Clover improvement for stout-smeared $2+1$ flavour SLiNC fermions: non-perturbative c_{sw} results

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– QCDSF-UKQCD Collaboration –

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[Lattice 2008, Williamsburg, USA]

- $O(a)$ Improvement
- The SLiNC action
- c_{sw}
- κ_c
- Z_V
- r_0 scale
- Conclusions

Introduction

- Gluon action has $O(a^2)$ corrections
- Naive fermion action has $O(a^2)$ corrections, but
  - Introduces 'doubling problem'
    'Cure'

$$\text{action} = \text{naive} + \text{Wilson mass term}$$

but has $O(a)$ corrections, so eg

$$\frac{m_H}{m_{H'}} = \# + \# O(a)$$

- Symanzik:
  - Systematic improvement to $O(a^n)$ $n = 2$
    Add basis of irrelevant operators and tune coefficients to remove completely $O(a^{n-1})$ effects Asymptotic series ??
  - Restrict to on-shell $\Rightarrow$
    equation of motion reduce the set of operators
    - in action
    - in matrix elements
- $O(a)$ improvement $\Rightarrow$ only one additional operator in action required

$$\mathcal{L}_{addit} \propto a c_{sw}(g_0^2) \sum \overline{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi = \text{clover term}$$

If can improve one on-shell quantity to $O(a^2)$:

- Fixes $c_{sw}(g_0^2)$
- Then all other physical on-shell quantities are automatically improved to $O(a^2)$, ie

$$\frac{m_H}{m_{H'}} = \# + \# O(a^2)$$

Matrix Elements:

- Require additional $O(a)$ operators, for example

$$\begin{aligned} \mathcal{A}_\mu &= (1 + b_A a m_q)(A_\mu + c_A a \partial_\mu^{LAT} P), \\ \mathcal{P} &= (1 + b_P a m_q) P \end{aligned}$$

with

$$A_\mu = \overline{q}\gamma_\mu\gamma_5 q, \qquad P = \overline{q}\gamma_5 q$$

PCAC:

- Find quark mass m_q^{WI} from *PCAC* relation

$$m_{qR}^{WI} = \underbrace{\frac{Z_A(1 + b_A a m_q)}{Z_P(1 + b_P a m_q)}}_{\text{numerical factor}} m_q^{WI} \qquad m_q^{WI} = \frac{\langle \partial_0^{LAT}(A_4(x_0) + c_A a \partial_0^{LAT} P(x_0)) O \rangle}{2\langle P(x_0) O \rangle}$$

- Choosing different { boundary conditions, O } gives different determinations of quark mass $m_q^{(i)\,WI}$, $i = 1, 2 \ldots$
- If improved then errors are $O(a^2)$.
  So find improvement coefficients, $c_{sw}, \ldots$, by determining point where

$$m_q^{(1)\,WI} = m_q^{(2)\,WI}$$

ALPHA Collaboration:

- Achieve this by means of 'Schrödinger Functional'
- Dirichlet boundary conditions on time boundaries:
  - Gluons fields fixed $\Longrightarrow$ constant chromo-electric background field
    - Can simulate with $m_q \sim 0$ with no zero mode problems
  - Quark fields fixed $\Longrightarrow$ sinks/sources $(\rho, \overline{\rho})$
    - for correlation functions can then choose $\rho, \overline{\rho} \in O$, eg

$$O^{(i)} = \sum_{\vec{y},\vec{z}} \overline{\rho}^{(i)}(\vec{y})\gamma_5 \rho^{(i)}(\vec{z}) \qquad \begin{cases} \text{i} = 1 \text{ lower boundary } x_0 = 0 \\ \text{i} = 2 \text{ upper boundary } x_0 = T \end{cases}$$

so can look at PCAC behaviour at different distances from boundary

- Redefine quark mass (slightly, but coincides to $O(a^2)$ in improved theory) to eliminate (unknown) c_A ($m_q^{WI} \to M$)
  Aim for improvement when

$$(M, \Delta M) = (0, 0) \qquad \text{giving} \quad c_{sw}^*, \kappa_c^* \ldots$$

where

$$M \equiv M^{(1)} \qquad \Delta M \equiv M^{(1)} - M^{(2)}$$

(chosen at suitable x_0)

In a little more detail

$$m_q^{(i)\,WI} = r^{(i)}(x_0) + c_A\, s^{(i)}(x_0) \qquad i = 1, 2$$

with

$$r^{(i)}(x_0) = \frac{\partial_0^{LAT} f_A^{(i)}(x_0)}{2 f_P^{(i)}(x_0)} \qquad s^{(i)}(x_0) = a\,\frac{\partial_0^{2\,LAT} f_P^{(i)}(x_0)}{2 f_P^{(i)}(x_0)}$$

where

$$f_A^{(1)}(x_0) = -\frac{1}{3}\,\langle A_0(x_0) O^{(1)}\rangle \qquad f_P^{(1)}(x_0) = -\frac{1}{3}\,\langle P(x_0) O^{(1)}\rangle$$

$$f_A^{(2)}(T - x_0) = +\frac{1}{3}\,\langle A_0(x_0) O^{(2)}\rangle \qquad f_P^{(2)}(T - x_0) = -\frac{1}{3}\,\langle P(x_0) O^{(2)}\rangle$$

$$O^{(i)} = \sum_{\vec{y},\vec{z}} \overline{\rho}^{(i)}(\vec{y})\gamma_5\,\rho^{(i)}(\vec{z}) \qquad \left\{ \begin{array}{l} \text{i} = 1 \text{ lower boundary } x_0 = 0 \\ \text{i} = 2 \text{ upper boundary } x_0 = T \end{array} \right.$$

Redefine quark mass (slightly, coincides to $O(a^2)$ in improved theory) to eliminate (unknown) c_A:

$$M^{(i)}(x_0, y_0) = r^{(i)}(x_0) - \widehat{c}_A s^{(i)}(x_0) \qquad \widehat{c}_A = -\frac{r^{(1)}(y_0) - r^{(2)}(y_0)}{s^{(1)}(y_0) - s^{(2)}(y_0)}$$

$$M \equiv M^{(1)}(T/2, T/4) \qquad \Delta M \equiv M^{(1)}(3T/4, T/4) - M^{(2)}(3T/4, T/4)$$

Aim for

$$(M, \Delta M) = (0, 0) \qquad \text{giving} \quad c_{sw}^*, \kappa_c^* \ldots$$

(Small) Ambiguities

- Infinite volume expect $O(a^2\Lambda_{QCD}^2)$ — in chiral limit, otherwise additional $O(a^2 m_q^2)$ term
- Finite volume additional $O(a^2/L_s^2)$

So

- $O(a^2\Lambda_{QCD}^2) \to 0$ as a (or g_0^2) $\to 0$
- $O(a^2/L_s^2) \sim O(1/N_s^2) \not\to 0$ — $L_s = aN_s$
  - Keep L_s fixed in physical units as $a \to 0$ (but then fine tuning for β), 'constant physics condition': $O(a^2/L_s^2) \to 0$
  - Simulate for several values of N_s and extrapolate to $N_s \to \infty$: $O(a^2/L_s^2) \sim O(1/N_s^2) \to 0$
  - Poor man's solution:
    Simulate at $\beta = \infty$ and subtract $O(1/N_s^2)$ terms
    Practically, does it matter?
    - c_{sw} negligble
    - Z_V, a 1% effect

SLiNC fermions $2+1$ flavours

Stout LinkNon-perturbative Clover = SLiNC

$$
\begin{aligned}
S_F &= \sum_x \Big\{ \kappa\overline{\psi}(x)\tilde{U}_\mu(x+\hat{\mu})[\gamma_\mu - 1]\psi(x-\hat{\mu}) - \kappa\overline{\psi}(x)\tilde{U}_\mu^\dagger(x-\hat{\mu})[\gamma_\mu + 1]\psi(x+\hat{\mu}) \\
&\quad + \overline{\psi}(x)\psi(x) + \frac{1}{2} c_{sw}(g_0^2)\overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}
\end{aligned}
$$

- The hopping terms use a stout smeared link ('fat link')

Dirac kinetic term and Wilson mass term

$$
\begin{aligned}
\tilde{U}_\mu &= \exp\{iQ_\mu(x)\}\, U_\mu(x) \\
Q_\mu(x) &= \frac{\alpha}{2i}\left[VU^\dagger - UV^\dagger - \tfrac{1}{3}\mathrm{Tr}(VU^\dagger - UV^\dagger)\right]
\end{aligned}
$$

V_μ is the sum of all staples around U_μ

- Clover term built from thin links
  (already length $4a$ do not want fermion matrix too extended)

Why stout smearing?

- Need smearing at present lattice spacings
- Analytic
  - can take derivative (so HMC force well defined)
  - perturbation expansions

H. Perlt talk

To complete action:

- Gluon action: Symanzik tree–level (plaquette + rectangle)

$$S_G = \frac{6}{g_0^2}\left\{c_0 \sum_{Plaquette} \frac{1}{3}\mathrm{Re\,Tr}(1 - U_{Plaquette}) + c_1 \sum_{Rectangle} \frac{1}{3}\mathrm{Re\,Tr}(1 - U_{Rectangle})\right\}$$

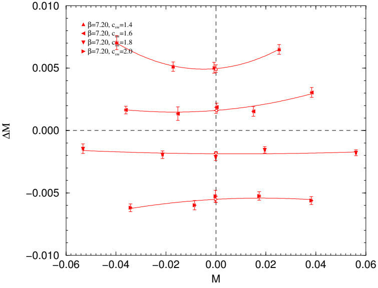
with

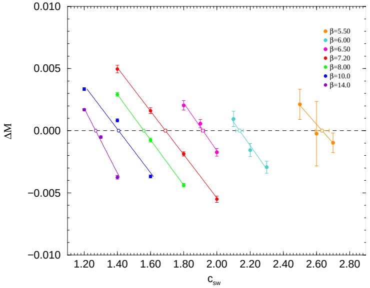
$$\beta = \frac{6c_0}{g_0^2} = \frac{10}{g_0^2} \qquad \text{with} \quad c_0 = \frac{20}{12}\,,\ c_1 = -\frac{1}{12}$$

Programme:

- Chroma – R. Edwards and B. Joo, arXiv:hep-lat/0409003
  [for BG/L additions P.A. Boyle,
  http://www.ph.ed.ac.uk/˜ paboyle/bagel/Bagel.html]
- SF details follow T. Klassen, arXiv:hep-lat/9705025
- Practically:
  - 'Mild smearing' $\alpha = 0.1$
  - $8^3 \times 16$ lattices
  - T. Kaltenbrunner initiated investigation

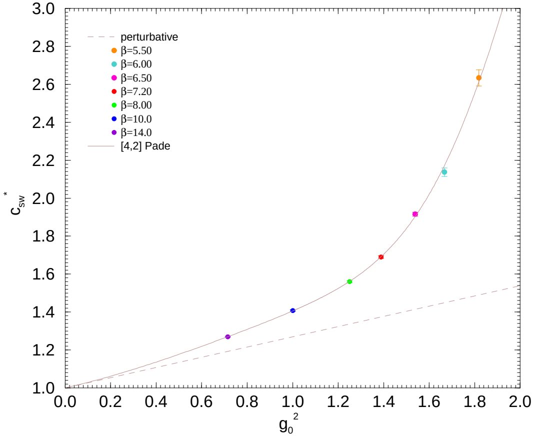
$\beta = 5.50$ not yet completed





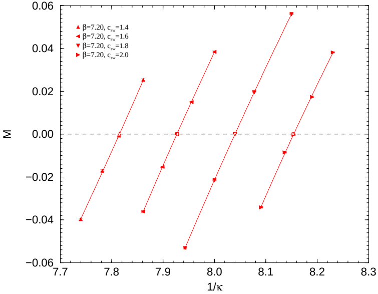
$M = 0$ gives
$\Delta M(c_{sw}, \kappa_c(c_{sw}))$

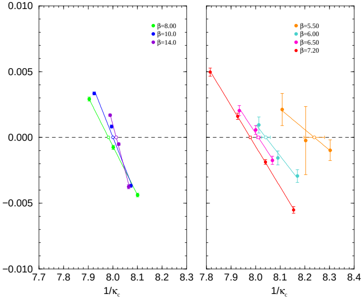
$\Delta M = 0$ gives
c_{sw}^*



c^*_{sw}

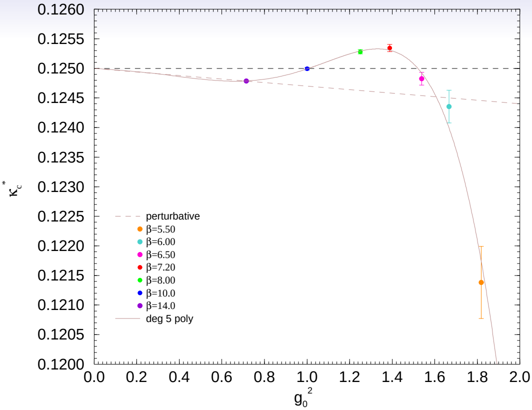
$$c_{sw}^{TI} = \frac{u_0^S}{u_0^4} \quad \text{cf } c_{sw}^{TI} = \frac{1}{u_0^3}$$





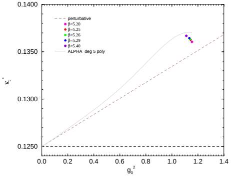
$M = 0$ gives
$\kappa_c(c_{sw})$

$\Delta M = 0$ gives
κ_c^*



κ_c^*

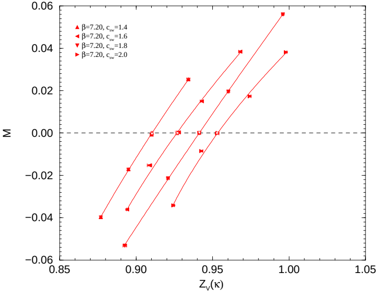
cf κ_c^* for $n_f = 2$

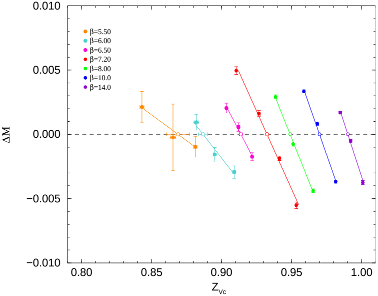


Z_V – same procedure

CVC gives (in chiral limit)

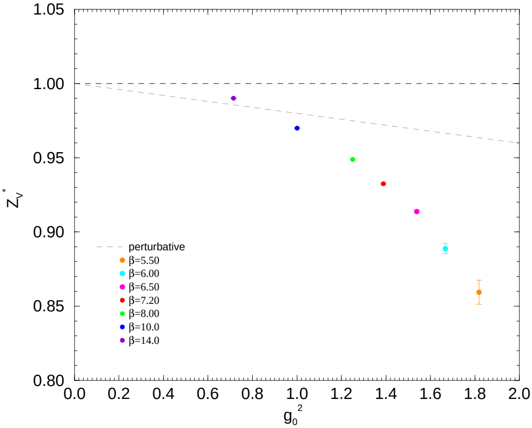
$$Z_V = \frac{f_1}{f_V(x_0)} \qquad f_1 = -\frac{1}{3}\langle O^{(2)}\, O^{(1)}\rangle \quad f_V(x_0) = \frac{i}{6}\langle O^{(2)} \sum_{\vec{x}} V_0(x) O^{(1)}\rangle$$





$M = 0$ gives
$Z_{Vc}(\kappa_c(c_{sw}))$

$\Delta M = 0$ gives
Z_V^*


1.05
1.00
0.95
0.90
0.85
0.80
Z*V
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
g0 2
perturbative
β=5.50
β=6.00
β=6.50
β=7.20
β=8.00
β=10.0
β=14.0

Z_V^*

Scales

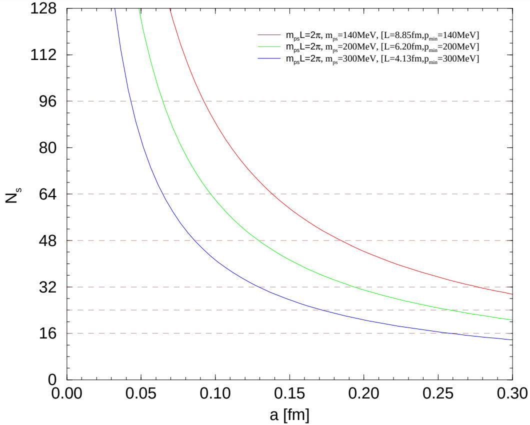
- What is a sensible region to work in?
- Short runs on $16^3 \times 32$ lattices:

(β, κ)	r_0/a	a
$(7.20, 0.1230)$	9.00	$0.056\,\mathrm{fm} \equiv (3.55\,\mathrm{GeV})^{-1}$
$(6.50, 0.1240)$	7.55	$0.066\,\mathrm{fm} \equiv (2.98\,\mathrm{GeV})^{-1}$
$(6.00, 0.1225)$	7.00	$0.071\,\mathrm{fm} \equiv (2.76\,\mathrm{GeV})^{-1}$

  3-flavour run r_0/a results using scale
  $r_0 = 0.500\,\mathrm{fm} = (394.6\,\mathrm{MeV})^{-1}$.
- Improvement (an asymptotic series) wins for smaller a, say $a \leq 0.1\,\mathrm{fm}$

A long way to go:

$p_{\min} \equiv \frac{2\pi}{L_s} = m_{ps}$



Either:

- Small a with 'large' m_{ps}
  no continuum extrapolation but chiral extrapolation
- 'Coarse' a with $m_{ps} \sim m_\pi$
  no chiral extrapolation but continuum extrapolation
- Mixture

Conclusions

- $O(a)$ improvement works for (stout) smeared actions
- Typical clover results obtained
  - as a decreases need a significant $c_{sw} \gg c_{sw}^{tree} \equiv 1$ for $O(a)$ improvement
  - Seeking/have found a region where $a \sim 0.05 - 0.1\,\text{fm}$