Exact Chiral Fermions and Finite Density on Lattice

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Introduction : Why Exact Chiral Fermions?

Overlap and Domain Wall Fermions

Our Results

Summary

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Introduction : Why Exact Chiral Fermions ?

- The finite temperature transition in our world, i.e., QCD with $2 + 1$ flavours of dynamical quarks, is widely accepted to be governed by chiral symmetry.

- Staggered fermions have dominated the area of nonzero temperatures and densities.

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- Staggered fermions have dominated the area of nonzero temperatures and densities.

- As I presented in Lattice 2006, hadronic screening lengths, advocated by DeTar & Kogut (PRD '87) to explore the large scale composition of QGP, illustrate their deficiency in the pionic screening length.

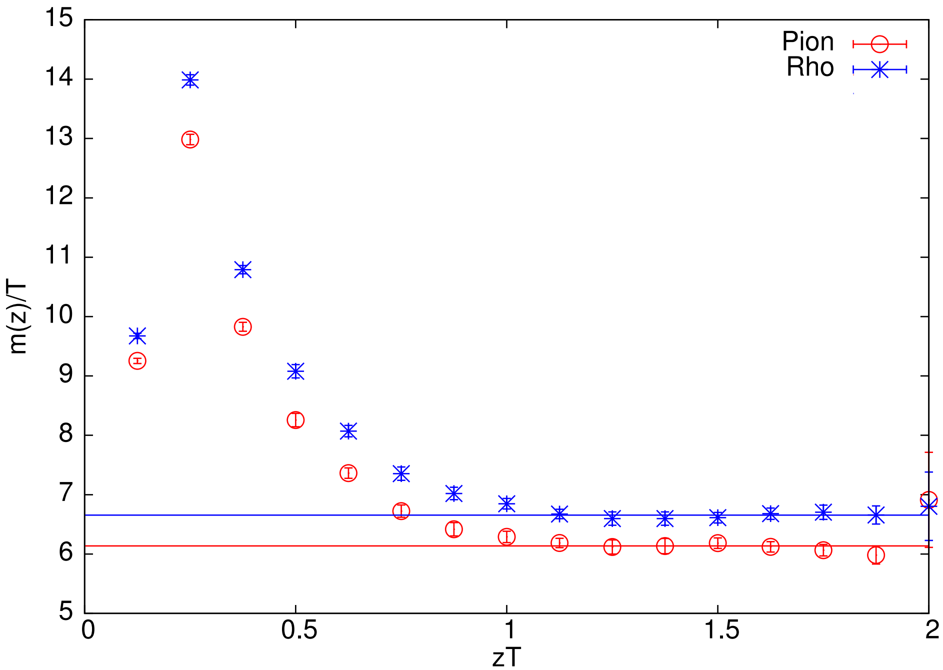
- Overlap fermions appear to do better.

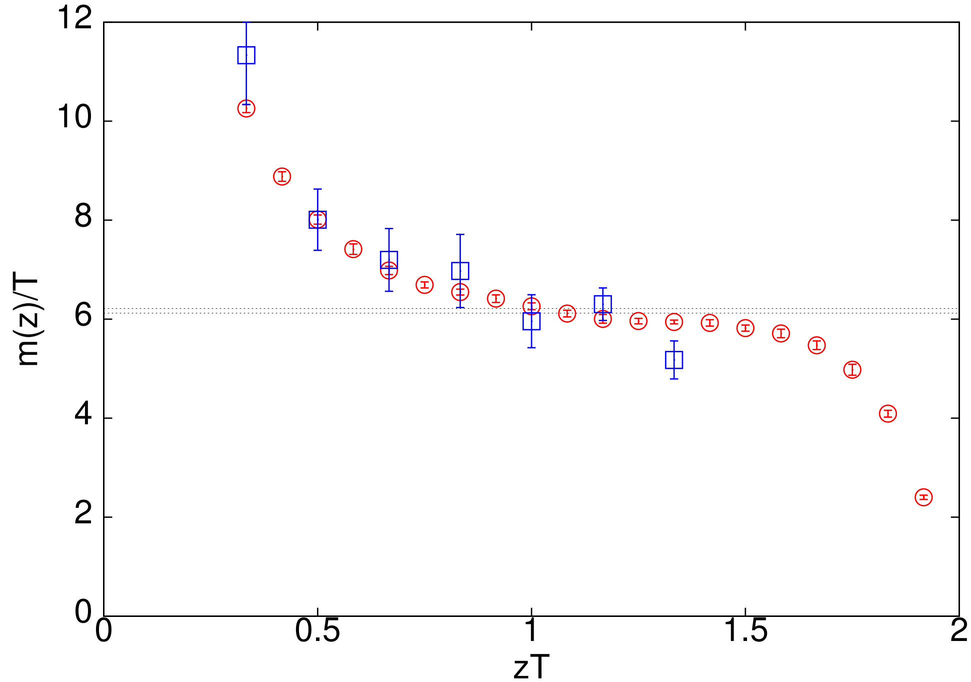
Overlap Compared with Staggered Fermions

♣ Local masses [$\sim \ln(C(r)/C(r+1)$] show nice plateau behaviour for pi & rho for Overlap (left) unlike staggered (right) fermions.

Gavai, Gupta, Lacaze PRD 2008

Gavai, Gupta PRD 2002





Screening Masses Compared

♣ The pionic screening length shows significant a^2 corrections for staggered (left) unlike Overlap (right) fermions.

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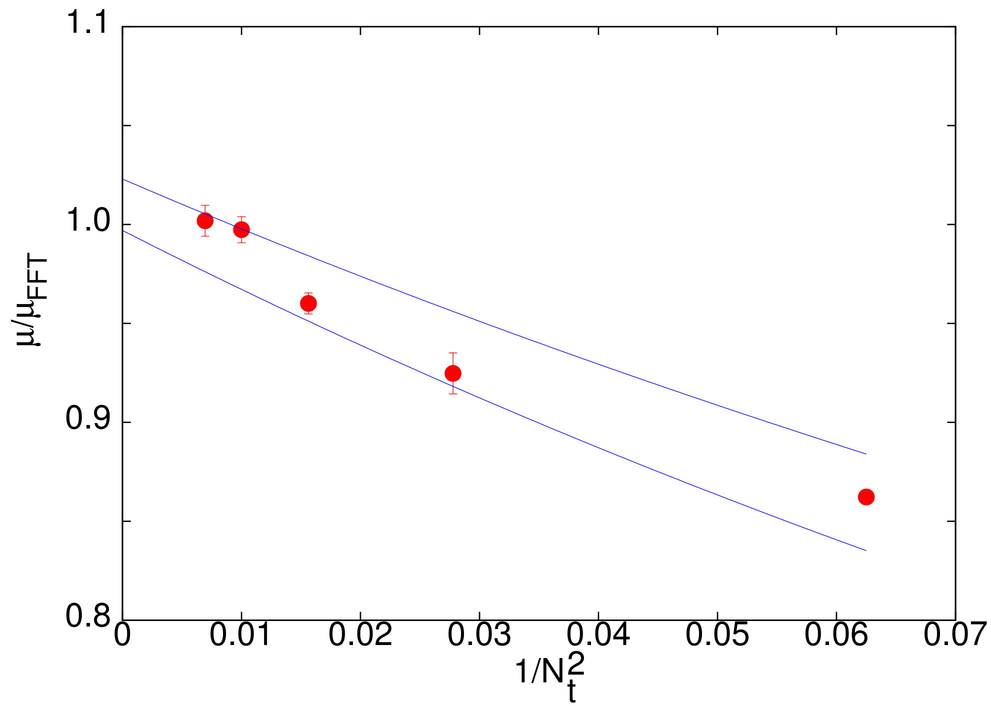

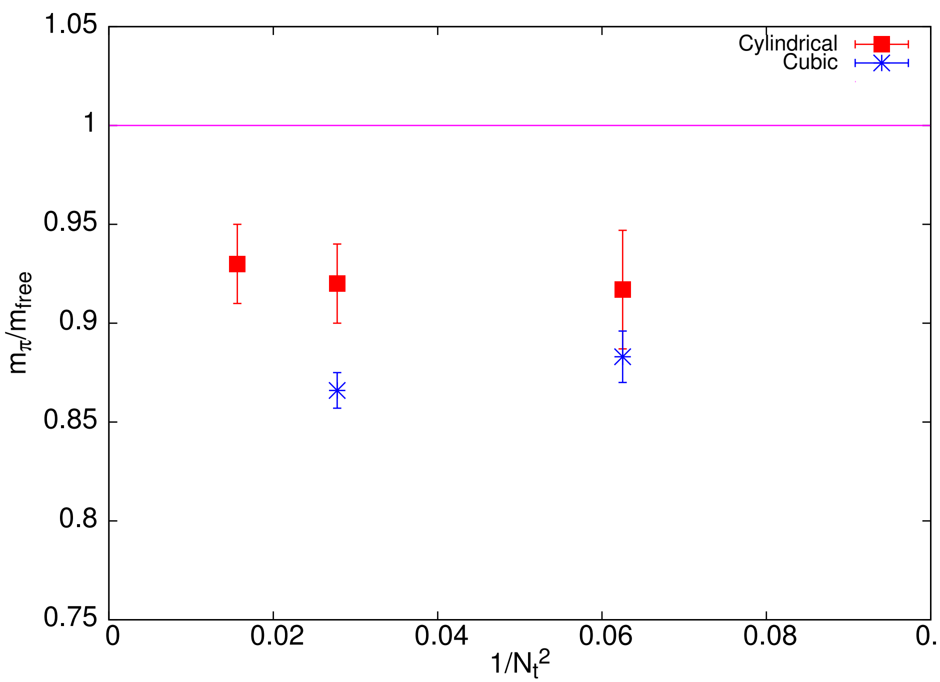

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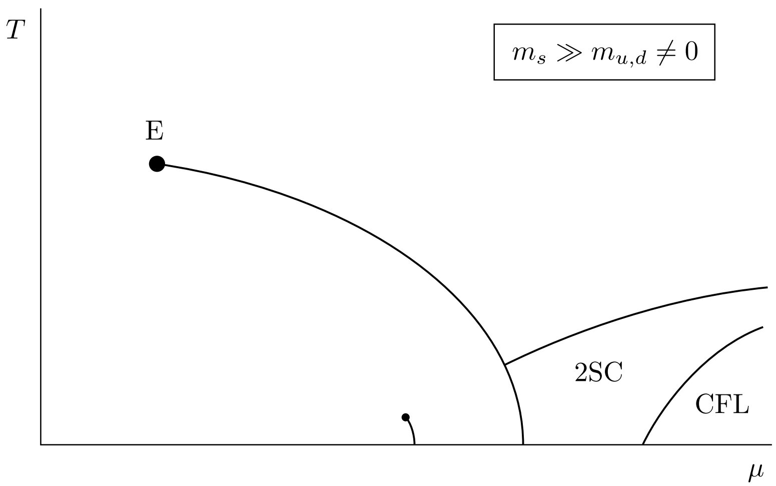
Expected QCD Phase Diagram



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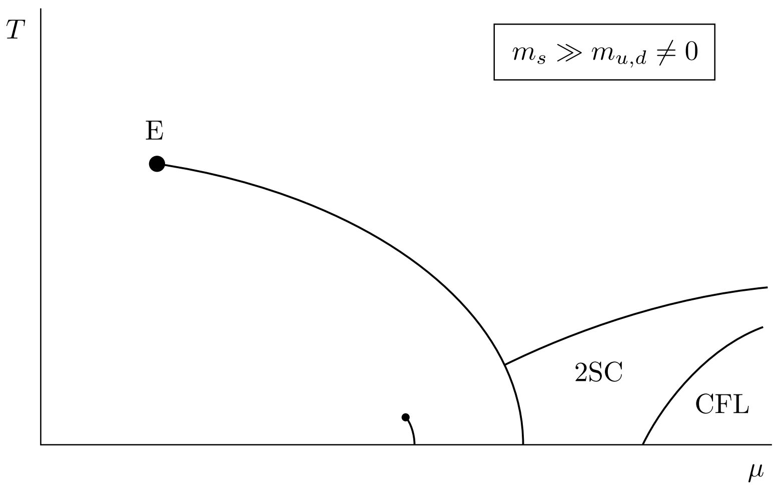

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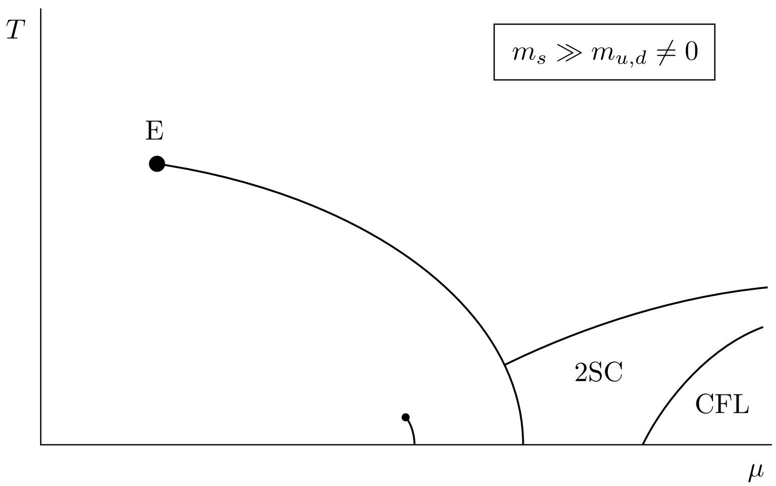

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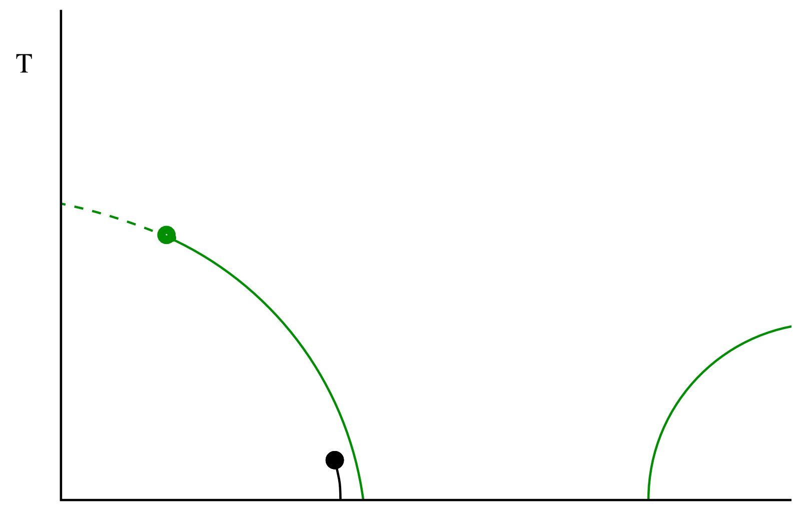

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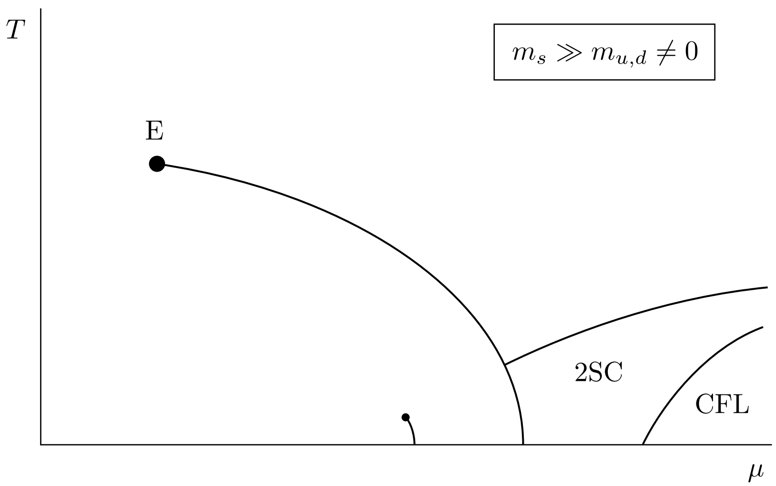

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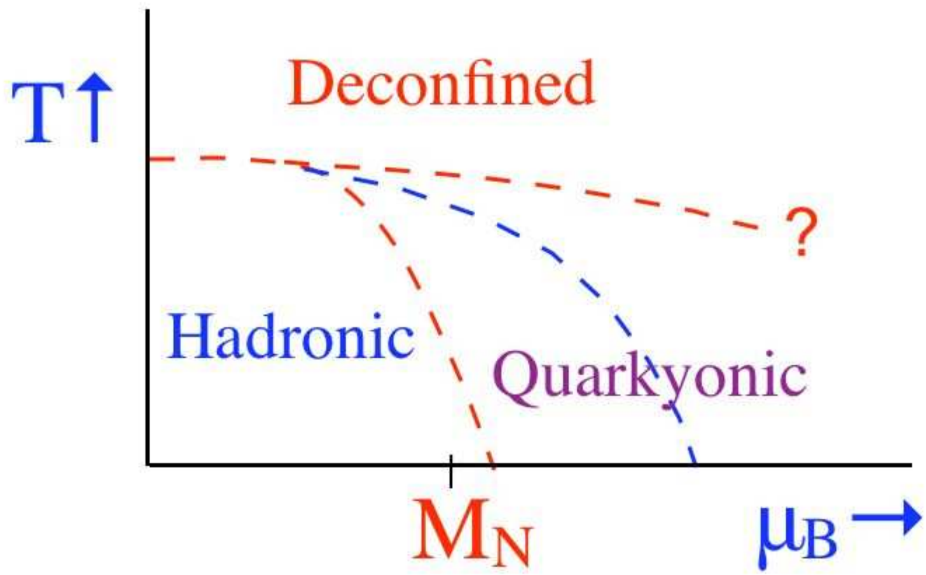

Overlap & Domain Wall Fermions

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♠ Overlap fermions, and Domain Wall fermions in the limit of large fifth dimension satisfy this relation.

Introducing Chemical Potential

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- Note $\gamma_5 D_w(a\mu)$ is no longer Hermitian, requiring an extension of the sign function. B & W proposal : For complex $\lambda = (x + iy)$, sign(λ) = sign (x).
- Gattringer-Liptak, PRD 2007, showed for $M = 1$ numerically that no μ^2 divergences exist for the free case (U =1).

- We show this to be true analytically and for all M as well. Furthermore, this holds for all functions such that $K(a\mu) \cdot L(a\mu) = 1$ for Overlap (Banerjee, Gavai, Sharma, PRD 2008) and Domain Wall Fermions (Gavai, Sharma 2008).

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- We claim that chiral invariance is lost for nonzero μ. Note that

$$\delta S = \alpha \sum_{x,y} \bar{\psi}_x \Big[\gamma_5 D(a\mu) + D(a\mu)\gamma_5 - \frac{a}{2} D(0)\gamma_5 D(a\mu) - \frac{a}{2} D(a\mu)\gamma_5 D(0) \Big]_{xy} \psi_y \ , \tag{2}$$

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- However, the sign function definition above merely ensures

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which is not sufficient to make $\delta S = 0$. True for both Overlap and Domain Wall fermions and any K,L.

Consequences

- Exact Chiral Symmetry on lattice lost for any $\mu \neq 0$: Real or Imaginary! Note $D_w(a\mu)$ is Hermitian for the latter case.
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- μ-dependent mass for even massless quarks.
- Only smooth chiral condensates : No (clear) chiral transition for any (large) μ possible. How small a, or large N_T may suffice ?
- All coefficients of a Taylor expansion in μ do have the chiral invariance but the series will be smooth and should always converge.

What if . . .

♠ the chiral transformations were $\delta\psi = \alpha\gamma_5(1 - \frac{a}{2}D(a\mu))\psi$ and $\delta\bar{\psi} = \alpha\bar{\psi}(1 - \frac{a}{2}D(a\mu))\gamma_5$?

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- Moreover, symmetry groups *different* at each μ. Recall we wish to investigate $\langle\bar{\psi}\psi\rangle(a\mu)$ to explore if chiral symmetry is restored.

- The symmetry group remains *same* at each T with $\mu = 0$ $\Longrightarrow \langle\bar{\psi}\psi\rangle(am = 0, T)$ is an order parameter for the chiral transition.

Our Results

- We investigated thermodynamics of free overlap and domain wall fermions with an aim to examine the continuum limit analytically and numerically.

- Analytically, we prove the absence of μ^2-divergences for general K and L. Our numerical results were for tuning the irrelevant parameter M to obtain small deviations from continuum limit on coarse lattices.

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- Dirac operator is diagonal in momentum space. Use its eigenvalues to compute $\mathcal{Z}$:
  $\lambda_\pm = 1 - [sgn\left(\sqrt{h^2+h_5^2}\right) h_5 \pm i\sqrt{h^2}]/\sqrt{h^2+h_5^2}$, with
  $h_i = -\sin ap_i$, i =1, 2 and 3, $h_4 = -a\ \sin(a_4 p_4)/a_4$ and
  $h_5 = M - \sum_{i=1}^{3}[1 - cos(ap_i)] - a[1 - cos(a_4 p_4)]/a_4$.

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- Hiding p_i-dependence in terms of known functions g, d and f, the energy density on an $N^3 \times N_T$ lattice is found to be

$$\epsilon a^4 = \frac{2}{N^3 N_T} \sum_{p_i, n} F(\omega_n) = \frac{2}{N^3 N_T} \sum_{p_i, n} \left[(g + \cos\omega_n) + \sqrt{d + 2g\cos\omega_n} \right] \times \left[\frac{(1 - \cos\omega_n)}{d + 2g\cos\omega_n} + \frac{\sin^2\omega_n (g + \cos\omega_n)}{(d + 2g\cos\omega_n)(f + \sin^2\omega_n)} \right] \quad (4)$$

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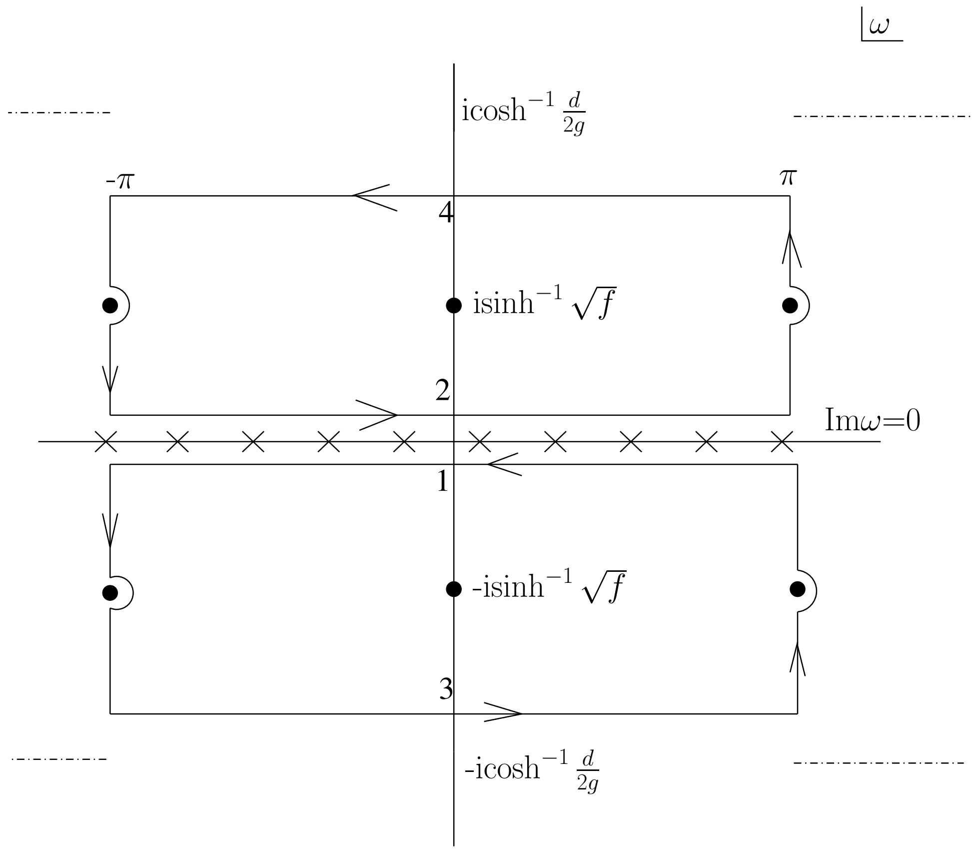
- Can be evaluated using the standard contour technique or numerically.

Analytic Evaluation : $\mu = 0$.



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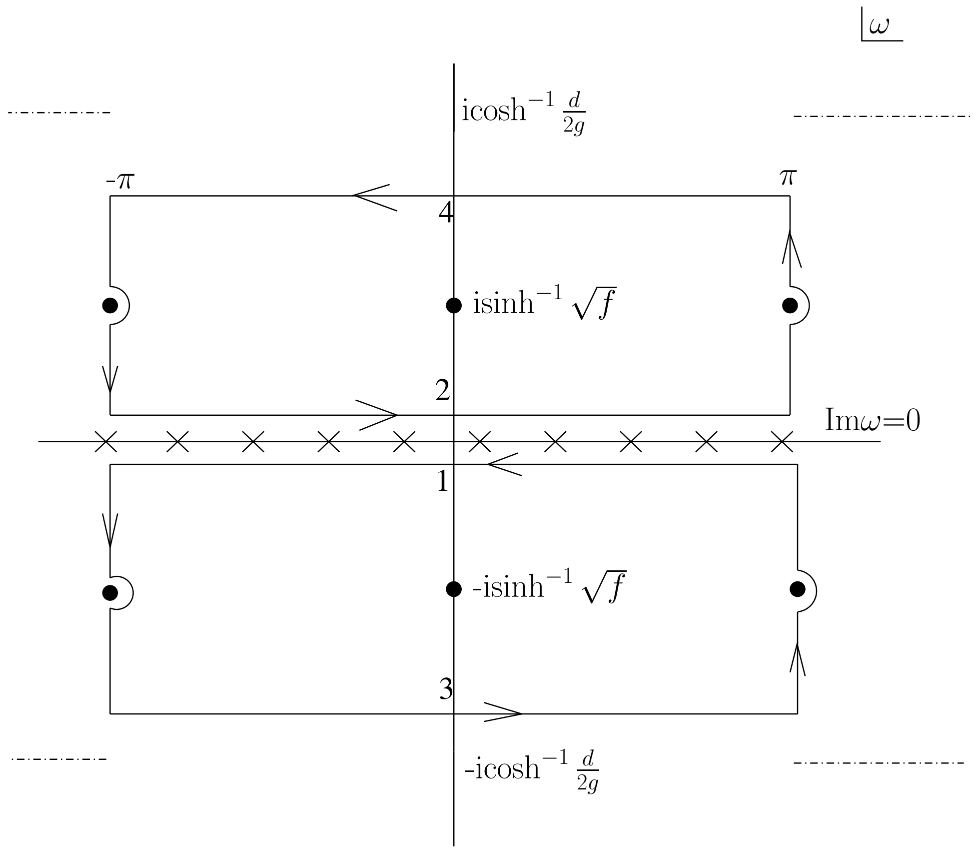

- Poles at $\omega = \pm i \ \sinh^{-1}\sqrt{f}$ and Poles (branch points) at $\pm i \ \cosh^{-1}\frac{d}{2g}$.

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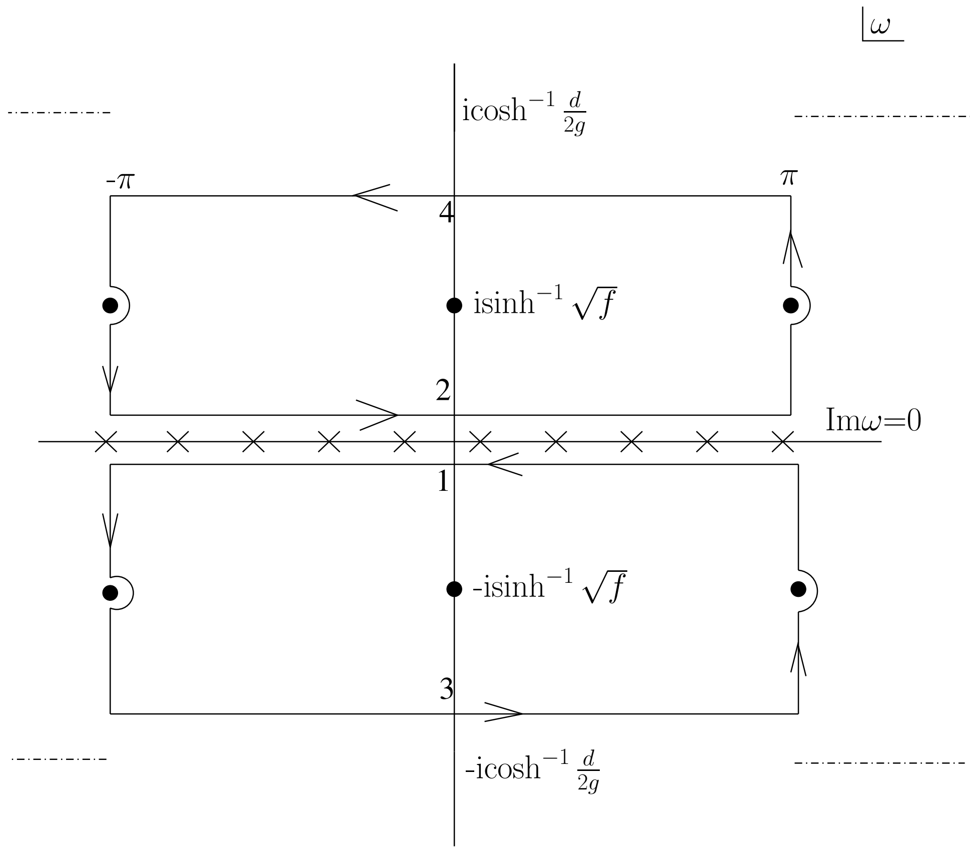

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- Evaluating integrals, $\epsilon a^4 = 4N^{-3}\sum_{p_j}\left[\sqrt{f/1+f}\right]$ $[\exp(N_T \sinh^{-1}\sqrt{f}) + 1]^{-1}$ $+\epsilon_3 + \epsilon_4$, where $f = \sum_i \sin^2(ap_i)$.

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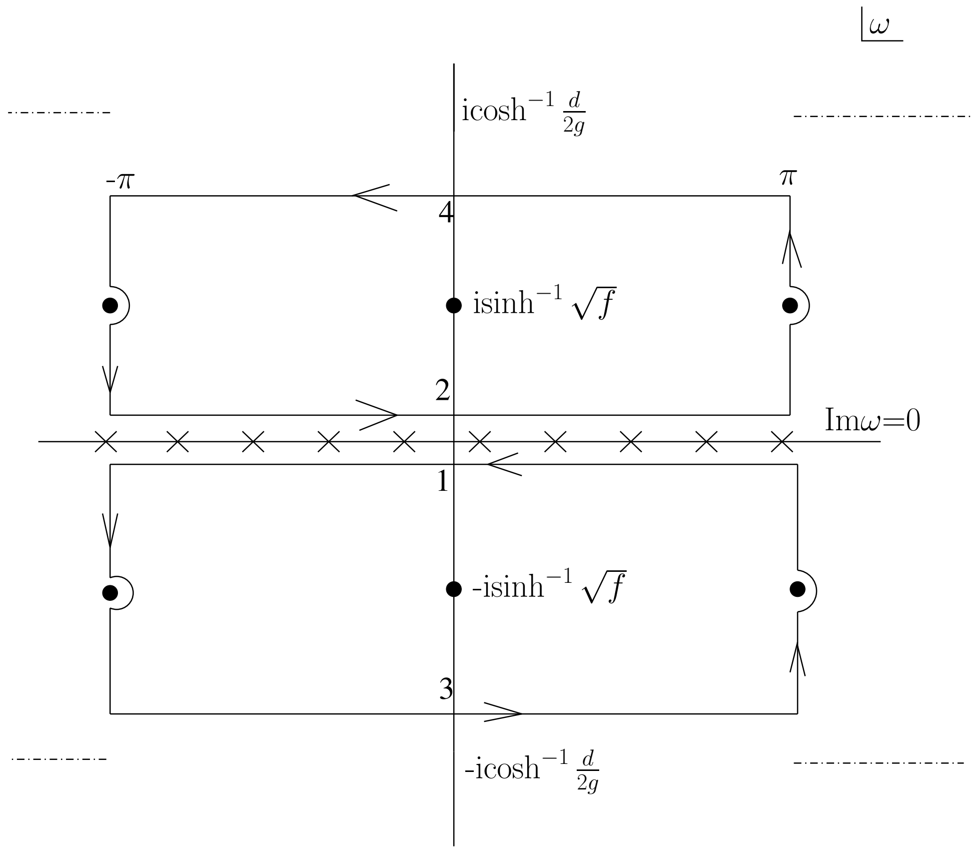

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- Can be seen to go to ϵ_{SB} as $a \to 0$ for all M.

More Details : $T = 0$, $\mu \neq 0$

- Defining $K(\mu) + L(\mu) = 2R\cosh\theta$ and $K(\mu) - L(\mu) = 2R\sinh\theta$, the same treatment as above goes through by substituting $\sin\omega_n \rightarrow R\sin(\omega_n - i\theta)$ and $\cos\omega_n \rightarrow R\cos(\omega_n - i\theta)$.

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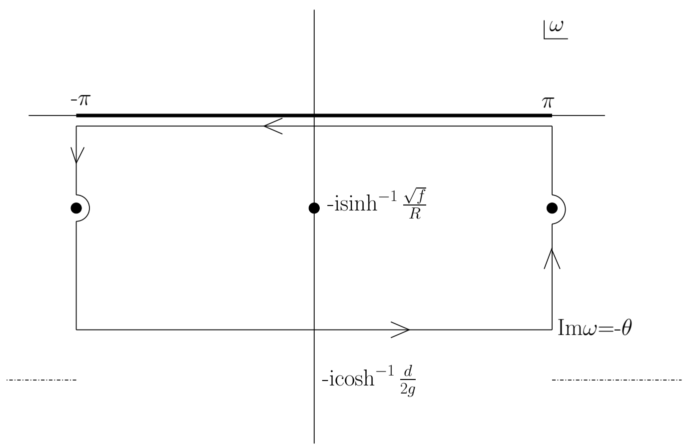
- Energy density is also functionally the same with $F(1, \omega_n) \to F(R, \omega_n - i\theta)$.

- Additional observable, number density : Has the same pole structure so similar computation.



Divergence Cancellation at $T = 0$, $\mu \neq 0$

- Doing the contour integral, the energy density turns out to be :

$$\epsilon a^4 = (\pi N^3)^{-1} \sum_{p_j} \Big[2\pi \mathrm{Res}\ F(R,\omega) \Theta \left(K(a\mu) - L(a\mu) - 2\sqrt{f} \right) + \int_{-\pi}^{\pi} F(R,\omega) d\omega - \int_{-\pi}^{\pi} F(1,\omega) d\omega \Big].$$

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- Similar derivation goes through for Domain Wall Fermions ($a_5 = 1$) as well.

Numerical Evaluation

$\diamondsuit$ Two Observables : $\Delta\epsilon(\mu, T) = \epsilon(\mu, T) - \epsilon(0, T)$ and Susceptibility, $\sim \partial^2 \ln \mathcal{Z} / \partial \mu^2$. Done for both Overlap and Domain Wall Fermions ($a_5 = 1$).

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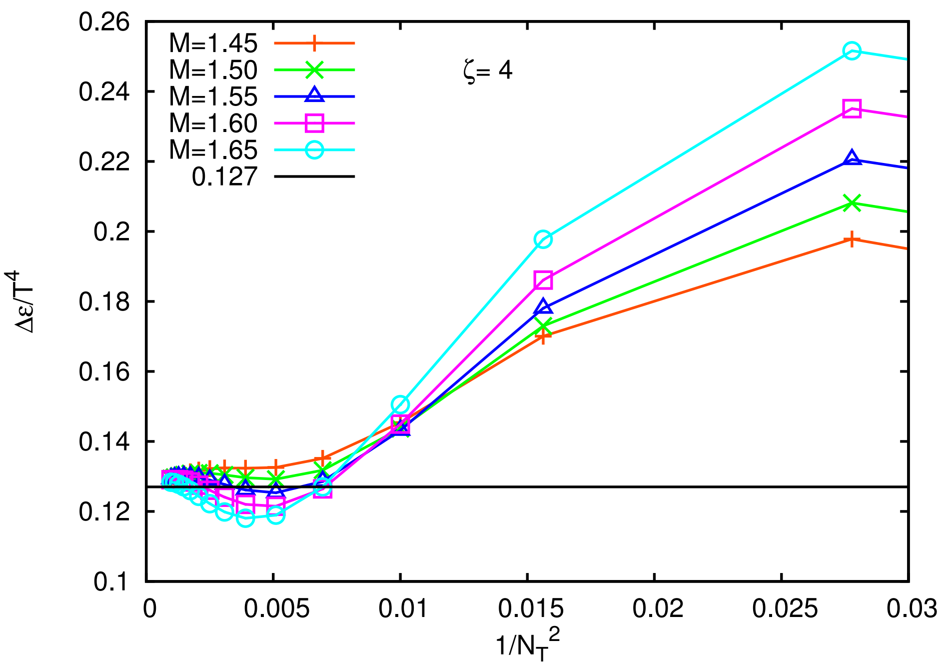
♢ Former computed for two $r = \mu/T = 0.5$ and 0.8 while latter for $\mu = 0$

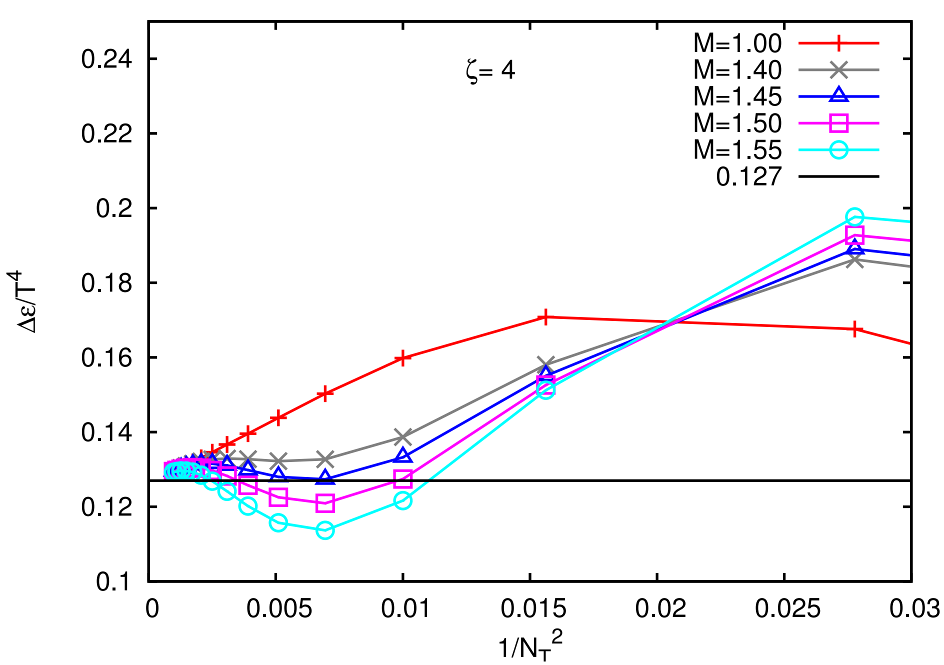
Numerical Evaluation

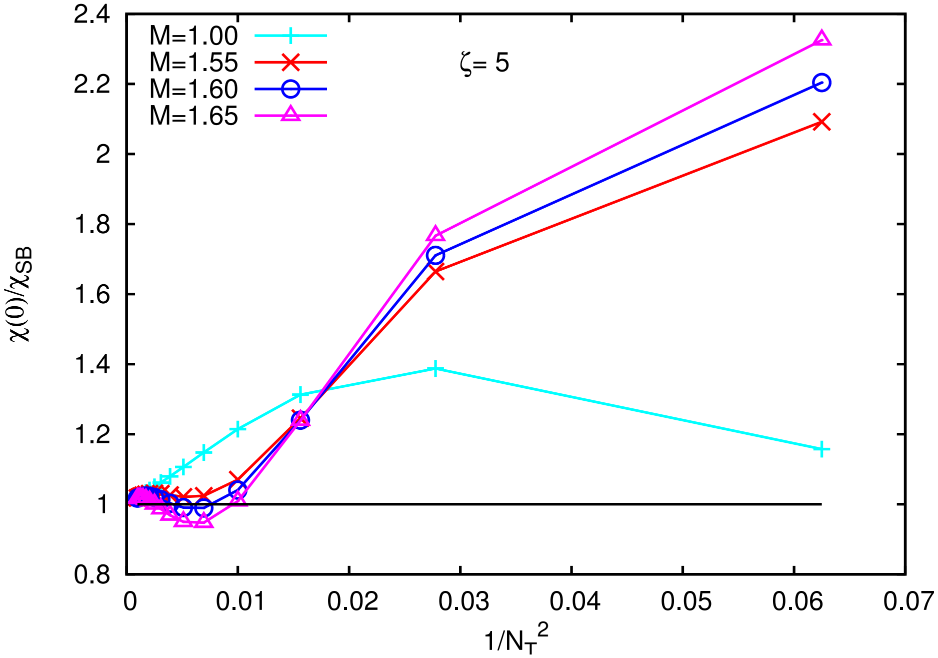
♢ Two Observables : $\Delta\epsilon(\mu, T) = \epsilon(\mu, T) - \epsilon(0, T)$ and Susceptibility, $\sim \partial^2 \ln \mathcal{Z}/\partial\mu^2$. Done for both Overlap and Domain Wall Fermions ($a_5 = 1$).

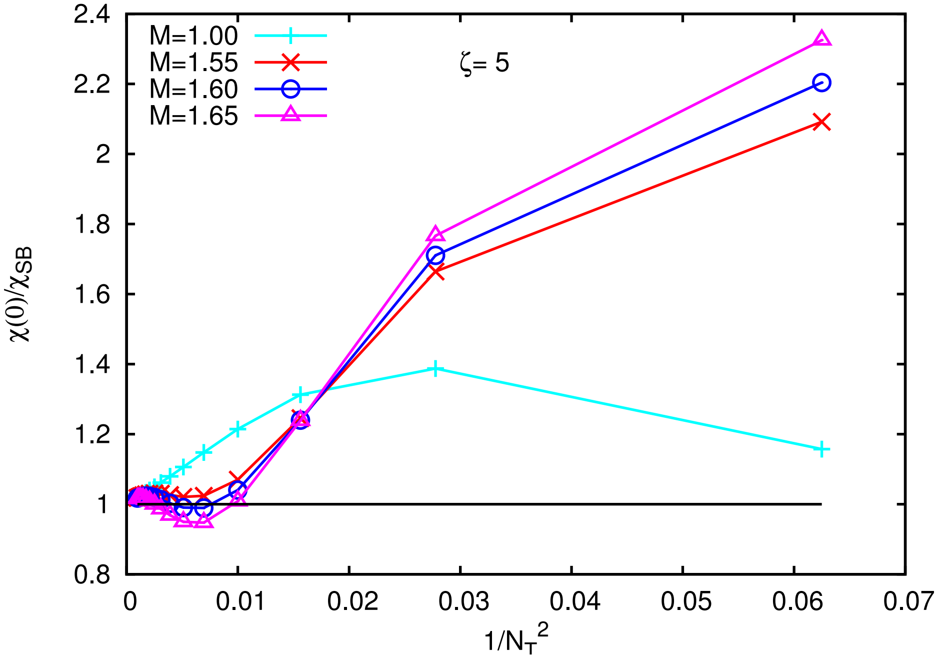
♢ For odd N_T and large enough μ the sign function is undefined as an eigenvalue becomes pure imaginary (Overlap).

♢ Former computed for two $r = \mu/T = 0.5$ and 0.8 while latter for $\mu = 0$

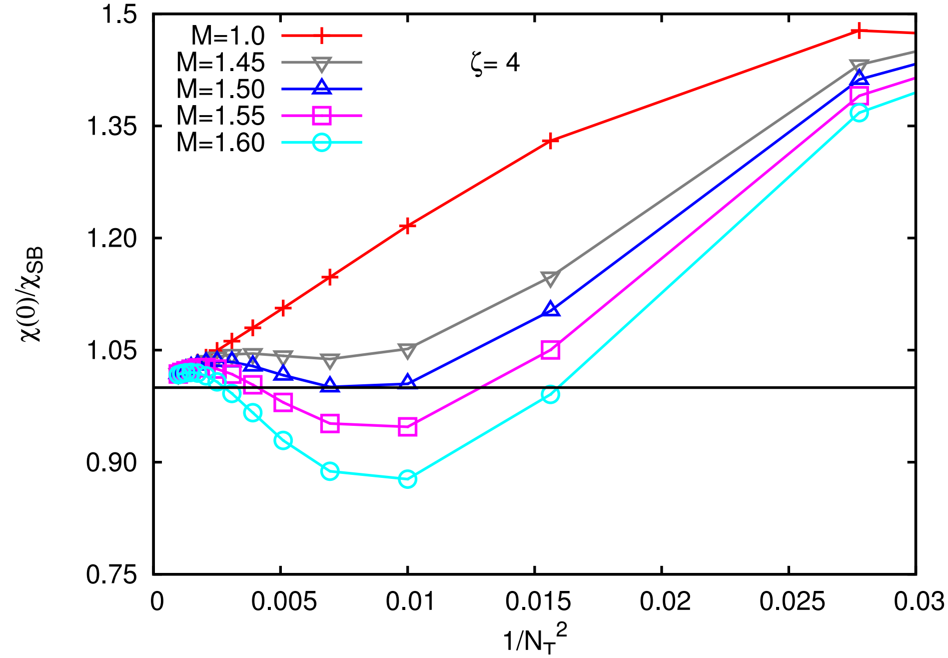





M=1.00
M=1.55
M=1.60
M=1.65
ζ= 5
χ(0)/χSB
1/NT2
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07

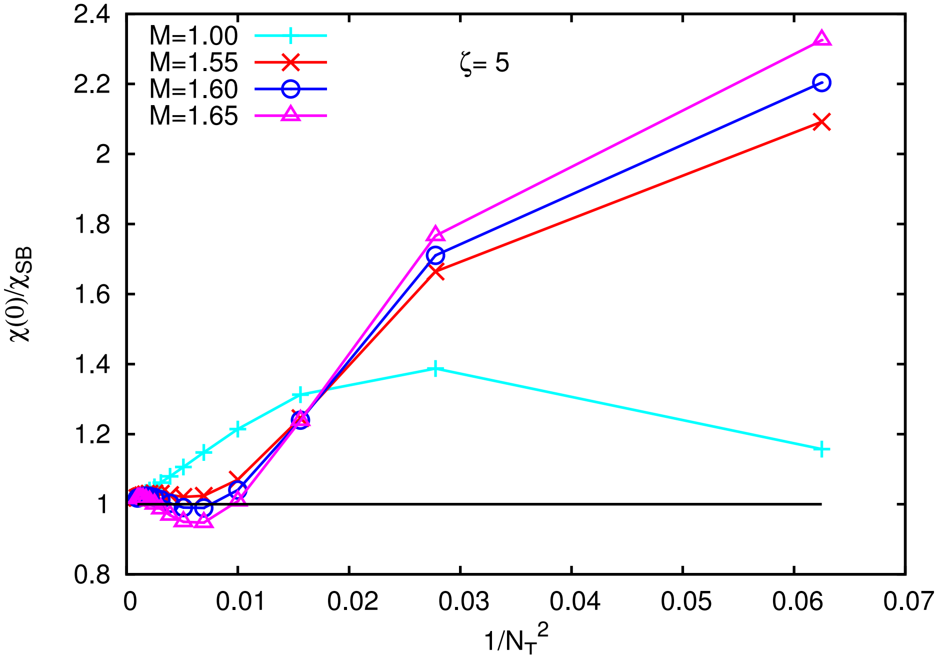


Overlap Fermions



Domain Wall Fermions

♡ Susceptibility too behaves the same way as the energy density.



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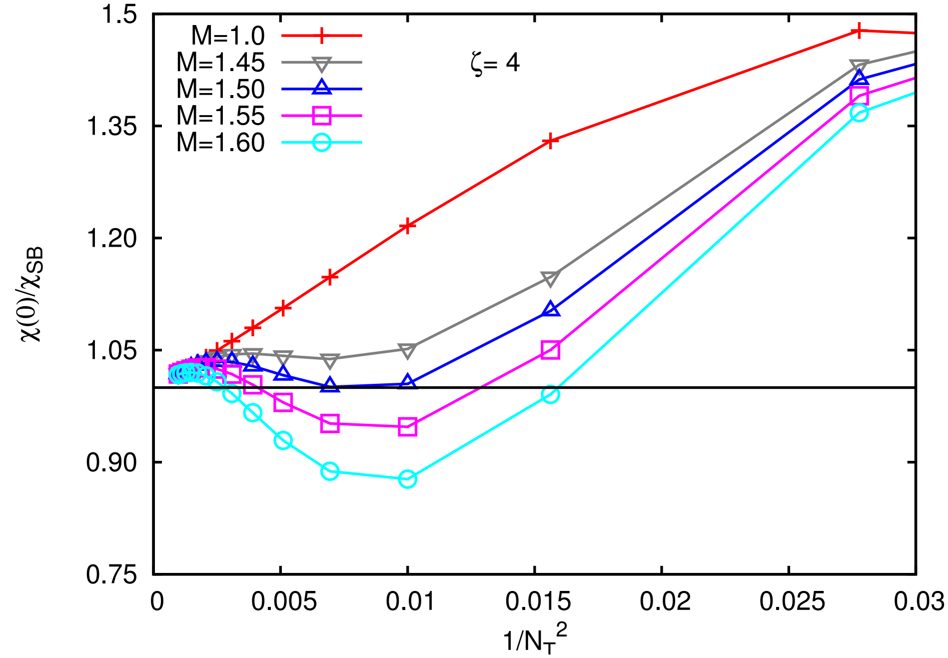
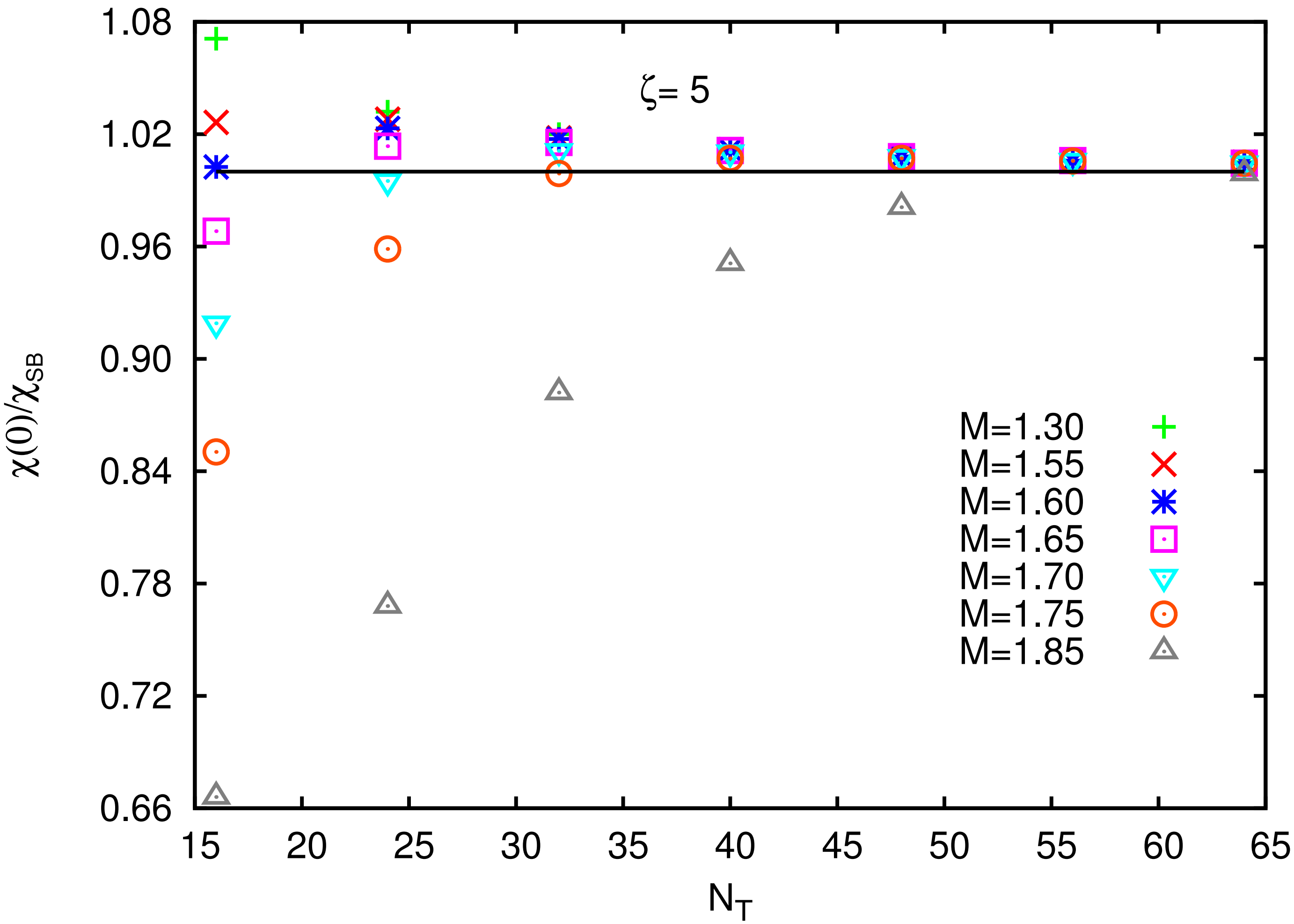


Domain Wall Fermions

♡ Susceptibility too behaves the same way as the energy density.

♡ $1.50 \leq M \leq 1.60$ seems optimal, with 2-3 % deviations already for $N_T = 12$ for overlap, while $1.40 \leq M \leq 1.50$ seems optimal for Domain Wall Fermions, with similar deviations for $N_T = 12$.



1.08
1.02
0.96
0.90
0.84
0.78
0.72
0.66
15
20
25
30
35
40
45
50
55
60
65
ζ= 5
$\chi(0)/\chi_{SB}$
N_T
M=1.30
M=1.55
M=1.60
M=1.65
M=1.70
M=1.75
M=1.85

Summary

- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

Summary

- Exact chiral symmetry without violation of flavour symmetry important for many studies on lattice, especially for the critical point and the QCD phase diagram in μ–T plane.

- Overlap and Domain wall fermions lose their chiral invariance on introduction of chemical potential in the Bloch-Wettig method and its generalizations.

- However, no μ^2-divergence exists in the continuum limit for both Overlap and Domain Wall Fermions for B & W and an associated general class of functions $K(\mu)$ and $L(\mu)$ with $K(\mu) \cdot L(\mu) = 1$.

- For the choice of $1.5 \leq M \leq 1.6$ $(1.4 \leq M \leq 1.5)$, both the energy density and the quark number susceptibility at $\mu = 0$ exhibited the smallest deviations from the ideal gas limit for $N_T \geq 12$ for Overlap (Domain Wall) Fermions.

Numerical Evaluation

♣ Zero temperature contribution : as $N_T \to \infty$, ω sum becomes integral which we estimated numerically.
♣ Continuum limit by holding $\zeta = N/N_T = LT$ fixed and increasing N_T.

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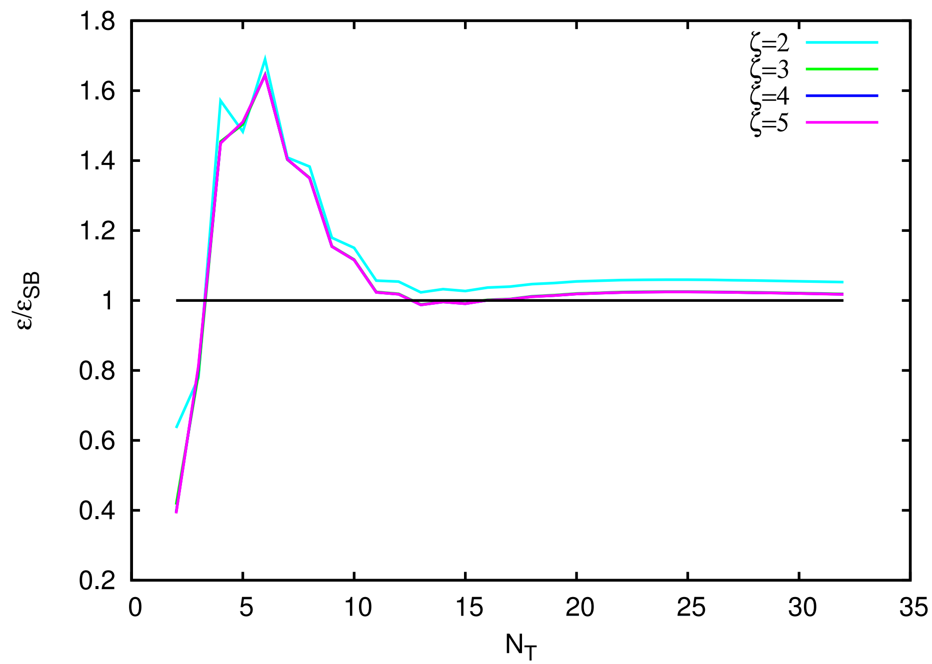

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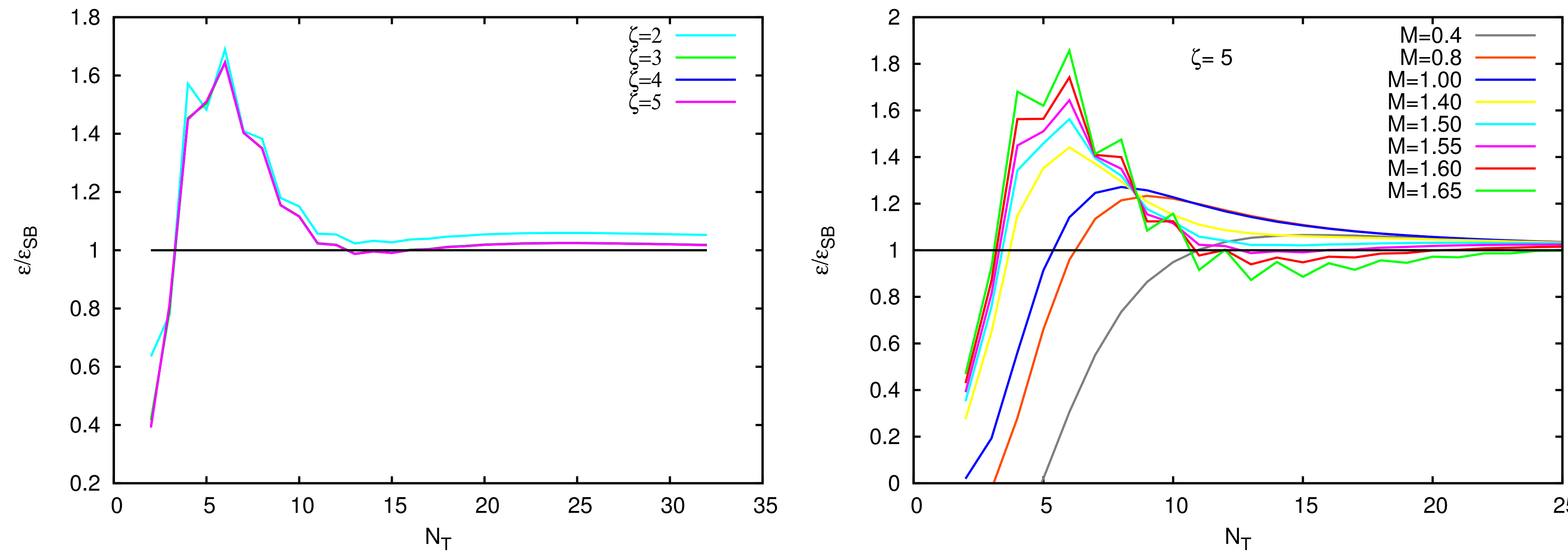
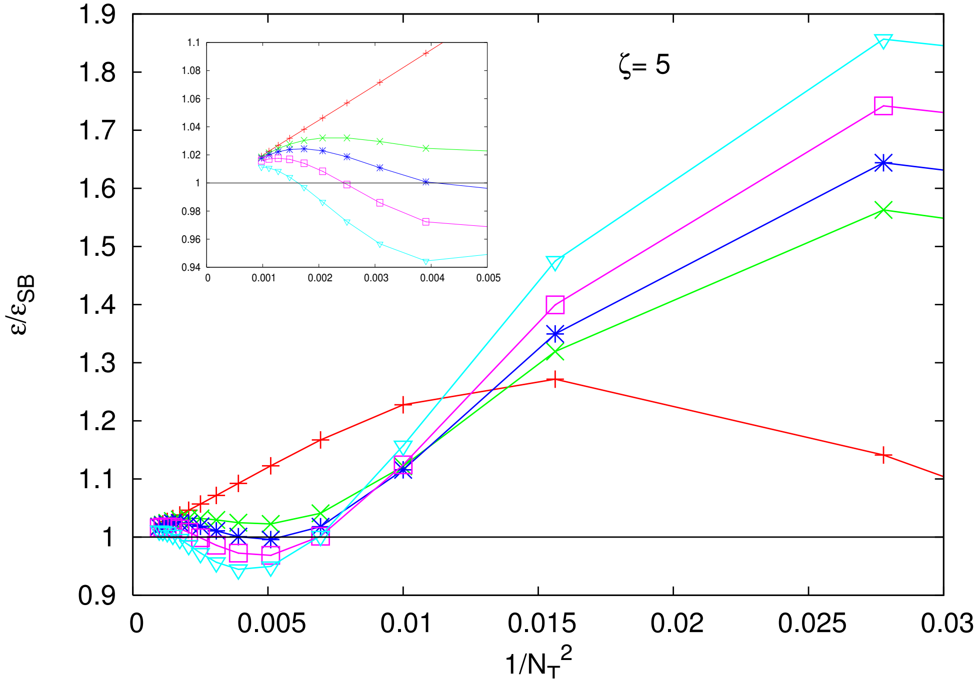


Approach to SB-Limit



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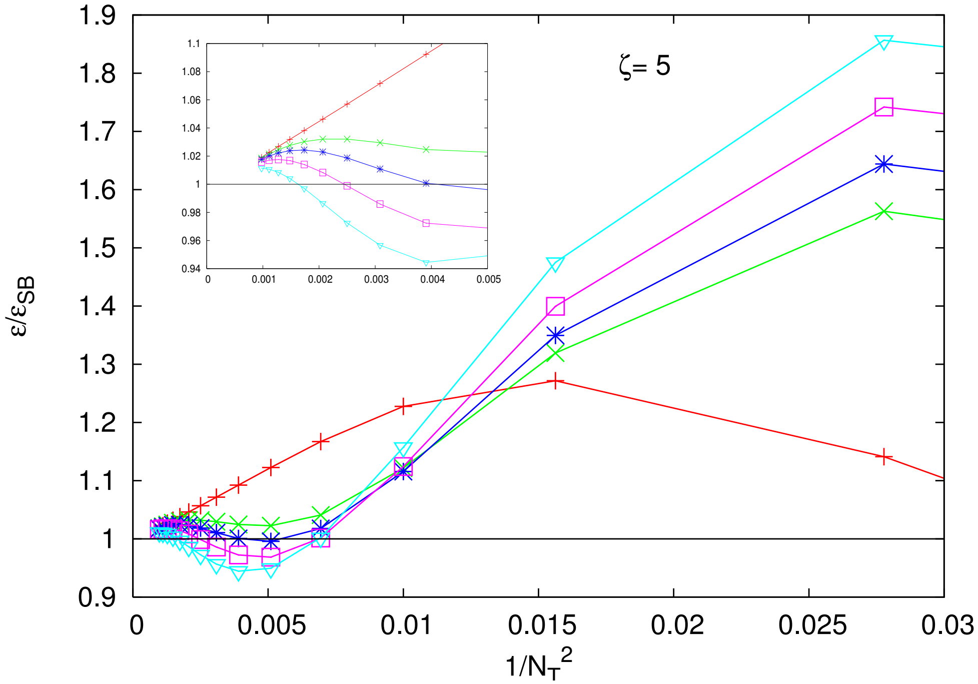
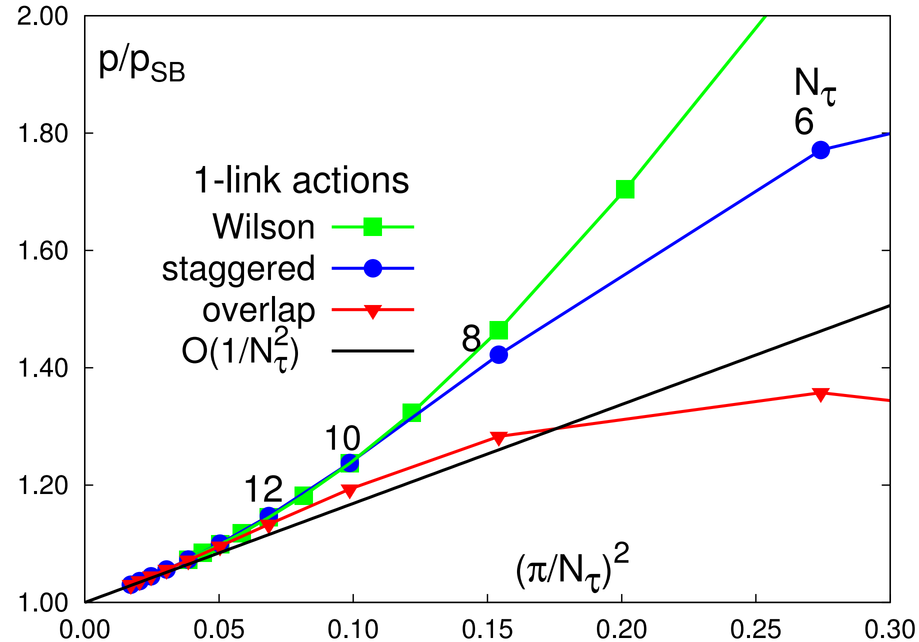


Banerjee, Gavai & Sharma , arXiv:0803.3925



Hegde, Karsch, Laermann & and Shcheredin, arXiv:0801.4883

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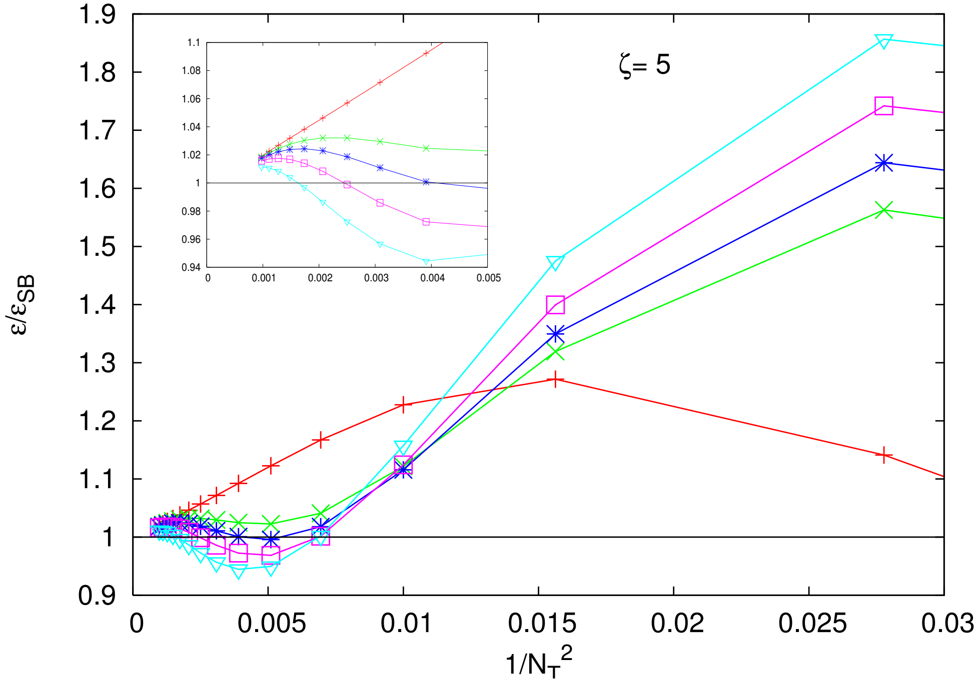

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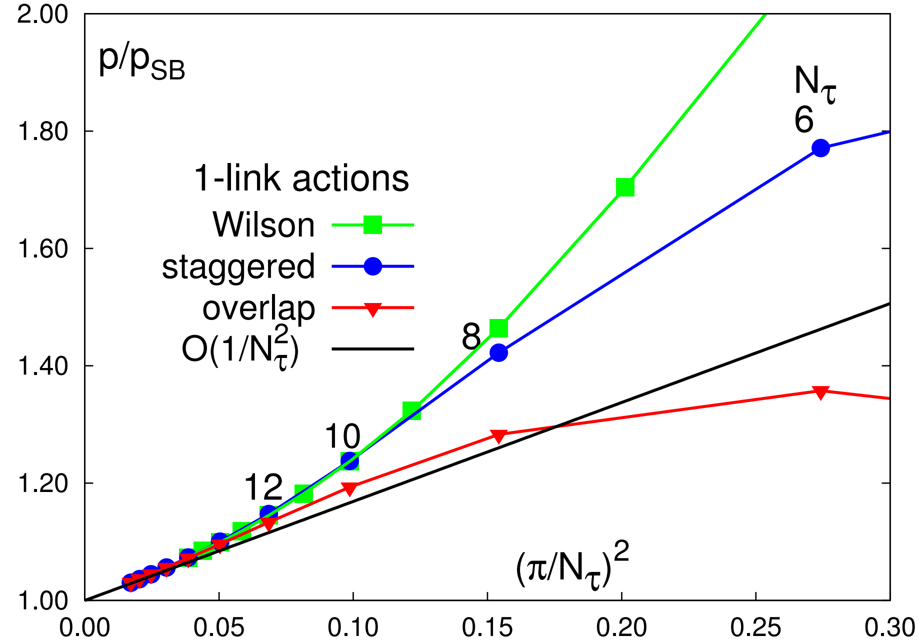

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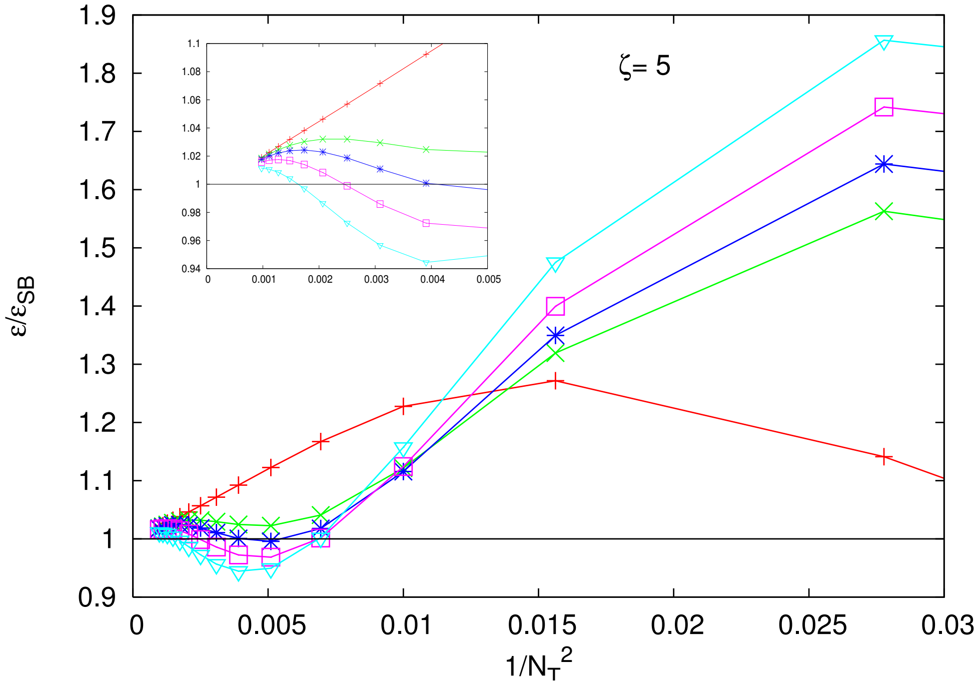

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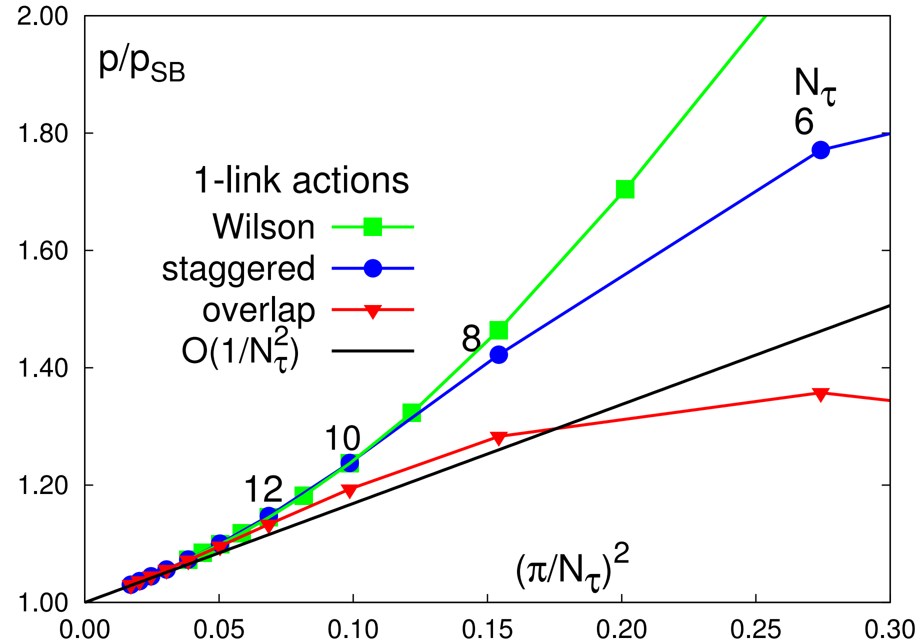

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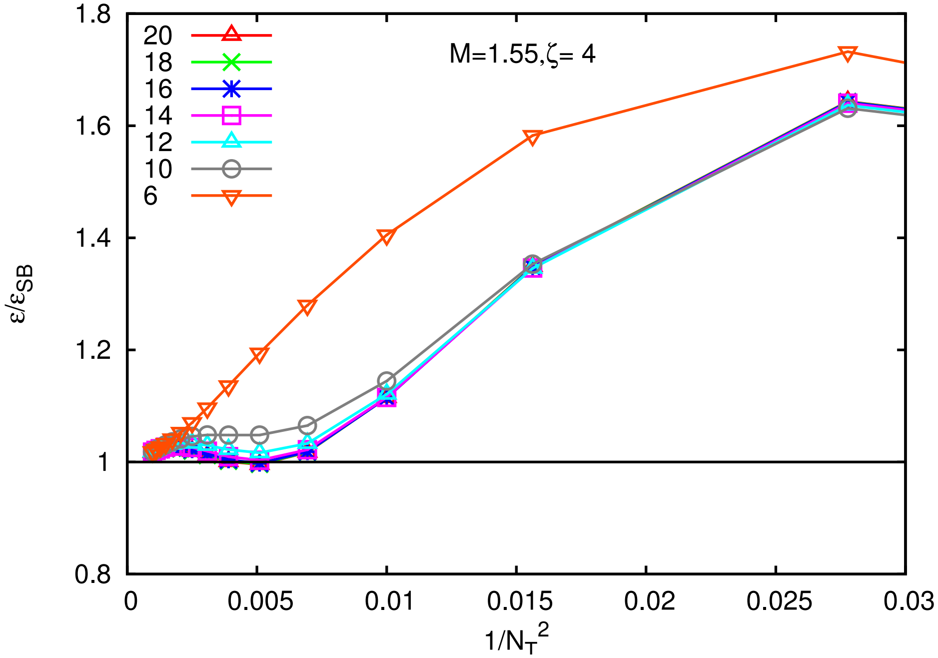
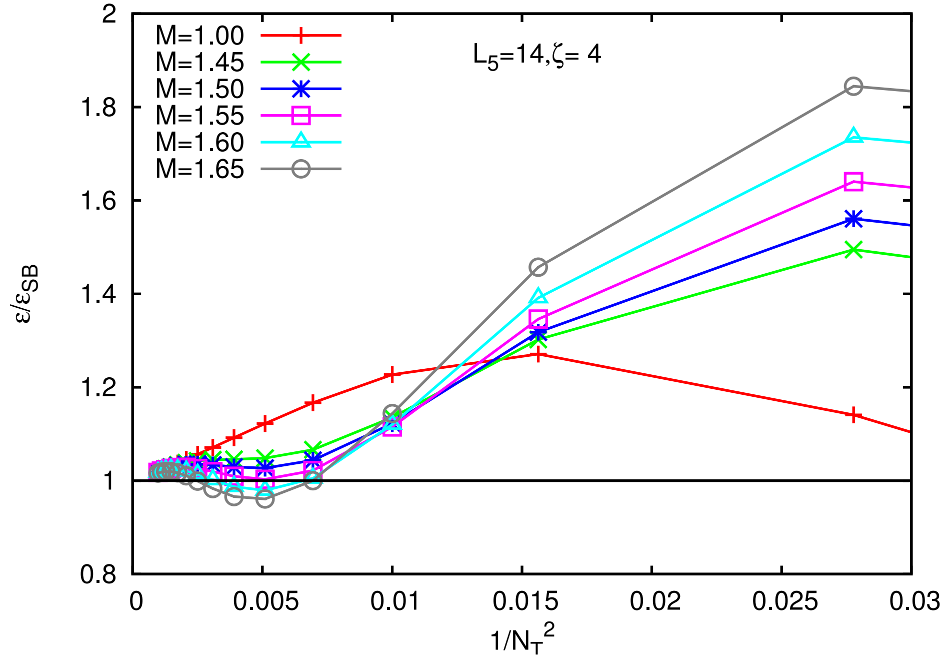
Domain Wall Fermions





Rajiv V. Gavai and Sayantan Sharma, in preparation.

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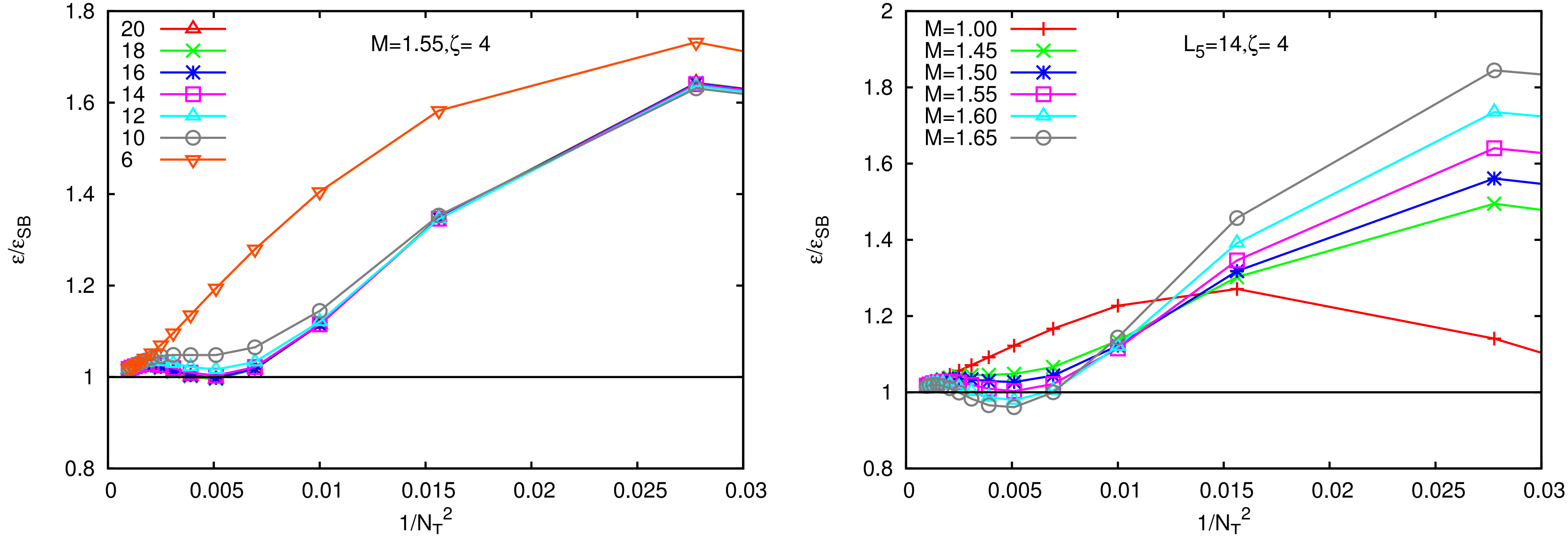

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◊ $L_5 \geq 14$ seems to be large enough to get L_5-independent results.

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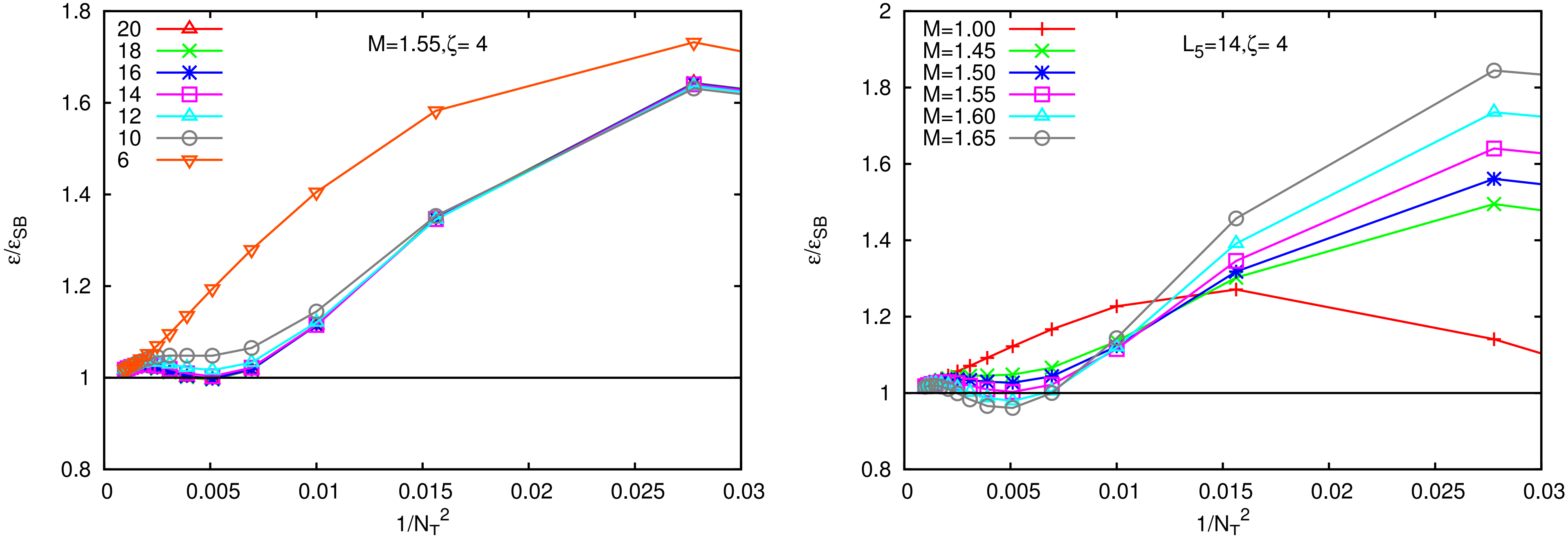

Rajiv V. Gavai and Sayantan Sharma, in preparation.

◇ $L_5 \geq 14$ seems to be large enough to get L_5-independent results.

◇ Optimal range again seems to be $1.50 \leq M \leq 1.60$; $M = 1.9$ used by Chen et al. (PRD 2001) in their study of order parameters of FTQCD.