Confining string beyond the free approximation: the case of random percolation

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INTRODUCTION

In this seminar we discuss about the properties of the flux tube in the gauge theory DUAL to the percolation model

Results about Polyakov-Polyakov correlation $\langle P(0)P^*(R)\rangle$

- BOND and SITE percolation;
- different lattices: Simple Cube (SC) and Body-Centered Cube (BCC);
- different values of critical occupation probability p_c;

We test different REGULARIZATIONS $\rightarrow$ results true in the continuum limit

PERCOLATION MODEL (1)

- We set the bonds (sites) of a 3d lattice on or off according to probabilities p and $1-p$:
  BOND (SITE) percolation
- Networks of connected on links appear (clusters)
- There exists a value p_c at wich an infinite cluster appears (second order critical point)

We have a chain of maps (in 3d):

S_q-gauge theory $\Leftrightarrow$ q-Potts model (Kramers-Wannier duality)

$(e^{\beta_{gauge}}-1)(e^{\beta_{potts}}-1)=q$

q-Potts model $\Leftrightarrow$ percolation model (Fortuin-Kasteleyn reformulation)

$p=1-e^{-\beta_{potts}}$

PERCOLATION MODEL (2)

Using them, one is led to define the Wilson loop $W(C)$, in a given configuration C, as:

$W = 1$ if no clusters are linked to the the loop; $W = 0$ otherwise

We study the $\lim_{q \to 1}$ of the theory, i.e. we study the gauge theory DUAL to the “random” percolation model

$p < p_c$ finite size clusters $\Rightarrow$ deconfinement

$p > p_c$ infinite cluster $\Rightarrow$ confinement

Finite temperature theory defined in a 2d infinite slice with thickness L=1/T

The same definitions $\Rightarrow$ Polyakov-Polyakov correlators

PERCOLATION MODEL (3)

Some properties of this gauge theory:

- Well defined string tension at $T = 0$
- Glueball spectrum at $T = 0$
- Finite temperature transition at T_c
- Well defined string tension at finite temperature
- Universality of the adimensional ratio $T_c/\sqrt{\sigma} \simeq 1.5$
- It is possible to study monopoles properties
  (See S.Lottini's Talk)

String picture

- The vacuum acts on the chromo-magnetic field as a "dual superconductor", keeping the flux between two color sources squeezed in a string-like tube
- The energy is (in first approximation) proportional to its length $\Rightarrow V(R) \propto \sigma R$
- In the *rough* phase (where the string fluctuates quantistically on any length scale) there are many predictions that can be tested on the lattice

NLO approximation

$\langle P(0)P^*(R)\rangle$ is expected to exhibit the following next-to-leading (NLO) functional form:

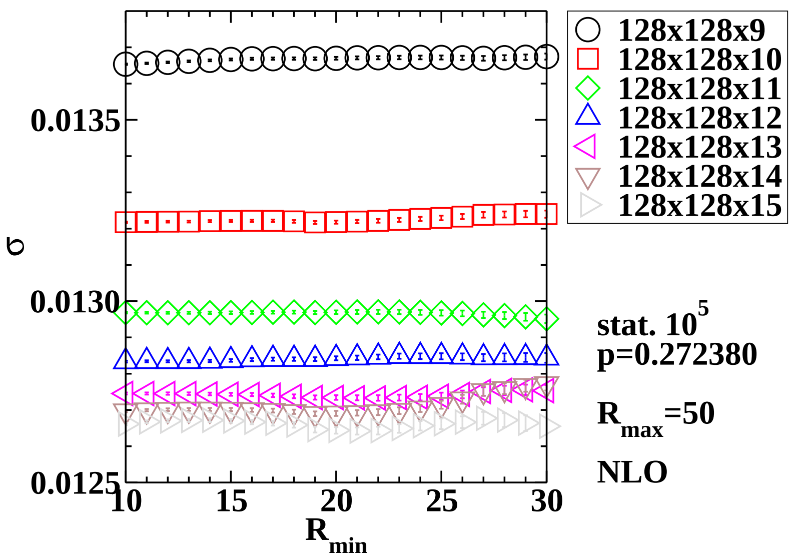
$$\langle P(0)P^*(R)\rangle = \frac{e^{-cL-\sigma RL-\frac{(d-2)\pi^2 L[2E_4(\tau)-E_2^2(\tau)]}{1152\sigma R^3}+O(1/R^5)}}{\eta(\tau)^{d-2}}$$

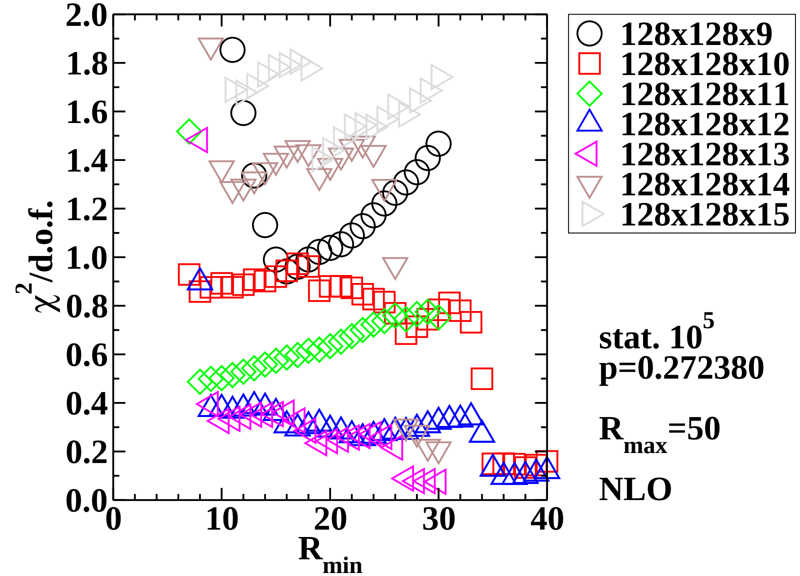
Simulations on $128^2 \times L$; $L = 1/T$ is chosen such that $0.3T_c \lesssim T \lesssim 0.8T_c$; 10^5 measures;

Fit in the range $R_{min} \lesssim R \lesssim R_{max} \sim 50$

RESULTS

BOND percolation; SC lattice; $p = 0.272380 \Rightarrow$ critical for $L = 6$;



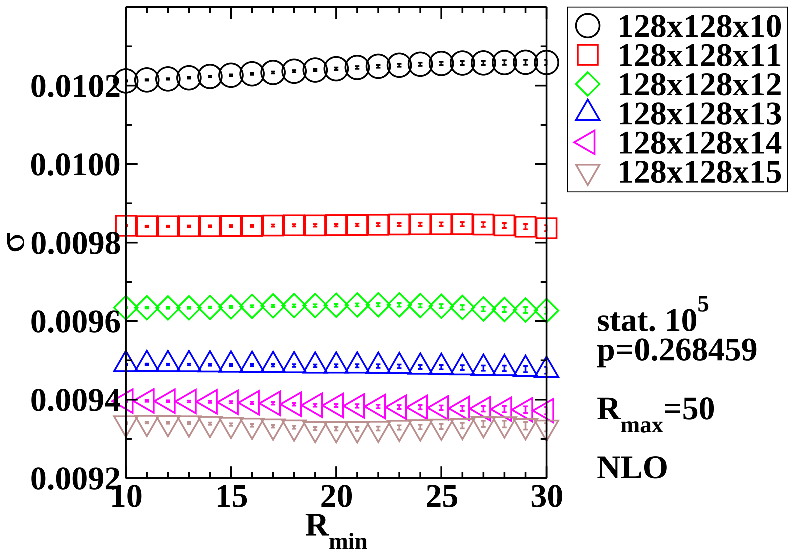


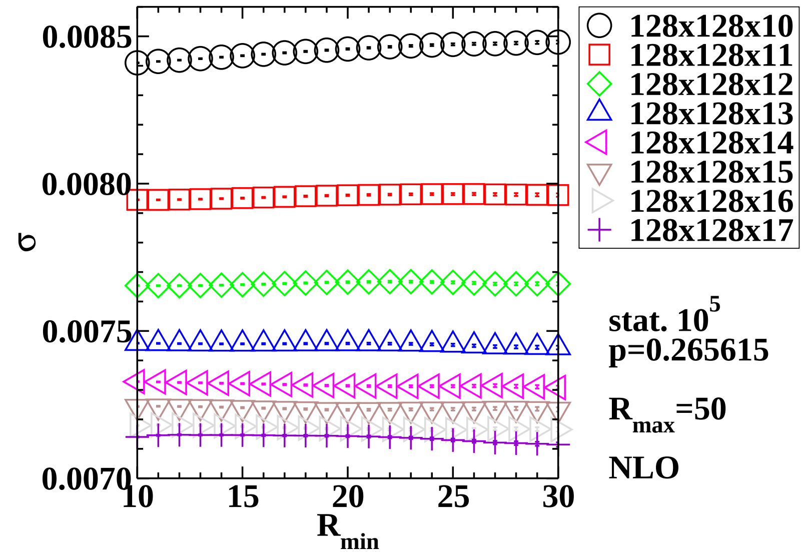
We see stable "plateau" for $R_{min} \geq 8$: the NLO is very good !!!

$\chi^2/dof \sim 1 \Rightarrow 10 \geq L \geq 15$

Note: different L, different $\tilde{\sigma}$ plateaux !!!

RESULTS

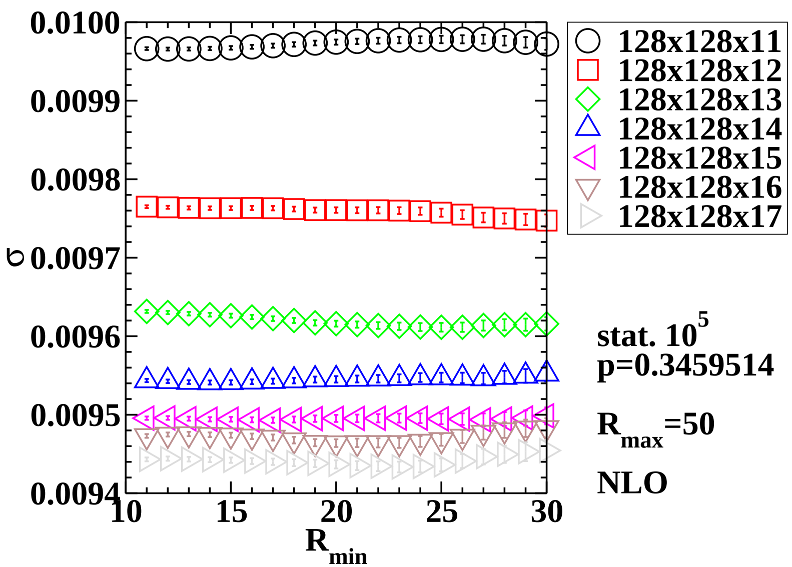


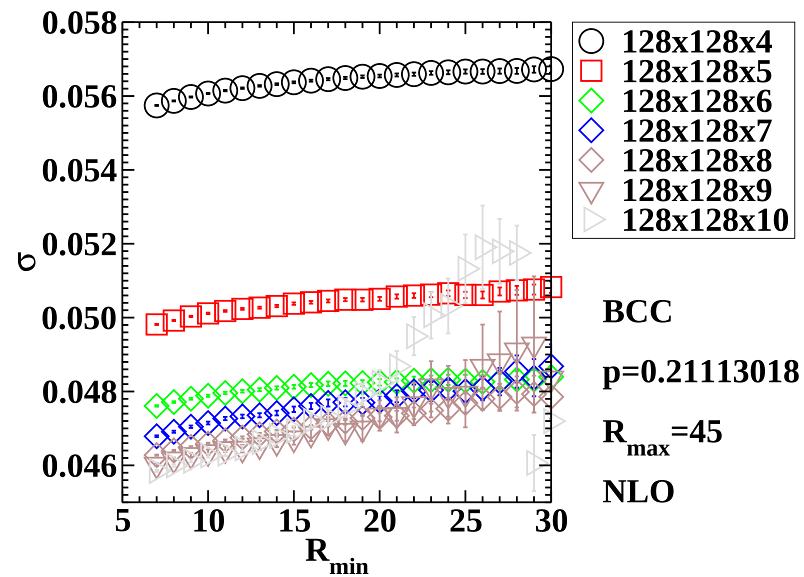


p is critical for $L = 7$;
BOND percolation; SC lattice;
$\chi^2/dof \sim 1 \Rightarrow 12 \geq L \geq 15$

p is critical for $L = 8$;
BOND percolation; SC lattice;
$\chi^2/dof \sim 1 \Rightarrow 13 \geq L \geq 17$

RESULTS





p is critical for $L = 7$;

SITE percolation; SC lattice;

$\chi^2/dof \sim 1 \Rightarrow 11 \geq L \geq 17$

p is critical for $L = 3$;

BOND percolation; BCC lattice;

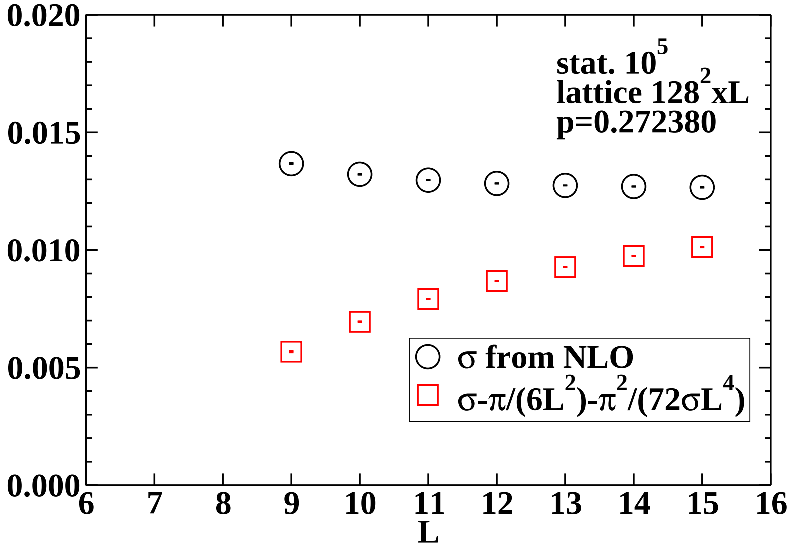
$\chi^2/dof \sim 1 \Rightarrow 4 \geq L \geq 10$

L dependence

$\tilde{\sigma}$ depends on L;
What is the dependence using the NLO approximation ?

$$\sigma(L,R)_{NLO} \xrightarrow{R\to\infty} \sigma(L) = \tilde{\sigma} - \frac{\pi}{6L^2} - \frac{\pi^2}{72\tilde{\sigma}L^4}$$

Using this relation we determine the behaviour of $\sigma(L)$:



Note: in this case p is critical for $L = 6 \Rightarrow \sigma(L = L_c) = 0$

Beyond NLO

Because the NLO terms are not enough to describe the correct behaviour, we introduce a new term $\propto L^{-6}$

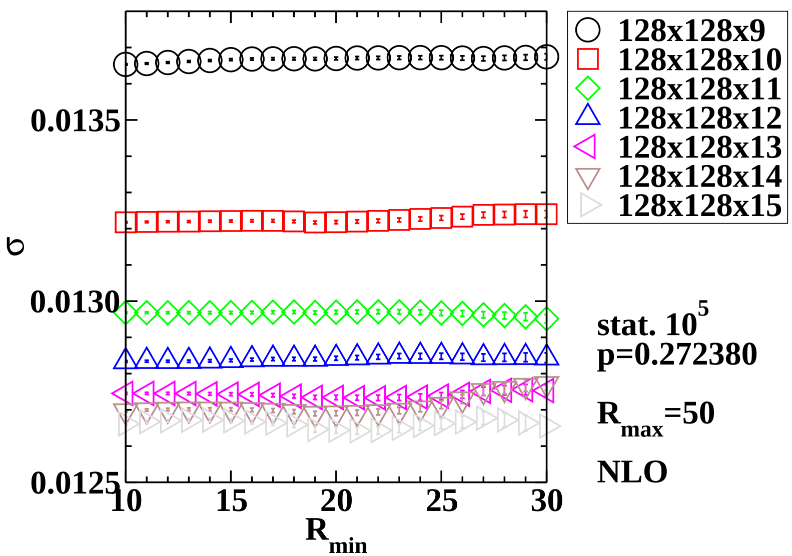
$$\sigma(L) = \sigma_0 - \frac{\pi}{6L^2} - \frac{\pi^2}{72\sigma_0 L^4} + \frac{\pi^3}{C\sigma_0^2 L^6}$$

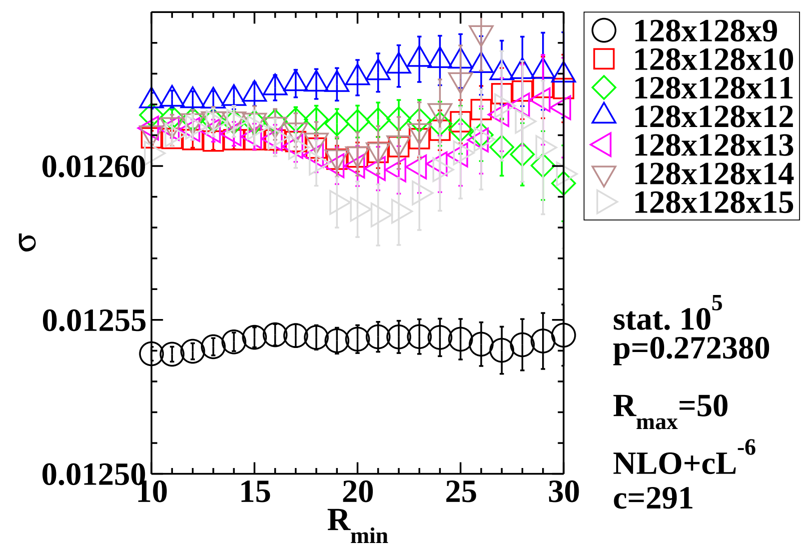
We determine the "correct" value of σ_0 and C

model	σ_0	C	χ^2/dof
$p_{L=6}$, SC, BOND	0.012612(6)	291(7)	0.15
$p_{L=7}$, SC, BOND	0.009234(5)	281(5)	1.20
$p_{L=8}$, SC, BOND	0.007059(5)	297(5)	0.38
$p_{L=7}$, SC, SITE	0.009399(8)	307(9)	0.20
$p_{L=3}$, BCC, BOND	0.0474(4)	295(14)	0.81

RESULTS

Example: $p_{L=6}$, BOND percolation; SC lattice.



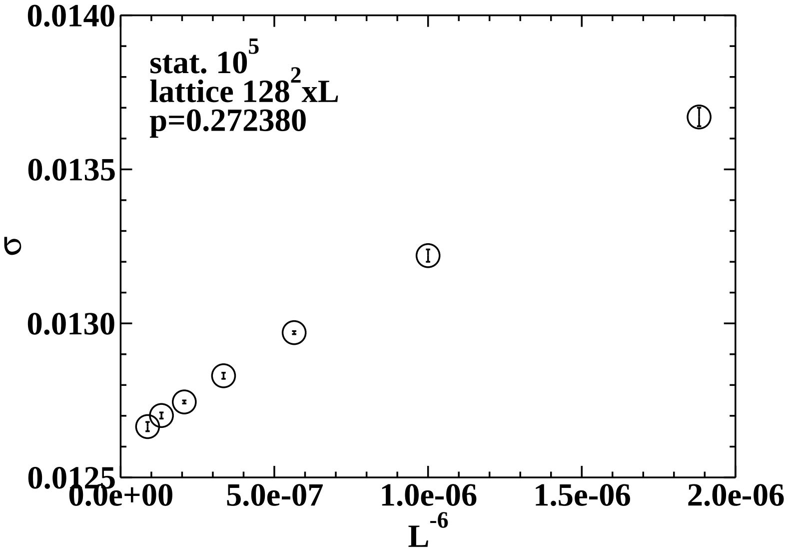


For $10 \lesssim L \lesssim 15$ data are on the same "plateau"

A different method to determine C

A different method to determine C
(useful to highlight sistematic effects):

We know $\tilde{\sigma}$ has L^{-2} and L^{-4} corrections, so: $\tilde{\sigma} = \sigma_0 + \frac{\pi^3}{C\sigma_0^2 L^6}$



model	σ_0	C
1	0.012612(6)[6]	291(7)[23]
2	0.009234(5)[8]	281(5)[26]
3	0.007059(5)[6]	297(5)[32]
4	0.009399(8)[7]	307(9)[30]
5	0.0474(4)[2]	295(14)[61]

$T_c/\sqrt{\sigma_0}$ comment

Bond percolation, SC:

	(method 1)	(method 2)
L	$T_c/\sqrt{\sigma_0}$	$T_c/\sqrt{\sigma_0}$
6	1.4841(4)	1.4837(4)
7	1.4866(5)	1.4860(5)
8	1.4878(5)	1.4871(5)

There is a small difference due to “correction-to-scaling” but with only 3 data we cannot determine the “correct” value of $T_c/\sqrt{\sigma_0}$

This correction tends to increase the value of $T_c/\sqrt{\sigma_0}$ so (since these simulations are more precise) we accept as percolation value the following:

$$T_c/\sqrt{\sigma_0} = 1.4878(5)[7]$$

Correction to $\tilde{\sigma}$

Bond percolation, SC, p_8, simulations on $N^2 \times 10$:

N	$\tilde{\sigma}$
128	0.00848(3)
194	0.00854(3)
256	0.00856(3)
320	0.00859(2)

The finite volume effects are evident!

We can determine the correction to $N \to \infty$:

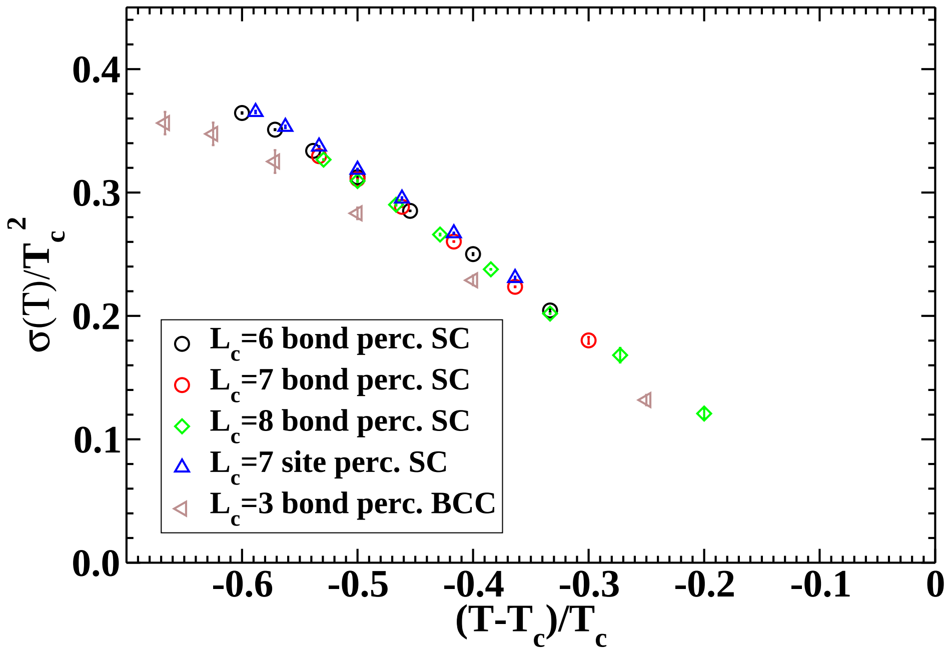
$$\tilde{\sigma}(1/N) = \tilde{\sigma}(0) - cN^{-1/\nu} \qquad \nu = 4/3$$

$\tilde{\sigma}(0) = 0.00870(5), \qquad c = 0.0083(27), \qquad \chi^2/dof = 0.04$

Same observations for $L = 11 \ldots$

Universality

We would like to know if our results are “regularization” independent...

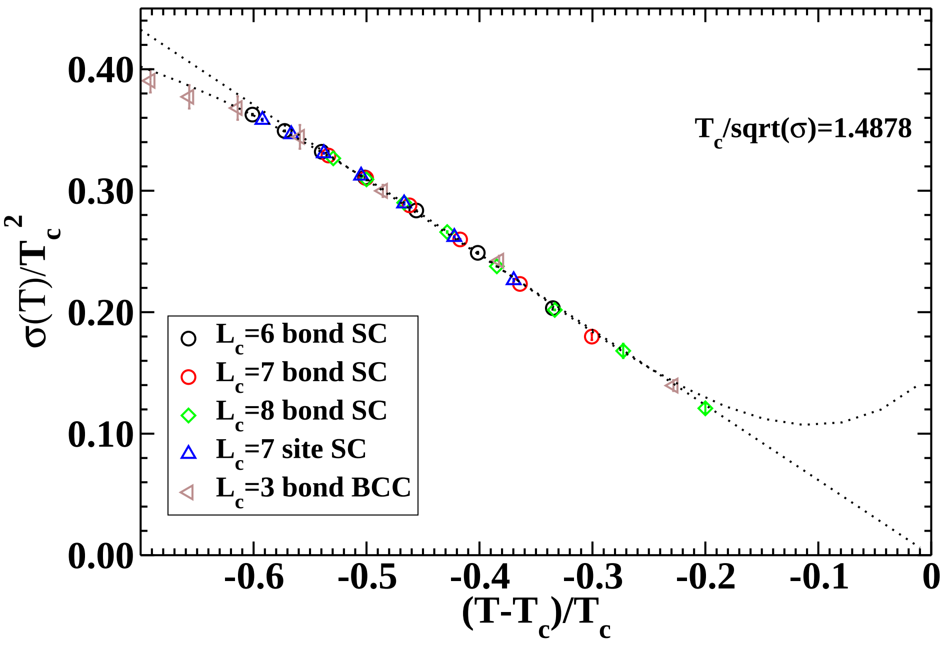


model	$T_c/\sqrt{\sigma_0}$
1	1.4841(4)
2	1.4866(5)
3	1.4878(5)
4	1.4735(6)
5	1.531(7)

We do not see a “true” universality because $T_c/\sqrt{\sigma_0}$ are different!

Universality

Now we impose the value of $T_c/\sqrt{\sigma_0}$:



- All data on the same curve!
- A linear behaviour
- The curve passes through the origin

Data are in the scaling region!!!

An indipendent way to determine C

We can determine C and $T_c/\sqrt{\sigma_0}$ with only two pieces of information:

- Scaling region in the range $-0.55 < t < -0.225$ ($t = \frac{T-T_c}{T_c}$)
- Linear behaviour with a straight line which goes through the origin

We know that ($S = \frac{\sigma_0}{T_c^2}$ and $x = \frac{T}{T_c}$):

$$\frac{\sigma(L)}{T_c^2} = S - \frac{\pi}{6}x^2 - \frac{\pi^2}{72S}x^4 + \frac{\pi^3}{CS^2}x^6$$

$$\frac{\sigma(L)}{T_c^2} = A(x-1)$$

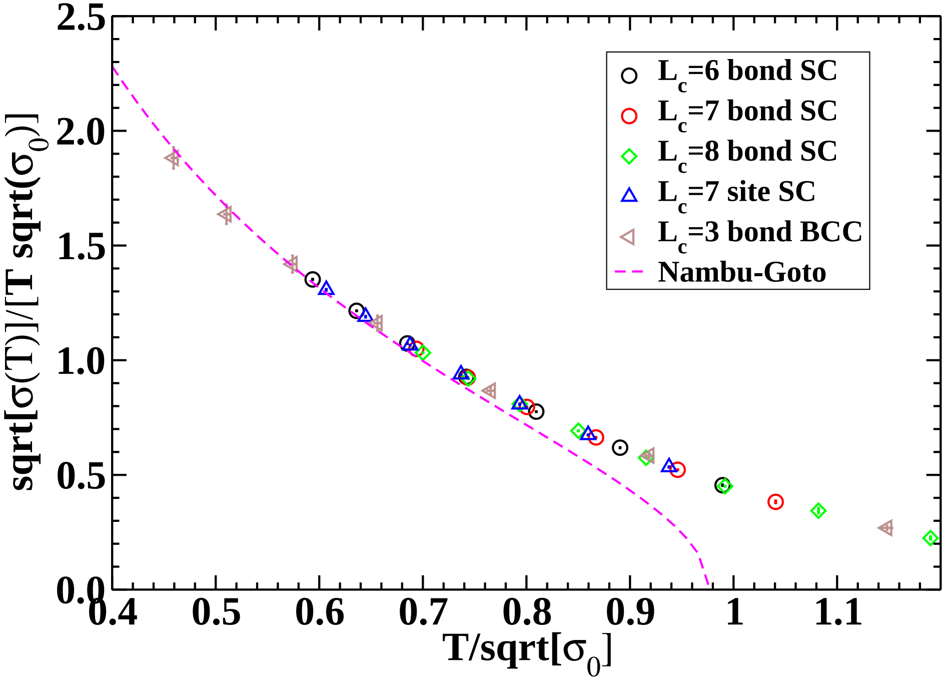
$$C = 290$$

$$S = 0.45138$$

$$\frac{T_c}{\sqrt{\sigma_0}} = 1.48843$$

Universality

A different way to show the universality of our results:



Nambu-Goto:

$$\sigma_{NG}(T) = \sigma_0 \sqrt{1 - \frac{T^2}{T_c^2}}$$

with $T_c^2 = \frac{3\sigma_0}{\pi} \Rightarrow$

$$\frac{T_c}{\sqrt{\sigma_0}} = \sqrt{\frac{3}{\pi}} \sim 0.9772\ldots$$

$$\frac{\sigma_{NG}(T)}{T\sqrt{\sigma_0}} = \frac{\sqrt{\sigma_0}}{T}\sqrt{1 - \frac{\pi}{3}\left(\frac{T}{\sqrt{\sigma_0}}\right)^2} \Rightarrow y = \frac{1}{x}\sqrt{1 - \frac{\pi}{3}x^2}$$

CONCLUSION

- It is possible to use NLO approximation in percolation
- We have determined the coeff. of the L^{-6} term: $C = 297(5)[32]$
- $T_c/\sqrt{\sigma} = 1.4878(5)[7]$ is far from NG result $\frac{T_c}{\sqrt{\sigma}_0} \sim 0.9772\ldots$
- $\frac{T_c}{\sqrt{\sigma_0}}$ and C are constrained each other in the “scaling region”
- We have shown our results do not depend on the REGULARIZATION used
  (they are well defined also in the continuum)

Once more, we can conclude that, also in the “gauge theory” DUAL to the percolation model, there exists a flux tube, that can be described by the effective string picture