Probing SU(3) Chiral Perturbation Theory fits to 2+1 flavor DWF QCD

Robert Mawhinney, Columbia University
For the RBC and UKQCD Collaborations
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- NLO SU(3) ChPT fits for decay constants and masses
- Adding another observable to the fits, $\langle \pi^+ | \bar{s} d | K^+ \rangle$
- First efforts at full NNLO fits

Collaboration Members

RBC members:

Y. Aoki, T. Blum, M. Cheng, N. Christ, S. Cohen, C. Dawson, T. Doi, K. Hashimoto, T. Ishikawa, T. Izubuchi, C. Jung, C. Kim, M. Li, S. Li, M. Lightman, M. Lin, R. Mawhinney, S. Ohta, S. Sasaki, E. Scholz, A. Soni, T. Yamazaki

UKQCD members:

C. Allton, D. Antonio, K. Bowler, P. Boyle, D. Brommel, M. Clark, M. Donnellan, J. Flynn, A. Hart, B. Joo, A. Juettner, C. Kelly, A. Kennedy, R. Kenway, C. Maynard, B. Pendleton, C. Sachrajda, A. Trivini, R. Tweedie, J. Wennekers, A. Yamaguchi, J. Zanotti

Zero Temperature Ensembles

Volume	a^{-1} (GeV) m_{res}	m_l m_1	MD time units	# wall sources
$16^3 \times 32 \times 8$	1.8(1) 0.0107(1)	(0.02, 0.04) (0.04, 0.04)	1797.5 1797.5	
$16^3 \times 32 \times 16$ (1.82 fm)^3	1.62(4) 0.00308(4)	(0.01, 0.04) (0.02, 0.04) (0.03,0.04)	4015 4045 7600	
$24^3 \times 64 \times 16$ (2.74 fm)^3	1.73(3) 0.00315(2)	(0.005,0.04) (0.01,0.04) (0.02,0.04) (0.03,0.04)	8200 8200 2850 2813	2×90 2×90
$32^3 \times 64 \times 16$ ($\approx$ 2.60 fm)^3	$\approx$ 2.42(4) $\approx$ 0.0006	(0.004, 0.03) (0.006, 0.03) (0.008, 0.03)	4948 4960 2470	170 105 110
$48^3 \times 64 \times 16$ ($\approx$ 3.91 fm)^3	$\approx$ 2.42(4) $\approx$ 0.0006	(0.002, 0.03)	250	

Analysis Overview

- Data from $24^3 \times 64 \times 16$ volumes, $(2.74\ \mathrm{fm})^3$, $1/a = 1.73(3)$ GeV
- RBC and UKQCD paper, arXiv:0804.0473
- Coulomb gauge fixed wall sources at t=5 and 59.
- Periodic and anti-periodic boundary conditions used to generate P+A propagators to remove "around the world" effects.
- Masses of Ω, π, and K fix lattice scale, $m_u = m_d$ and m_s using SU(2) ChPT for chiral extrapolation (Enno Scholz talk).
- Partially quenched pseudoscalar masses and decay constants calculated
- $m_\pi L$ for various valence masses given below

Light dynamical mass	Valence Mass		
	$m_x = 0.001$	$m_x = 0.005$	$m_x = 0.01$
0.005	3.36	4.60	5.76
0.01	3.44	4.65	5.81

Chiral Perturbation Theory

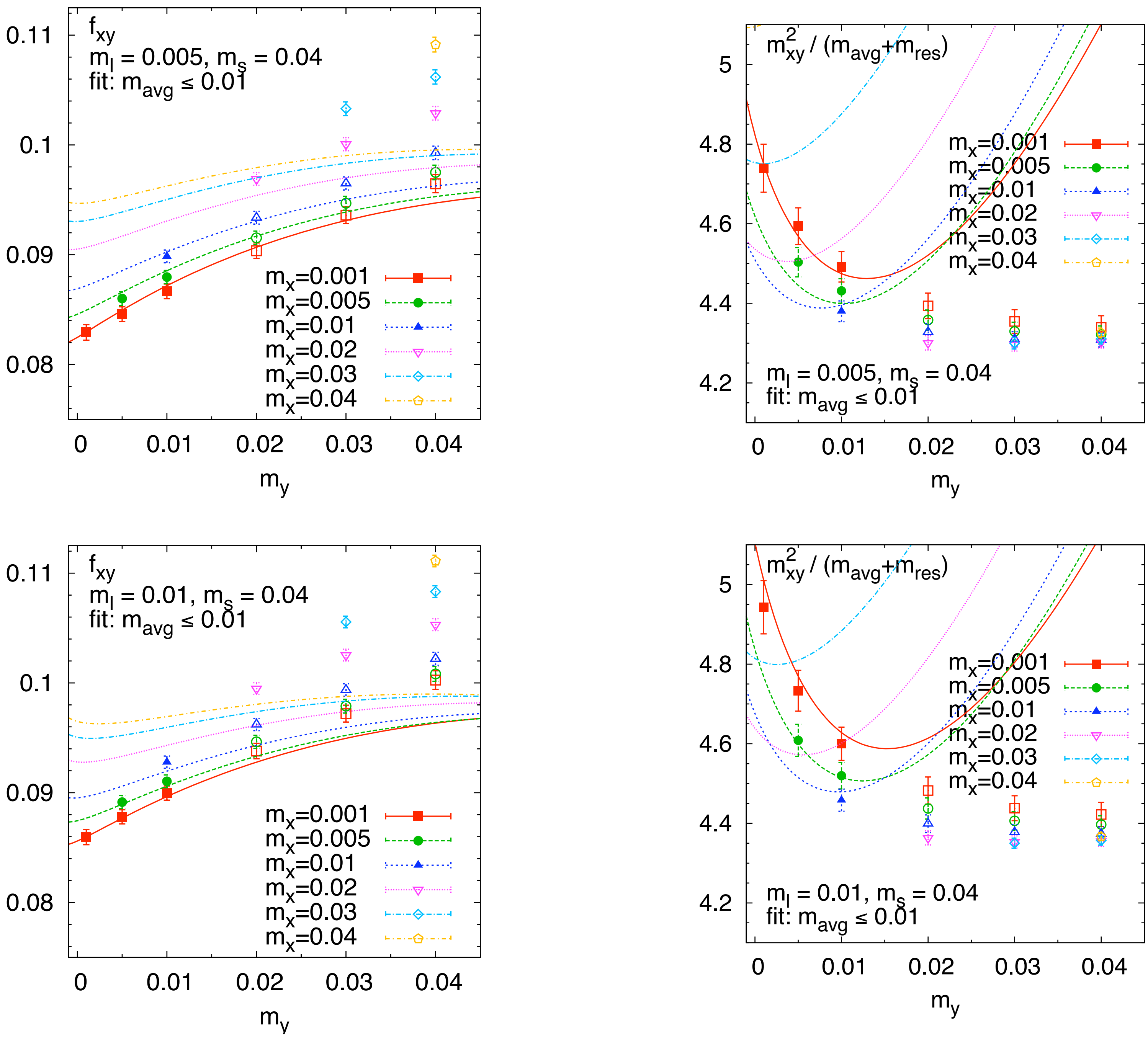
- Expansion in powers of

$$\frac{m_{PS}^2}{(4\pi f)^2} \qquad \frac{p^2}{(4\pi f)^2}$$

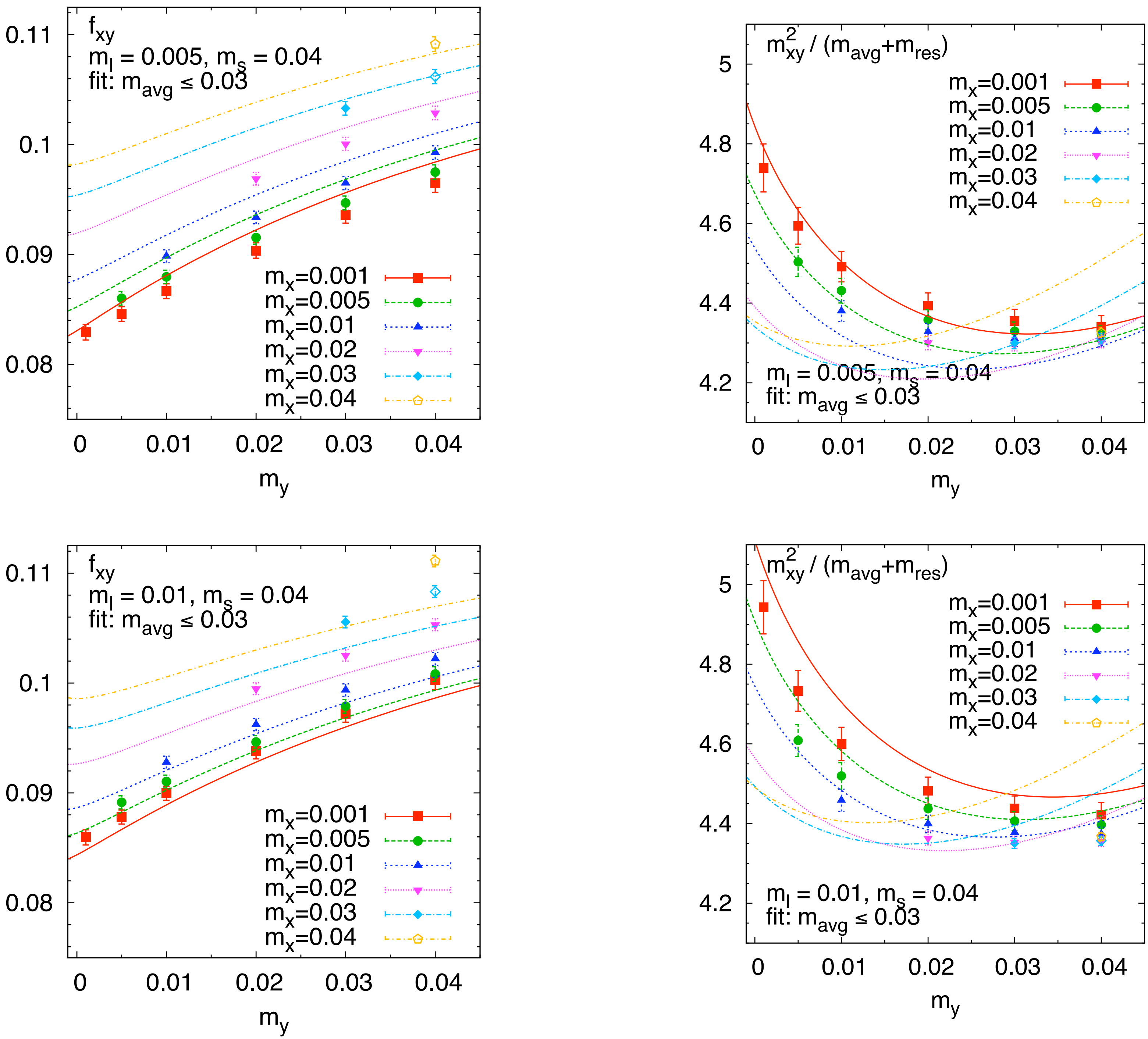
- SU(2) ChPT: m_l is light and only pion masses enter in logs
- SU(3) ChPT: m_l and m_s are considered light, and both enter logs
- Example (Sharpe and Shoresh, 2000) with six free parameters

$$\begin{aligned}
m_P^2 &= \chi_V \left\{ 1 + \frac{48}{f^2}(2L_6 - L_4)\bar{\chi} + \frac{16}{f^2}(2L_8 - L_5)\chi_V \right. \\
&\quad + \frac{1}{24 f^2 \pi^2}\left[\frac{2\chi_V - \chi_l - \chi_s}{\chi_V - \chi_\eta}\chi_V \log \chi_V - \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{(\chi_V - \chi_\eta)^2}\chi_V \log \chi_V \right. \\
&\quad \left.\left. + \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{\chi_V - \chi_\eta}(1 + \log \chi_V) + \frac{(\chi_\eta - \chi_l)(\chi_\eta - \chi_s)}{(\chi_V - \chi_\eta)^2}\chi_\eta \log \chi_\eta \right] \right\} \\
f_P &= f \left\{ 1 + \frac{8}{f^2}(3L_4\bar{\chi} + L_5\chi_V) \right. \\
&\quad \left. - \frac{1}{16\pi^2 f^2}\left[(\chi_V + \chi_l) \log \frac{\chi_V + \chi_l}{2} + \frac{\chi_V + \chi_s}{2} \log \frac{\chi_V + \chi_s}{2} \right] \right\} \\
\chi_i &= 2B_0(m_i + m_{\rm res})
\end{aligned}$$

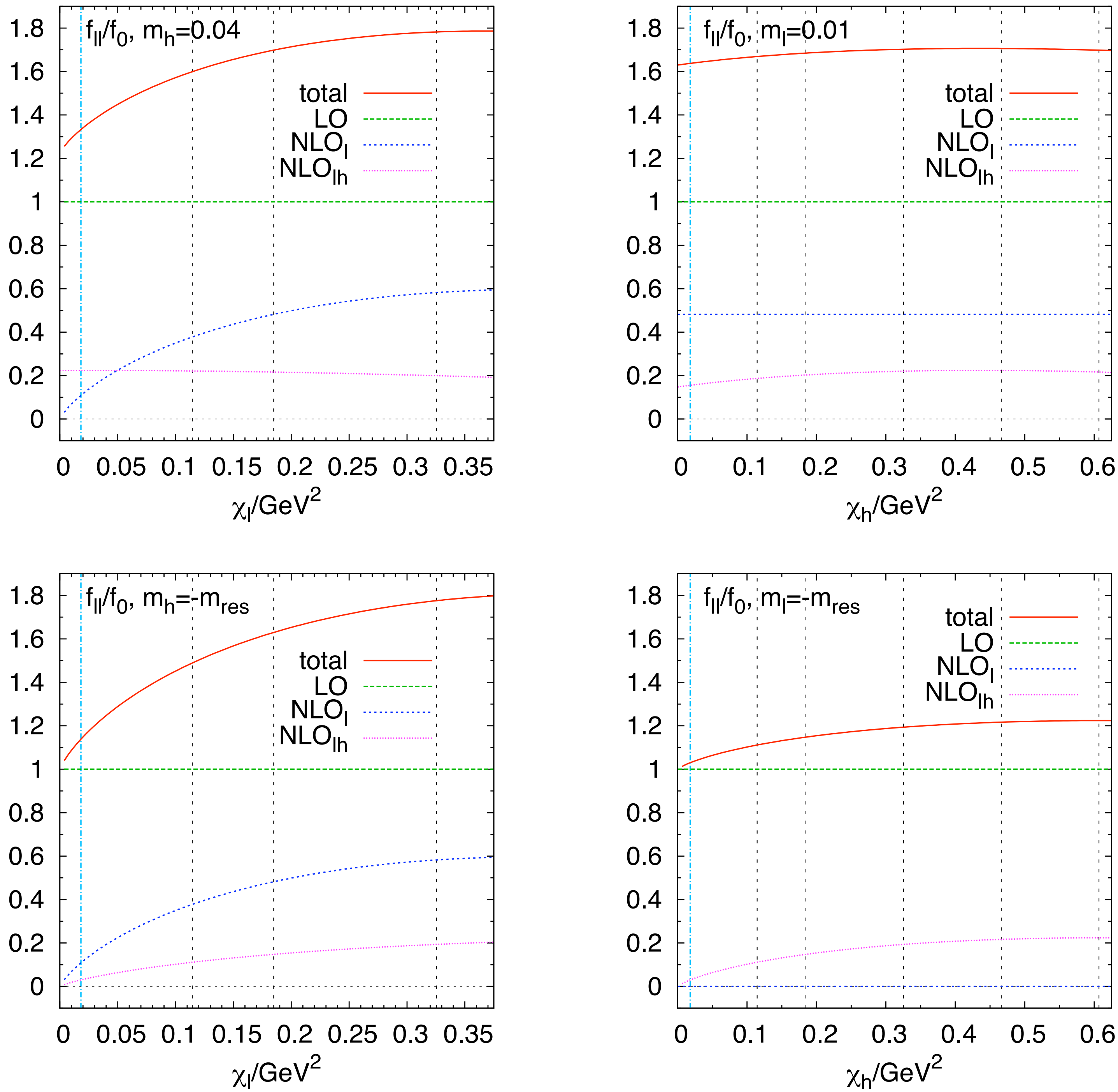
SU(3) ChPT for $m_{PS} < 420$ MeV



SU(3) ChPT for m_{PS} < 670 MeV



LO and NLO for SU(3) ChPT

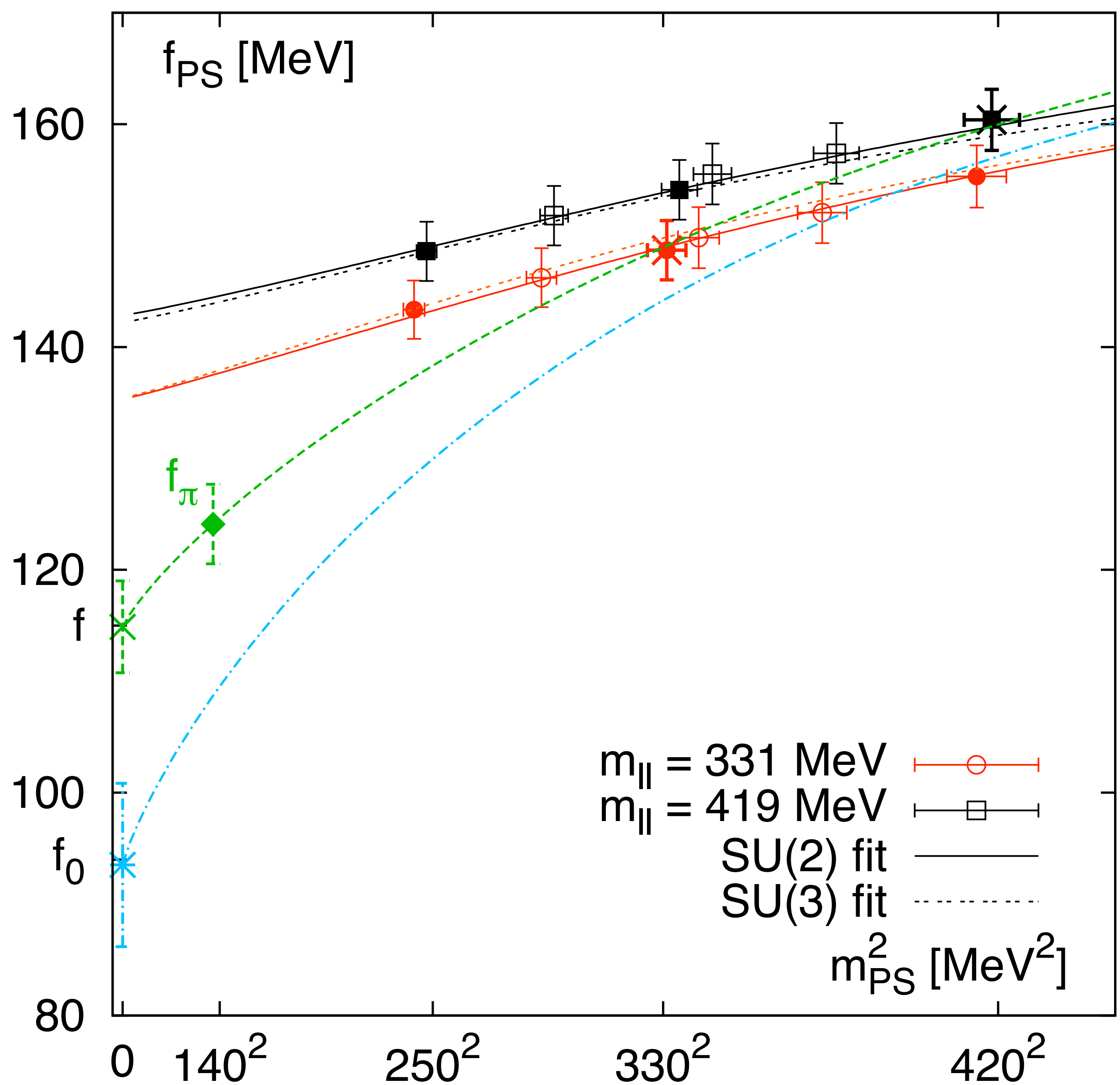


Comparison of SU(3) ChPT LECs

- Convergence of SU(3) for $m_{PS} < 420$ MeV poor
- Because of poor convergence, do not attempt a systematic error.
- Naively quote results for LECs from our fits
- Generally in good agreement with others

	$L_4^{(3)}$	$L_5^{(3)}$	$L_6^{(3)}$	$L_8^{(3)}$	$(2L_8^{(3)} - L_5^{(3)})$	$(2L_6^{(3)} - L_4^{(3)})$
this work[a]	1.4(0.8)(–)	8.7(1.0)(–)	0.7(0.6)(–)	5.6(0.4)(–)	2.4(0.4)(–)	0.0(0.4)(–)
Bijnens, NLO	$\equiv 0$	14.6	$\equiv 0$	10.0	5.4	$\equiv 0$
Bijnens, NNLO	$\equiv 0$	9.7(1.1)	$\equiv 0$	6.0(1.8)	2.3[b]	$\equiv 0$
MILC, 2007	$1.3(3.0)(^{+3.0}_{-1.0})$	$13.9(2.0)(^{+2.0}_{-1.0})$	$2.4(2.0)(^{+2.0}_{-1.0})$	7.8(1.0)(1.0)	2.6(1.0)(1.0)	$3.4(1.0)(^{+2.0}_{-3.0})$

f_{PS} comparison SU(2) and SU(3) ChPT



About Correlated Fits

- Partially quenched data is very correlated
- With 90 measurements, we can look at correlations of pseudoscalar masses for 3 lightest, degenerate pseudoscalars

Writing $C_{ij} = \sigma_{ik}\,\rho_{kl}\,\sigma_{lj}$, we have C_{ij} equal to

$$\begin{pmatrix} 0.7673 & 0.0 & 0.0 \\ 0.0 & 0.7018 & 0.0 \\ 0.0 & 0.0 & 0.6466 \end{pmatrix} \times 10^{-3} \times \begin{pmatrix} 1.0000 & 0.9075 & 0.7882 \\ 0.9075 & 1.0000 & 0.9568 \\ 0.7882 & 0.9568 & 1.0000 \end{pmatrix} \times \sigma$$

- Eigenvalues of correlation matrix show very small eigenvalues occurring

$$\begin{pmatrix} 2.77 \\ 0.2156 \\ 0.0143 \end{pmatrix}$$

Correlations for 6 Partially Quenched Masses

$$\begin{pmatrix} 1.0000 & 0.9705 & 0.9157 & 0.9075 & 0.8496 & 0.7882 \\ 0.9705 & 1.0000 & 0.9818 & 0.9791 & 0.9437 & 0.8927 \\ 0.9157 & 0.9818 & 1.0000 & 0.9873 & 0.9767 & 0.9392 \\ 0.9075 & 0.9791 & 0.9873 & 1.0000 & 0.9884 & 0.9568 \\ 0.8496 & 0.9437 & 0.9767 & 0.9884 & 1.0000 & 0.9881 \\ 0.7882 & 0.8927 & 0.9392 & 0.9568 & 0.9881 & 1.0000 \end{pmatrix}$$

- Eigenvalues now span 5 orders of magnitude
- Strong correlations mean much more data required to resolve correlation matrix accurately

$$\begin{pmatrix} 5.692 \\ 0.2713 \\ 0.0254 \\ 0.0107 \\ 0.000592 \\ 0.0000312 \end{pmatrix}$$

Fit Strategy

- A correlated χ^2 is useful for a quantitative statement about how well our fit anzatz agrees with the data
- Strong correlations in our data make such a statement problematic
- As our dataset gets arbitrarily large, the covariance matrix will resolve and the χ^2 will become very bad, since NLO ChPT is only accurate to a certain level for given quark masses.
- Can ask whether NLO ChPT is accurate at, for example, the 10% level for 400 MeV pseudoscalars.
- Even if NLO fits agree well with the data, how large are the NLO corrections?
- Important to include estimate of NNLO errors
- For our data, NLO fits agree well for 400 MeV pseudoscalars, but NLO terms are large, and convergence of series appears poor.

Adding Another Quantity to Fits

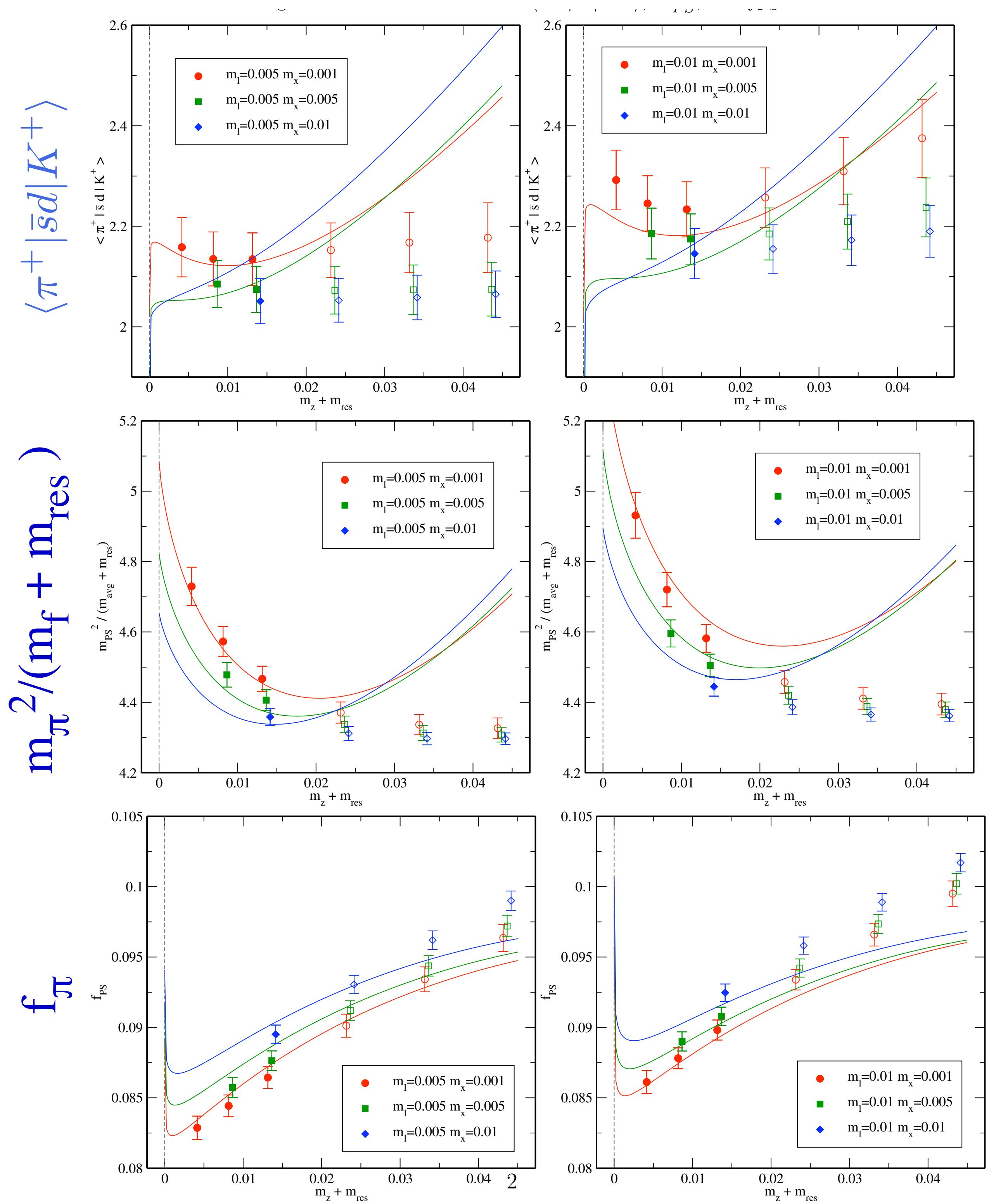
- Can ask about ChPT behavior of a new quantity $\langle \pi^+ | \bar{s} d | K^+ \rangle$
- To lowest order, we have

$$\langle \pi^+ | \bar{s} d | K^+ \rangle = \frac{m_\pi^2}{2 m_f} = B_0$$

- Aubin, Laiho, Li and Lin have calculated this to NLO
- It depends on f_0, B_0, L_5, L_6, L_8

$$\left\langle \pi^+ \left| \Theta^{(3,\bar{3})} \right| K^+ \right\rangle = -\frac{2}{f^2}\alpha^{(3,\bar{3})} - \frac{16 B_0}{f^2}\Big\{ L_5 m_X m_{xz} - 2L_8 \left(m_X^2 + m_{xz}^2 \right) - 2L_6 \left(2m_D^2 + m_S^2 \right) \Big\} + \left\langle \pi^+ \left| \Theta^{(3,\bar{3})} \right| K^+ \right\rangle_{logs}$$

- Plot results of simultaneous fit to f_π m_π and $\langle \pi^+ | \bar{s} d | K^+ \rangle$
- Can fit to pairs of these quantities
- 10-15% shift in f_0
- Data for $\langle \pi^+ | \bar{s} d | K^+ \rangle$ not well fit.
- Logs are largest for f_π and it may have the largest higher order corrections.



Results of fits

Fit	B_0	f_0	L_4	L_5	L_6	L_8	χ^2/dof
m_{PS}^2 (fix L_4, L_5, L_6)	2.377(33)	$6.3(1.0) \times 10^{-2}$	-6.7×10^{-5}	2.51×10^{-4}	-5.7×10^{-5}	$3.23(67) \times 10^{-4}$	$1.6(2.4) \times 10^{-1}$
$\bar{s}d$ (fix L_4, L_5, L_6)	2.374(81)	$9.5(1.5) \times 10^{-2}$	-6.7×10^{-5}	2.51×10^{-4}	-5.7×10^{-5}	$1.46(74) \times 10^{-4}$	$1.3(1.7) \times 10^{0}$
f_{PS} (fix L_4, L_5, L_6)	2.44(36)	$5.26(43) \times 10^{-2}$	-6.7×10^{-5}	2.51×10^{-4}			$5.5(9.8) \times 10^{-1}$
$\bar{s}d$ and m_{PS}^2	2.17(15)	$7.56(88) \times 10^{-2}$	$-0.6(3.0) \times 10^{-1}$	$1.6(3.5) \times 10^{-3}$	$-0.3(1.5) \times 10^{-1}$	$0.9(1.8) \times 10^{-3}$	$5.0(5.7) \times 10^{-1}$
$\bar{s}d$ and m_{PS}^2 (fix L_4, L_6)	2.394(27)	$7.24(78) \times 10^{-2}$	-6.7×10^{-5}	$2.5(2.9) \times 10^{-3}$	-5.7×10^{-5}	$1.4(1.5) \times 10^{-3}$	$9.3(9.2) \times 10^{-1}$
m_{PS}^2 and f_{PS}	2.33(15)	$5.44(44) \times 10^{-2}$	$-6.4(6.2) \times 10^{-5}$	$2.0(1.2) \times 10^{-4}$	$-5.3(4.4) \times 10^{-5}$	$3.52(41) \times 10^{-4}$	$6.4(5.6) \times 10^{-1}$
simu (all)	2.13(16)	$5.95(46) \times 10^{-2}$	$-1.10(90) \times 10^{-4}$	$2.6(1.2) \times 10^{-4}$	$-3.4(6.7) \times 10^{-5}$	$3.32(51) \times 10^{-4}$	$1.09(70) \times 10^{0}$
m_{PS}^2 and f (M&E)	2.35(16)	$5.41(40) \times 10^{-2}$	$-6.7(8.0) \times 10^{-5}$	$2.51(99) \times 10^{-4}$	$-5.7(8.5) \times 10^{-5}$	$3.9(1.6) \times 10^{-4}$	

- f_0 larger for fits not involving f_π
- f_0 ranges from 0.053 to 0.095

Preliminary NNLO Fits

Bijnen's NNLO formula for m_π^2 and f_π involve the following constants

Order	Constants	Number at order	Total number
LO	f_0, B_0	2	2
NLO	L_i, for $i = 4, 5, 6, 8$	4	6
NNLO	L_i, for $i = 0, 9$	10	
	K_i, for $i = 17 - 23, 25 - 27, 39, 40$	12	
	Two linear comb. of K_i not determined		
	Set $K_{39} = K_{40} = 0$	10	22

From Bijnens [1], the NNLO analytic terms correcting m_π^2 (for 2 valence quarks and 3 dynamical quarks) is

$$
\begin{aligned}
\delta_{\rm ct}^{(6)23} \sim \; & - 2\,\chi_{13}^2\,(K_{17}^r + K_{19}^r - 3K_{25}^r - K_{39}^r) \\
& + \chi_1\chi_3\,(K_{19}^r - K_{23}^r - 3K_{25}^r) \\
& - 6\,\bar\chi\,\chi_{13}\,(K_{18}^r + K_{20}^r/2 - K_{26}^r - K_{40}^r) \\
& - 9\,\bar\chi^2\,(K_{21}^r + K_{22}^r - K_{26}^r - 3K_{27}^r) \\
& + 6\,\chi_\pi\chi_\eta\,(K_{21}^r - K_{26}^r)\,,
\end{aligned}
$$

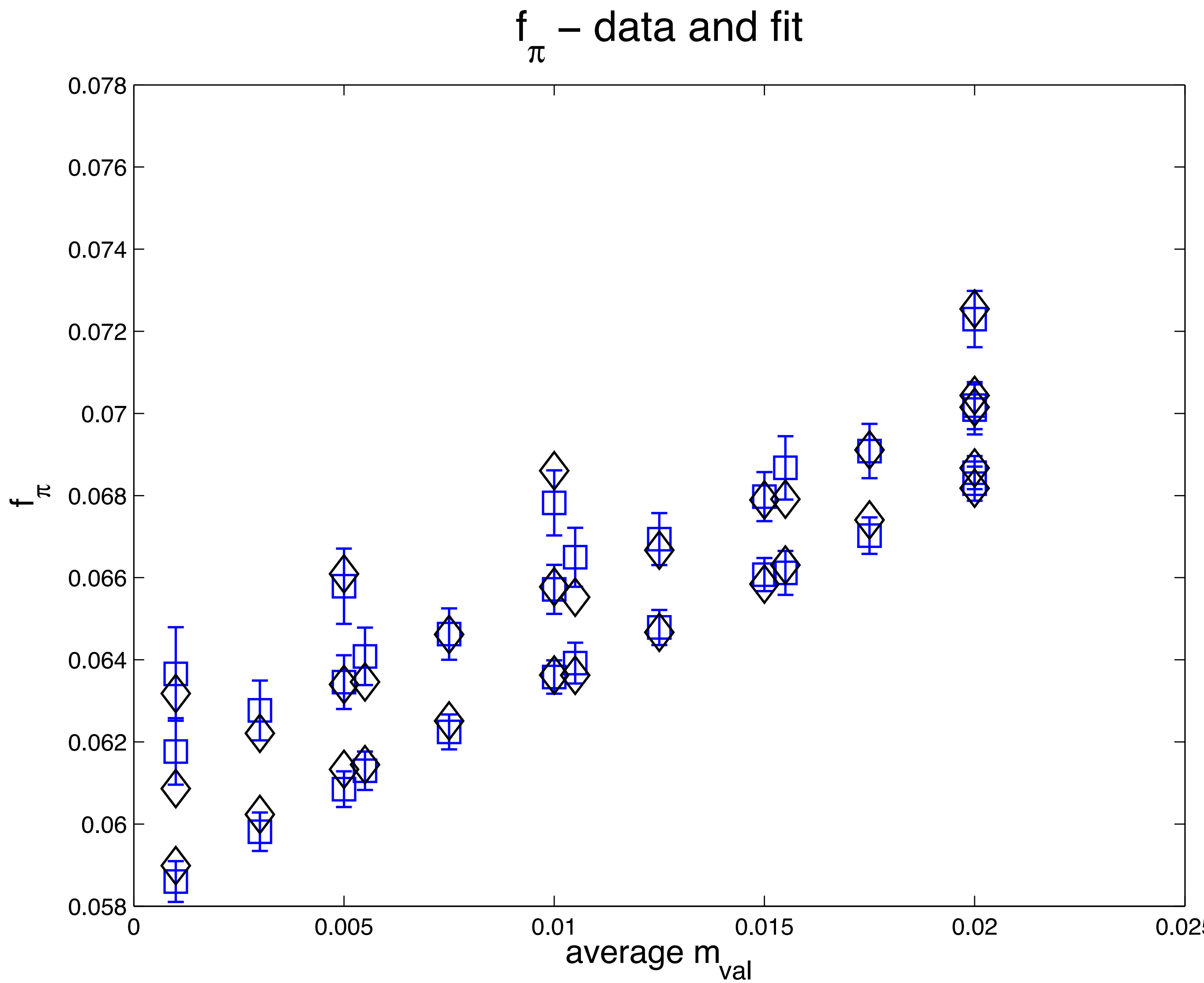
Also from Bijnens [1], the NNLO analytic terms correcting f_π (for 2 valence quarks and 3 dynamical quarks) is

$$
\begin{aligned}
f_{\rm ct}^{(6)23} \sim \; & 2\,\chi_{13}^2\,K_{19}^r \;-\; \chi_1\chi_3\,(K_{19}^r - K_{23}^r) \\
& + 3\,\bar\chi\,\chi_{13}\,K_{20}^r \;+\; 9\,\bar\chi^2\,(K_{21}^r + K_{22}^r) \\
& - 6\,\chi_\pi\chi_\eta\,K_{21}^r,
\end{aligned}
$$

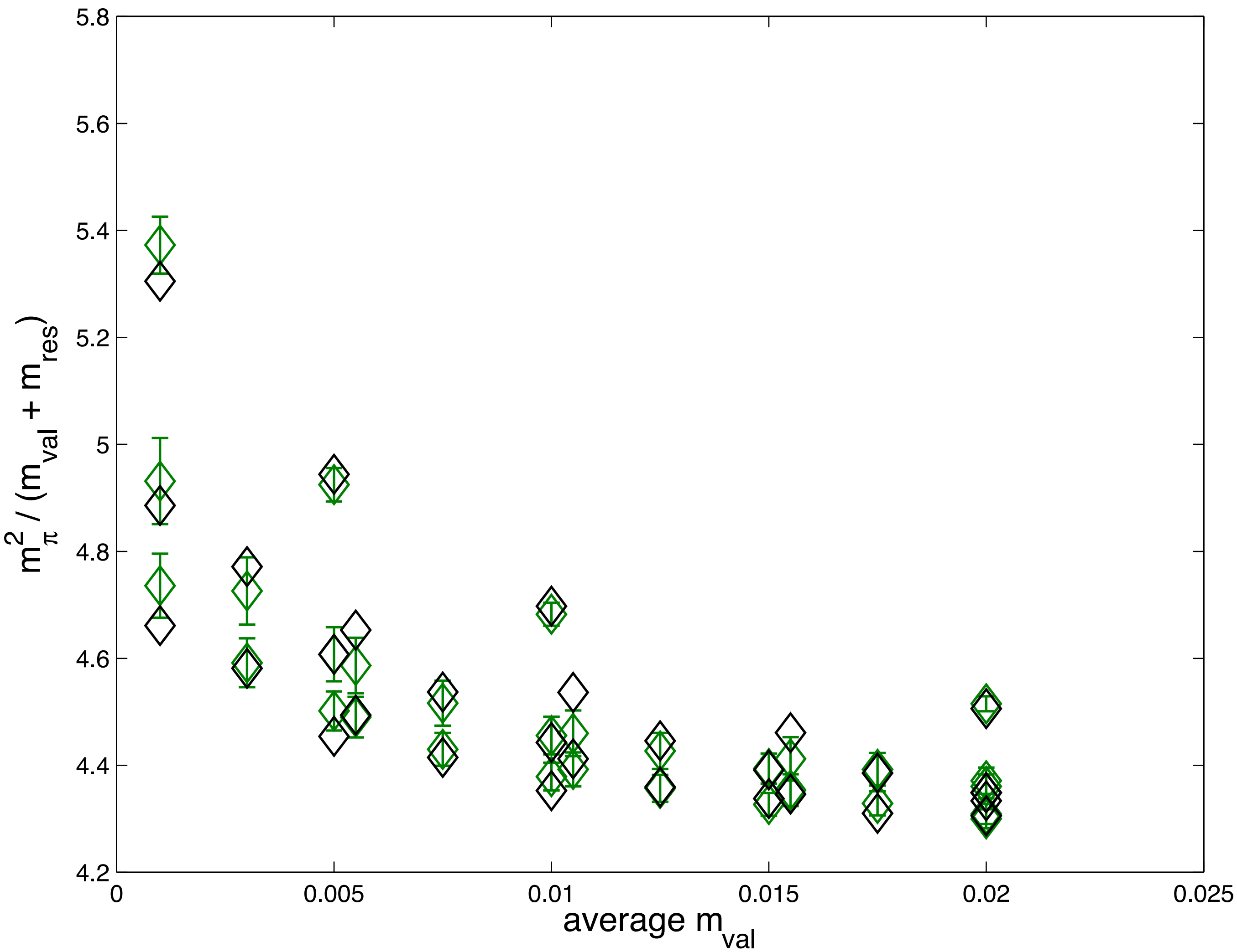
NNLO Preliminaries

- Using calculations of Bijnens, Danielsson and Lahde, PRD 73 (2006) 074509
- Fortran code provided by Bijnens
- Basic tests of NLO Fortran code versus other fit codes successful
- First simple attempts to assess numerical stability and graph results
- Used constrained minimizer, since there may be undamped directions in parameter space with current data set
- Add $m_l = 0.02$ ensemble and increase valence quark masses to 0.02

NNLO fit to f_π



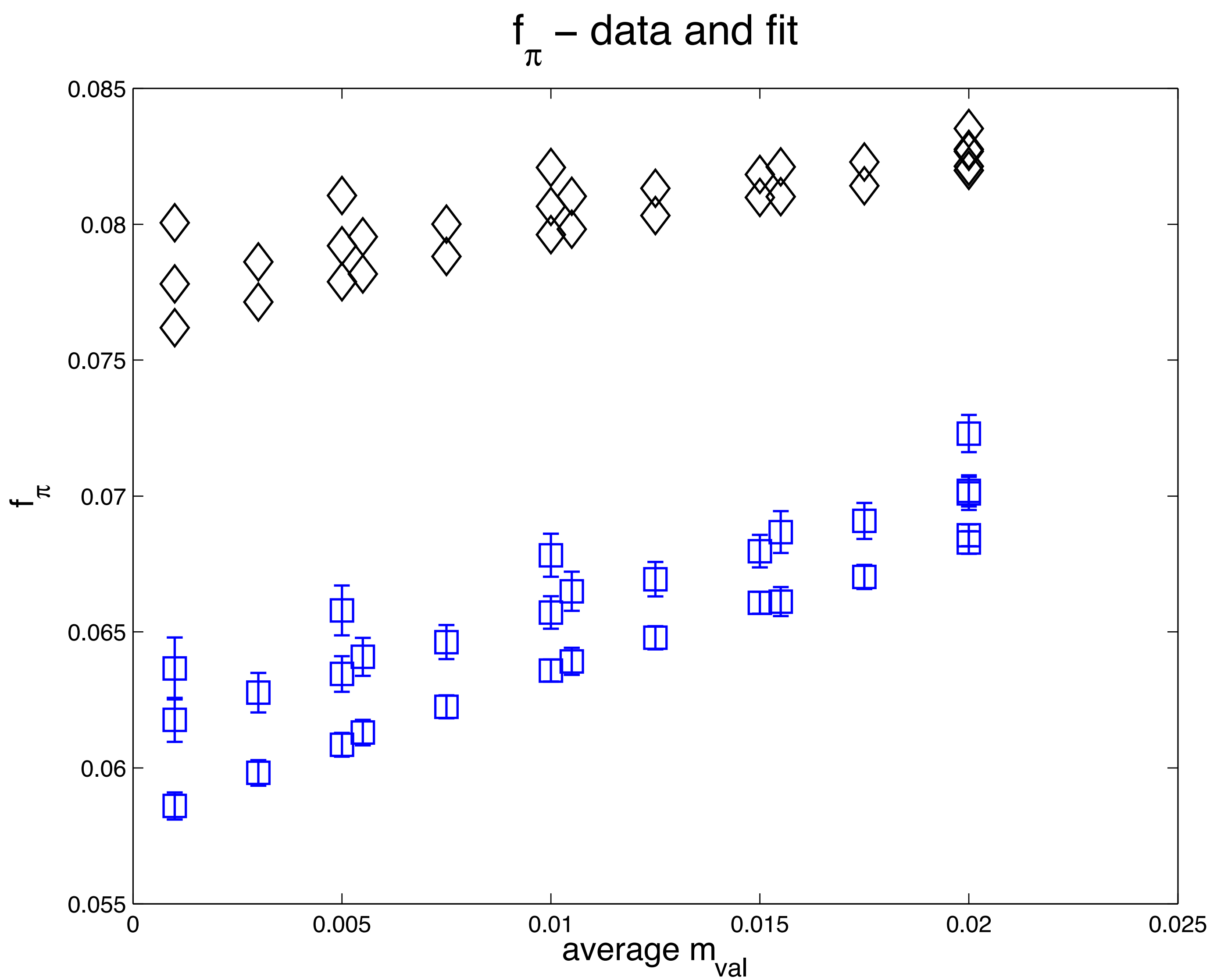
NNLO fit to m_π^2



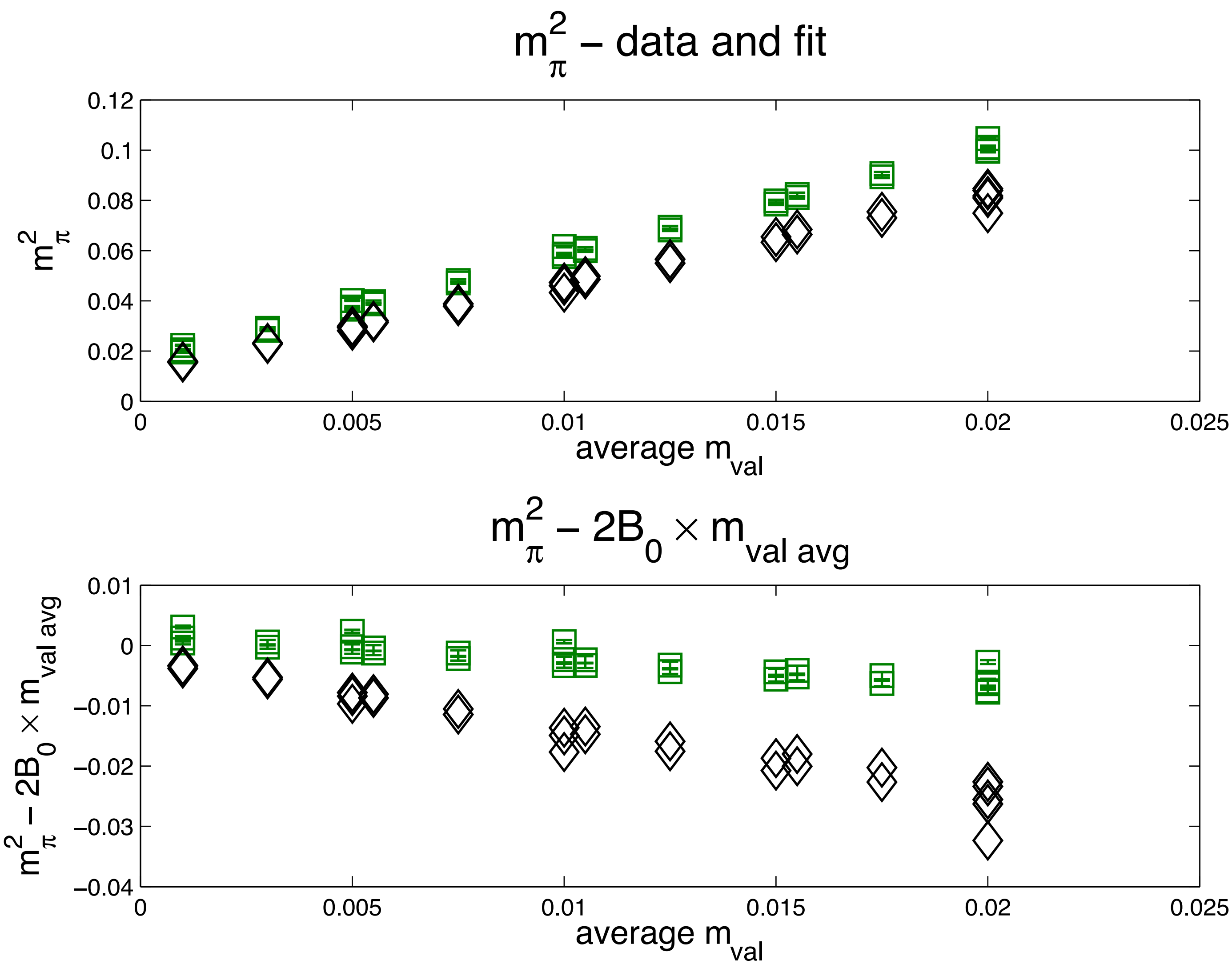
constant	NLO initial	NLO lb	NLO ub	NLO final	NNLO initial	NNLO lb	NNLO ub	NNLO final
$\tilde{f}_0$	0.07	0.03	0.1	0.0378	0.0378	0.02	0.07	0.0619
f_0	0.099	0.042	0.14	0.0534	0.0534	0.028	0.099	0.0875
B_0	2.2	1.0	3.0	2.352	2.352	2.0	3.0	2.32
L_i in units of 10^{-3}								
L_0					0.0	-1.0	1.0	0.38
L_1					0.43	0.31	0.54	0.31
L_2					0.73	0.61	0.85	0.61
L_3					-2.53	-2.9	-2.16	-2.65
L_4	1.0	-10.0	10.0	0.11	0.11	-0.4	0.4	0.18
L_5	1.0	-10.0	10.0	0.62	0.62	0.4	0.8	0.8
L_6	1.0	-10.0	10.0	0.049	0.049	-0.1	0.1	-0.058
L_7					-0.31	-0.45	-0.17	-0.19
L_8	1.0	-10.0	10.0	0.47	0.47	0.42	0.48	0.42
L_9					0.0	-0.1	0.1	0.0
K_i in units of 10^{-6}								
K_{17}					0.0	-100	100	-6.8
K_{18}					0.0	-100	100	-11
K_{19}					0.0	-100	100	4.0
K_{20}					0.0	-100	100	-0.38
K_{21}					0.0	-100	100	-15
K_{22}					0.0	-100	100	3.8
K_{23}					0.0	-100	100	1.7
K_{25}					0.0	-100	100	1.1
K_{26}					0.0	-100	100	-1.7
K_{27}					0.0	-100	100	2.2
K_{39}					0.0	0.0	0.0	0.0
K_{40}					0.0	0.0	0.0	0.0

Table 3: Values for NLO and NNLO constants. For NLO, all masses less than 0.01 were used in fit. For NNLO, all masses less than 0.02 were used. These results are very preliminary.

Using NNLO values in NLO fit to f_π



Using NNLO values in NLO fit to m_π



Summary

- SU(3) ChPT fits our data well for pseudoscalar masses below 400 MeV
- Accuracy near the kaon mass scale is not good, and is even worse for $\Delta S = 1$ matrix elements needed for non-leptonic kaon decays
- Adding $\langle \pi^+ | \bar{s}d | K^+ \rangle$ into the simultaneous fits indicates higher order terms are important
- Preliminary NNLO fits show LO parameters varying in range expected from NLO fits to m_π f_π and $\langle \pi^+ | \bar{s}d | K^+ \rangle$
- New data on 32^3 will provide more data for NNLO fits and continued investigation into convergence of SU(3).