Higgs mass bounds from a chirally invariant Higgs-Yukawa model with overlap fermions

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Organization of the talk

- 1. Introduction and motivation
- 2. A chirally invariant $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ HY-model
- 3. Conceptual considerations

  $\longrightarrow$ Sign-problem? Simulation strategy
- 4. Results for lower Higgs mass bounds

  $\longrightarrow$ Dependence on cutoff, finite volume, top/bottom mass-splitting
- 5. Results for upper Higgs mass bounds (Preliminary)

  $\longrightarrow$ Dependence on cutoff, finite volume
- 6. Outlook

1. Introduction and motivation

- LHC will explore Higgs-Sector.
  $\longrightarrow$ Interest in theoretical predictions on Higgs properties.

- Due to triviality of Higgs-Sector, cutoff Λ cannot be removed.
  $\longrightarrow$ Only cutoff-dependent Higgs mass bounds $m_H^{up}(\Lambda)$, $m_H^{low}(\Lambda)$.

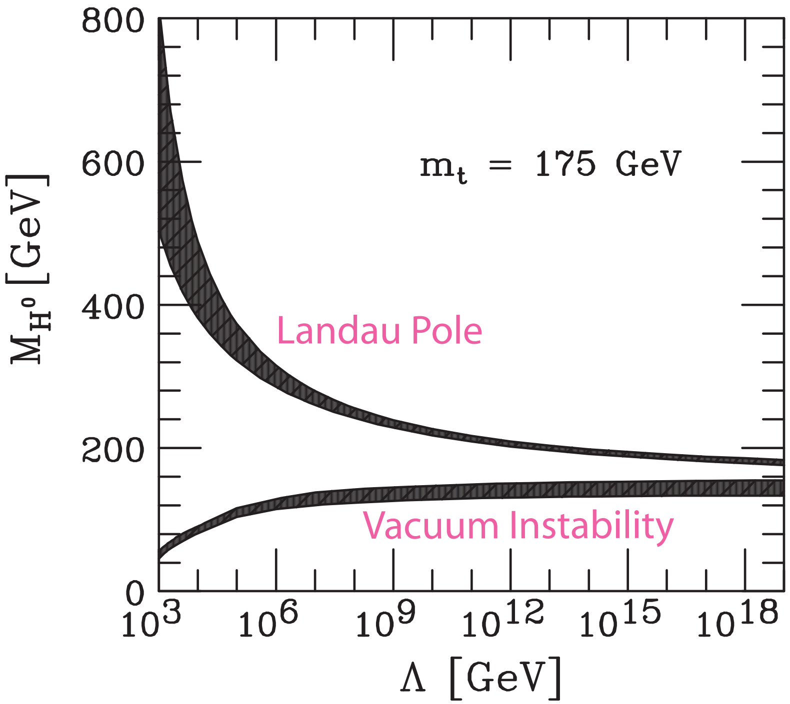
- With requirement of minimal value for Λ (e.g. by experiment) cutoff-dependent bounds translate into absolute Higgs mass bounds.

- Absolute and Λ-dependent Higgs mass bounds are important for...
  1. narrowing the possible energy range, where to expect the Higgs.
  2. determining the energy scale, where new physics sets in,
     once the Higgs is actually discovered at the LHC.

- Higgs mass bounds have been derived from perturbation theory.
  $\longrightarrow$ Upper bound: Landau pole
  $\longrightarrow$ Lower bound: Vacuum instability

Picture taken from Hagiwara *et al*. (Particle Data Group)

Phys. Rev. D66 (2002)



- Problem: Validity of $m_H^{up}(\Lambda)$ doubtful,
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- Solution: Study Higgs-Sector non-perturbatively, e.g. on the lattice.
- Lattice Higgs- and Higgs-Yukawa-models investigated in 1990's, but...
  $\longrightarrow$ ...Higgs-models do not include fermions.
  $\longrightarrow$ ...earlier Higgs-Yukawa models explicitly broke chiral symmetry
  although indispensable for chiral theories.

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  although indispensable for chiral theories.

Study lattice Higgs-Yukawa model with built-in chiral symmetry.

2. The model

- In pure Higgs-Sector of SM the Higgs-Fermion coupling is

$$L_Y = -y_b \cdot (\bar{t},\bar{b})_L \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} b_R \; - y_t \cdot (\bar{t},\bar{b})_L \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix} t_R \; + \; h.c.$$

- Higgs-dynamics dominated by coupling to heaviest fermions.
  $\longrightarrow$ Consider only the heaviest, *i.e.* the top-bottom doublet,
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Requirement of chirally invariant lattice fermions

$\Rightarrow$ Use Neuberger's **overlap fermions**.

Establish chiral symmetry

- Equivalent notation by rewriting complex Higgs doublets into 2×2 matrix ϕ:

$$L_Y = y_t \cdot (\bar{t}, \bar{b}) \left[P_+ \ \mathrm{diag}\left(1, \frac{y_b}{y_t}\right) \phi^\dagger P_+ + P_- \phi \ \mathrm{diag}\left(1, \frac{y_b}{y_t}\right) P_- \right] \begin{pmatrix} t \\ b \end{pmatrix}$$

- Idea [Lüscher]: Replace projectors $P_\pm$ on righthanded side with modified projectors $\hat{P}_\pm$ based on Neuberger overlap operator $\mathcal{D}^{(ov)}$:

$$P_\pm = \frac{1}{2}\left(1 \pm \gamma_5\right), \quad \hat{P}_\pm = \tfrac{1}{2}\left(1 \pm \hat{\gamma}_5\right), \quad \hat{\gamma}_5 = \gamma_5 \left(1 - \frac{a}{\rho} \mathcal{D}^{(ov)}\right)$$

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- Result is a chirally invariant lattice Higgs-Yukawa coupling

$$L_Y = y_t \cdot (\bar{t}, \bar{b}) \underbrace{\left[P_+ \text{ diag}\left(1, \frac{y_b}{y_t}\right) \phi^\dagger + P_- \phi \text{ diag}\left(1, \frac{y_b}{y_t}\right) \right]}_{\text{B}} \left(1 - \frac{1}{2\rho}\mathcal{D}^{(ov)}\right) \begin{pmatrix} t \\ b \end{pmatrix}$$

- Contents of model:
  $\longrightarrow$ One 4-component, real Higgs field Φ ($\equiv$ complex doublet in SM),
  $\longrightarrow$ N_f (mass-degenerated) fermion generations $\psi^{(i)} = \binom{t^{(i)}}{b^{(i)}}$:

$$Z = \int D\Phi \prod_{i=1}^{N_f} \left[D\psi^{(i)} \, D\bar{\psi}^{(i)} \right] \exp\left(-S_F - S_\Phi\right)$$

$$S_F = \sum_{i=1}^{N_f} \bar{\psi}^{(i)} \left[\mathcal{D}^{(ov)} + y_t \cdot B \left(1 - \frac{1}{2\rho} \mathcal{D}^{(ov)} \right) \right] \psi^{(i)}$$

$$S_\Phi = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\hat{\mu}} + \Phi_{x-\hat{\mu}} \right] + \sum_x \Phi_x^\dagger \Phi_x + \lambda \sum_x \left(\Phi_x^\dagger \Phi_x - 1 \right)^2$$

- Four-dimensional space-time.

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$$

- Four-dimensional space-time.
- Exact global $SU(2)_L \times SU(2)_R$ symmetry with $\Omega_L, \Omega_R \in \mathrm{SU}(2)$

$$
\psi \longrightarrow \Omega_L \hat{P}_- \psi + \Omega_R \hat{P}_+ \psi \qquad \bar{\psi} \longrightarrow \bar{\psi} P_+ \Omega_L^\dagger + \bar{\psi} P_- \Omega_R^\dagger
$$

$$
\phi \longrightarrow \Omega_R \phi \Omega_L^\dagger \qquad \phi^\dagger \longrightarrow \Omega_L \phi^\dagger \Omega_R^\dagger
$$

3.1 Sign - Problem ?

- For $y_b = y_t$ spectrum of fermionic matrix

$$\mathcal{M} = \mathcal{D}^{(ov)} + y_t \cdot B \left(1 - \frac{1}{2\rho}\mathcal{D}^{(ov)}\right)$$

is complex conjugate, since

$$T^{-1}\mathcal{M}T = \mathcal{M}^*, \quad \text{with}\, T = \gamma_0\gamma_2\gamma_5\tau_2$$

- Thus, $\det(\mathcal{M}) \in \mathbb{R}$, however, negative $\det(\mathcal{M})$ not excluded.

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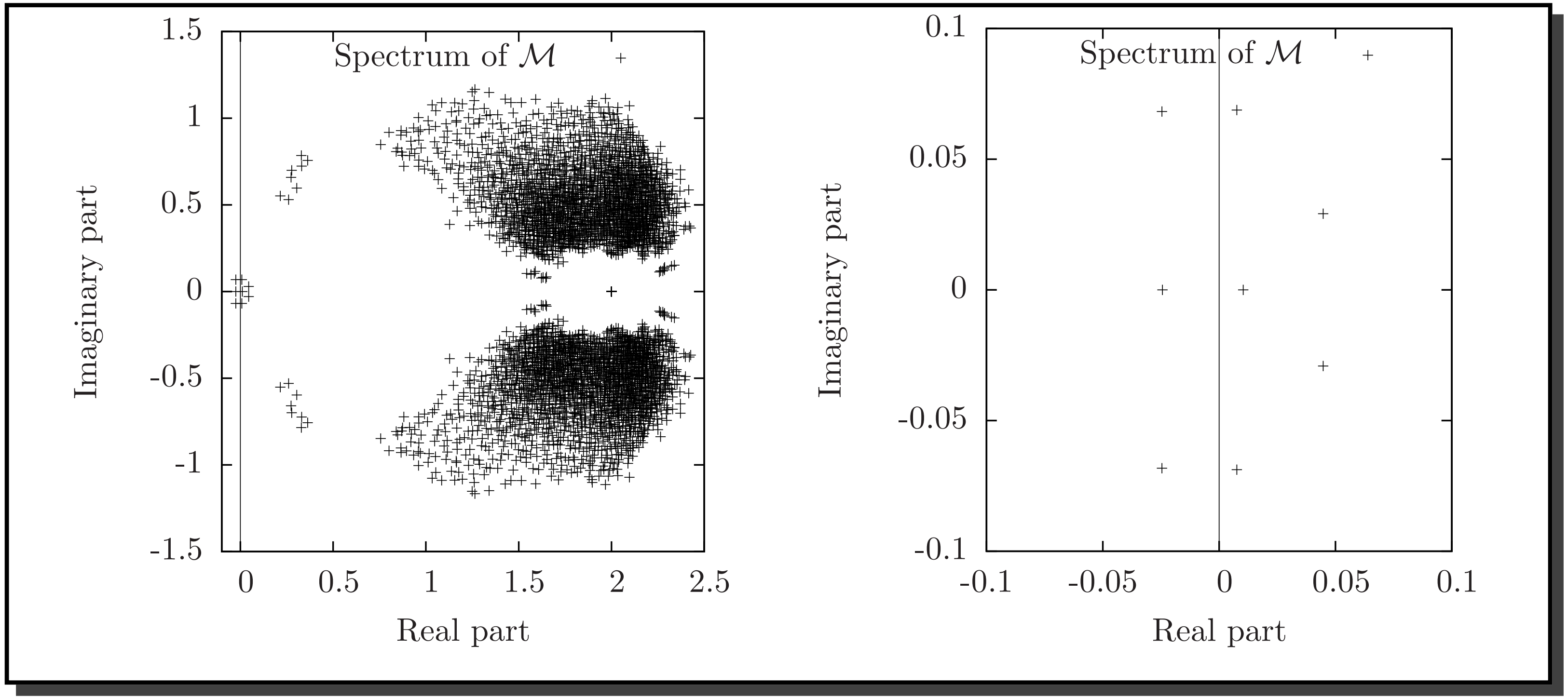
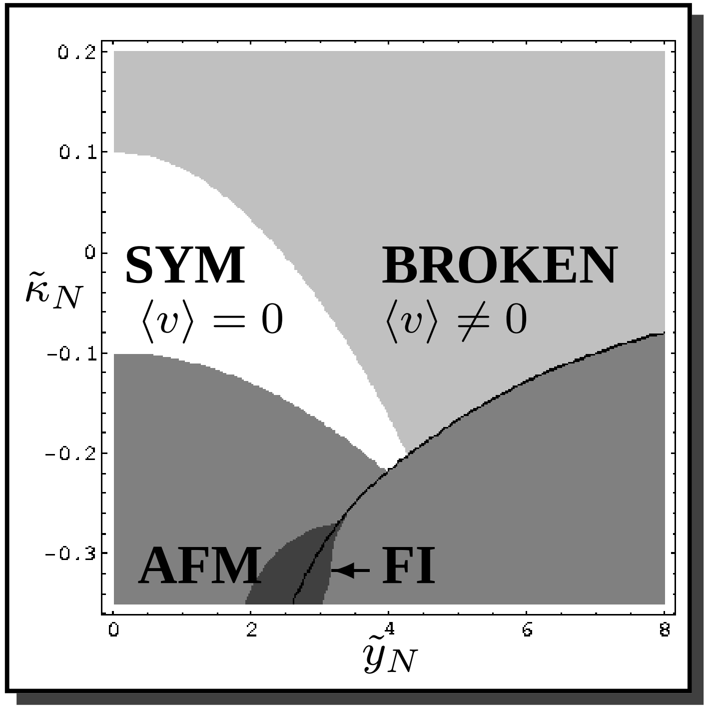


3.2 Simulation strategy

- To access odd N_f use PHMC-algorithm.
- Determine phase structure:
  $\rightarrow$ Locate symmetric ($\langle v\rangle = 0$) and broken ($\langle v\rangle \neq 0$) phases.
- Strategy for finding cutoff-dependent Higgs mass bound $m_H^{up,low}(\Lambda)$:
  $\rightarrow$ Fix physical scale and cutoff Λ by phenomenological value $\langle v_r\rangle = 246\,\text{GeV}$.
  $\rightarrow$ Simulate model close to phase transition in broken phase at several values of Λ.
  $\rightarrow$ Tune Yukawa coupling parameter y by fixing top quark mass $m_{top} = 175\,\text{GeV}$.
  $\rightarrow$ Consider weak quartic couplings λ for lower Higgs mass bounds.
  $\rightarrow$ Consider strong quartic couplings λ for upper Higgs mass bounds.



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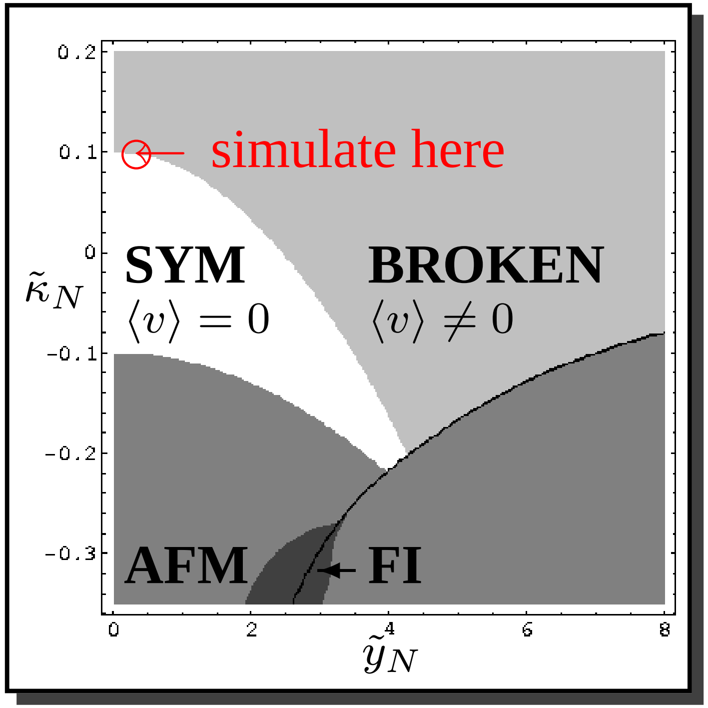
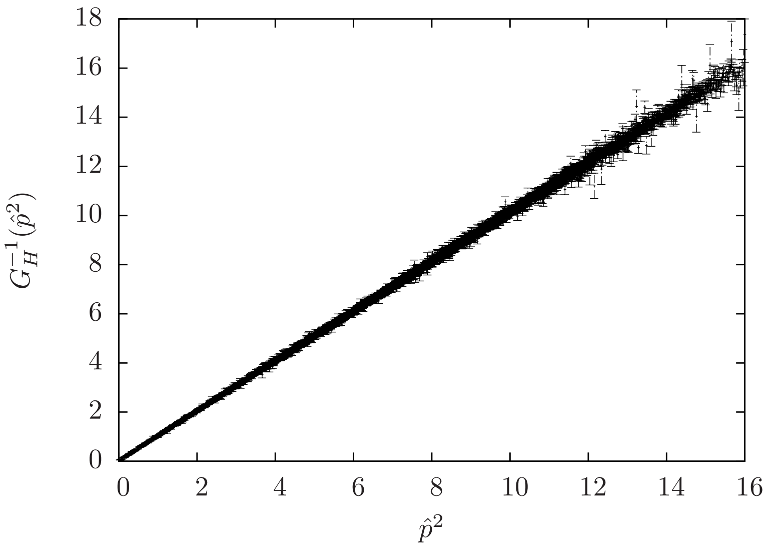


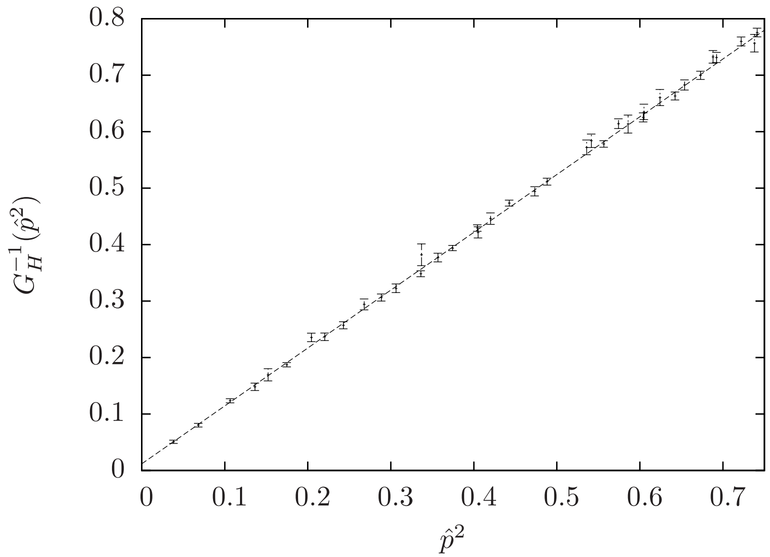
4. Results for lower mass bound

- Fix physical scale by: $246\,\mathrm{GeV} = \frac{\langle v \rangle}{\sqrt{Z_G} \cdot a}$
- Obtain Z_G from Goldstone propagator $G_G(\hat{p}^2)$, $\hat{p}$: lattice momenta
- Obtain m_H, Z_H from Higgs propagator

$$G_H^{-1}(\hat{p}^2) = \frac{\hat{p}^2 + m_H^2}{Z_H}$$

Higgs propagator on $24^3 \times 32$-lattice, $\lambda_0 = 0$, $y_0 = 0.711$





$$Z_H = 0.9979 \pm 0.0007$$

$$m_H^{phys} = (43.1 \pm 0.5)\,\mathrm{GeV}$$

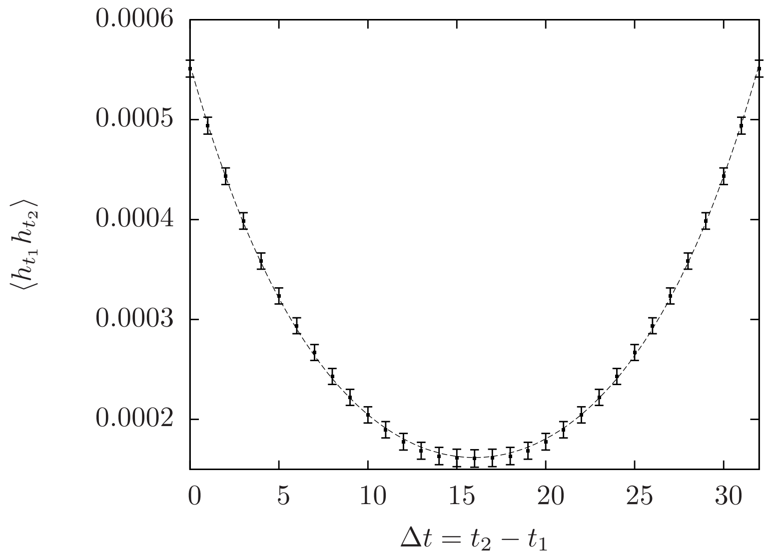
$$Z_G = 0.9940 \pm 0.0002$$

$$a^{-1} = (396.5 \pm 0.3)\,\mathrm{GeV}$$

- Compare Higgs mass from Higgs propagator $m_H^{prop} = (43.1 \pm 0.5)\,\text{GeV}$ to mass from exponential decay of Higgs time-slice correlator
- Check adjustment of Yukawa coupling constant by comparing m_{top} to its phenomenological value $175\,\text{GeV}$

$$24^3 \times 32,\ \lambda_0 = 0,\ y_0 = 0.711$$

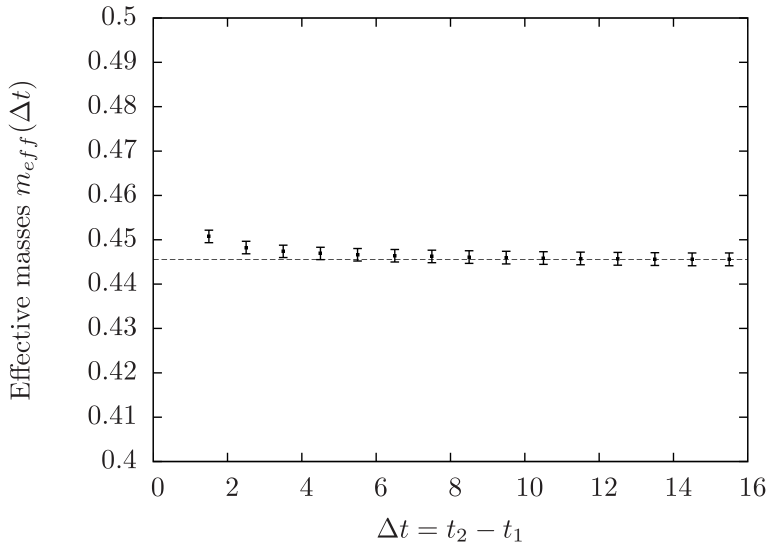
Higgs time-slice correlator



$$m_H^{lat} = 0.107 \pm 0.003$$

$$m_H^{phys} = (42.3 \pm 1.3)\,\text{GeV}$$

Effective top masses

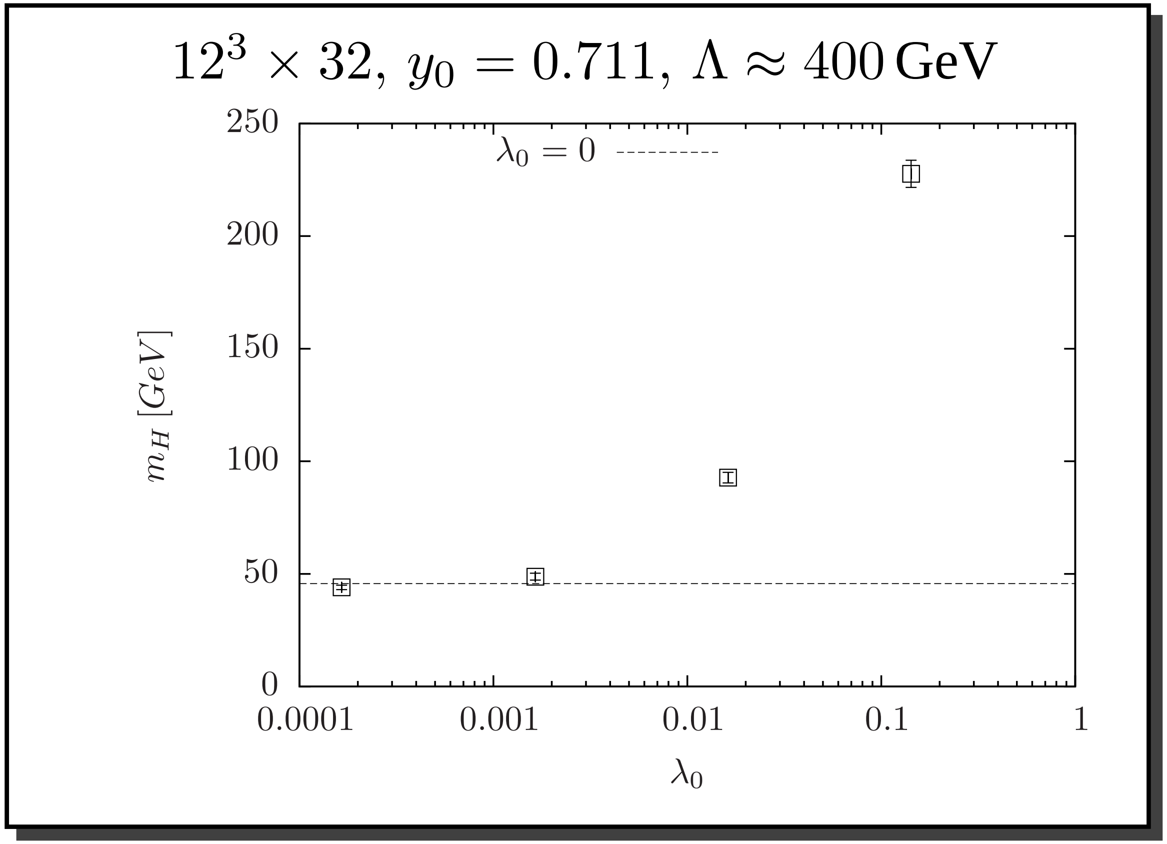


$$m_{top}^{lat} = 0.440 \pm 0.002$$

$$m_{top}^{phys} = (174.5 \pm 0.9)\,\text{GeV}$$

4.1 Dependence on quartic coup.

- At what coupling λ_0 is Higgs mass minimal? $\to$ Two competing effects:
  1) From PT: $\delta m_H^2 \propto \left(\lambda_0 - y_0^2\right) \cdot \Lambda^2$
  $\Rightarrow$ Higgs mass m_H increases with increasing λ_0
  2) Phase transition moves to larger κ (smaller m_0) as λ_0 increases
  $\Rightarrow$ Higgs mass m_H decreases with increasing λ_0 (when holding Λ const.)

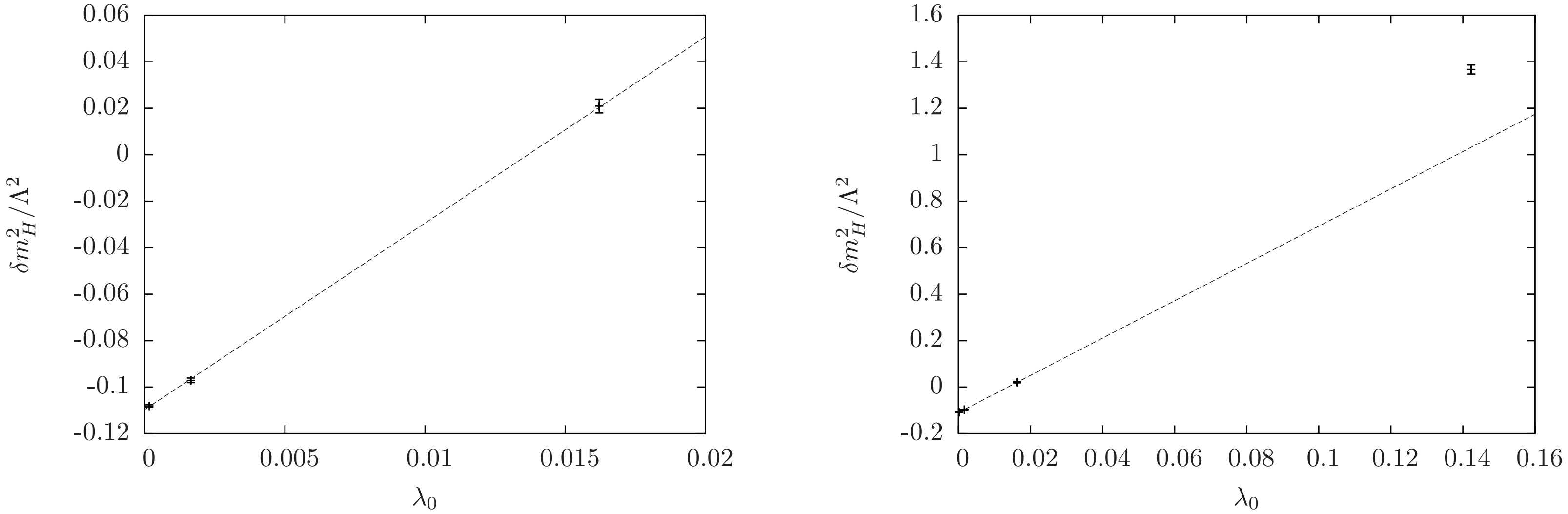


- The squared mass shift $\delta m_H^2 = m_H^2 - m_0^2$ can be compared to expected behaviour derived from PT:

$$\delta m_H^2/\Lambda^2 \propto \lambda_0 - y_0^2$$

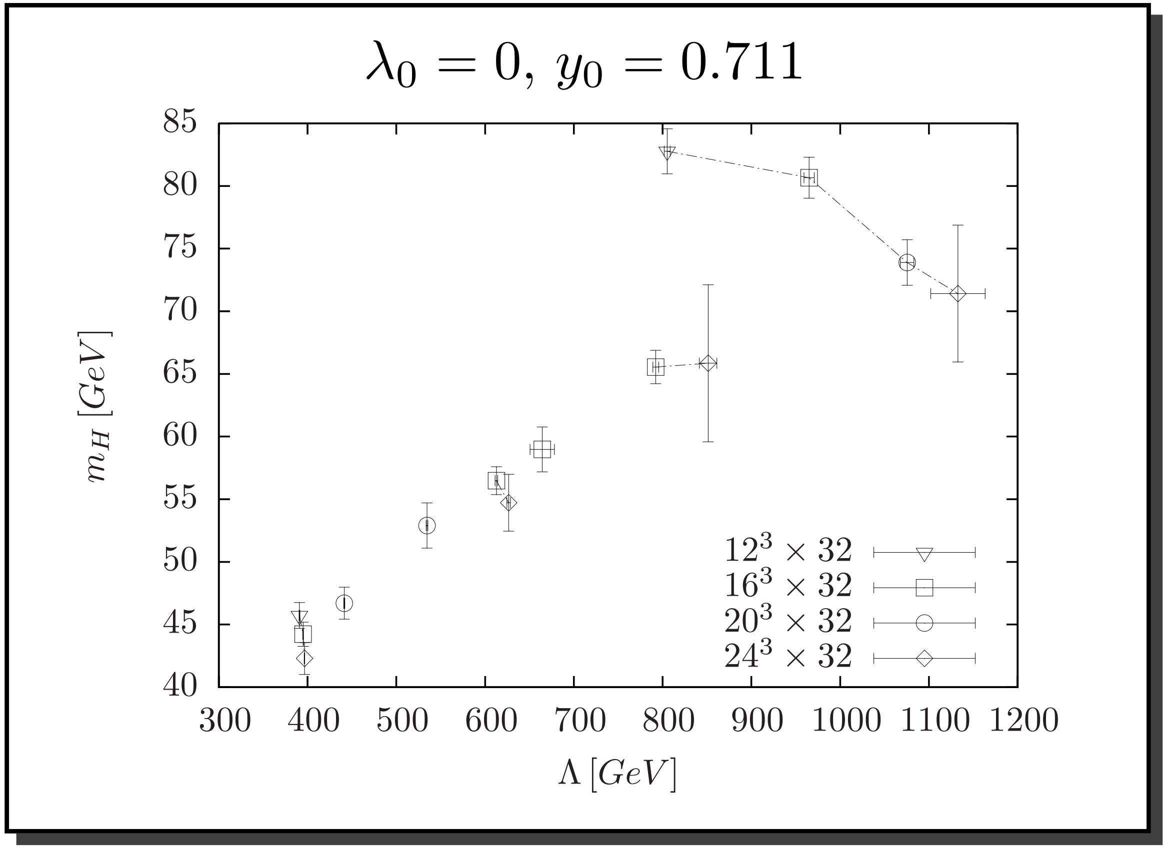
- Corresponding calculations in lattice-PT in progress to compare prefactors.

$12^3 \times 32,\ y_0 = 0.711,\ \Lambda \approx 400\,\mathrm{GeV}$



4.2 Cutoff - dependence

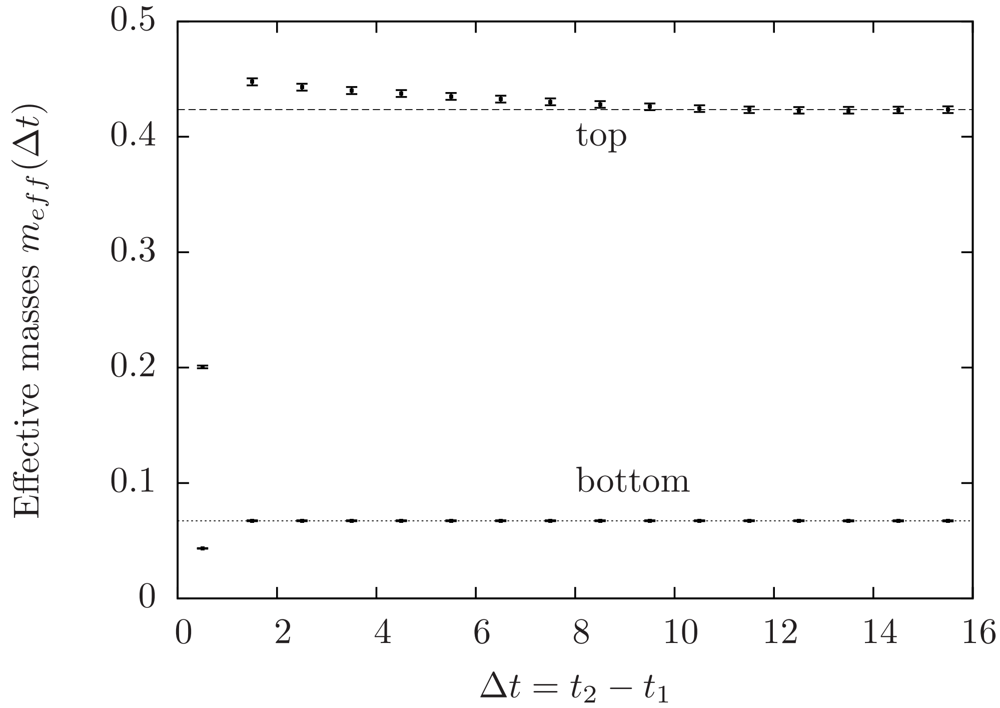
- Repeat simulation at various cutoffs.
- Check finite volume effects by comparing different lattice sizes.
  $\rightarrow$ Demand $\Lambda \geq 2 \cdot m_{top} \approx 350\,\text{GeV}$ to avoid cutoff-effects.
  $\rightarrow$ Stop increasing Λ when finite volume effects become strong.



4.3 Dep. on top-bottom splitting

- Allow for $y_t \neq y_b$ for moderate mass splittings y_b/y_t to avoid strong finite volume effects
- Physical situation is $y_b/y_t = 0.024 \rightarrow$ not reachable with our resources

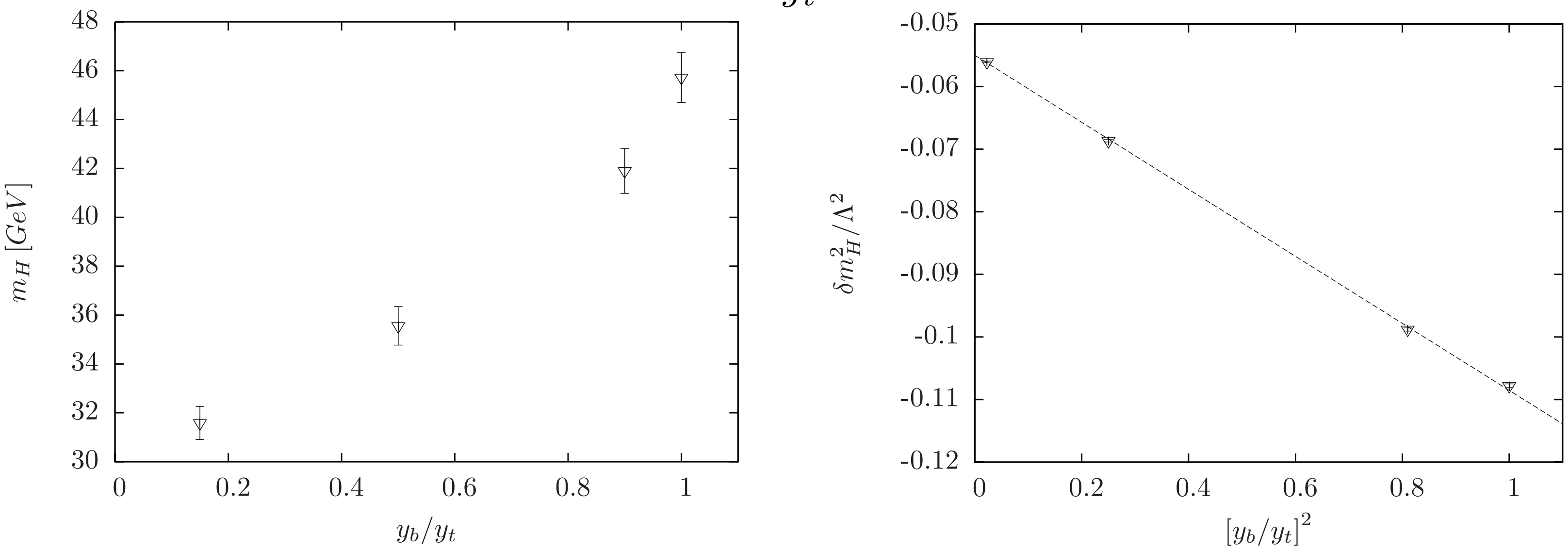
$12^3 \times 32,\ y_0 = 0.711,\ y_b/y_t = 0.15,\ \Lambda \approx 400\,\mathrm{GeV}$



$m_{top} = (168.5 \pm 1.8)\,\mathrm{GeV} \qquad m_{bot} = (26.7 \pm 0.3)\,\mathrm{GeV}$

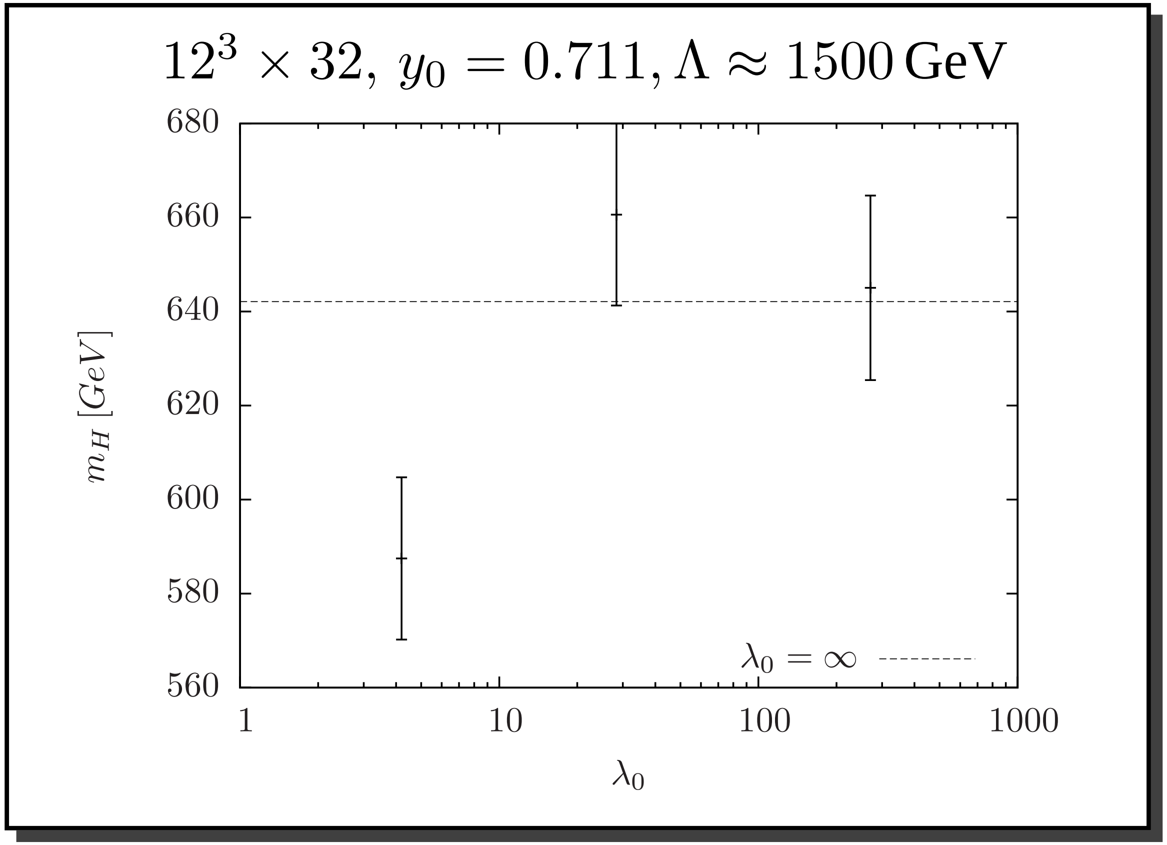
- What is influence on Higgs mass? $\rightarrow$ Again two competing effects:
  1) From PT: $\delta m_H^2 \propto \left(\lambda_0 - y_0^2\right) \cdot \Lambda^2$
  $\Rightarrow$ Higgs mass m_H increases with decreasing y_b/y_t
  2) Phase transition moves to larger κ (smaller m_0) as y_b/y_t decreases
  $\Rightarrow$ Higgs mass m_H decreases with decreasing y_b/y_t (when holding Λ const.)
- Compare behaviour of squared mass shift δm_H^2 with expectation from PT

$12^3 \times 32,\ y_0 = 0.711,\ \frac{y_b}{y_t} = 0.15,\ \Lambda \approx 400\,\mathrm{GeV}$



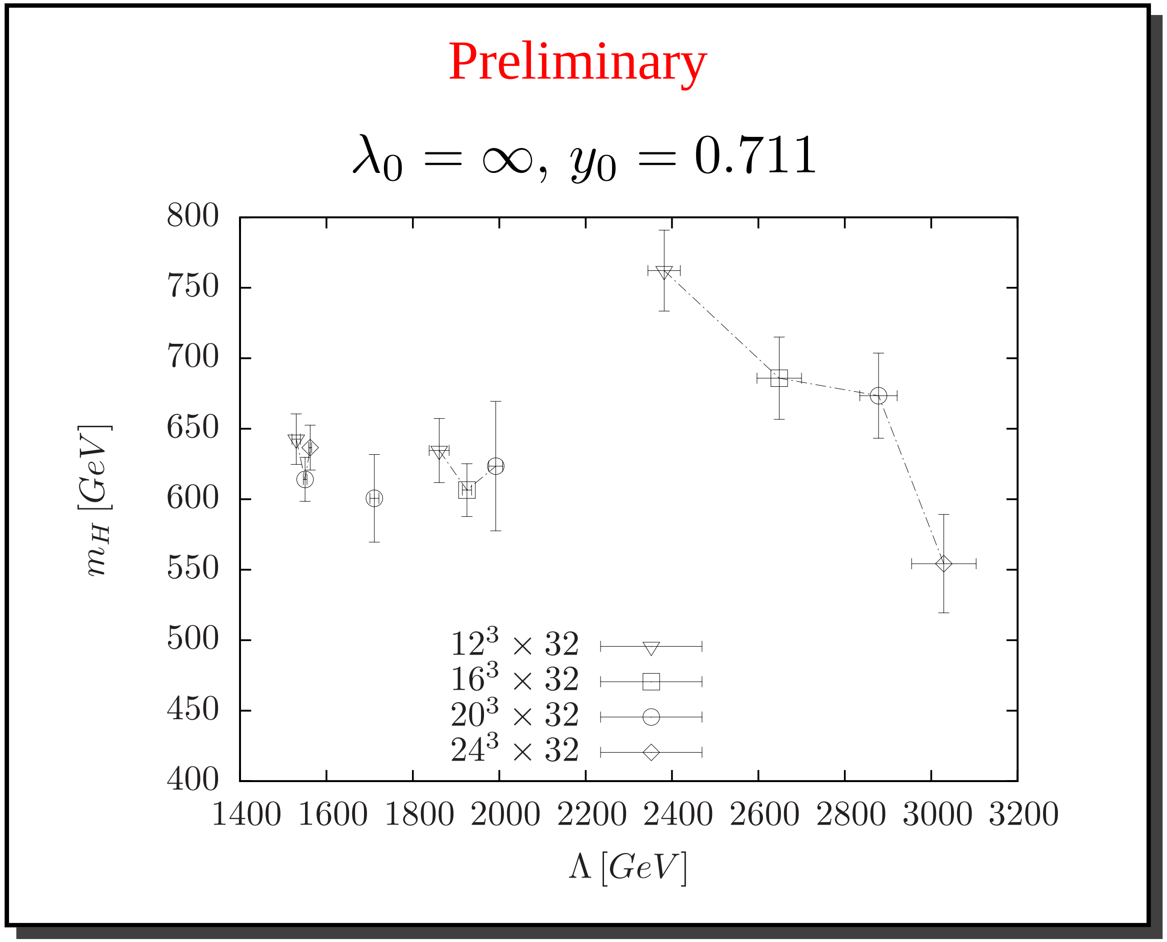
5. Results for upper bound

- At what quartic coupling λ_0 is Higgs mass m_H maximal?
  $\rightarrow$ Check that m_H rises monotonously with increasing λ_0.
- Conclude: Upper Higgs mass bounds can be computed at $\lambda_0 = \infty$.



5.1 Cutoff dependence

- Check finite volume effects by comparing different lattice sizes.
  $\rightarrow$ Demand $\Lambda \geq 2 \cdot m_H \approx 1300\,$GeV to avoid cutoff-effects.
  $\rightarrow$ Stop increasing Λ when finite volume effects become strong.



5. Summary and Outlook

- Cutoff-dependent lower Higgs mass bounds m_H^{low} have been presented.
- Dependence of m_H^{low} on λ_0, volume, cutoff Λ, and top-bottom mass-splitting has been investigated.
- Preliminary results for upper mass bounds have been shown.
- Larger lattices needed to control finite volume effects and to go to larger cutoffs Λ.
- Studying the decay properties of the Higgs ($h \rightarrow 2$ Goldstones) will become possible on a $32^3 \times 64$ lattice. This aim seems to be reachable with the available resources.