High-Precision Masses and Couplings Update from HPQCD Collaboration

G. Peter Lepage

Cornell University

Why Fundamental Parameters?

- QCD Phenomenology
- Input to beyond the Standard Model models
- High-precision tests of lattice QCD

High-Precision Lattice QCD

- Improved gluon action + improved staggered quarks (MILC)
  - a ranging between 0.06 and 0.18 fm.
  - u,d,s vacuum polarization ($n_f=3$) with ASQTAD quarks
  - realistic m_s
  - $m_u = m_d = m_s/10$ to $m_s/2.5$ (small enough for chiral extrapolation)

- Highly-improved staggered quarks (HISQ) for valence u,d,s,c
  - 3x smaller scaling violations for u,d,s
  - Relativistic formalism for c quarks: conserved currents, etc
  - Unified treatment of c with u,d,s

Current-Current Correlators
c Quarks

High-Precision Charm-Quark Mass from Current-Current Correlators in Lattice and Continuum QCD

I. Allison,[1] E. Dalgic,[2] C. T. H. Davies,[3] E. Follana,[4] R. R. Horgan,[5] K. Hornbostel,[6] G. P. Lepage,[7,*] J. Shigemitsu,[4] H. Trottier,[2] and R. M. Woloshyn[1]
(HPQCD Collaboration)

K. G. Chetyrkin,[8] J. H. Kühn,[8] M. Steinhauser,[8] and C. Sturm[9]

[1]*TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, V6T 2A3, Canada*
[2]*Physics Department, Simon Fraser University, Vancouver, British Columbia, Canada*
[3]*Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK*
[4]*Physics Department, The Ohio State University, Columbus, Ohio 43210, USA*
[5] *Department of Applied Mathematics and Theoretical Physics,*
Cambridge University, Wilberforce Road, Cambridge CB3 0WA, UK
[6]*Southern Methodist University, Dallas, Texas 75275, USA*
[7]*Laboratory for Elementary-Particle Physics, Cornell University, Ithaca, NY 14853, USA*
[8]*Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany*
[9]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, U.S.A.*
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Updated:
- 0.06 fm lattices
- new 4-loop c_ns

Pseudoscalar Correlator

Compute

$\overline{\psi}_c \gamma_5 \psi_c$

$$G(t) \equiv a^6 \sum_{\mathbf{x}} (am_{0c})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle$$

- Mass factors imply UV finite (PCAC because HISQ)
- Implies:

$$\boxed{G_{\text{cont}}(t) = G_{\text{lat}}(t) + \mathcal{O}(a^2) \quad \text{for all } t}$$

Moments

Low *n* moments perturbative (*E*$_{\text{threshold}}$ = 2*m*$_c$):

$$G_n = \sum_t (t/a)^n G(t)$$

$$\rightarrow \frac{\partial^n}{\partial E^n} \Pi(E=0)$$

Implies:

from lattice simulations

from continuum perturbation th.

$$G_n = \frac{g_n(\alpha_{\overline{\mathrm{MS}}}(\mu), \mu/m_c)}{(am_c(\mu))^{n-4}}$$

gives *m*$_c$
(*m*$_c$ only scale)

Refinements — Reduced Moments

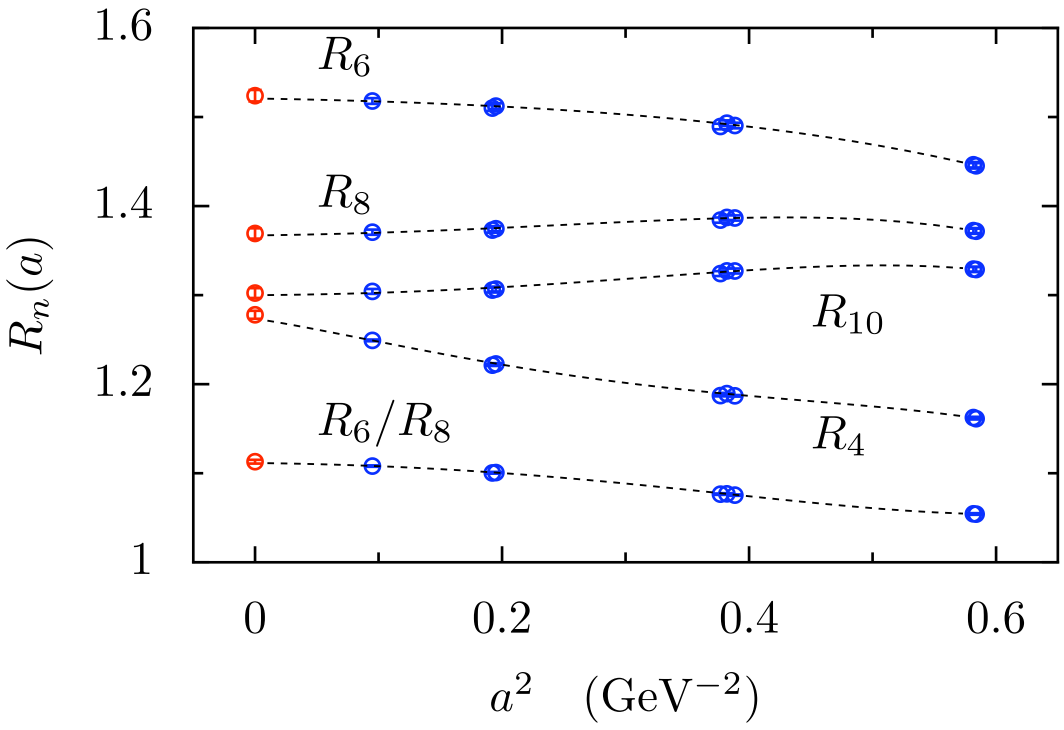
$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4. \\ \dfrac{am_{\eta_c}}{2am_{0c}} \left(G_n/G_n^{(0)}\right)^{1/(n-4)} & \text{for } n \geq 6. \end{cases}$$

meson/quark mass ratio
reduces mass tuning error

tree-level LQCD
removes explicit *a* and
cancels tree-level *O*(*a*^{n}) errors

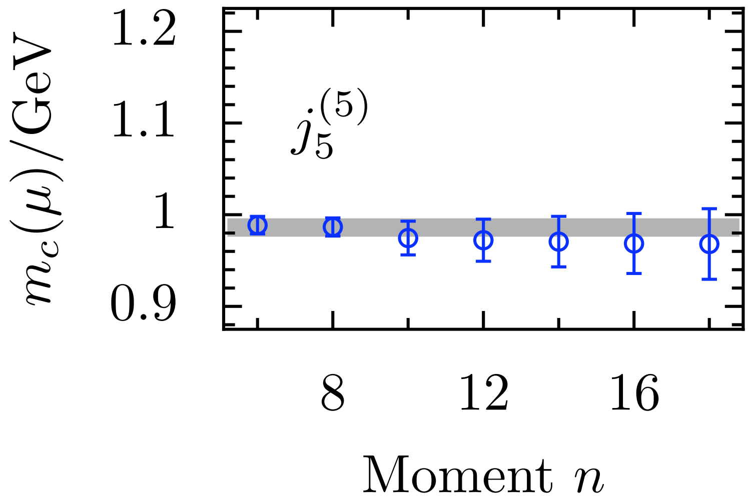
$$\Rightarrow \qquad R_n \equiv \begin{cases} r_4(\alpha_{\overline{\mathrm{MS}}}, \mu/m_c) & \text{for } n = 4. \\ \dfrac{r_n(\alpha_{\overline{\mathrm{MS}}}, \mu/m_c)}{2m_c(\mu)/m_{\eta_c}} & \text{for } n \geq 6. \end{cases}$$

Refinements — a^2, m_q Extrapolation



$$R_n(a) = R_n(0)\big(1 + c_1\alpha_s(am_c)^2 + c_2\alpha_s(am_c)^4 \\ + c_3\alpha_s(am)^6 + \cdots\big)\big(1 + d_1(2m_{u/d} + m_s)/m_c\big)$$

Results (Preliminary)



$$m_c(3\,\mathrm{GeV}) = 0.988\,(10)\,\mathrm{GeV}$$

$$m_c(m_c) = 1.269\,(9)\,\mathrm{GeV}$$

	R_6	R_8	R_{10}
a^2 extrapolation	0.2%	0.3%	0.2%
pert'n theory	0.4	0.3	1.3
$\alpha_{\overline{\mathrm{MS}}}$ uncertainty	0.3	0.4	1.0
gluon condensate	0.3	0.0	0.3
statistical errors	0.0	0.0	0.0
relative scale errors	0.4	0.4	0.4
overall scale errors	0.6	0.6	0.7
sea quarks	0.3	0.3	0.3
finite volume	0.1	0.1	0.3
Total	1.0%	1.0%	1.9%

Compare with continuum determination from vector current + $R(e^+e^-)$:

$m_c(3\mathrm{GeV})=0.986(13)$ GeV

Kuhn et al, Nucl. Phys. B778, 192 (2007) [hep-ph/0702103]

Variations — Vector, Axial-Vector Correlators

$$j_\mu^{(1)} \equiv \overline{\psi}_c(x+a\hat{\mu})\gamma_\mu\psi_c(x)$$
$$j_\mu^{(\mu)} \equiv \overline{\psi}_c(x)\gamma_\mu\psi_c(x)$$
$$j_{5\mu}^{(5\mu)} \equiv \overline{\psi}_c(x)\gamma_5\gamma_\mu\psi_c(x)$$

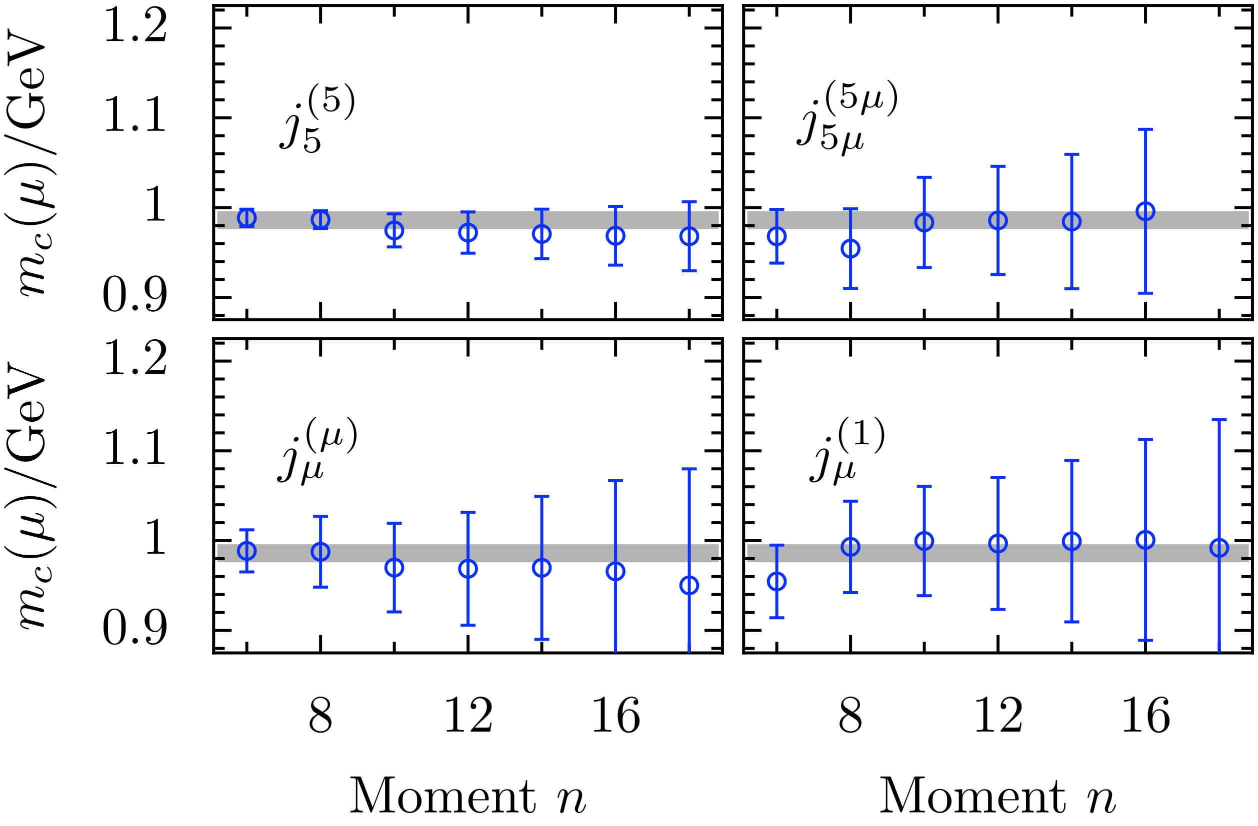
- Three different tastes
- Not conserved
- Hence $j_{\mathrm{cont}} = Z_j\, j_{\mathrm{lat}} + O(a\text{^}2)$

Redefine reduced moments to remove Z_j:

$$R_n^{(j)} \equiv \frac{am^{(j)}}{2am_{0c}}\left(\frac{G_n^{(j)}}{G_{n-2}^{(j)}}\,\frac{G_{n-2}^{(j0)}}{G_n^{(j0)}}\right)^{1/2}$$
$$\equiv \frac{r_n^{(j_{\mathrm{cont}})}(\alpha_{\overline{\mathrm{MS}}},\mu/m_c)}{2m_c(\mu)/m^{(j)}}$$

Compare quenched study: Bochkarev & de Forcrand, Nucl Phys B477, 489 (1996)

Results (Preliminary)



“Nonperturbative” Z_j

Use ratios of G_ns to determine current renormalizations Z_j

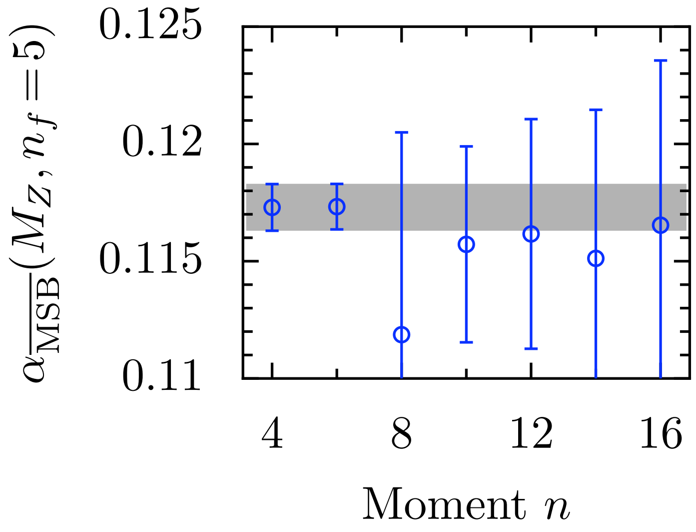
- Eg, for pseudoscalar:

  $$Z_{\mathrm{ps}}(a) = 1.057(8) \quad 1.045(7) \quad 1.031(7) \quad 1.019(7) \longrightarrow 1.007(10)$$

- Eg, for vectors, get correct leptonic width for psi to within 2-3% (Christine Davies talk).

Coupling from Ps. Correlator (Preliminary)

- R_4, R_6/R_8 ... dimensionless
- Compare lattice with pert'n theory to get coupling (at 3 GeV)



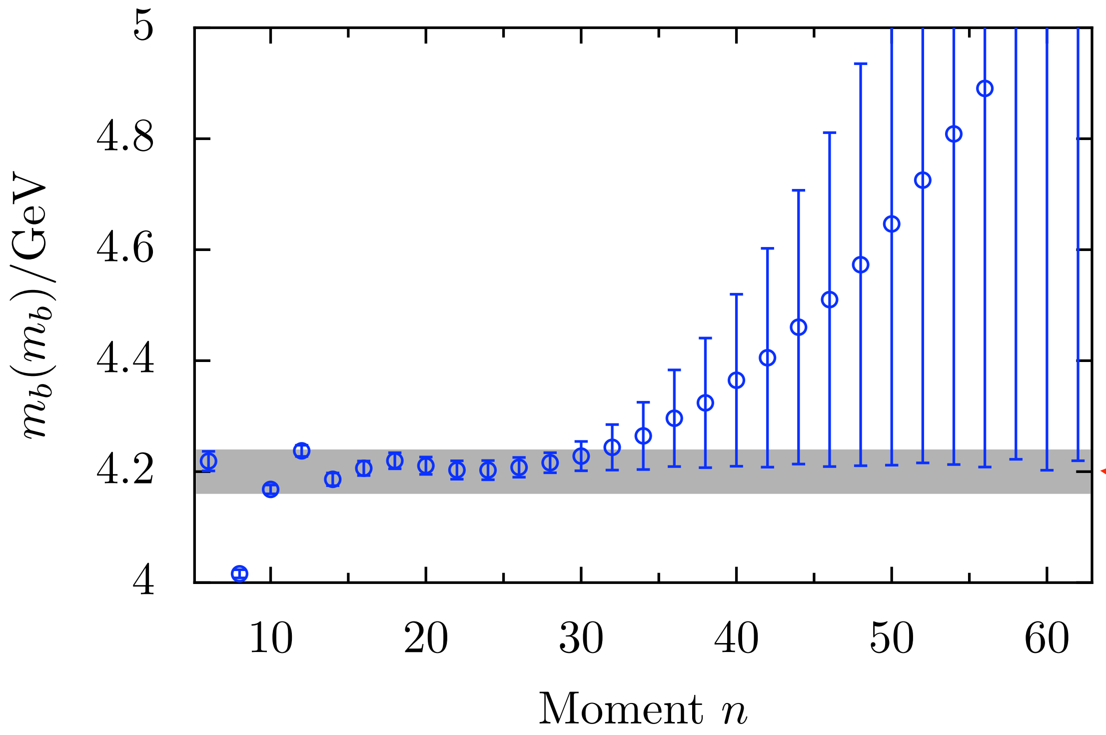
$$\boxed{\alpha_{\overline{\mathrm{MSB}}}(M_Z, n_f{=}5) = 0.1173\,(10)}$$

	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$
a^2 extrapolation	0.4%
pert'n theory	0.7
$\alpha_{\overline{\mathrm{MS}}}$ uncertainty	0.0
gluon condensate	0.0
statistical errors	0.1
relative scale errors	0.0
overall scale errors	0.1
sea quarks	0.3
finite volume	0.0
Total	0.9%

Compare PDG 2006
which gives 0.1176(20)

Current-Current Correlators
b Quarks

m_b from NRQCD Correlator (HPQCD Prelim.)



$m_b(m_b)$ = 4.20(4) GeV

$O(v^n,a^n)$

nonpert've

Compare continuum result (Kuhn et al): $m_b(m_b)$ = 4.16(3) GeV

QCD Coupling from Wilson Loops, etc

Update: Accurate Determinations of α_s from Realistic Lattice QCD

C. T. H. Davies,[1] I. D. Kendall,[1] G. P. Lepage,[2,*] C. McNeile,[1] J. Shigemitsu,[3] and H. Trottier[4]
(HPQCD Collaboration)
[1] *Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK*
[2] *Laboratory for Elementary-Particle Physics, Cornell University, Ithaca, NY 14853, USA*
[3] *Physics Department, The Ohio State University, Columbus, Ohio 43210, USA*
[4] *Physics Department, Simon Fraser University, Vancouver, British Columbia, Canada*

(Dated: July 8, 2008)

arXiv:0807.1687

Wilson Loops

- UV singular ⇒ perturbative

- a^n errors highly suppressed (IR nonperturbative)

- a-dependent coupling scale

$$\log(W) = \sum_{n=1}^{\infty} c_n \alpha_V(d/a)$$
$$\times \left(1 + w_m a(2m_{u/d} + m_s) + \cdots\right)$$

nonperturbative corrections

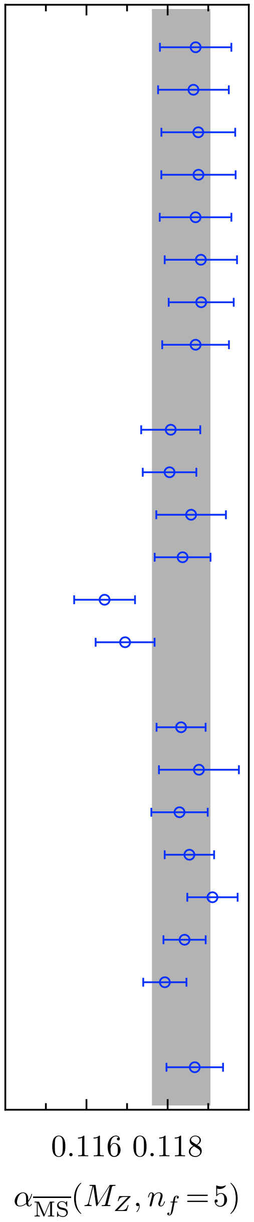
- c_1, c_2, c_3 from numerical evaluation of Feyman diagrams

- $c_4 \ldots c_{10} \ldots$ from fit to a dependence

What’s new?

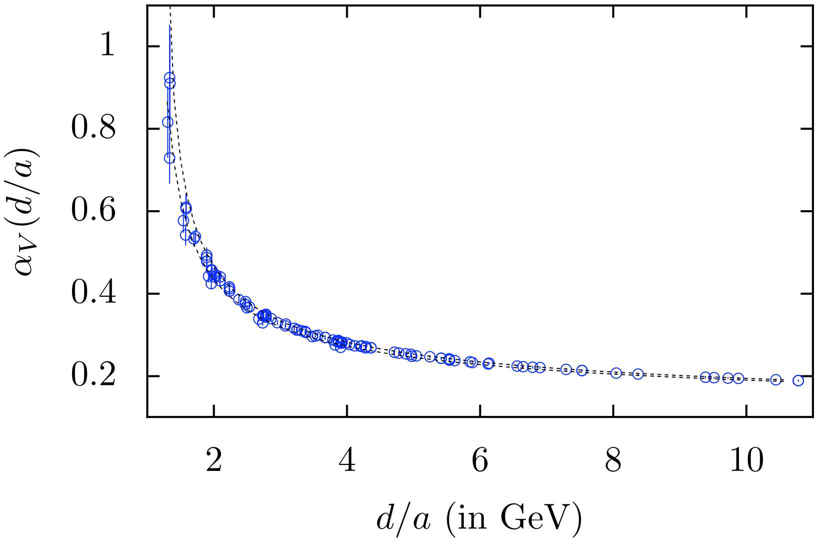
- 11 configurations, covering wide range of sea-quark masses and 5 lattice spacings, including 40% smaller lattice spacing.
- Better use of MILC’s r_1/a (smaller errors for ratio of scales), including a^n and chiral corrections to r_1.
- Fit nonperturbative chiral corrections to log(W).
- New result is one sigma above old result.

Results



0.1187(9)	$\log W_{11}$
0.1186(9)	$\log W_{12}$
0.1188(9)	$\log W_{\mathrm{BR}}$
0.1188(9)	$\log W_{\mathrm{CC}}$
0.1187(9)	$\log W_{13}$
0.1188(9)	$\log W_{14}$
0.1188(8)	$\log W_{22}$
0.1187(8)	$\log W_{23}$
0.1181(7)	$\log W_{13}/W_{22}$
0.1180(7)	$\log W_{11}W_{22}/W_{12}^2$
0.1186(9)	$\log W_{\mathrm{CC}}W_{\mathrm{BR}}/W_{11}^3$
0.1184(7)	$\log W_{\mathrm{CC}}/W_{\mathrm{BR}}$
0.1164(7)	$\log W_{14}/W_{23}$
0.1170(7)	$\log W_{11}W_{23}/W_{12}W_{13}$
0.1183(6)	$\log W_{12}/u_0^6$
0.1188(10)	$\log W_{\mathrm{BR}}/u_0^6$
0.1183(7)	$\log W_{\mathrm{CC}}/u_0^6$
0.1185(6)	$\log W_{13}/u_0^8$
0.1191(6)	$\log W_{14}/u_0^{10}$
0.1184(5)	$\log W_{22}/u_0^8$
0.1179(5)	$\log W_{23}/u_0^{10}$
0.1187(7)	$\alpha_{\mathrm{lat}}/W_{11}$

$$\boxed{\alpha_{\overline{\mathrm{MS}}}(M_Z, n_f = 5) = 0.1183\,(7)}$$



Summary (Preliminary)

- *c*-quark HISQ correlators imply (n_f=5):

$$m_c(m_c) = 1.269\,(9)\,\mathrm{GeV}$$
$$\alpha_{\overline{\mathrm{MSB}}}(M_Z) = 0.1173\,(10)$$

← HISQ *cc* correlators agree with continuum to <1%

- *b*-quark NRQCD correlator implies (n_f=5):

$$m_b(m_b) = 4.20\,(4)\,\mathrm{GeV}$$

- New analysis of coupling (n_f=5):

$$\alpha_{\overline{\mathrm{MSB}}}(M_Z) = 0.1183\,(7)$$