to ~12% accuracy** with a simultaneous fit to the lattice and 12-bin BABAR data using the z-expansion
  - With this method, can extract $|V_{ub}|$ to improved accuracy using the lattice $B\to\pi\ell\nu$ form factor numerical data that we already have!!!