

Static Exotic Potentials GQQ and GGG



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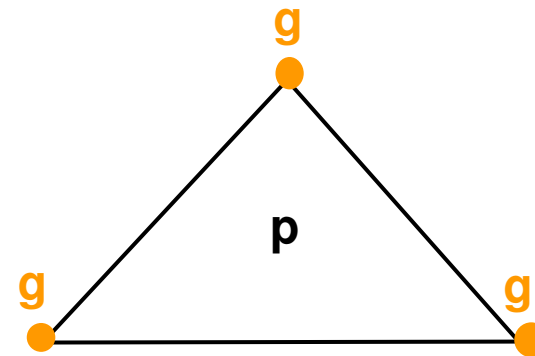
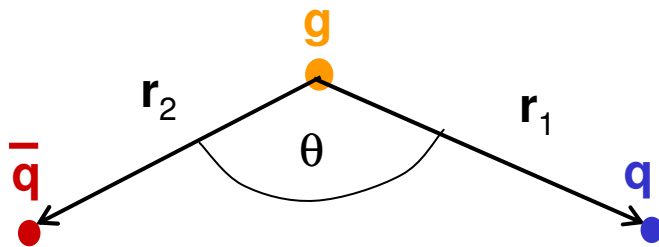
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- Motivation
- The Wilson loops for GQQ and $G\bar{G}\bar{G}$
- Results for the Hybrid GQQ
- Results for the Glueball $G\bar{G}\bar{G}$

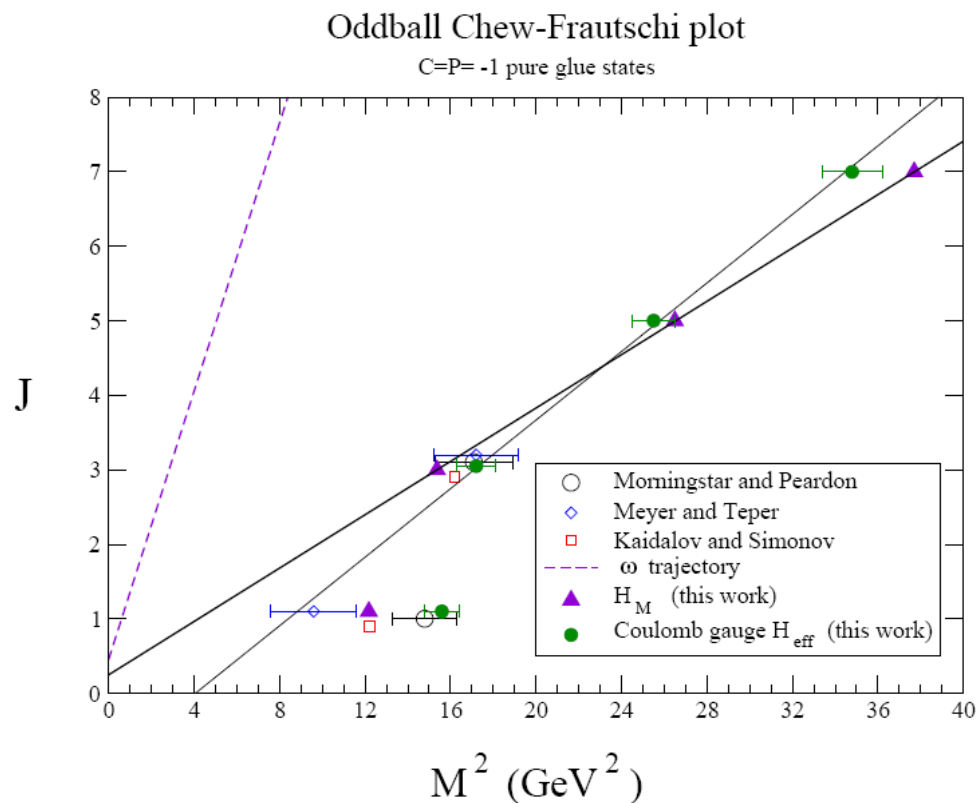
Static Exotic Potentials GQQ and GGG

Utilizing Wilson Loops, we study the static potentials of GQQ and GGG



Motivation

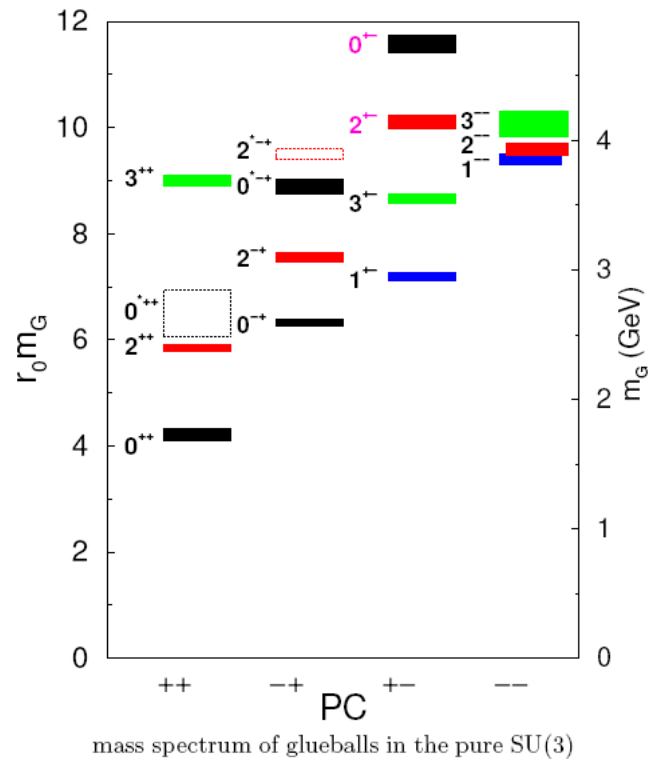
The **experiments** BESIII at IHEP in Beijing, LHC at CERN, GLUEX at JLab and PANDA at GSI in Darmstadt, will scan the mass range of GQQ hybrids and GGG glueballs. The odderon might also depend on GGG glueballs...



F. Llanes-Estrada, P. Bicudo
and S. Cotanch,
Phys.Rev.Lett.96, 081601(2006).

Motivation

- The **computations** of GQQ hybrids and GGG glueballs are performed,
- with constituent quark-gluon models, say for excited states,
 - in Lattice QCD,

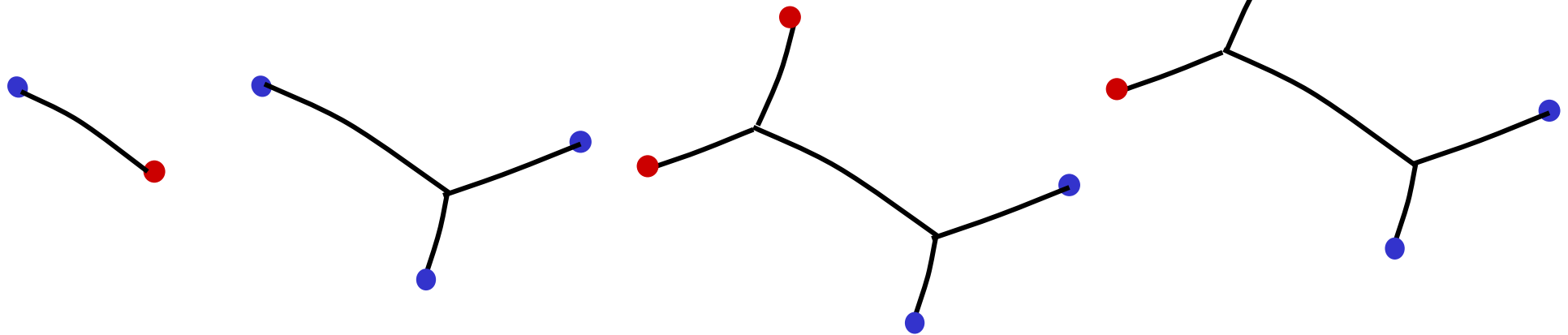


C. Morningstar and M. Peardon,
 Phys. Rev. D **60**, 034509 (1999)

Motivation

Previously, Static potentials have been studied in Lattice QCD to

- be applied in constituent quark-gluon models
- understand confinement
- understand spin and temperature
- *mostly for mesons, some for baryons, a little for tetraquarks, pentaquarks, 2-gluon glueballs...*
- **but not** for GQQ hybrids and GGG glueballs



The Wilson Loops for GQQ and GGG

We utilize **Wilson loops** to measure the **static** potentials in Lattice QCD. The positions of the quarks are connected by gauge paths composed of fundamental links, to maintain gauge invariance, while the positions of gluons are connected by adjoint paths,

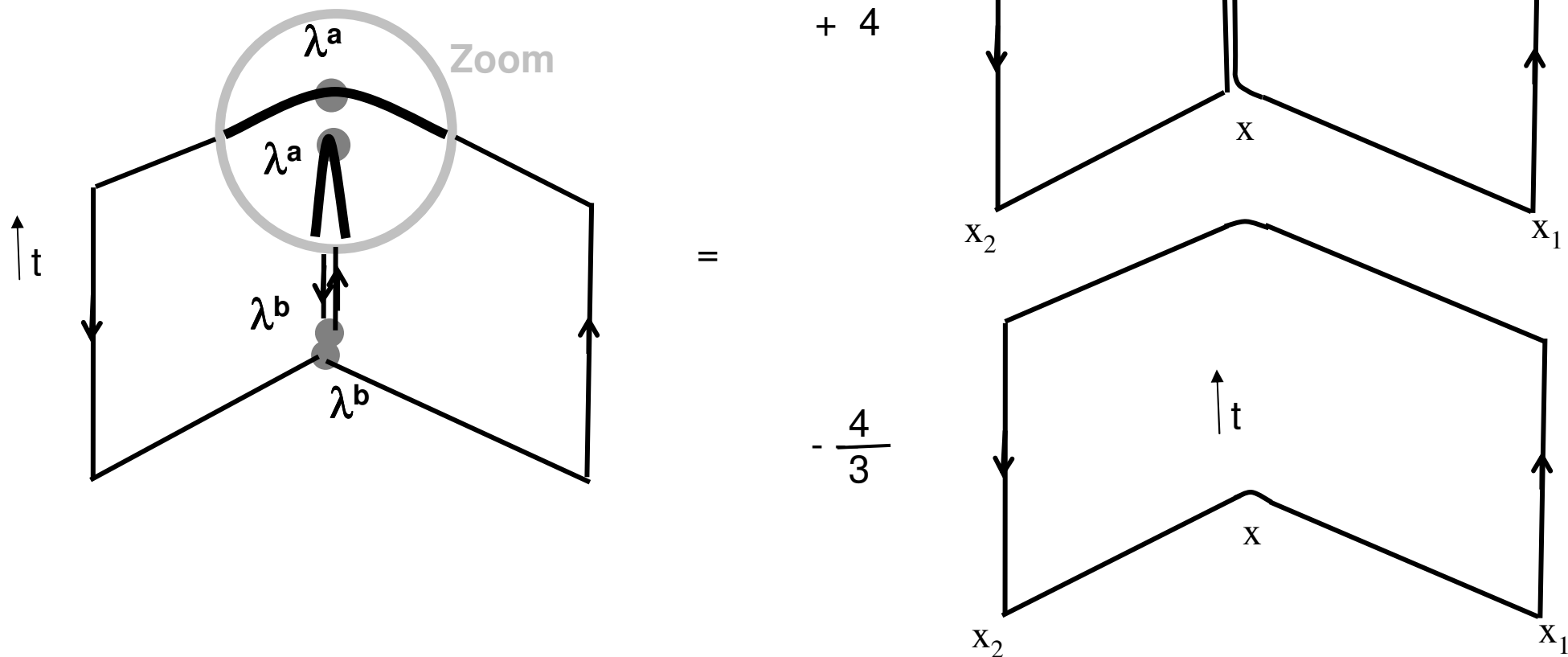
$$\tilde{U}^{ab} = \lambda^a \begin{array}{c} \xrightarrow{U^+} \\ \xleftarrow{U} \end{array} \lambda^b$$

The Fierz relation can be used to re-write the adjoint paths with fundamental paths.

$$\Sigma_a \begin{array}{c} \lambda^a \\ \lambda^a \end{array} \begin{array}{c} \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} = 2 \begin{array}{c} \nearrow \\ \searrow \end{array} \begin{array}{c} \searrow \\ \nearrow \end{array} - (2/3) \begin{array}{c} \searrow \\ \nearrow \end{array} \begin{array}{c} \searrow \\ \nearrow \end{array}$$

The Wilson Loops for GQQ and GGG

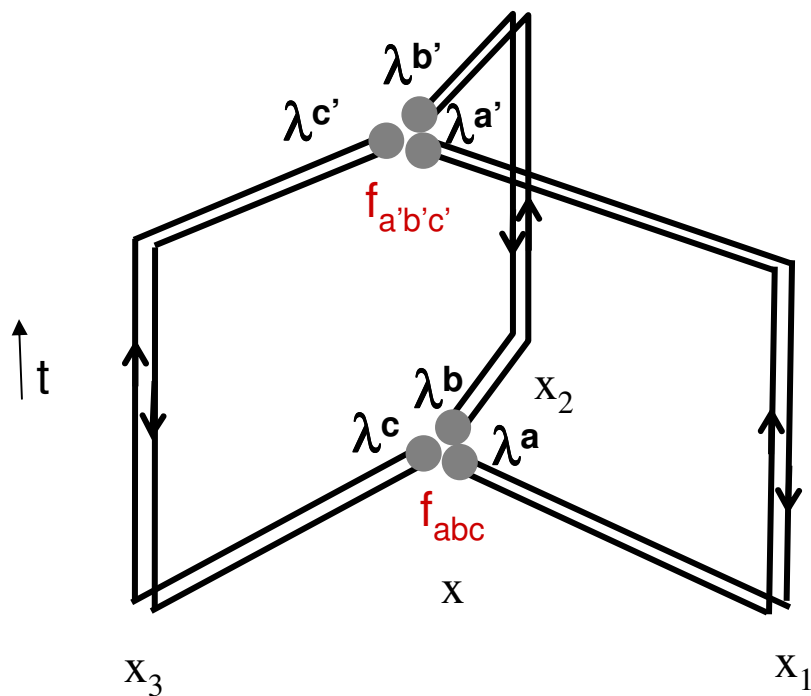
For instance, for the **hybrid GQQ** Wilson loop we get,



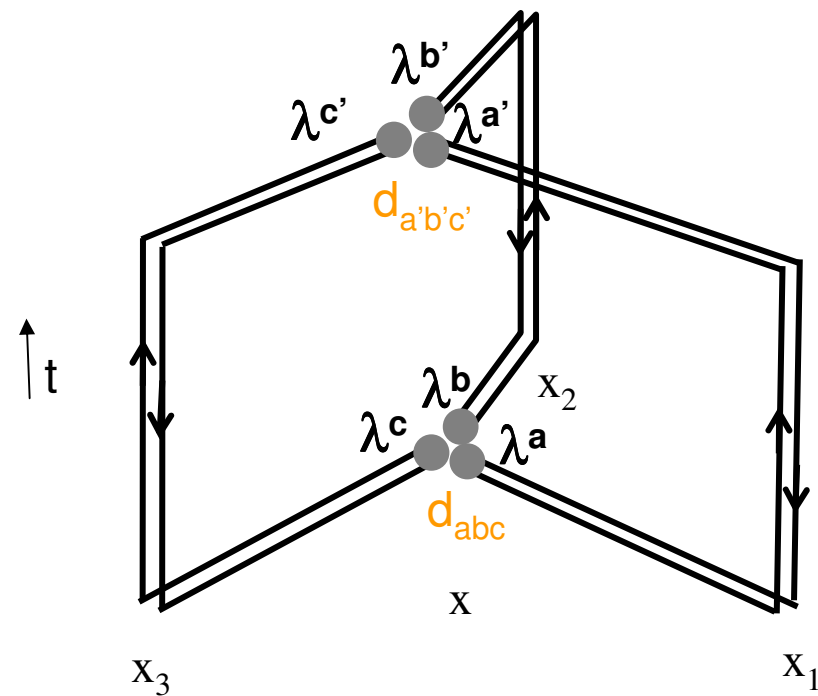
The Wilson Loops for GQQ and GGG

Whereas for the glueball GGG , we have two possible wavefunctions, **antisymmetric** or **symmetric**, with Wilson loops,

antisym.



symmetric



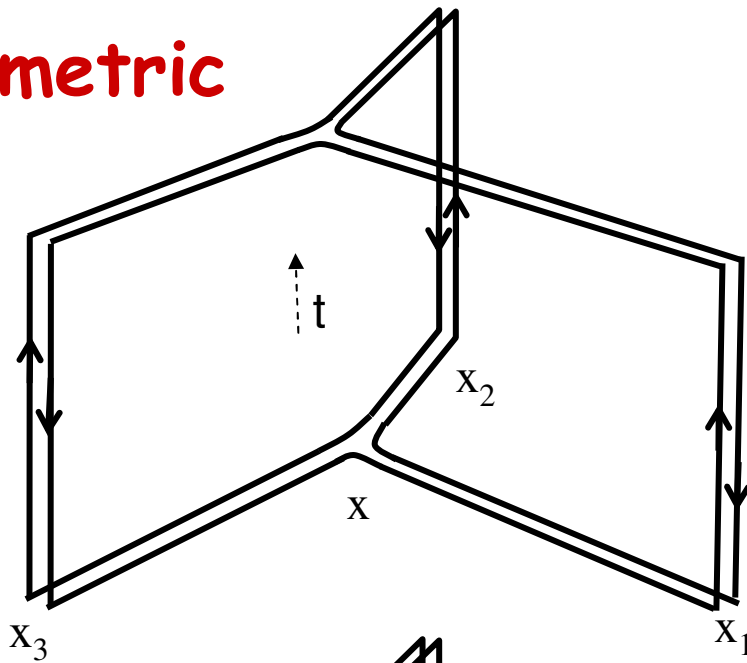
The Wilson Loops for GQQ and $G\bar{G}\bar{G}$

The two $G\bar{G}\bar{G}$ Wilson loops can be rewritten with the Fierz relation,

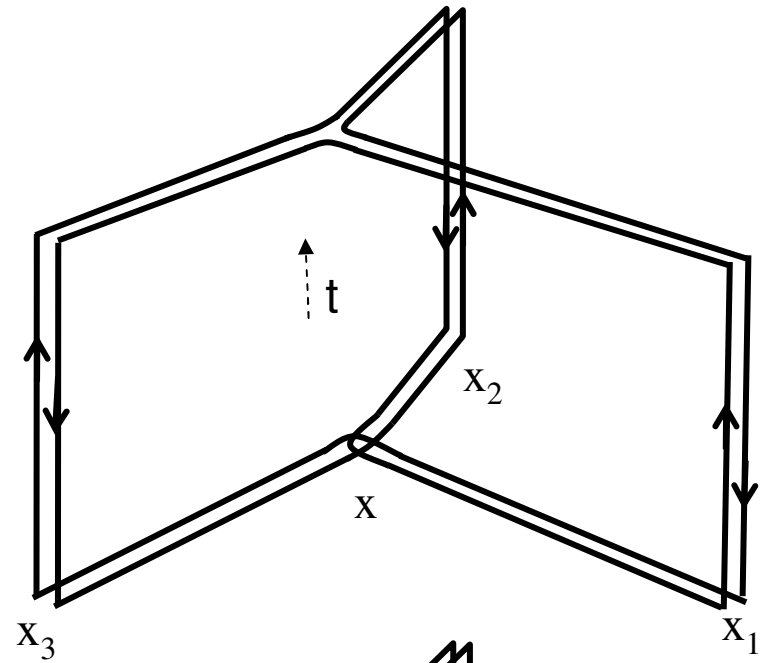
$$\begin{aligned}
 & \text{Loop with vertices } \lambda^c, \lambda^c, \lambda^b, \lambda^a, \lambda^a \\
 &= \frac{16}{9} \text{ (Term 1)} - \frac{8}{3} \text{ (Term 2)} \\
 &- \frac{8}{3} \text{ (Term 3)} - \frac{8}{3} \text{ (Term 4)} + 8 \text{ (Term 5)}
 \end{aligned}$$

antisymmetric

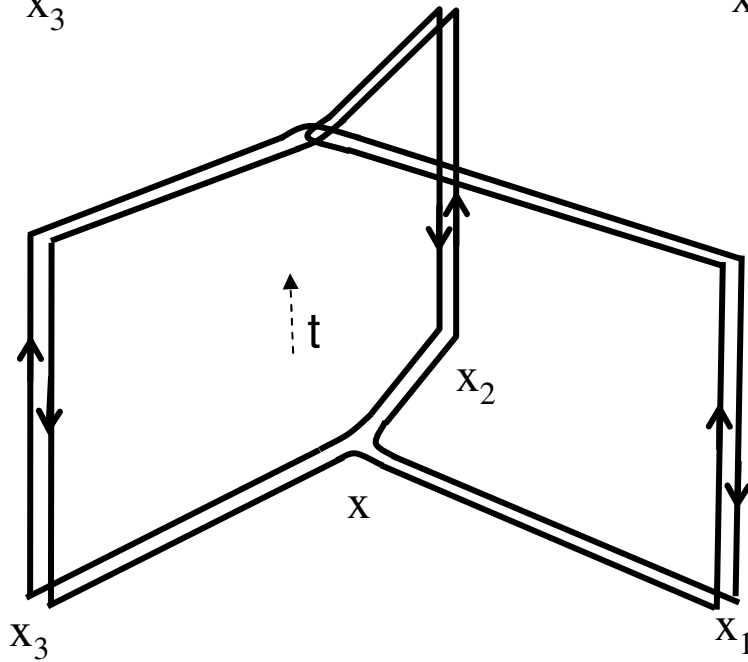
$$W_{3g}^A = 4$$



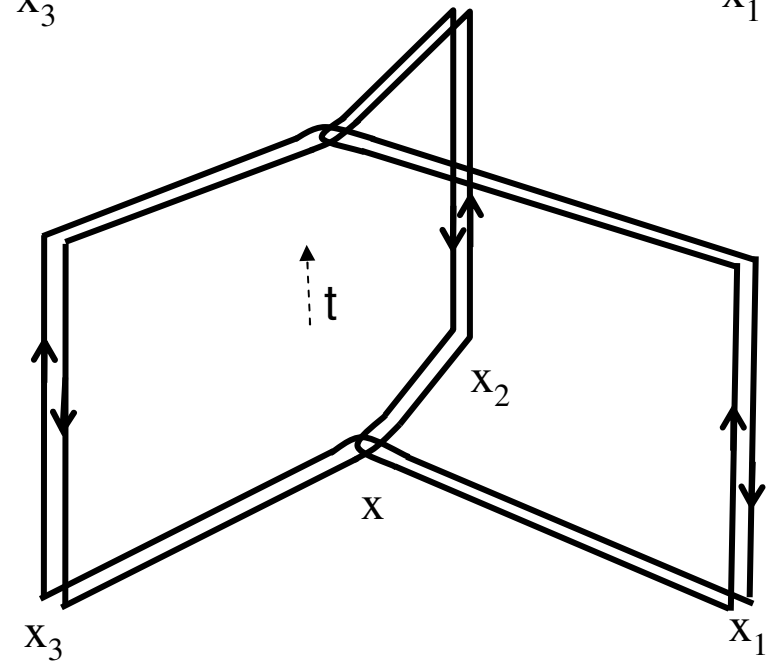
$$- 4$$



$$- 4$$

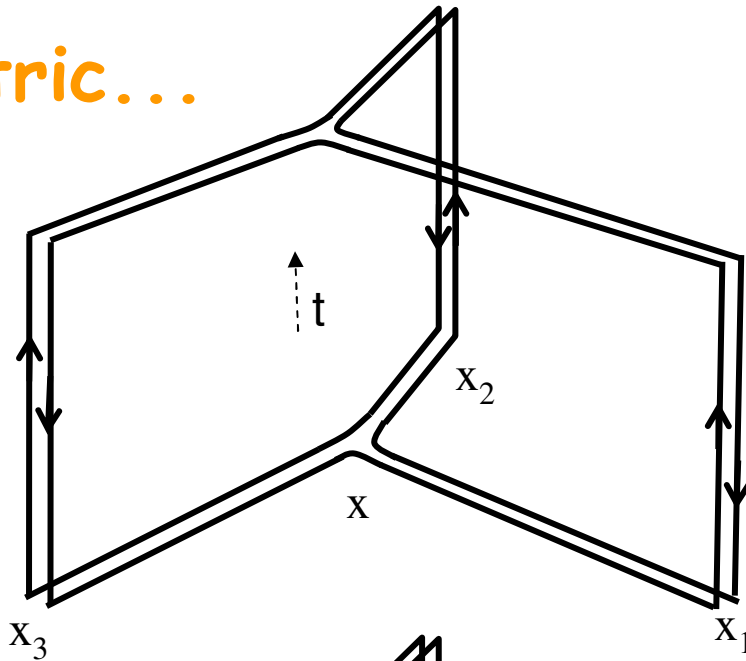


$$+ 4$$

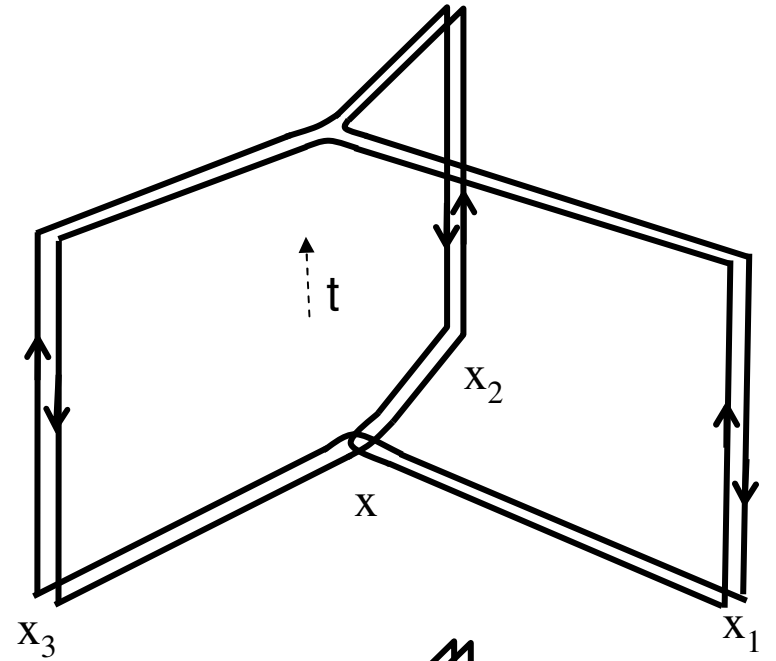


symmetric...

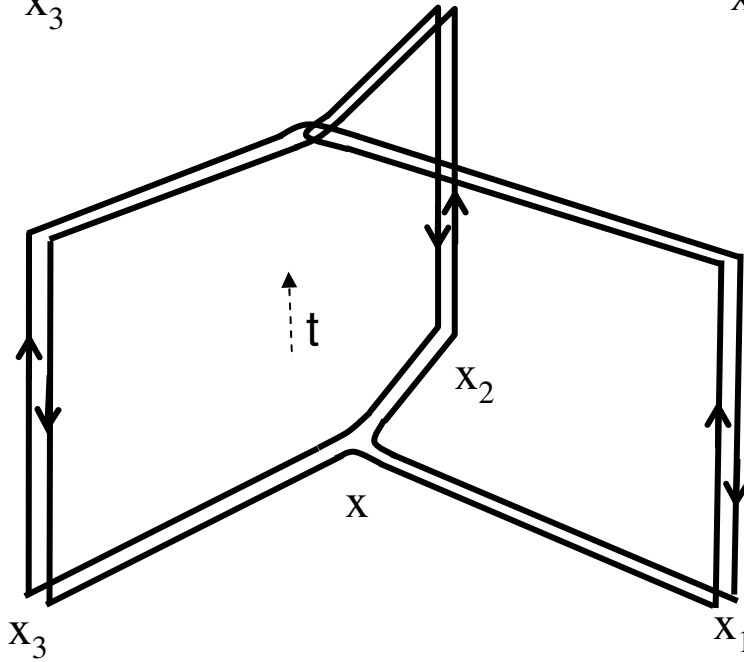
$$W_{3g}^S = 4$$



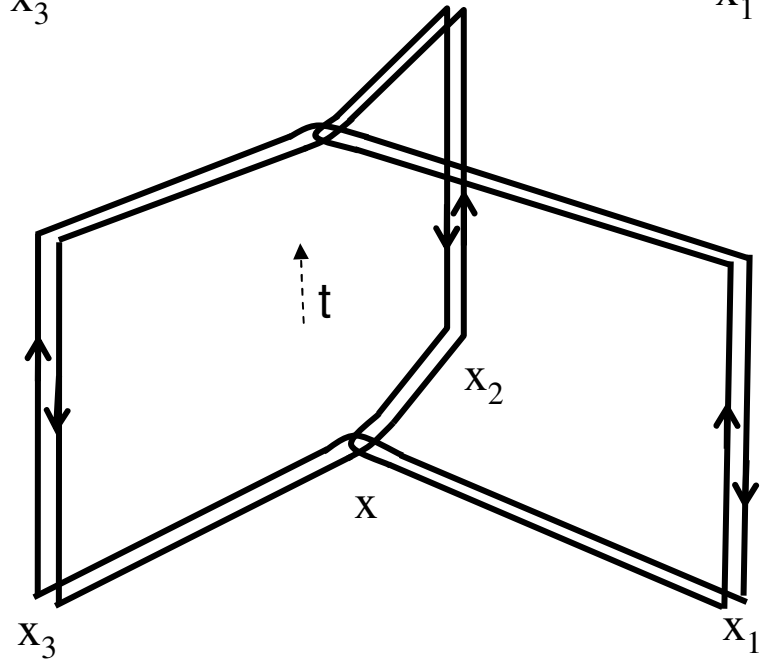
+ 4



+ 4

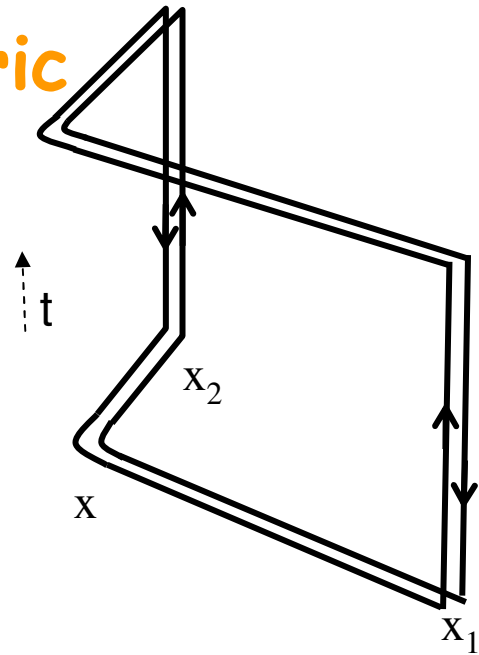


+ 4

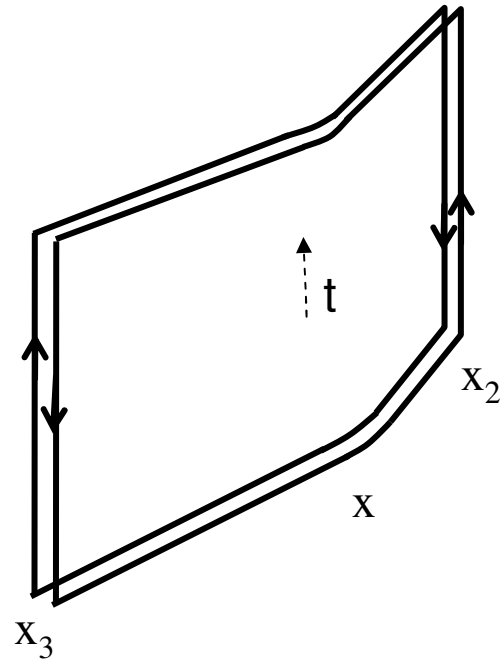


...symmetric

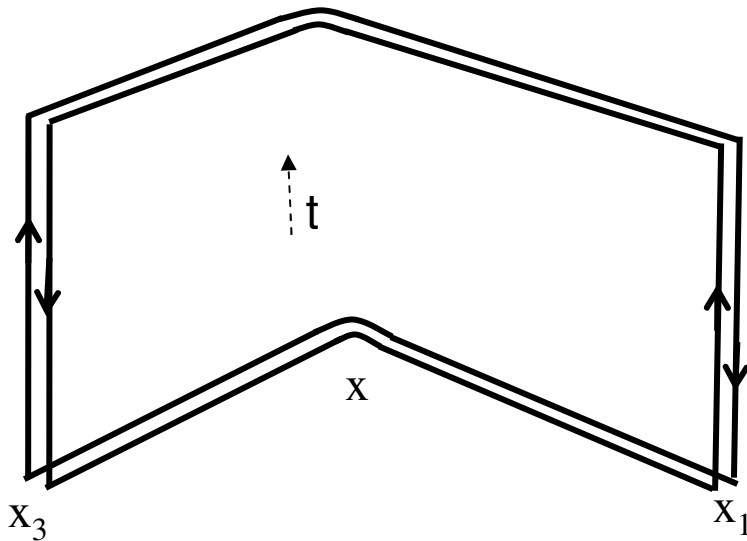
$$-\frac{16}{3}$$



$$-\frac{16}{3}$$



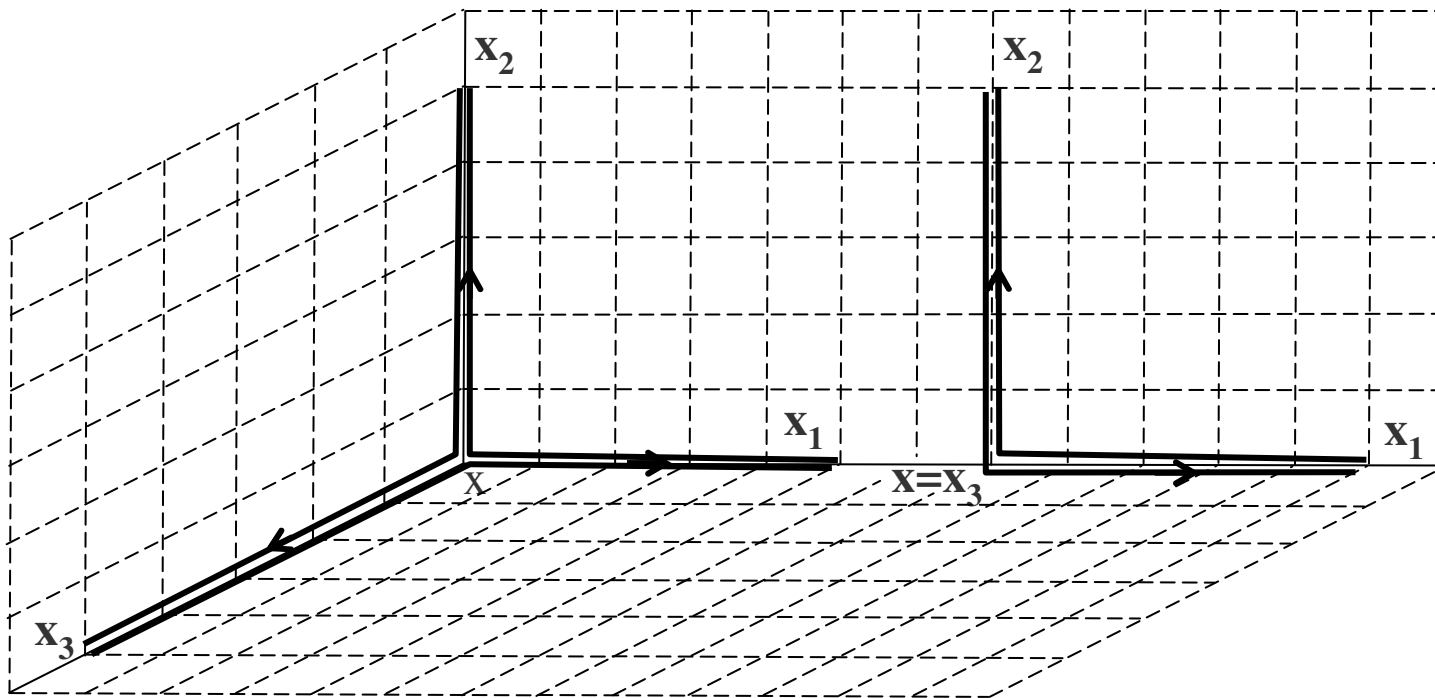
$$-\frac{16}{3}$$



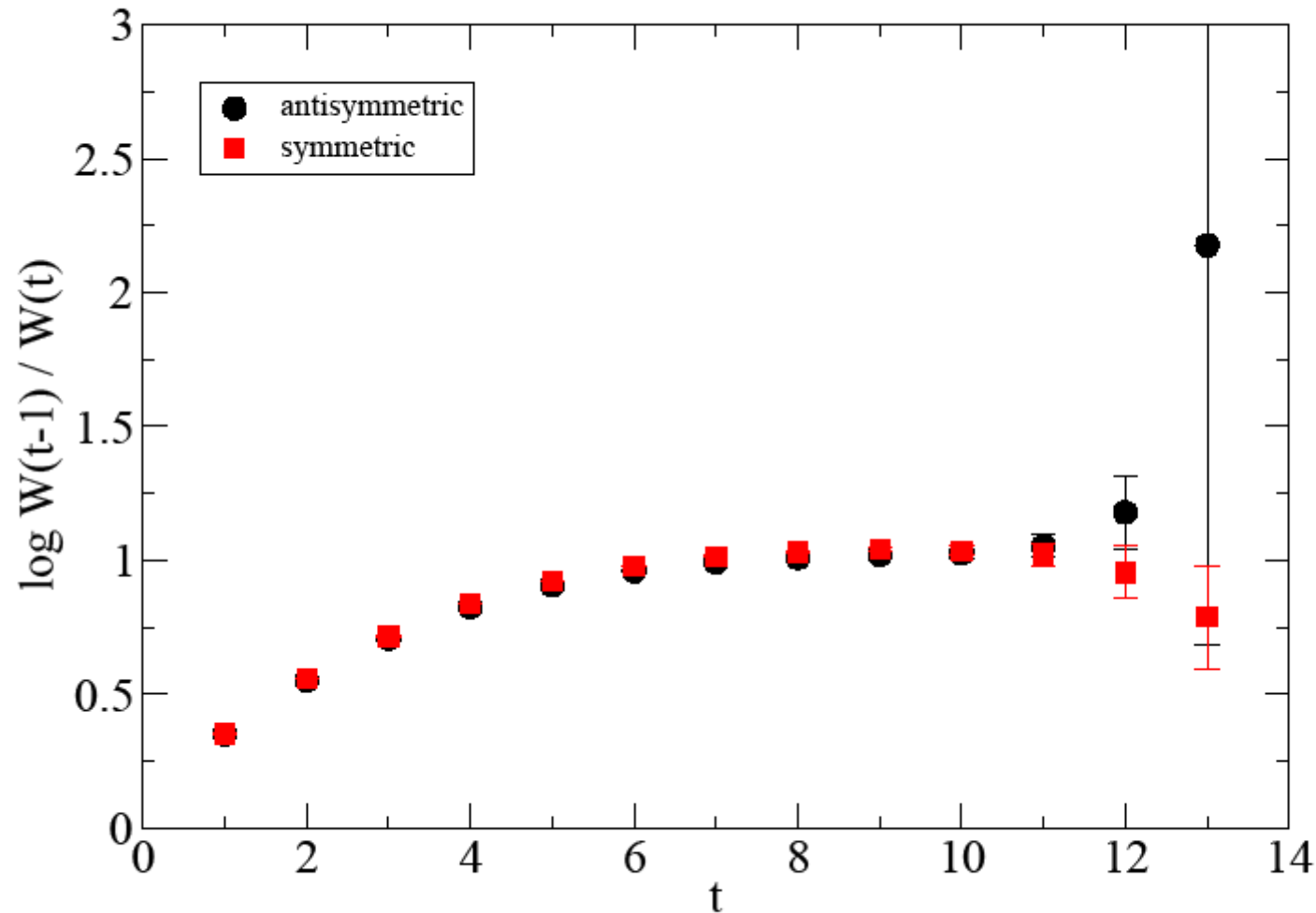
$$+\frac{32}{3}$$

The Wilson Loops for GQQ and $G\bar{G}\bar{G}$

In what concerns the spatial geometry of the $G\bar{G}\bar{G}$ we first use the simplest spatial paths



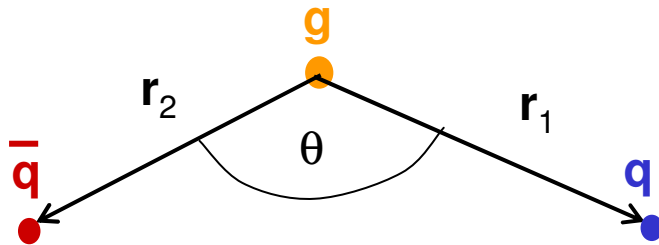
Then the **static potential** V is obtained fitting the exponential euclidian time t decay of the Wilson loop W , $W = cst. \text{Exp} (- V t)$



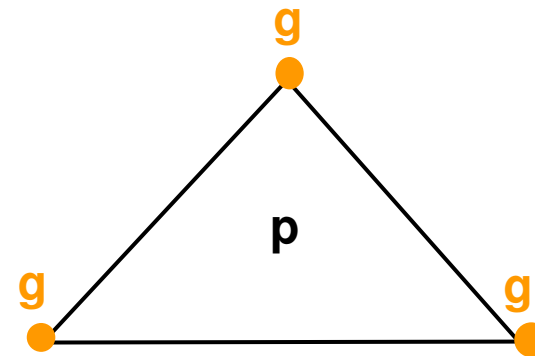
Results for the Hybrid GQQ

We compute the static potentials as a function of the **variables**,

for the hybrid GQQ
distance r_1 , distance r_2 and angle θ



for the glueball GGG
perimeter p



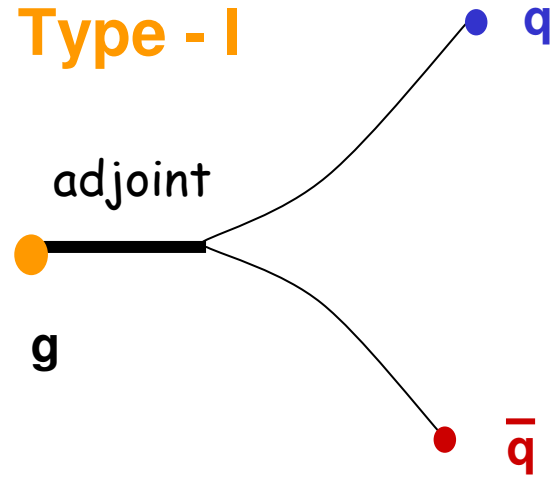
We want to compare with different **models of confinement**:

Casimir

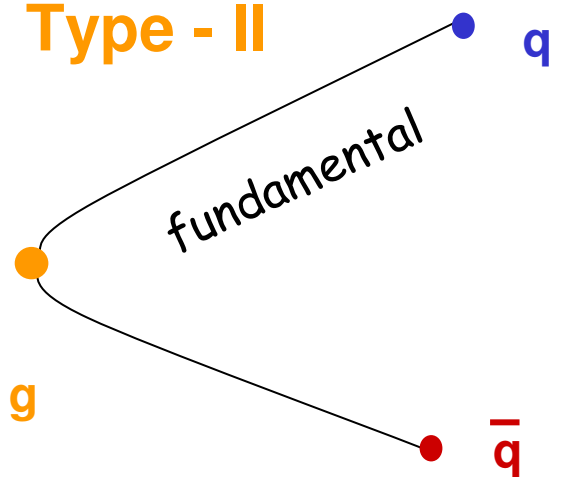
The potentials are a sum of 2-body potentials,

$$V_{ij} \propto \lambda_i \cdot \lambda_j$$

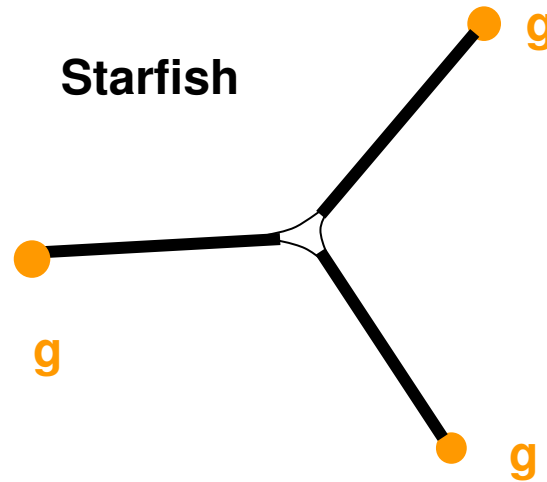
Type - I



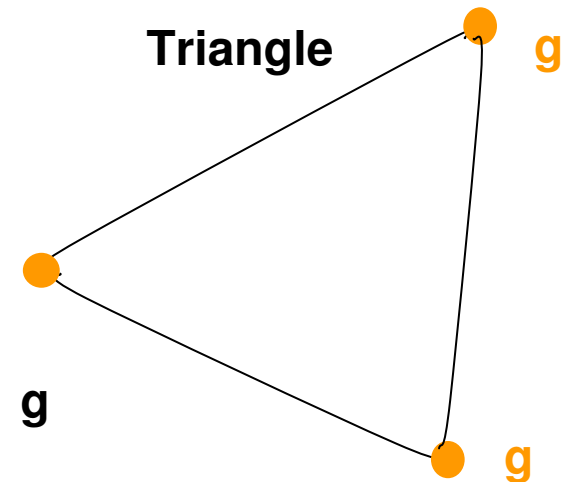
Type - II



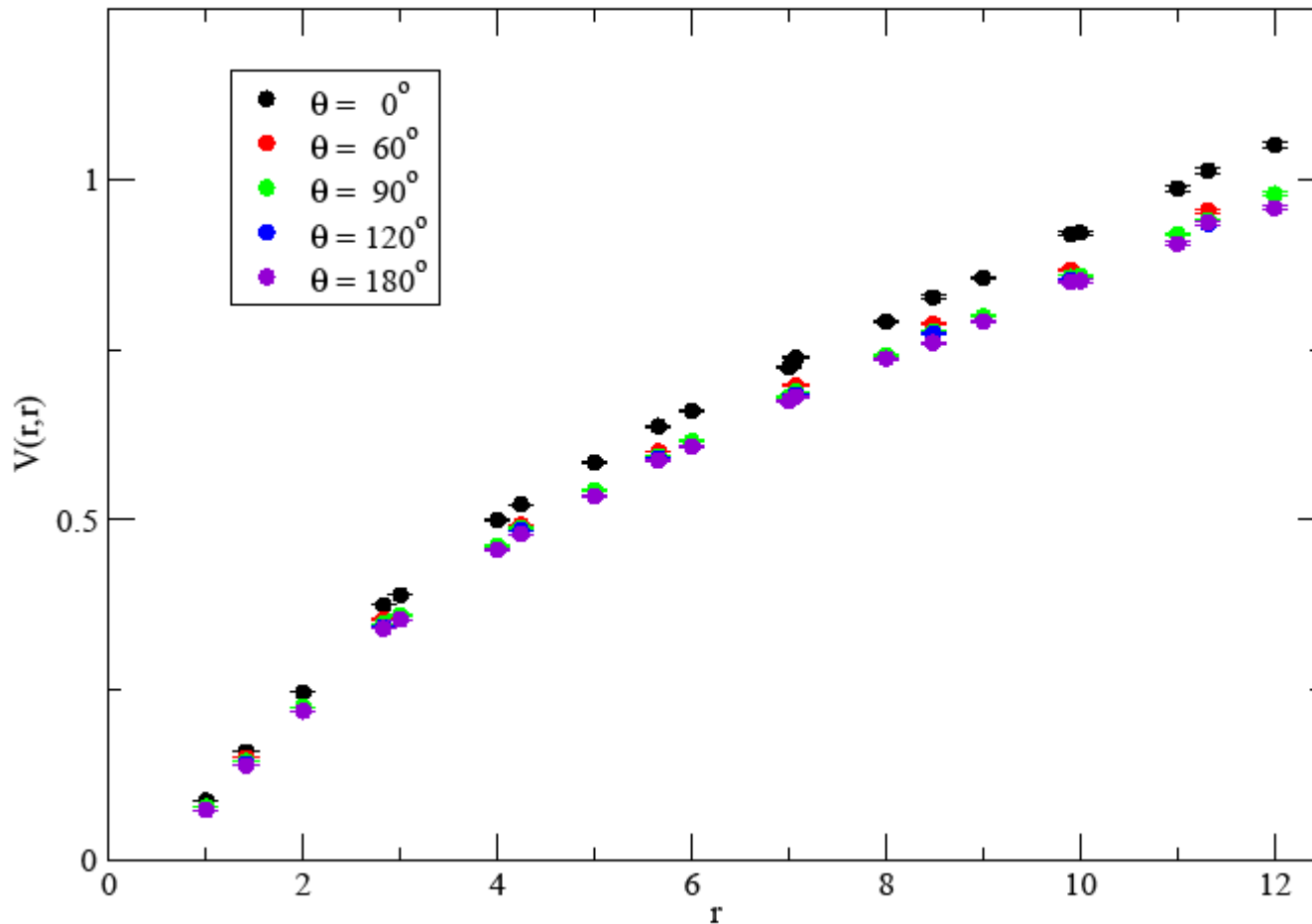
Starfish



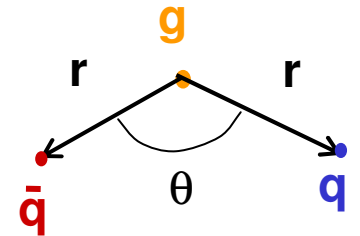
Triangle



Results for the Hybrid GQQ

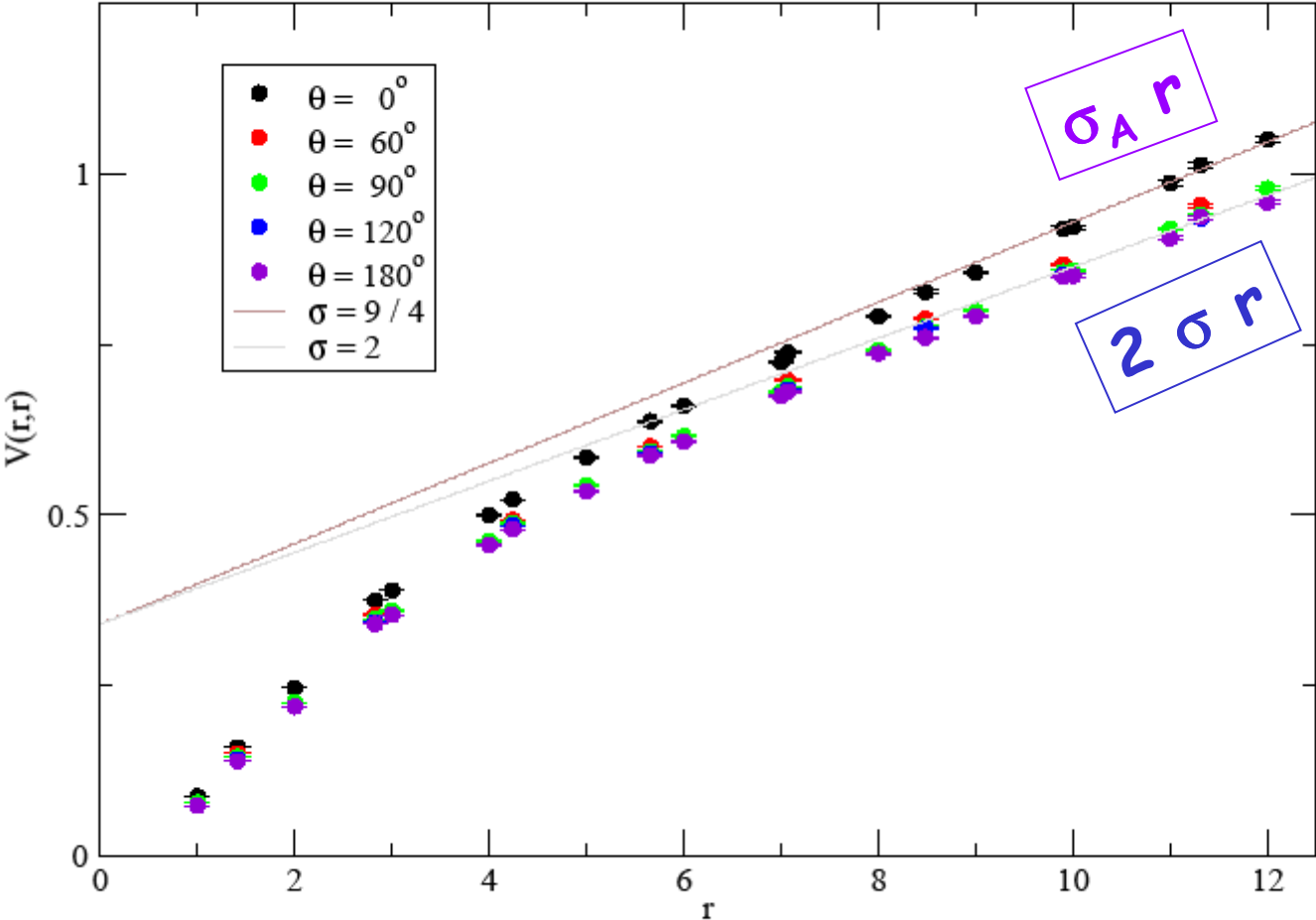


variables:

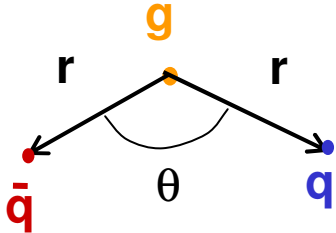


units: $a = 0.072$ fm ($24^3 \times 48$, $\beta = 6.2$, 141 config.)

Results for the Hybrid GQQ

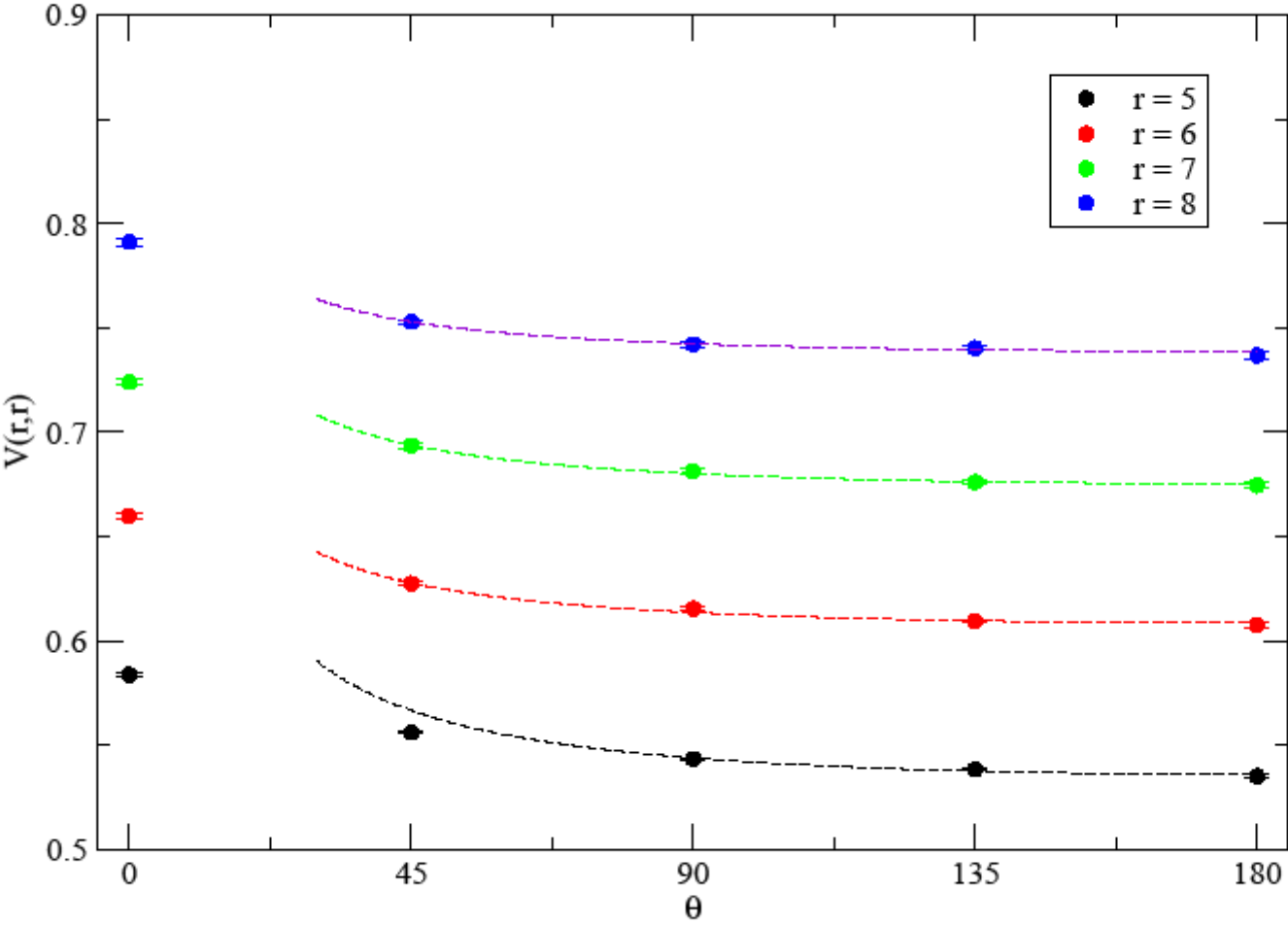


variables:

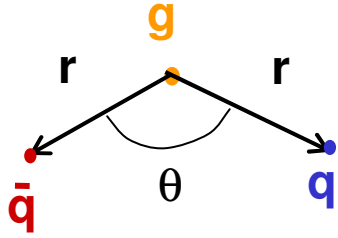


units: $a = 0.072 \text{ fm}$ ($24^3 \times 48, \beta = 6.2, 141 \text{ config.}$)

Results for the Hybrid GQQ

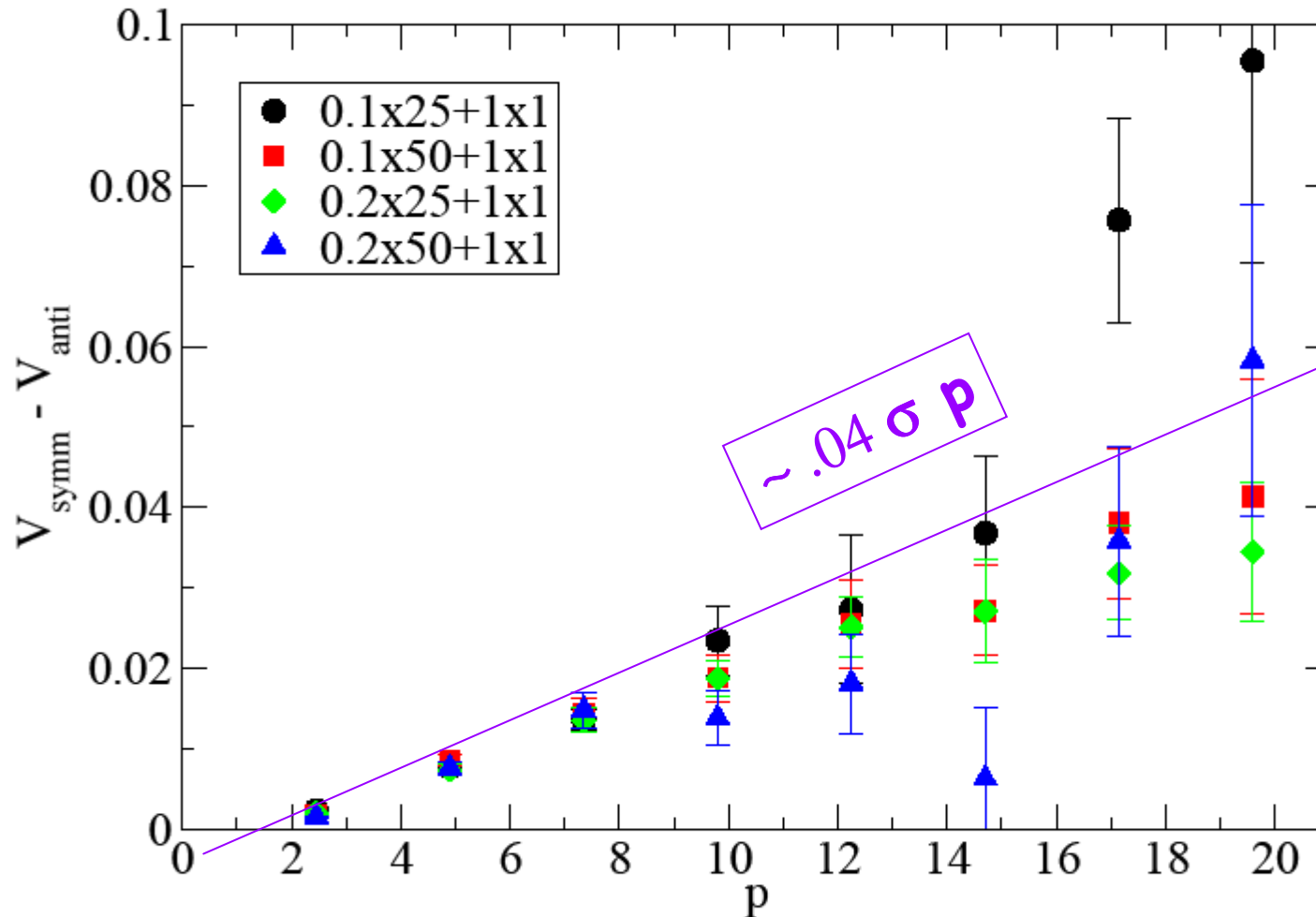


variables:

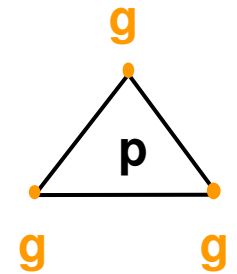


units: $a = 0.072$ fm ($24^3 \times 48, \beta = 6.2, 141$ config.)

Results for the Glueball $G\bar{G}G$

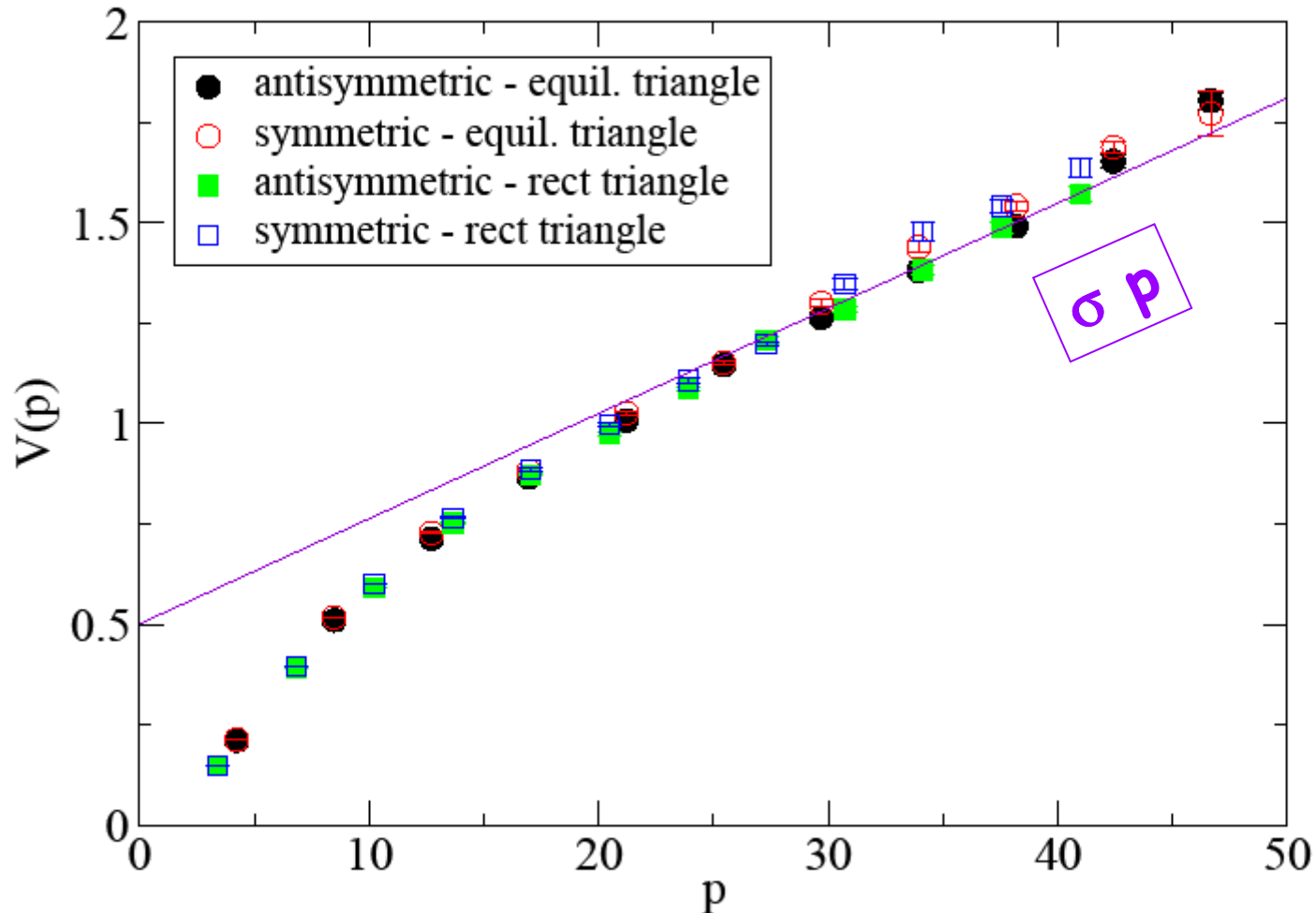


variables:



units: $a = 0.072$ fm $(24^3 \times 48, \beta = 6.2, 141$ config.)

Results for the Glueball GGG



units: $a = 0.072$ fm $(24^3 \times 48, \beta = 6.2, 141$ config.)

Conclusion & Outlook on GQQ and GGG

➔ The GQQ and GGG are confined by *fundamental* strings, as in a *Type-I* superconductor

➔ although simple, this result matters for constituent quark-gluon models.

The subtle nuances are,

➔ in the GQQ, the 2 quark strings repel and when superposed they reproduce the *Casimir scaling* observed by G. Bali in GG,

➔ In the GGG, the symmetric potential is *slightly larger* than the antisymmetric one. Both reproduce the GG potential when 2 G superpose.

• P Bicudo, M Cardoso and O. Oliveira, PRD (r) 77, 091504 (2008), arXiv:0704.2156 [hep-lat].

• M Cardoso and P. Bicudo, arXiv:0807.1621 [hep-lat].

Please expect more results soon!