

Axial coupling and momentum fraction of the nucleon with twisted mass fermions

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17 July 2008, Lattice 2008



Collaboration

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Outline

3-point functions

Results

Why ?

Long term motivation : compute the nucleon observables with twisted mass

- Form factors (we start with g_A)
- Parton distributions PDF ($\langle x \rangle$)
- Generalized parton distributions GPD (nothing yet)

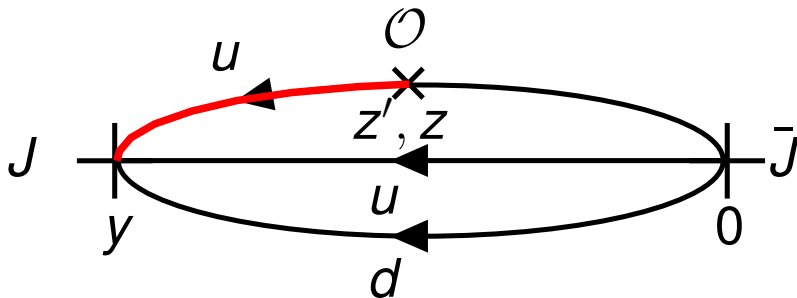
Intense experimental program at JLab and CERN

How to compute

Form of the matrix element we are interested in

$$R = \sum_{\vec{y}, \vec{z}} e^{i\vec{p}\cdot\vec{y}} e^{i\vec{p}'\cdot\vec{z}} \langle 0 | J_\gamma(y) O(z) \bar{J}_\rho(0) | 0 \rangle$$

$\propto \langle N(p, s) | O(z) | N(p', s') \rangle$ if J is a nucleon interpolating field



- Look for a plateau on t_z with $t_y \gg t_z \gg 0$

Generalized source

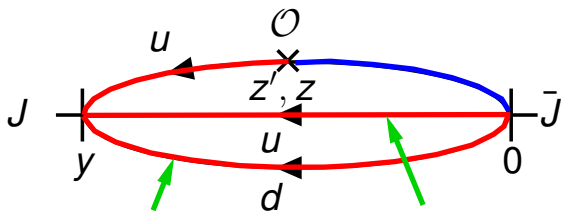
$$R = \sum_{\vec{y}, \vec{z}} e^{i\vec{p} \cdot \vec{y}} e^{i\vec{p}' \cdot \vec{z}} \langle 0 | J_\gamma(y) O(z) \bar{J}_\rho(0) | 0 \rangle$$

- After Wick contraction R has the form :

$$R = B^\dagger \gamma_5 \Lambda S_u$$

- With B solution of Dirac equation $DB = \Sigma_G$ where Σ_G is a combination of quarks propagators.
- Σ_G is called generalized or sequential source
- New inversions are necessary
- To create Σ_G
 - Fix time of sink (ie t_y , 12 in our case)
 - Fix momentum at sink. Operator momentum inserted when doing contractions
 - Choose how to project states

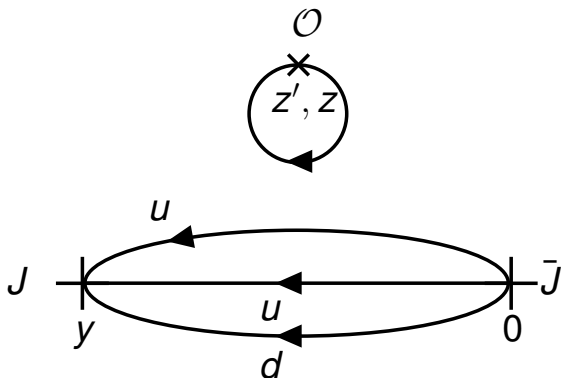
Contractions



- Green arrows : 2 propagators used in sequential source
- Red part : generalized propagator
- Blue part : normal propagator

Just need to combine correctly Dirac, color, and space indices.

Quark disconnected diagrams



- Evaluation of disconnected diagram numerically demanding (all to all propagator)
- Consider only non singlet observables

Renormalization issues

- We will use the RI-MOM scheme to renormalize non-perturbatively the bare quantities
- Z_{44} computed for some quark masses, but not yet extrapolated to chiral limit (Z.Liu, V.Morenas)
- Hypercubic artefacts reduced using [arXiv :0705.3523](#) technique
- Ideally (no mixing) multiplicative renormalization using scheme \mathcal{S} at scale μ

$$\mathcal{O}^{\mathcal{S}}(\mu) = Z_{\mathcal{S}}^{\mathcal{O}}(\mu)\mathcal{O}_{bare}$$

- Some renormalization constants have already been computed by other members of the collaboration ($Z_A = 0.76(1)$ from [arXiv :0710.0975](#))

g_A

- g_A is one of the first check point for nucleon structure calculation
- Well known experimentally from neutron beta decay ($g_A \simeq 1.269$)
- Can be extracted with $O(z) = A_\mu^{u-d} = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$
- We will need to use chiral perturbation theory, to check for finite size effects, for cutoff effects

$\langle X \rangle$

- $\langle X \rangle =$ momentum fraction carried by the quarks
- Twist-2 operators

$$\mathcal{O}^{\{\mu_1 \dots \mu_n\}} = \left(\frac{i}{2}\right)^{n-1} G_{ff'} \bar{\psi}_f \gamma^{\{\mu_1} \overleftrightarrow{D}^{\mu_2} \dots \overleftrightarrow{D}^{\mu_n\}} \psi_{f'} - \text{traces},$$

$\{\dots\}$ means symmetrization on the Lorentz indices.

- In our calculation, we use the operator

$$\mathcal{O}_{44}(x) = \frac{1}{2} \bar{u}(x) [\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3} \sum_{k=1}^3 \gamma_k \overleftrightarrow{D}_k] u(x),$$

where $D_\mu = \frac{1}{2}(\nabla_\mu + \nabla_\mu^*)$.

$$\langle X \rangle$$

- Then the bare lowest moment of the quark distribution functions $\langle x \rangle$ is given by

$$\langle x \rangle = \frac{1}{m_N^2} \langle N, \vec{0} | \mathcal{O}_{44} | N, \vec{0} \rangle = \frac{1}{m_N} \frac{C_{44}(t)}{C_N(t_y)} \quad (0 \ll t \ll t_y).$$

Here

$$C_{44}(t) = \sum_{\vec{y}, \vec{z}} \langle J(t_y, \vec{y}) \mathcal{O}_{44}(t, \vec{z}) \bar{J}(0, 0) \rangle,$$

$$C_N(t_y) = \sum_{\vec{y}} \langle J(t_y, \vec{y}) \bar{J}(0, 0) \rangle,$$

and $J(x)$ is the interpolating field for the nucleon.

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Ensembles

$\beta = 3.9$, $a = 0.0855(6)$ fm from f_π .

$a\mu$	$m_\pi(\text{GeV})$	$L^3 \times T$	$m_\pi L$	Nmeas
0.0100	0.483(1)	$24^3 \times 48$	5	173
0.0085	0.446(1)	$24^3 \times 48$	4.7	161
0.0064	0.389(1)	$24^3 \times 48$	4.1	131
0.0040	0.312(2)	$24^3 \times 48$	3.2	392

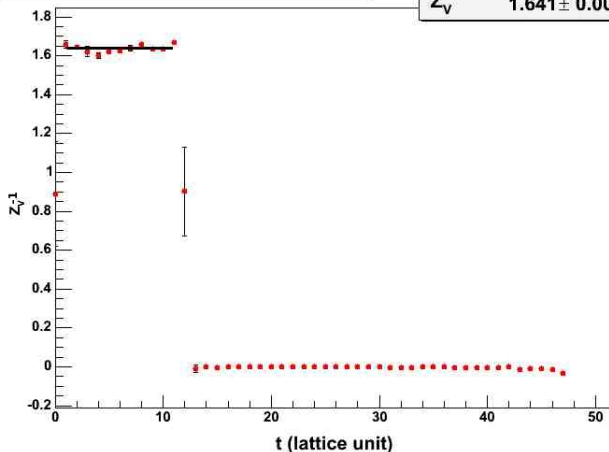
- APE and Gaussian smearings were used

Vector current

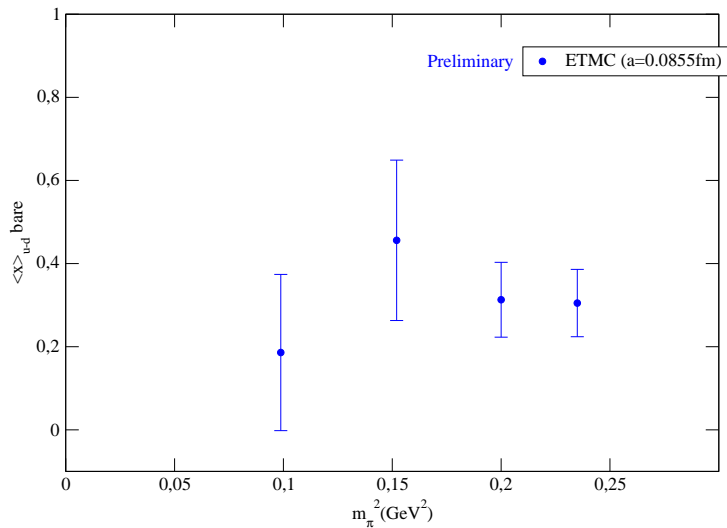
- Preliminary (10 configurations only)

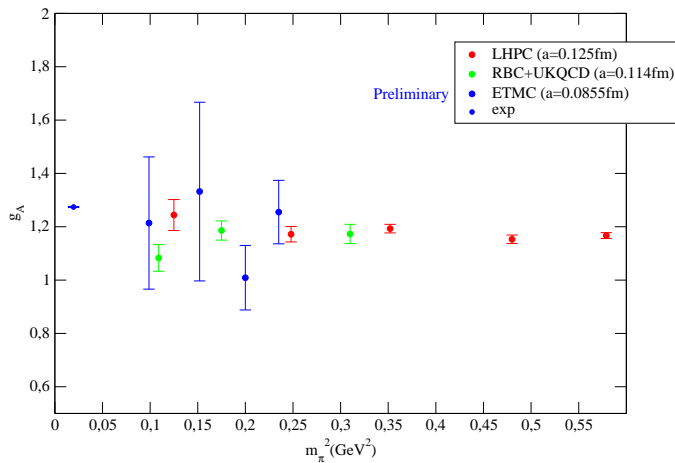
$Z_V^{-1} - \beta = 3.9$ - $L=24$, $T=48$ - $\mu=0.0085$

χ^2 / ndf 33.87 / 10
 Z_V^{-1} 1.641 ± 0.002366



- Close to the value found by the collaboration
 $(Z_V = 0.6104(2)(3))$

$\langle X \rangle$ 

g_A 

Summary

- First results for g_A and $\langle x \rangle$
- Error bars will be strongly reduced in the near future
- Planned work
 - Higher moments of GPDs
 - Renormalization for more complicated operators
 - Partially twisted boundary conditions for low transfer
 - Disconnected diagrams