The curvature of the critical surface $(m_{u,d}, m_s)^{\rm crit}(\mu)$, on finer and bigger lattices

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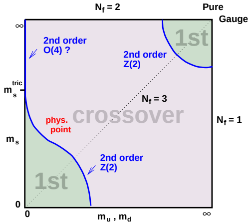
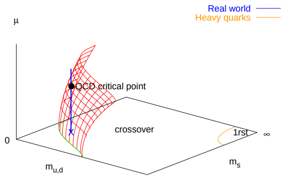
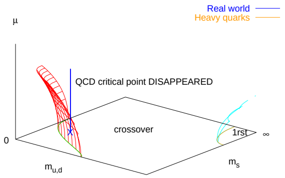
The issue





$c_1 > 0$



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Only derivatives at $\mu = 0$ are reliable: $\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$

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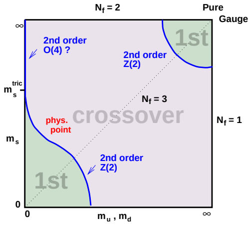

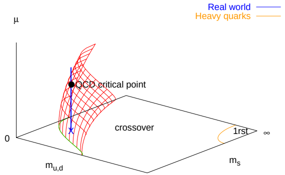

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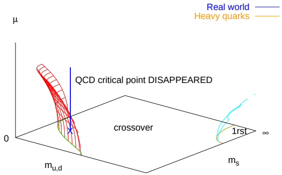

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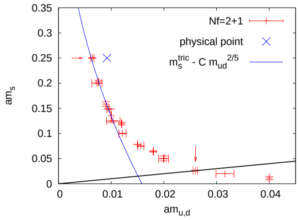

This year:

- $N_t = 4,\ N_f = 3 \quad (m_s = m_{u,d})$: $8^3 \longrightarrow 12^3$
  higher-order terms
- $N_t = 4,\ N_f = 2+1 \quad (m_s = m_s^{\text{physical}})$: 16^3
- $N_t = 6,\ N_f = 3$: 18^3

The two methods

Measure $B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2} = \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first}-\text{order} \\ 3 & \text{crossover} \end{cases}$ for $V \to \infty$

$$\frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am}$$, hard / easy

1. Finite-μ: MC at $\mu = i\mu_i$, fit $B_4(\mu_i)$ with truncated Taylor series in μ^2

truncation error?

2. Derivative: MC at $\mu = 0$, reweight to small $\mu = i\mu_i$, measure $\frac{\Delta B_4}{\Delta \mu^2}$

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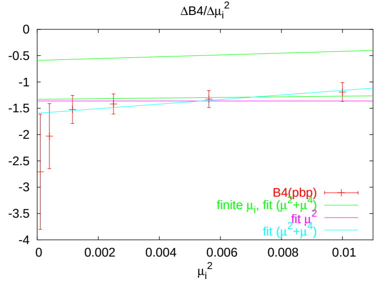
Comparison $8^3 \times 4, N_f = 3$:

- consistent value for $\frac{\partial B_4}{\partial(a\mu)^2}$
- also for NLO $\frac{\partial^2 B_4}{\partial(a\mu)^4}$
- **Derivative** method superior

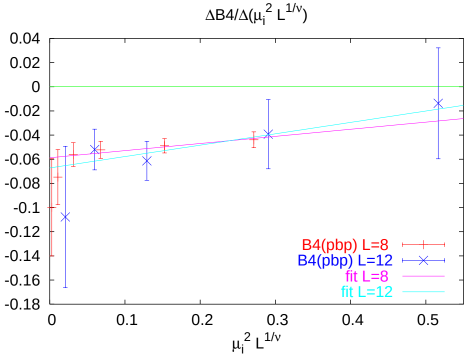
5 million traj., 2 weeks Grid computing



$N_t = 4, N_f = 3$, larger volume

$\frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am}$; scaling $\rightarrow$ each factor $\propto L^{1/\nu}$, $\nu = 0.63$

Compare $8^3 \times 4$ and $12^3 \times 4$ (Derivative method):

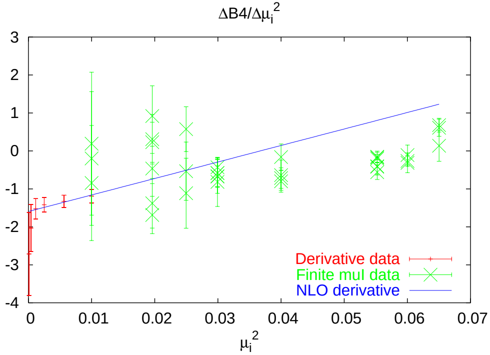


- Consistency of leading and subleading terms
- Subleading term $\sim \left(\frac{\mu}{\pi T}\right)^4$ weakens curvature for imaginary μ

$\Longrightarrow$ reinforces exotic scenario for real μ

$N_t = 4, N_f = 3$: combining the two methods

Methods 1 and 2 cover different ranges of μ_i $\longrightarrow$ combine them



$$\frac{B_4(\mu_i) - B_4(0)}{\mu_i^2} = \underbrace{b_1}_{<0} + \underbrace{b_2}_{>0} \mu_i^2$$

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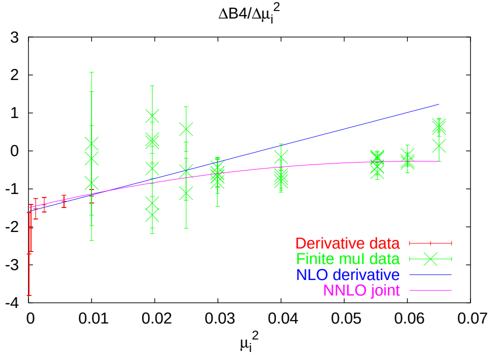

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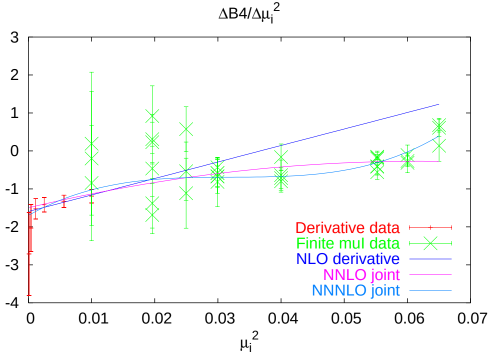

$$\frac{B_4(\mu_i)-B_4(0)}{\mu_i^2} = \underbrace{b_1}_{<0} + \underbrace{b_2}_{>0}\mu_i^2 + \underbrace{b_3}_{<0}\mu_i^4 + \underbrace{b_4}_{>0}\mu_i^6$$

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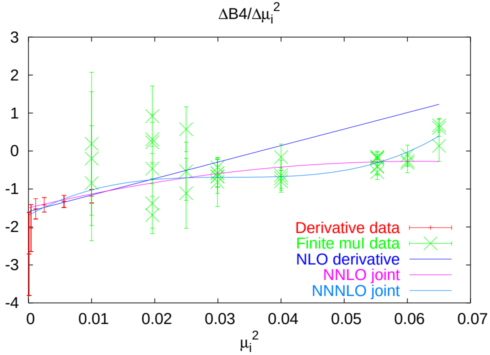
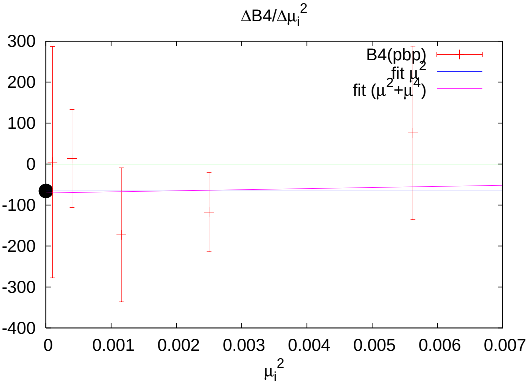


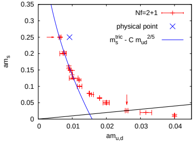
$$\frac{B_4(\mu_i)-B_4(0)}{\mu_i^2} = \underbrace{b_1}_{<0} + \underbrace{b_2}_{>0}\mu_i^2 + \underbrace{b_3}_{<0}\mu_i^4 + \underbrace{b_4}_{>0}\mu_i^6$$

Real μ: $B_4(\mu) = B_4(0) + \underbrace{(-b_1)}_{>0}\mu^2 + \underbrace{(+b_2)}_{>0}\mu^4 + \underbrace{(-b_3)}_{>0}\mu^6 + \underbrace{(+b_4)}_{>0}\mu^8$

B_4 increases with $\mu \rightarrow$ crossover: all terms reinforce exotic scenario!

$N_t = 4, N_f = 2+1$: moving along the critical line





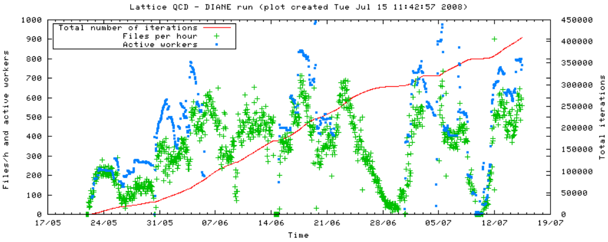
- $16^3 \times 4, am_s = 0.25, am_{u,d} = 0.005$, *lighter than in nature*
  350k trajectories, 5 weeks of Grid computing
- $b_1 = -66(41)$ (μ^2 fit) $\;\longrightarrow\; \partial am^c / \partial(a\mu^2) = -0.64(39)$

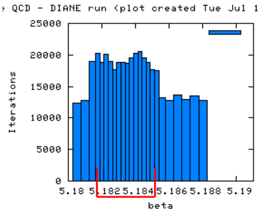
[or $b_1 = -71(75)$ ($\mu^2 + \mu^4$ fit)]

- $c_1 = -80(50)$, ie. $\frac{m_c(\mu)}{m_c(0)} = 1 - \mathbf{80(50)} \left(\frac{\mu}{\pi T}\right)^2$ not conclusive yet

LQCD on the Computing Grid

- 725k trajectories (2 quark masses) in 2 months $\longrightarrow$ 115 CPU years
- on average 700 CPUs active at all times
- 330k files = 3 TB of data transferred
- computing support provided by CERN IT/GS: *thanks a lot!*





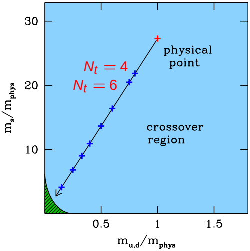
prioritized scheduling

- calculations on EGEE Grid
- resources provided by CERN, CYFRONET (Poland), CSCS (Switzerland), NIKHEF (Holland) + 10 more across Europe

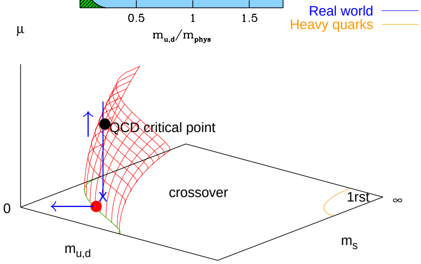
$N_t = 6, N_f = 3$: towards the continuum limit

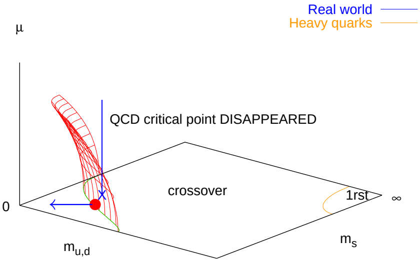
1. $\mu = 0$: re-tune the quark mass for 2nd-order transition at $T = T_c$

$\longrightarrow$ At $T=0$, $\frac{m_\pi}{T_c} = 0.954(12)$ instead of $1.680(4)$ $(N_t = 4)$



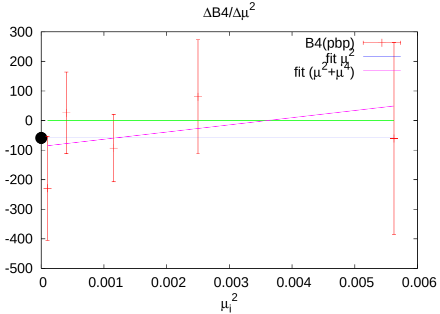
cf. Endrodi, Fodor et al., arXiv:0710.0998





$N_t = 6, N_f = 3$: towards the continuum limit

2. Measure $\frac{\partial B_4}{\partial(am)}$ (easy) and $b_1 \equiv \frac{\partial B_4}{\partial(a\mu)^2}$ (hard)



- $18^3 \times 6, am = 0.003, m_\pi = 0.95 T_c \sim$ 170 MeV
  120k trajectories, 6 months of SX-8
- $b_1 = -58(49)$ (μ^2 fit) $\rightarrow c_1 = -28(23)$, ie. $\frac{m_c(\mu)}{m_c(0)} = 1 - \mathbf{28(23)} \left(\frac{\mu}{\pi T}\right)^2$

[or $b_1 = -88(75)$ ($\mu^2 + \mu^4$ fit)]

- Assume $c_1 = +18$, ie. 2 sigmas away; then $\frac{\mu_E}{T_E} = 1 \Rightarrow \frac{m_c(\mu_E)}{m_c(0)} \sim 3$, insufficient to reach physical point

Conclusions

- $N_t = 4$: - exotic scenario established for $N_f = 3$
  - reinforced by subleading terms

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(5)\left(\frac{\mu}{\pi T}\right)^2 - 20(8)\left(\frac{\mu}{\pi T}\right)^4 - \ldots$$

  - no qualitative change so far for $N_f = 2+1$ (in progress)

- $N_t = 6$: - sign undetermined, but curvature not large
  $\longrightarrow$ already disfavors standard scenario
  - more statistics needed...

- to be continued...