Proton decay matrix elements from chirally symmetric lattice QCD

▷ Paul Cooney, The University of Edinburgh
▷ The XXVI International Symposium on Lattice Field Theory

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- Experiments such as Super–Kamiokande are searching for proton decay
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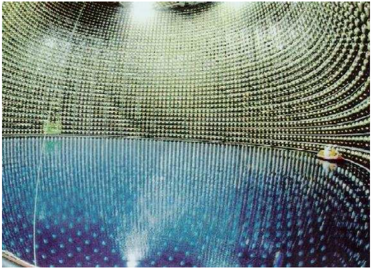

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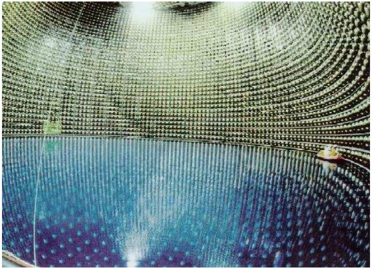

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For a generic decay channel, the partial decay width is:

$$\Gamma(p \to m + \bar{l}) = \left[\frac{m_p}{32\pi^2} \left(1 - \left(\frac{m_m}{m_p} \right)^2 \right) \right] \left| \sum_i C^i W_0^i(p \to m + \bar{l}) \right|^2$$

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The form factors can be related to a matrix element

$$P_L \left[W_0^i(q^2) - i\not{q} W_q^i(q^2) \right] u(k,s) = \langle m | \mathcal{O}^i | N \rangle$$

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The operators $\mathcal{O}^i$ are given by

$$\begin{aligned}\mathcal{O}^{RL} &= \epsilon^{abc}u^a(x,t)CP_R d^b(x,t)P_L u^c(x,t)\\ \mathcal{O}^{LL} &= \epsilon^{abc}u^a(x,t)CP_L d^b(x,t)P_L u^c(x,t)\end{aligned}$$

Define a general operator of the form

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where Γ_i are matrices with two spin indices, labelled by,

$$\begin{array}{cc} S = 1 & P = \gamma_5 \\ V = \gamma_\mu & A_\mu = \gamma_\mu \gamma_5 \\ T = \frac{1}{2}\{\gamma_\mu, \gamma_\nu\} & \tilde{T} = \gamma_5 \frac{1}{2}\{\gamma_\mu, \gamma_\nu\} \\ R = P_R = \frac{1}{2}(1+\gamma_5) & L = P_L = \frac{1}{2}(1-\gamma_5) \end{array}$$

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Operators with this structure are also used later in nucleon correlation functions and in the non-perurbative renormalization

We could measure the matrix elements $\langle m|\mathcal{O}^i|N\rangle$ directly

- Known as the *direct* method
- Three-point functions are required
- Computationally expensive

Alternatively can relate the three-point functions to two-point functions using Chiral Perturbation Theory

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For $p \to \pi^0 + e^+$, the chiral perturbation theory gives

$$\begin{aligned} W_0^{RL}(p \to \pi^0 + e^+) &= \alpha(1 + D + F)/\sqrt{2}f + \mathcal{O}(m_l^2/m_N^2) \\ W_0^{LL}(p \to \pi^0 + e^+) &= \beta(1 + D + F)/\sqrt{2}f + \mathcal{O}(m_l^2/m_N^2) \end{aligned}$$

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α and β are low energy constants from the chiral lagrangian
They can be calculated from two–point functions

$$\langle 0|\mathcal{O}^{RL}|N\rangle = \alpha P_L u(k,s)$$
$$\langle 0|\mathcal{O}^{LL}|N\rangle = \beta P_L u(k,s)$$

Define a class of two–point functions

$$f_{\Gamma_1\Gamma_2,\Gamma_3\Gamma_4}(t) = \sum_x \mathrm{tr}\left[\langle \mathcal{O}^{\Gamma_1\Gamma_2}\bar{\mathcal{O}}^{\Gamma_3\Gamma_4}\rangle P\right]$$

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Example: the proton correlation function

$$\sum_x \langle J_p(x,t)\bar{J}_p(0)\rangle = f_{PS,PS}(t)$$

Strategy:

- First find m_N from a correlated fit to the effective mass

$$m_{\text{eff}}(t) = \log\left(\frac{f_{PS,PS}(t)}{f_{PS,PS}(t+1)}\right) \rightarrow m_N \quad t \gg 0$$

- Then find G_N from a correlated fit to an effective amplitude

$$G_{N,\text{eff}} = \sqrt{2 f_{PS,PS}\, \mathrm{e}^{m_N t}} \rightarrow G_N \quad t \gg 0$$

- Finally to calculate α and β we use a ratio of two–point functions

$$R_\alpha(t) = 2G_N \frac{f_{RL,PS}(t)}{f_{PS,PS}(t)} \rightarrow \alpha \quad R_\beta(t) = 2G_N \frac{f_{LL,PS}(t)}{f_{PS,PS}(t)} \rightarrow \beta$$

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  - Iwasaki gauge action ($\beta = 2.13$)
  - Fifth dimension size $L_s = 16$
  - Inverse lattice spacing $a^{-1} = 1.73(3)$ GeV
- Two different lattice volumes
  $V = 16^3 \times 32$ and $24^3 \times 64$
- Two degenerate light quarks with masses
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- Oversampling and binning of correlation functions
- Multiple sources per configuration
- Local Smearing (L), Gaussian Smearing (G) / (G*) and Hydrogen-Like Smearing (H) of operators

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Fitting

Fit by minimising a correlated χ^2

$$\chi^2(p) = \sum_{t,t'} \left[p_{\text{eff}}(t) - p\right] C^{-1}_{tt'} \left[p_{\text{eff}}(t') - p\right]$$

With correlation Matrix

$$C_{tt'} = \frac{1}{N_{\text{boot}}} \sum_{n=1}^{N_{\text{boot}}} \left[p^{(n)}_{\text{eff}}(t) - \bar{p}_{\text{eff}}(t)\right] \left[p^{(n)}_{\text{eff}}(t') - \bar{p}_{\text{eff}}(t')\right].$$

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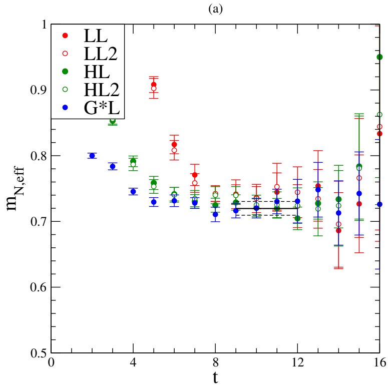
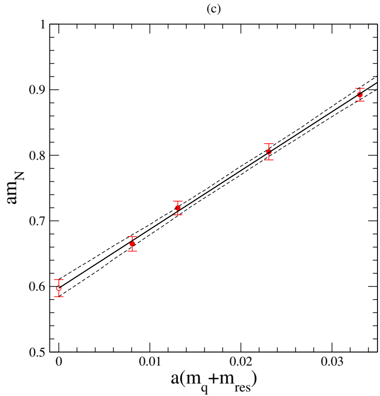
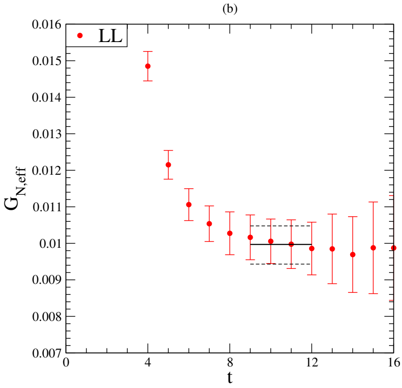
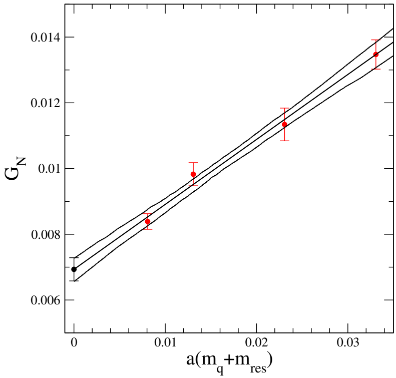
Nucleon Mass



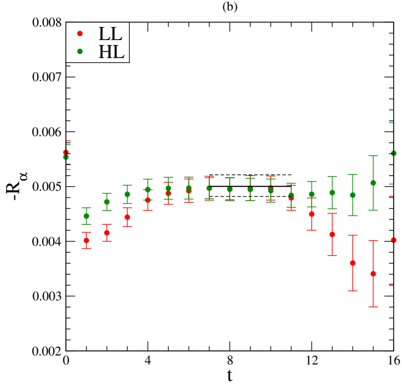


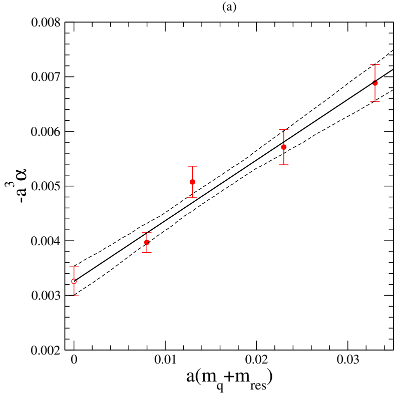
Nucleon Amplitude



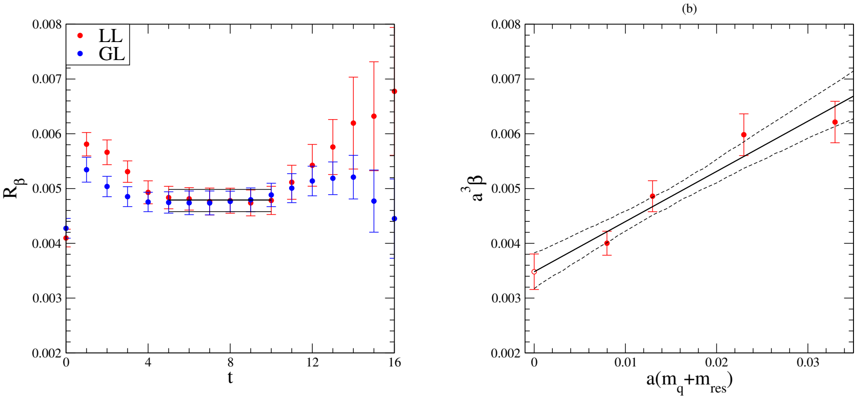


Low energy constant: α





Low energy constant: β



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  $\Rightarrow$ shown previously ($\approx 10\%$)
- Finite volume errors
- Extrapolation errors
- Errors in renormalisation
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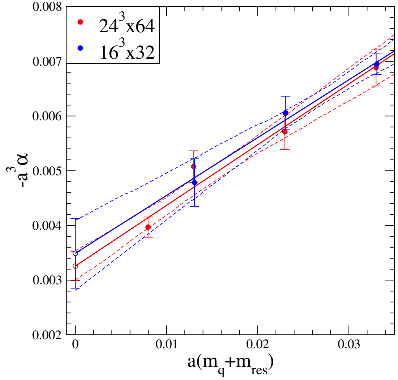
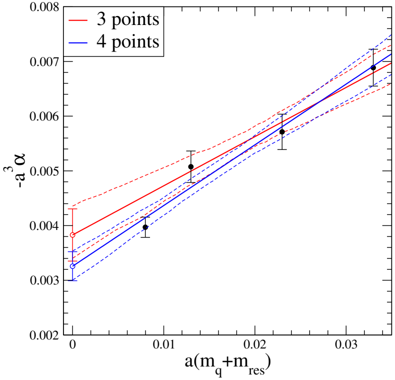
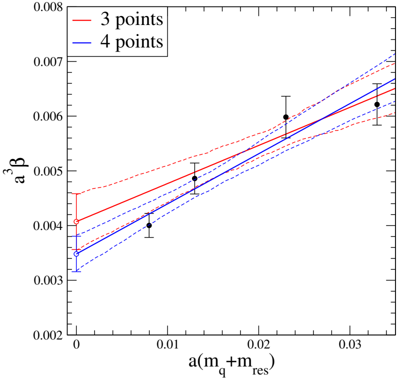


- No noticeable effect

Extrapolation Error



18%



17%

NPR

- Non-perturbative MOM scheme renormalisation of the Rome-Southampton group
- The renormalised operators are

$$\mathcal{O}^{A}_{\mathrm{ren}} = Z^{AB}\mathcal{O}^{B}_{\mathrm{latt}}$$

- A and B label the spin structure, eg LL
- Z^{AB} is the mixing matrix
- $\mathcal{O}^{LL}$ and $\mathcal{O}^{RL}$ mix with a 3rd operator $\mathcal{O}^{A(LV)}$
  $\Rightarrow$ Z^{AB} is a 3×3 matrix
- Exponentially accurate chiral symmetry from Domain Wall Fermions should suppress operator mixing

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- We define the parity basis of operators SS-SP, PP-PS, AA+AV
- These are related to the chirality basis of operators we are interested in via

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\begin{aligned}
LL &= \frac{1}{4}(SS+PP) - \frac{1}{4}(SP+PS) \\
RL &= \frac{1}{4}(SS-PP) - \frac{1}{4}(SP-PS) \\
A(LV) &= \frac{1}{2}AA - \frac{1}{2}(-AV)
\end{aligned}
$$

- Hence $Z_C = T Z_P T^{-1}$, where

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T = \begin{pmatrix} 1/4 & 1/4 & 0 \\ 1/4 & \text{-}1/4 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}
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\end{aligned}
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We want to calculate the non-perturbative amputated 3-quark vertex function of these operators

$$\mathcal{G}^{A}_{abc,\alpha\beta\gamma\delta}(p^2) = \epsilon^{abc}(C\Gamma)_{\alpha'\beta'}\Gamma'_{\delta\gamma'}\langle Q^{a'a}_{\alpha'\alpha}(p)Q^{b'b}_{\beta'\beta}(p)Q^{c'c}_{\gamma'\gamma}(p)\rangle$$

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- The renormalization condition in the RI–Mom Scheme is

$$Z_q^{-3/2} Z^{BC} M^{CA} = \delta^{BA}$$

- Where the matrix M is,

$$M^{AB} = \mathcal{G}^{A}_{abc,\alpha\beta\gamma\delta}(p^2) P^{B}_{abc,\beta\alpha\delta\gamma}$$

- and the projection matrices $P^{A}_{abc,\beta\alpha\delta\gamma}$ are chosen so that the renormalization condition is satisfied in the free field case where $Z_q = 1$ and $Z^{BC} = \delta^{BC}$.
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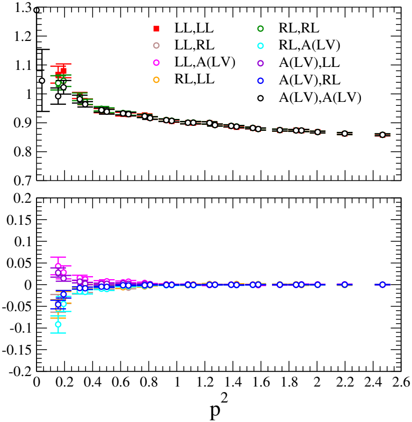
M^{AB}


LL,LL
LL,RL
LL,A(LV)
RL,LL
RL,RL
RL,A(LV)
A(LV),LL
A(LV),RL
A(LV),A(LV)
1.3
1.2
1.1
1
0.9
0.8
0.7
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
p^2

- Rotate the basis
- Perform a chiral extrapolation
- We match to the $\overline{\mathrm{MS}}$ scheme at 2GeV
- This gives
  $U^{\overline{\mathrm{MS}}\leftarrow\mathrm{latt}}(2GeV)_{LL} = 0.662(10)$
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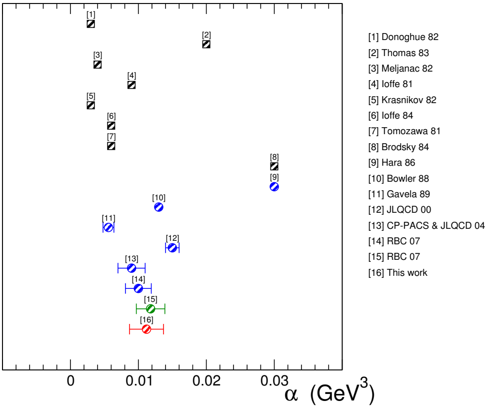

Putting all these pieces together we get

- α = -0.0112(12)(22)
- β = 0.0120(13)(23)

- The direct calculation is currently underway
- Example: Preliminary results for the $W_0^{LL}(p \to \pi^+ + \nu)$, on the $16^3 \times 32$ lattice, with valence quark mass $am_u = 0.03$

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