Strange quark content of the nucleon

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Overview

- Motivation
  - Experimental status of $G_E^s(Q^2), G_M^s(Q^2), G_A^s(Q^2)$
  - f_{Ts} and dark matter detection
- Method
  - Calculating the disconnected insertion
  - Extracting the form factors
- Preliminary results
- Conclusion

Basic problem

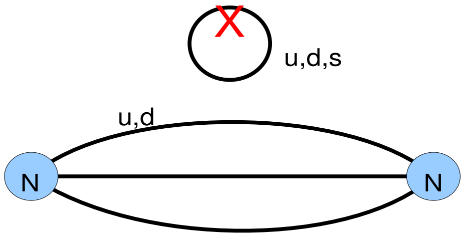
- Our basic goal is to calculate matrix elements of the form

$$\langle N(t)|\bar{s}\Gamma s(t')|N(t_0)\rangle$$

 corresponding to the strange quark contribution to form factors of the nucleon:

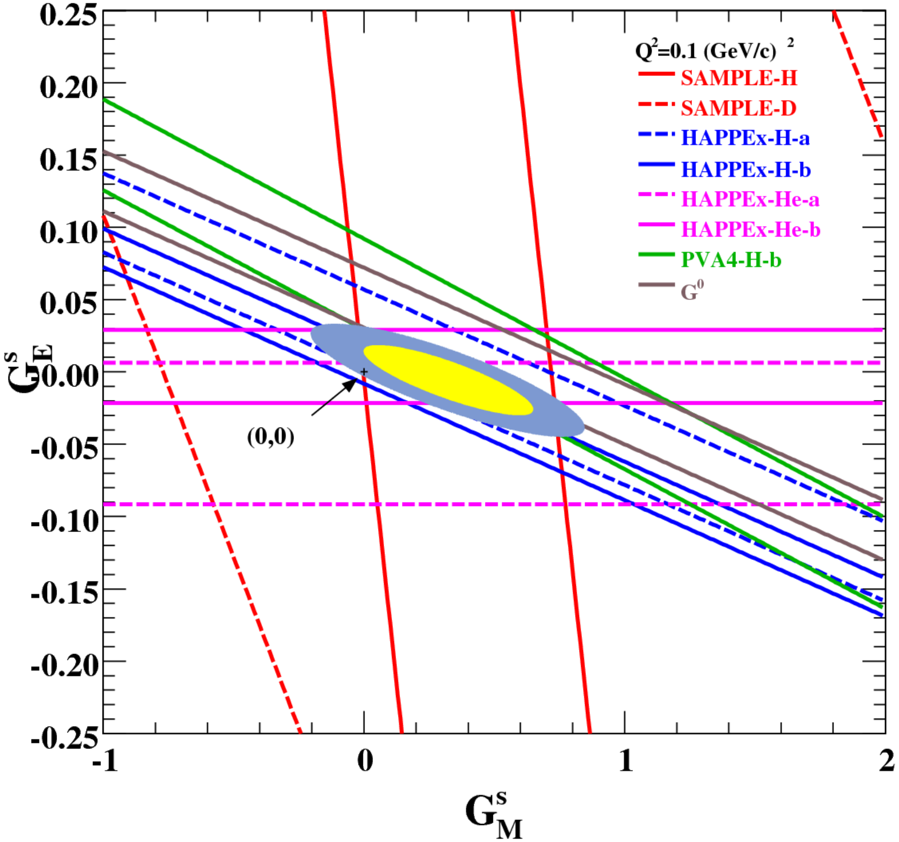
$$\Gamma = \begin{cases} \gamma_\mu & \to G_E^s(Q^2),\ G_M^s(Q^2) \\ \gamma_\mu\gamma_5 & \to G_A^s(Q^2) \\ I & \to G_S^s(Q^2) \end{cases}$$

- For $Q^2=0$,
  - $G_E^s(0) = 0, \quad G_M^s(0)$ is the strange magnetic moment
  - $G_A^s(0) = \Delta s$ is the strange contribution to the nucleon spin
  - $G_S^s(0) = \langle N|\bar{s}s|N\rangle$ is discussed below
- Strange form factors are purely disconnected:



Experiment

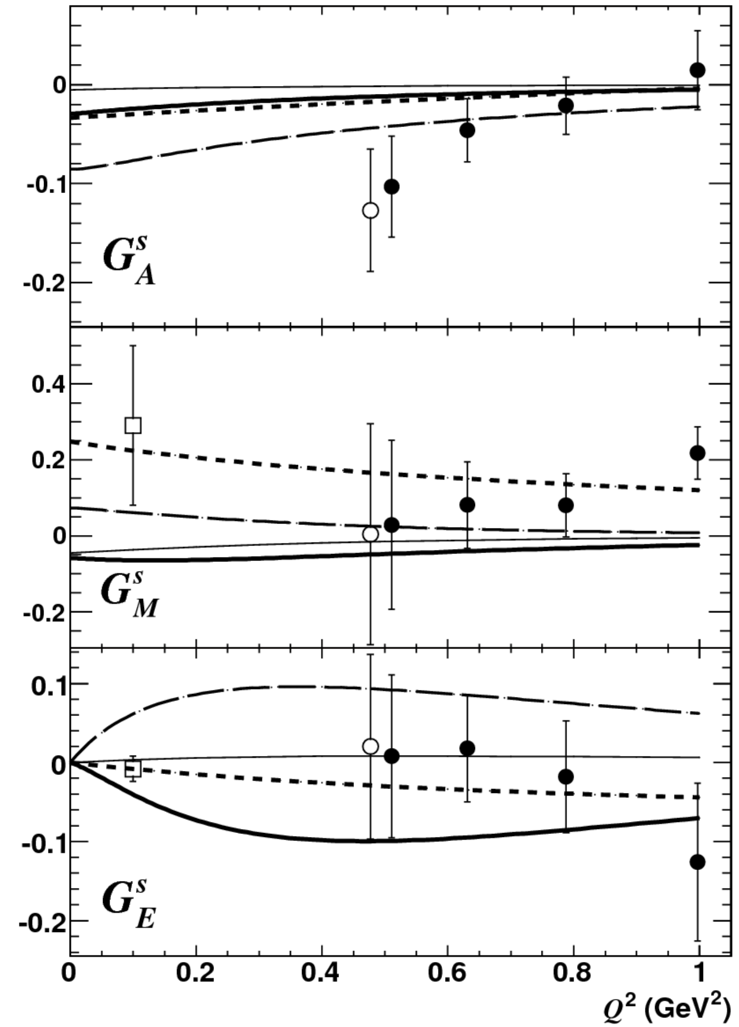
Parity-violating electron scattering
(SAMPLE, HAPPEx, PVA4, G0)



J. Liu et al., arXiv:0706.0226 [nucl-ex]

(see also Young et al., nucl-ex/0605010)

PVES + BNL E734 (νp scattering)



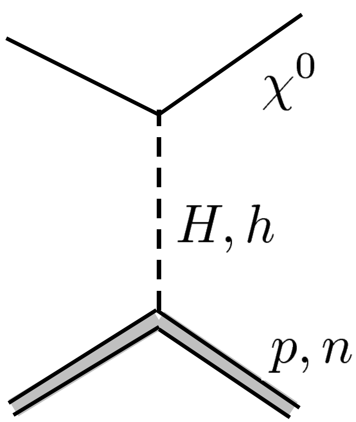
Pate et al., arXiv:0805.2889 [hep-ex]

Direct detection of dark matter

- The strange scalar matrix element is a major uncertainty in the interpretation of dark matter experiments, entering through the parameter

$$f_{Ts} = \frac{m_s \langle N | \bar{s} s | N \rangle}{M_N}$$

- In SUSY, the neutralino scatters from a nucleon via Higgs exchange:



$$\sigma^2 \sim \lambda^2_{Hpp} \sim f^2_{Ts}$$

- Uncertainty in f_{Ts} gives up to a factor of 4 uncertainty in the cross-section! (Bottino et al., hep-ph/0111229; Ellis et al., hep-ph/0502001)

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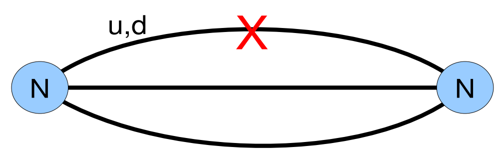
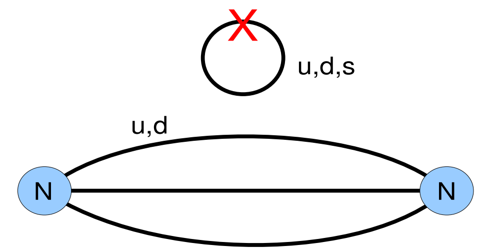
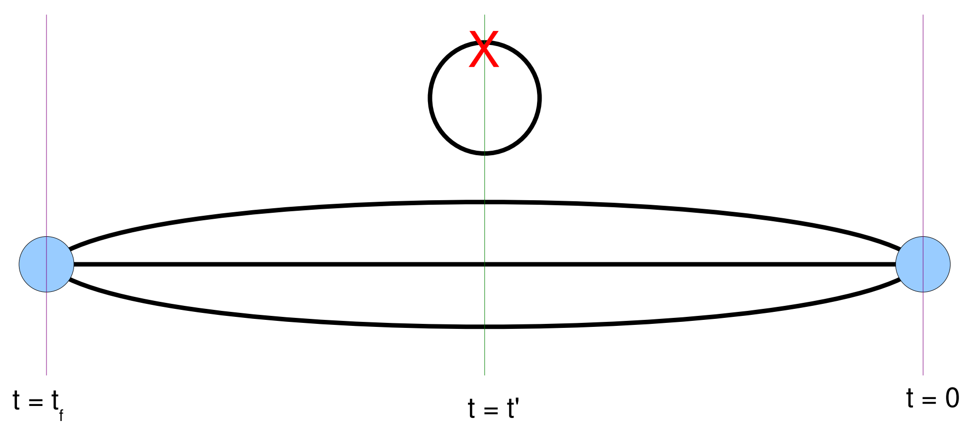
Disconnected Diagrams

Connected vs. Disconnected





Our matrix elements have the general form



$$\langle N(t_f,\vec{q})| \qquad \sum_{\vec{x}} e^{-i\vec{q}\cdot\vec{x}}\,\bar{s}(t',\vec{x})\,\Gamma\, s(t',\vec{x}) \qquad |N(0,\vec{0})\rangle$$

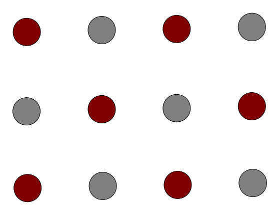
Trace estimation

- Standard method is stochastic:

$$\mathrm{Tr}(\Gamma D^{-1}) \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^\dagger \Gamma D^{-1} \eta_i, \quad \langle \eta_{(x)}^\dagger \eta_{(y)} \rangle = \delta_{x,y}$$

- Induces error due to off-diagonal terms.
- Error may be greatly reduced by employing “dilution,” i.e. by dividing the stochastic source into subsets and inverting on these to reduce contamination.

e.g. even/odd:

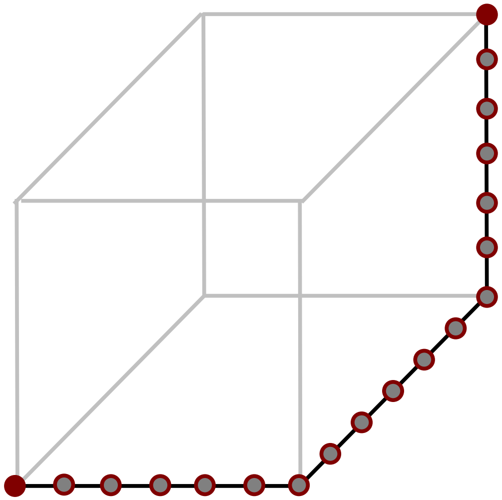


(Foley et al., hep-lat/0505023, generalization of color/spin “partitioning” used earlier)

$$\mathrm{Tr}(\Gamma D^{-1}) \approx \frac{1}{N}\sum_{i=1}^{N} \eta_i^{(e)\dagger} \Gamma D^{-1} \eta_i^{(e)} + \frac{1}{N}\sum_{i=1}^{N} \eta_i^{(o)\dagger} \Gamma D^{-1} \eta_i^{(o)}$$

Trace estimation

- Two sources of error: gauge noise and error in trace. In this calculation, we largely eliminate the second source by calculating a "nearly exact" trace on four time-slices.
- 864 sources (x12 for color/spin). A given source is nonzero on 4 sites on each of 4 time-slices.
- Minimal spatial separation between sites is $6\sqrt{3}a_s$. Small residual contamination is gauge-variant and averages to zero.
- Equivalent to using a single stochastic source with "extreme dilution."



Simulation parameters

- Configurations were provided by the LHPC "Spectrum Collaboration"
- $24^3 \times 64$ anisotropic lattice with $a_s = 0.108(7) \approx 3a_t$
- 2 dynamical flavors, Wilson fermion and gauge actions
- 863 configurations
- $M_\pi \approx 400 \text{ MeV}$

- 64 (x 12) inversions per configuration at the light quark mass, for the nucleon correlators
- 864 (x 12) inversions per configuration at the strange mass, for the trace

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Basic ingredients

- We define the usual nucleon correlator,

$$G^{(2)}(t,t_0;\vec{q}) = (1+\gamma_4)^{\alpha\beta}\sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}}\langle P^\beta(\vec{x},t)\bar{P}^\alpha(\vec{0},t_0)\rangle\,,$$

- as well as a three-point function for the disconnected scalar form factor,

$$G_S^{(3)}(t,t',t_0;\vec{q}) = (1+\gamma_4)^{\alpha\beta}\sum_{\vec{x},\vec{x}'} e^{i\vec{q}\cdot\vec{x}'}\langle P^\beta(\vec{x},t)[\bar{\psi}\psi(\vec{x}',t') - \langle\bar{\psi}\psi(\vec{x}',t')\rangle]\bar{P}^\alpha(\vec{0},t_0)\rangle,$$

- likewise for the electric,

$$G_E^{(3)}(t,t',t_0;\vec{q}) = (1+\gamma_4)^{\alpha\beta}\sum_{\vec{x},\vec{x}'} e^{i\vec{q}\cdot\vec{x}'}\langle P^\beta(\vec{x},t)[V_4(\vec{x}',t') - \langle V_4(\vec{x}',t')\rangle]\bar{P}^\alpha(\vec{0},t_0)\rangle,$$

- and for the axial,

$$G_A^{(3)}(t,t',t_0;\vec{q}) = \frac{1}{3}\sum_{i=1}^{3}\sum_{\vec{x},\vec{x}'} e^{i\vec{q}\cdot\vec{x}'}[-i(1+\gamma_4)\gamma_i\gamma_5]^{\alpha\beta} \times\langle P^\beta(\vec{x},t)[A_i(\vec{x}',t') - \langle A_i(\vec{x}',t')\rangle]\bar{P}^\alpha(\vec{0},t_0)\rangle$$

Basic ingredients

$$G_S^{(3)}(t,t',t_0;\vec{q}) = (1+\gamma_4)^{\alpha\beta} \sum_{\vec{x},\vec{x}'} e^{i\vec{q}\cdot\vec{x}'} \langle P^\beta(\vec{x},t)[\bar{\psi}\psi(\vec{x}',t') - \langle\bar{\psi}\psi(\vec{x}',t')\rangle]\bar{P}^\alpha(\vec{0},t_0)\rangle,$$

$$G_E^{(3)}(t,t',t_0;\vec{q}) = (1+\gamma_4)^{\alpha\beta} \sum_{\vec{x},\vec{x}'} e^{i\vec{q}\cdot\vec{x}'} \langle P^\beta(\vec{x},t)[V_4(\vec{x}',t') - \langle V_4(\vec{x}',t')\rangle]\bar{P}^\alpha(\vec{0},t_0)\rangle,$$

$$G_A^{(3)}(t,t',t_0;\vec{q}) = \frac{1}{3}\sum_{i=1}^{3}\sum_{\vec{x},\vec{x}'} e^{i\vec{q}\cdot\vec{x}'}[-i(1+\gamma_4)\gamma_i\gamma_5]^{\alpha\beta} \times\langle P^\beta(\vec{x},t)[A_i(\vec{x}',t') - \langle A_i(\vec{x}',t')\rangle]\bar{P}^\alpha(\vec{0},t_0)\rangle$$

- V_i and A_i are the point-split currents for the Wilson action.
- We always use the vacuum-subtracted value of the current, $[J(\vec{x},t) - \langle J(\vec{x},t)\rangle]$, even though this is only strictly necessary for the scalar density (since $\langle J(\vec{x},t)\rangle = 0$ for the others).
  - Empirically, we find that it makes little difference at $q^2=0$, but statistical errors are noticeably reduced for the vacuum-subtracted quantity at higher momenta.

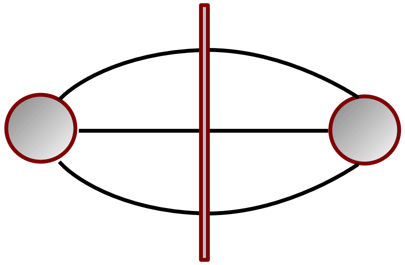
Ratio approach

- Conventionally, one extracts the (e.g. zero-momentum) form factor from the large *t* behavior of the ratio

$$R_X(t, t', t_0; q^2 = 0) = \frac{G_X^{(3)}(t, t', t_0; \vec{0})}{G^{(2)}(t, t_0; \vec{0})} \to G_X^s(q^2), \qquad X = S, E, A$$

(or from a similar expression integrated over time).

- Instead, we fit the numerator directly, since this allows us
  - to avoid contamination from backward-propagating states, which are problematic due to the short temporal extent of our lattice ($64a_t \approx 21a_s$).
  - to explicitly take into account the contribution of (forward-propagating) excited states.
- In the following, we always treat the system symmetrically with $\Delta t \equiv (t - t') = (t' - t_0)$



Direct fit

- First, we perform a fit to the nucleon two-point function, of the form

$$G^{(2)}(t,0;\vec{0}) \approx c_1^2 e^{-m_1 t} + c_2^2 e^{-m_2 t} + c_b^2 e^{-m_b(L_t - t)}\,.$$

  - The coefficients and masses are very well-determined, since we are required to calculate correlators from all initial times (a total of 863 x 64 = 55,232).

- Next, we perform a fit to the three-point function,

$$G^{(3)}(\Delta t, 0, -\Delta t;\vec{0}) \approx j_1 c_1^2 e^{-2m_1\Delta t} + 2\,\mathrm{Re}(j_{12}) c_1 c_2 e^{-(m_1+m_2)\Delta t} + j_2 c_2^2 e^{-2m_2\Delta t}\,.$$

- Here j_1 and j_2 are the form factors for the proton and its first excited state, and j_{12} is a transition matrix element between them. In practice, we expect j_2 and j_{12} to absorb the contribution of still higher states, and trust only j_1 to be reliable.

Direct fit

- Finally, for the purpose of plotting our results, we normalize them by the fit to the proton correlator, with the backward-propagating term removed,

$$G^{(2)}(t,0;\vec{0}) \approx c_1^2 e^{-m_1 t} + c_2^2 e^{-m_2 t} + \cancel{c_b^2 e^{-m_b(L_t - t)}}\,.$$

- Such a plot should exhibit a plateau at large times where only the ground state contributes.
- In practice, the plateau may be short due to the onset of statistical errors. This motivates the two-state fit, which we find to be robust.

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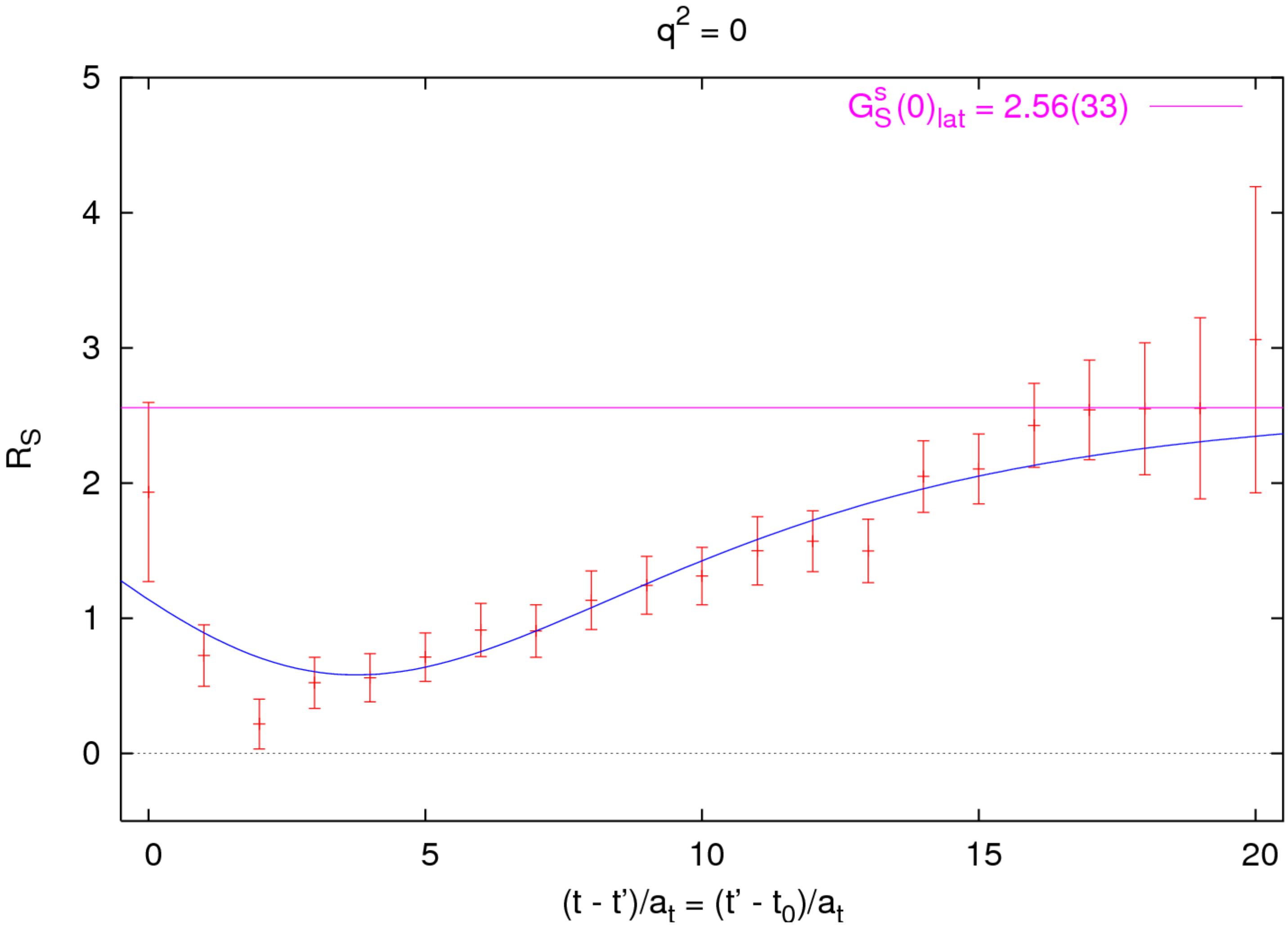
Strange scalar form factor



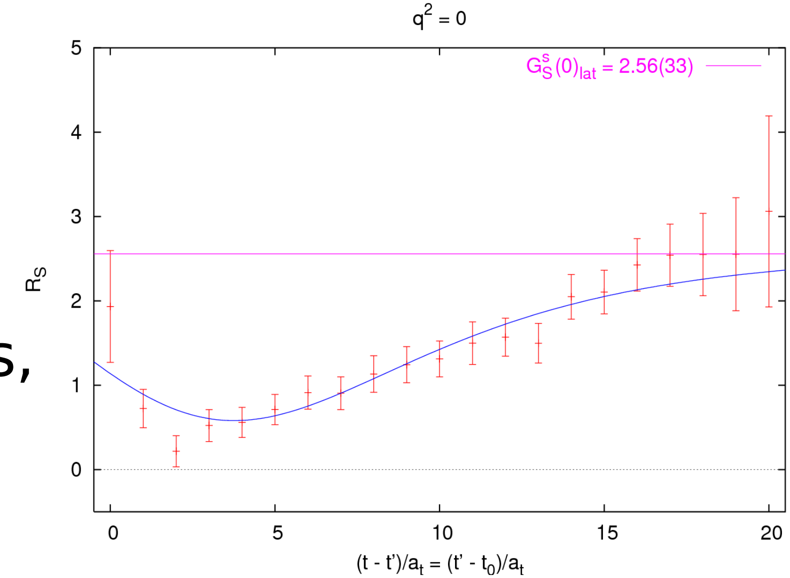
Strange scalar form factor

- For the renormalization-invariant quantity f_{Ts}, we estimate

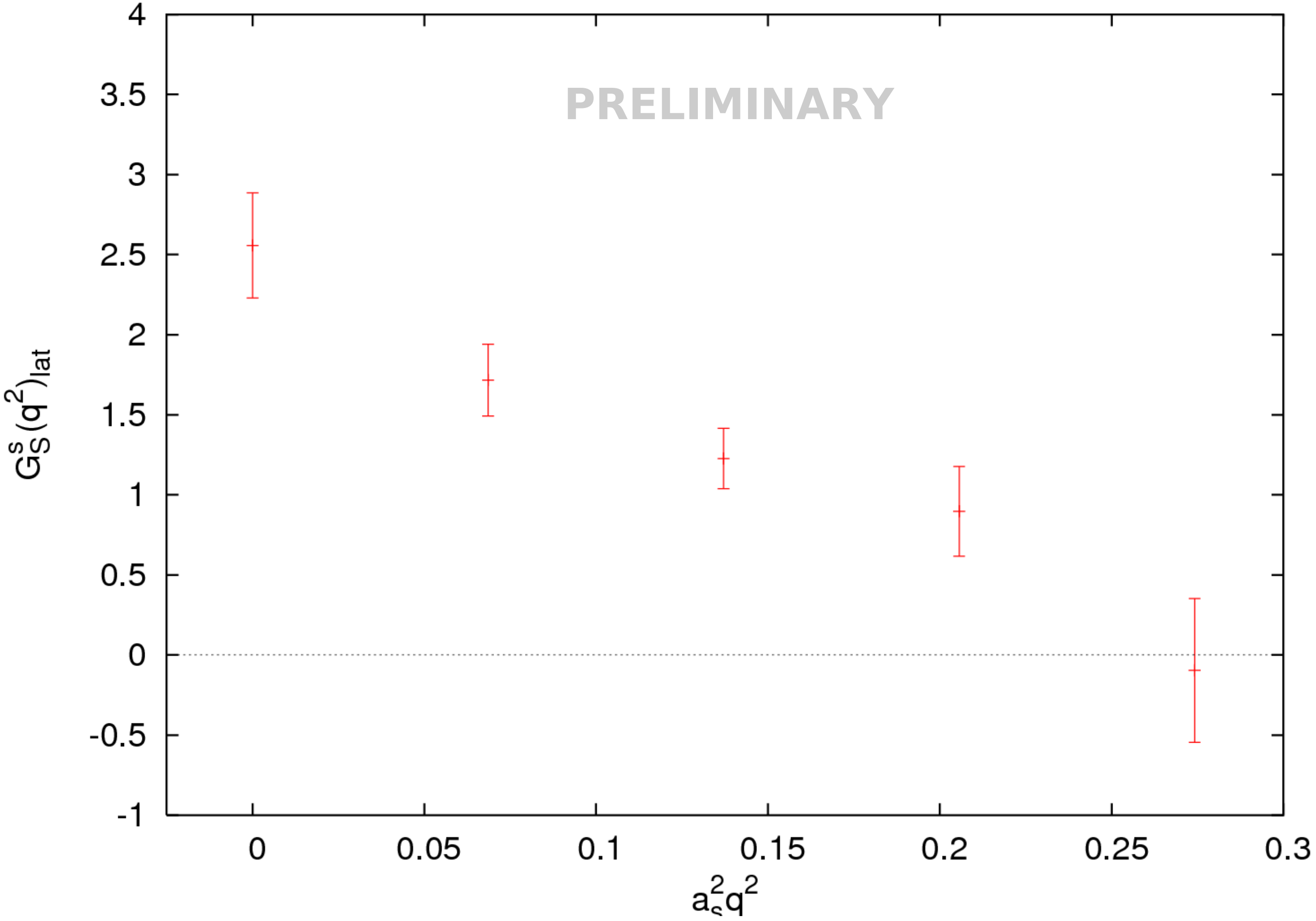
$$f_{Ts} = \frac{m_s \langle N | \bar{s}s | N \rangle}{M_N} = 0.48(7)(3)$$

  where we have inserted the physical nucleon mass. The second error is the uncertainty in relating this mass to the lattice scale, the first error is statistical, and no other systematics are included.

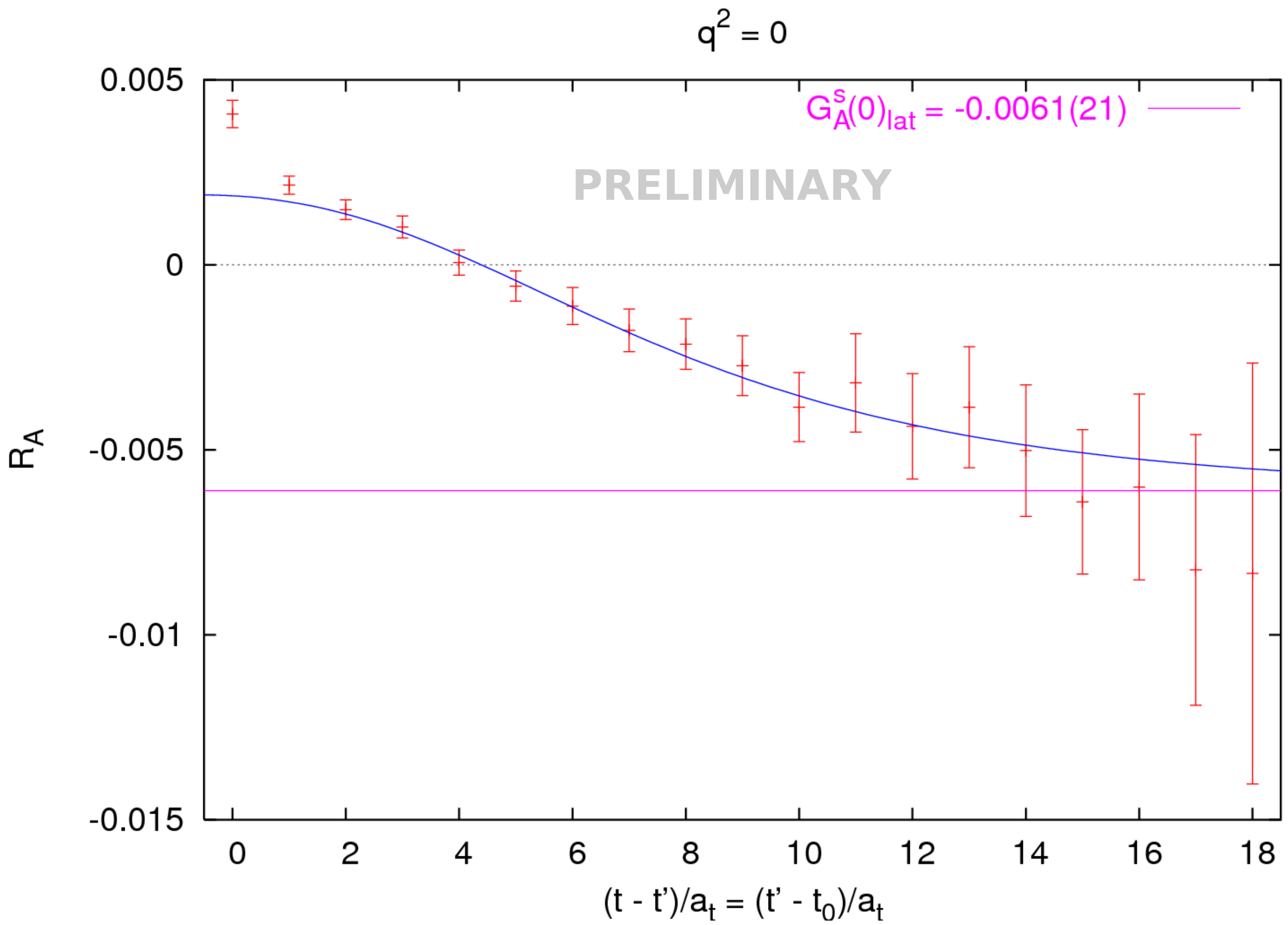
- Note that the matrix element in the numerator was calculated for a world with a 400 MeV pion. If we work consistently in such a world by inserting our calculated nucleon mass, the scale dependence drops out, and we find $f_{Ts} = 0.40(6)$.



Momentum dependence of $G_S^s(q^2)$

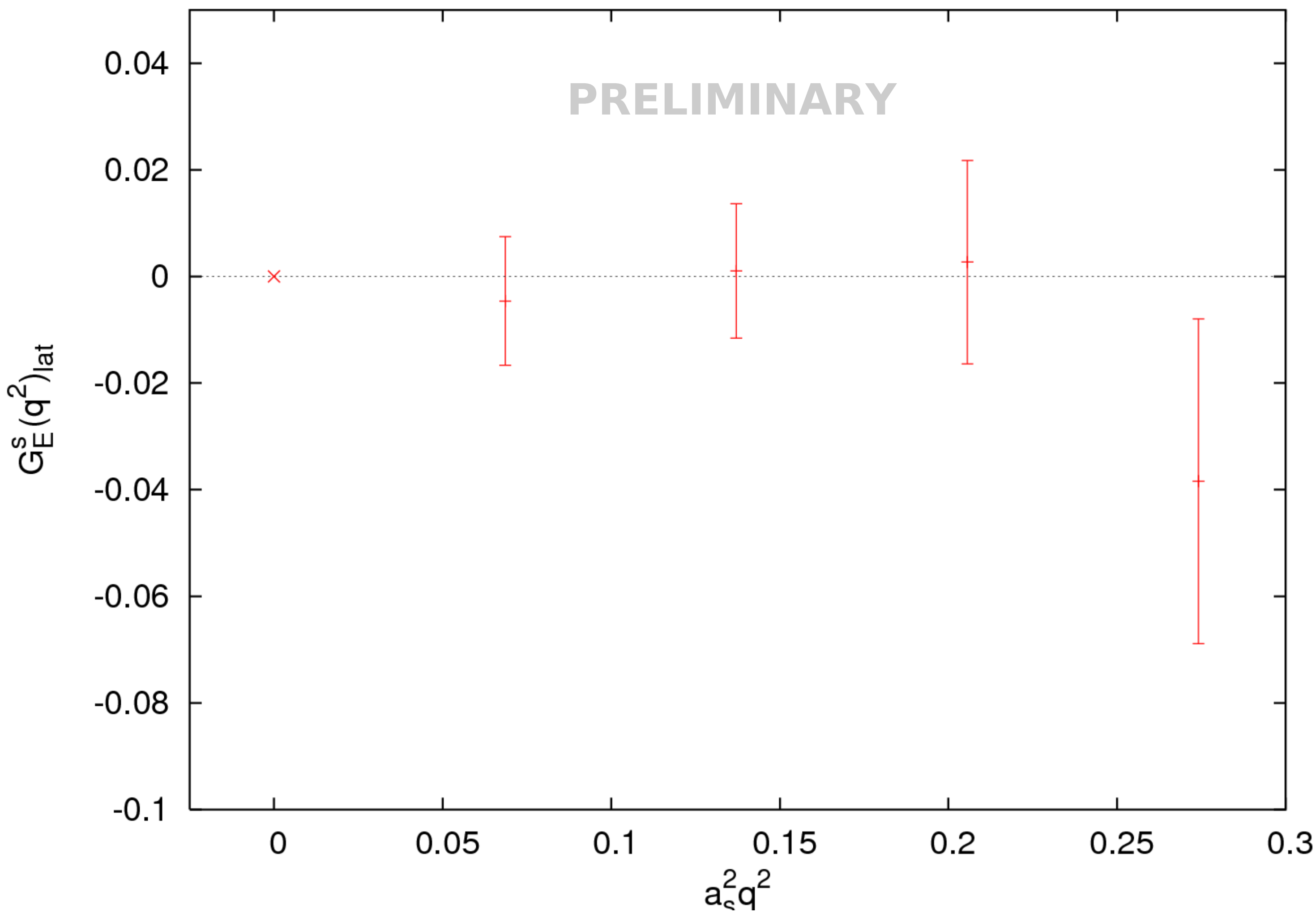


Strange axial form factor



- Results have not been renormalized.
- Calculated value is distinct from zero at the 3-σ level.

Strange electric form factor



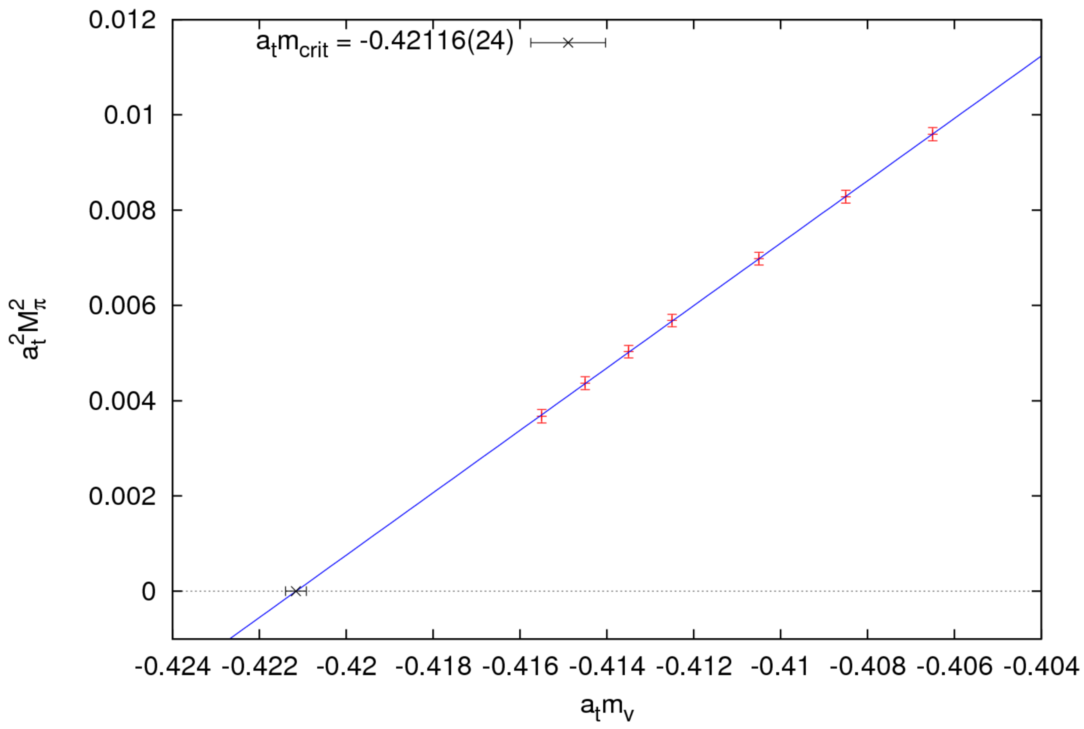
Outlook

- Preparing to repeat calculation on 2+1 flavor, clover-improved lattices.
  - strange quark in the sea
  - longer time extent: $24^3 \times 128$
  - multiple pion masses will allow chiral extrapolation
- Experimenting with new techniques.
  - eigenvalue deflation for improved performance of the inverter and possibly for variance reduction
  - adaptive multigrid (talk by M. Clark) for variance reduction (short-term) and as a preconditioner (longer-term)
  - improved nucleon sources for earlier isolation of the ground state

Bonus slides

Quark mass determination

- We determine the strange quark mass m_s that enters the parameter f_{Ts} by subtracting the critical mass m_{crit} from the naïve mass that appears in the Wilson action.
- The critical mass is determined from the quark mass dependence of the (partially quenched) pseudoscalar.



$$a_t m_0 = -0.38922$$
$$a_t m_{crit} = -0.42116(24)$$

$$a_t m_s = a_t m_0 - a_t m_{crit} = 0.03194(24)$$

Explicit noise vs. gauge averaging

- A common approach for disconnected diagrams is to employ a set of noise vectors η_i whose components are elements of Z_2, Z_4, $U(1)$, etc.
- These are optimal is the sense that each individual vector satisfies $\eta^\dagger_{(x)}\eta_{(x)} = 1$ (Dong & Liu, 1994). This does not hold for, e.g., gaussian noise.
- Our approach is equivalent to using three noise vectors over color and space that are mutually orthogonal in color, together with dilution.
  - Consider a random gauge transformation,

$$\sum_a D^{-1}_{aa}(x,x) \to \sum_a g^\dagger_{ab}(x) D^{-1}_{bc}(x,x) g_{ca}(x)$$

Identifying $(\eta_b)_a = g_{ab}$, we have $\eta^\dagger_a(x)\eta_b(x) = \delta_{ab}$