Fractionally charged Wilson loops as a probe of θ-dependence in $CP^{(N-1)}$ sigma models

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Outline

- Background - techniques for calculation at nonzero θ
- Observation: In d=2 U(1) gauge theory a Wilson loop with fractional charge $q=\theta/2\pi$ is equivalent to including a θ-term within the loop.
- Fractionally charged Wilson loops on the lattice
- Predicted behavior of vacuum free energy $\varepsilon(\theta)$ vs N
- Calculated value of topological susceptibility vs N
- Lattice results: free energy density $\varepsilon(\theta)$ for CP^1, CP^5 and CP^9
- Conclusions

The $CP^{(N-1)}$ model with a θ-term

N scalar fields z_i with $z_i^* z^i = 1$

$$\mathcal{L} = \beta N (D_\mu z_i)^* (D_\mu z^i) - i(\theta/2\pi)\varepsilon_{\mu\nu}\partial_\mu A_\nu$$

$$D_\mu = \partial_\mu + iA_\mu$$

$$A_\mu = (i/2)(z_i^* \partial_\mu z^i - \partial_\mu z_i^* z^i)$$

$$\mu = 0..1 \qquad i = 1..N$$

Lattice calculation at nonzero θ

$$\mathcal{L} = \beta N (D_\mu z_i)^* (D_\mu z^i) - i(\theta/2\pi)\varepsilon_{\mu\nu}\partial_\mu A_\nu$$

- Imaginary θ term makes usual MC simulation impossible
- Some alternative approaches:
  - Burkehalter, Imachi, Shinno and Yoneyama, "CP^{N-1} Models with a θ Term and a Fixed Point Action"
  - Azcoiti, Di Carlo, Galante and Laliena, "θ dependence of the CP^9 model"
  - Beard, Pepe, Riederer and Wiese, "Study of CP(N-1) θ-Vacua by Cluster-Simulation of SU(N) Quantum Spin Ladders"
  - Imachi, Kambayashi, Shinno and Yoneyama, "The Theta-term, CP(N-1) model and the inversion approach to the imaginary Theta method"
- Recent review paper by Vicari and Panagopoulos, "θ-dependence of SU(N) gauge theories in the presence of a topological term", arXiv:0803.1593v3, April 17, 2008.

Fractionally charged Wilson Loop

We calculate the expectation of a Wilson loop with fractional charge $q \in [0:1]$

$$\langle W_C(q) \rangle = \int dA \; exp[-iq \oint A \cdot dx] \;\; e^{-S[A]}$$

Using Stokes' theorem with $\theta = 2\pi q$

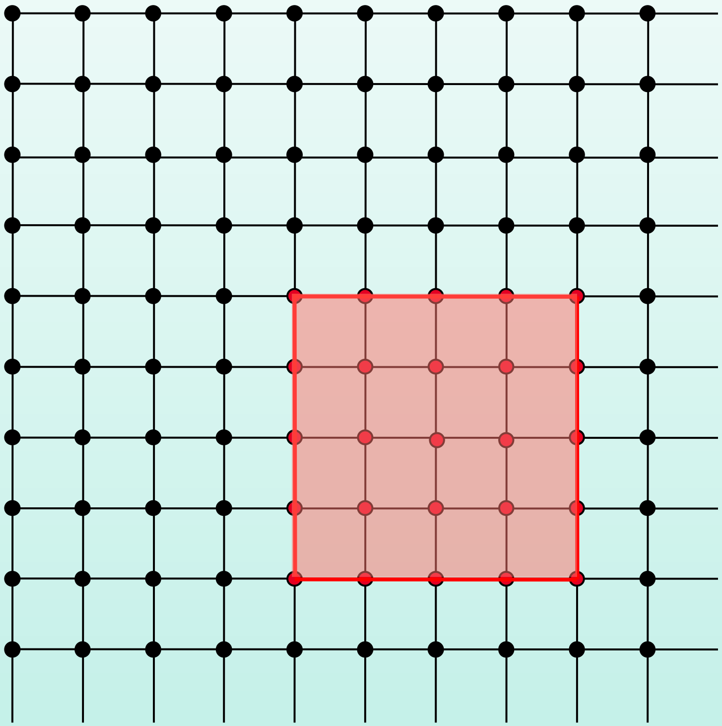
$$q \oint A \cdot dx = q \int d^2x \; \varepsilon_{\mu\nu} \partial_\mu A_\nu = \theta \int d^2x \; Q(x)$$

The expectation of the Wilson loop is proportional to the partition function $Z(\theta)$ for the volume V within the loop.

$$\langle W_C(q) \rangle = \int dA \; exp[-i\theta \int d^2x \; Q(x)] \;\; e^{-S[A]}$$

$$\sim exp[\varepsilon(\theta) - \varepsilon(0)] V$$

Wilson loops on the lattice



Use the log-plaquette definition of topological charge density

$$Q_v = (-i/2\pi) ln\, P_v$$

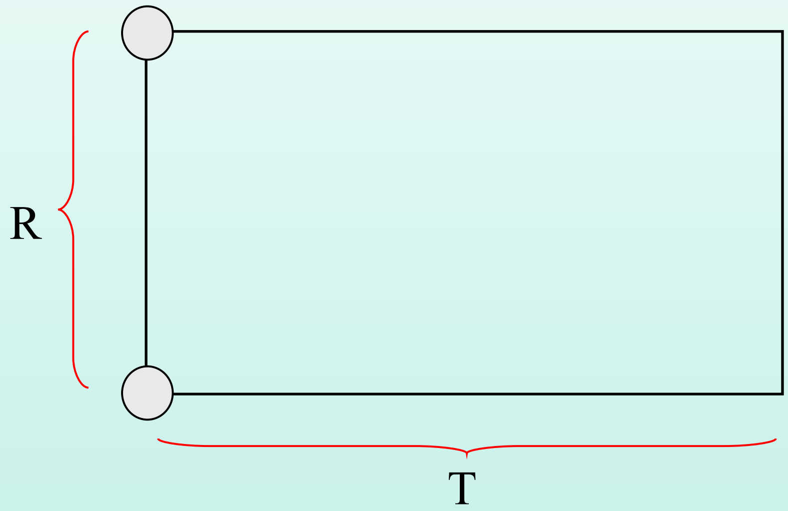
$$\langle W_C(q) \rangle = \langle exp[q \Sigma_{v \in V} lnP_v] \rangle$$

$$□ \sim exp[\varepsilon(\theta) - \varepsilon(0)]V$$

The Wilson loop is proportional to the partition function of a θ vacuum within the loop.

Wilson loops and confinement

A Wilson loop can be thought of as the correlator of a pair of strings of spatial extent R connecting test charges.



In d=2 $CP^{(N-1)}$ the potential is linear in R, and so a fractionally charged Wilson loop will have the form

$$e^{-V(R)T} = e^{-\varepsilon(\theta)RT} = e^{-\varepsilon(\theta)V}$$

Predictions for $\varepsilon(\theta)$ and N

- $\varepsilon(\theta) = \varepsilon(\theta + 2\pi k) \qquad k \in Z$

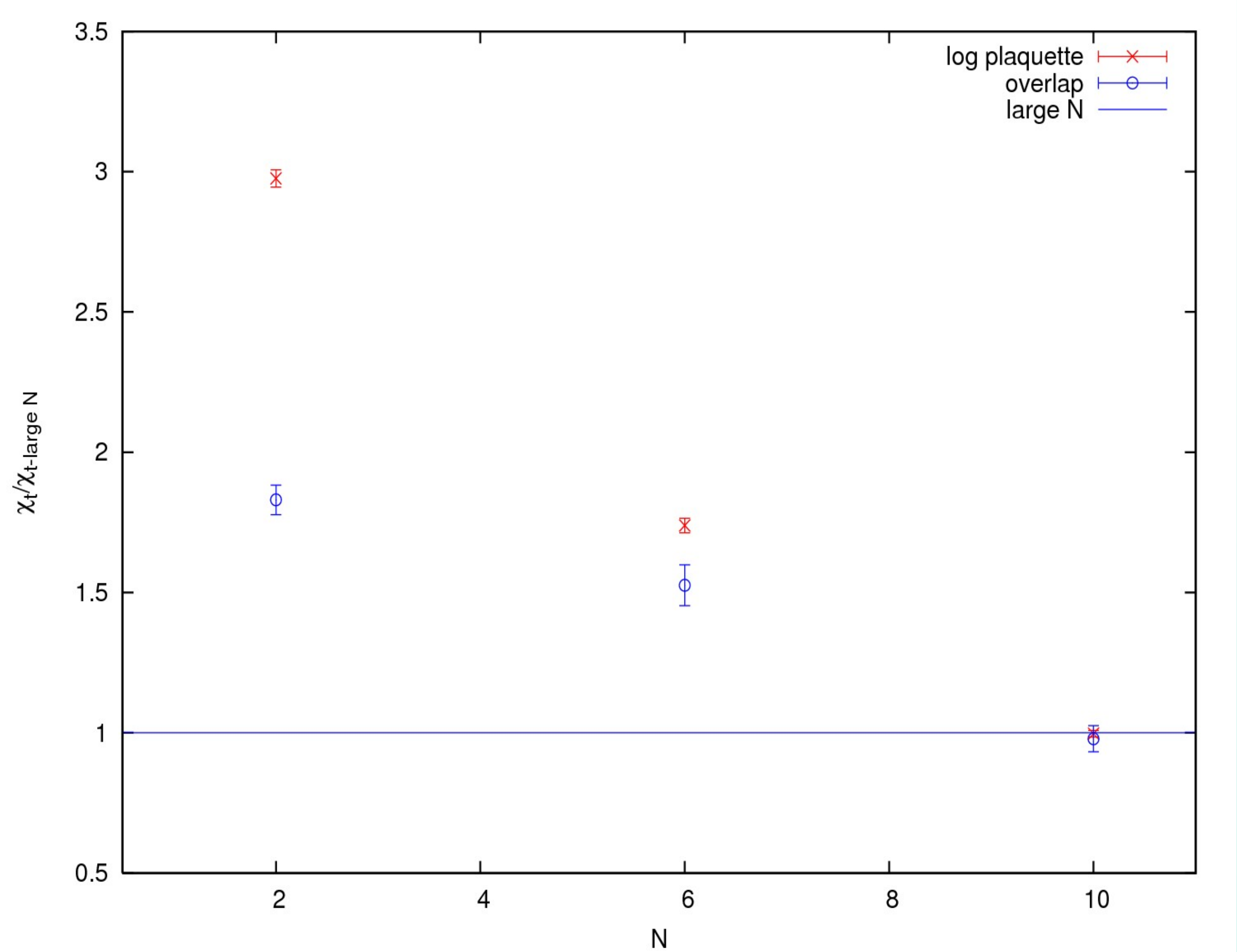
- Small N (N < 4) - dilute instanton gas

$$\varepsilon(\theta) - \varepsilon(0) = \chi_t(1 - \cos\theta)$$

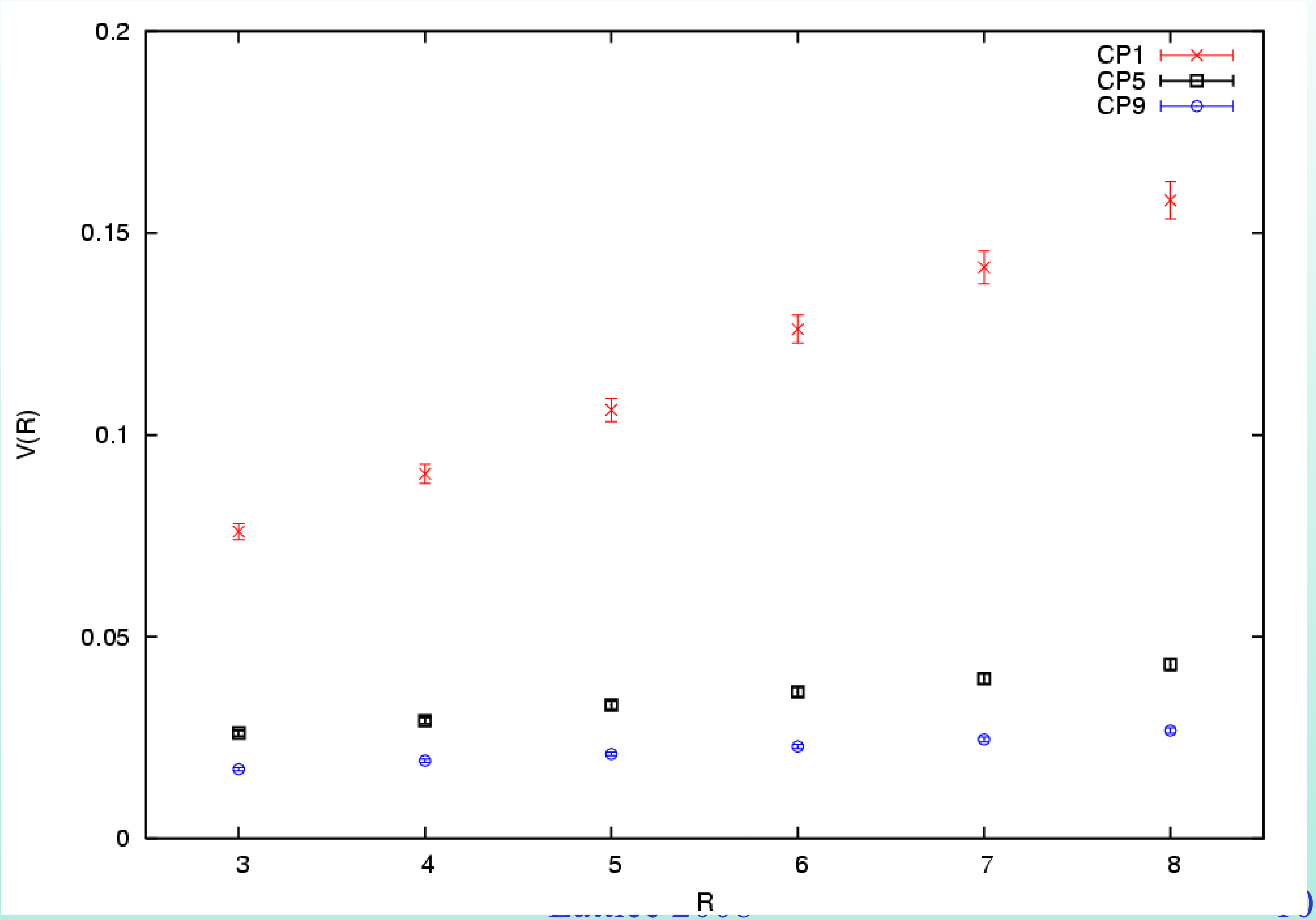
- Large N (N > 4) - instantons disappear, symmetry about $\theta = \pi$ requires string breaking

$$\varepsilon(\theta) - \varepsilon(0) = (1/2)\chi_t \min_{k \in Z}(\theta - 2\pi k)^2$$

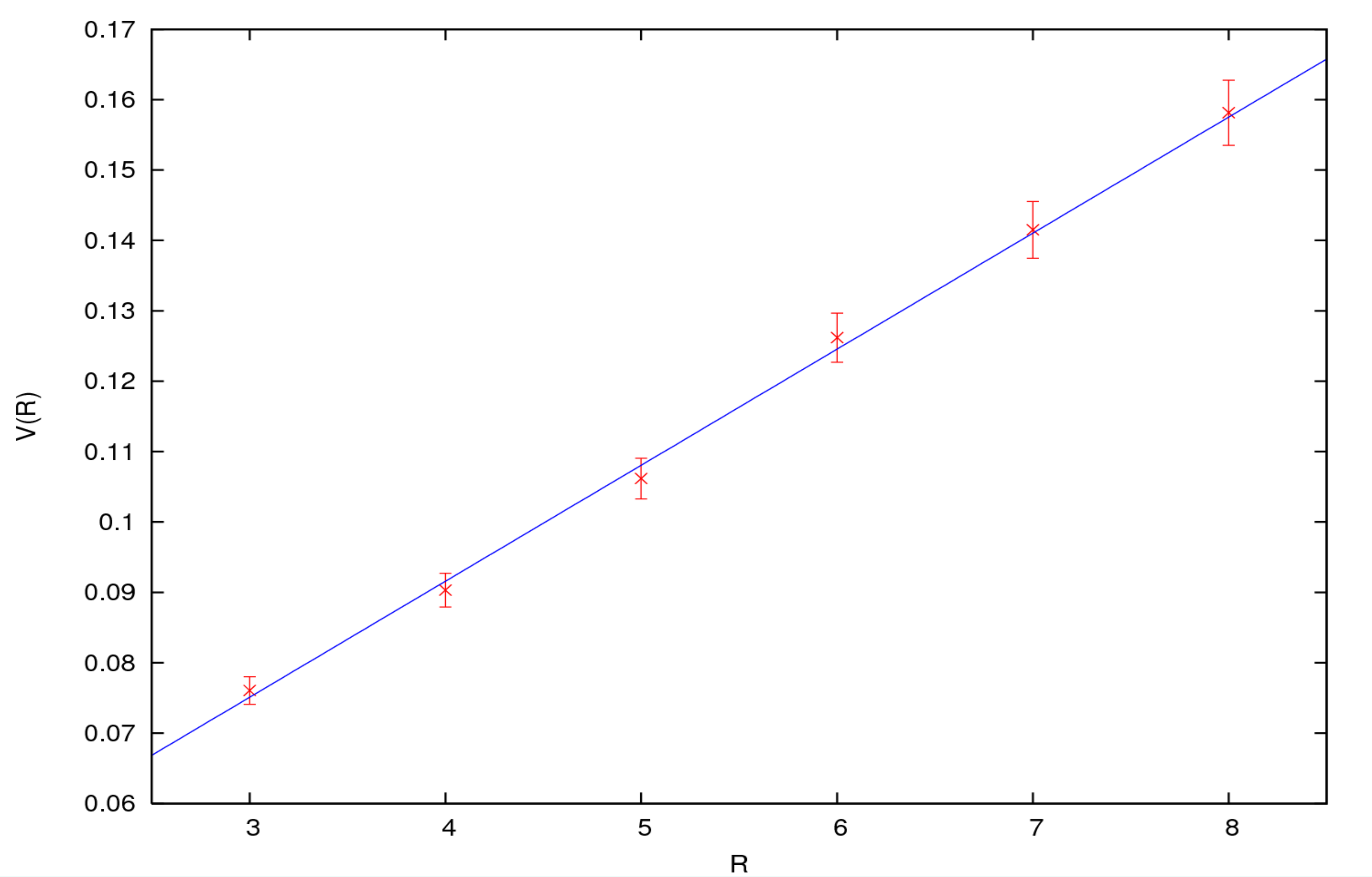
Measured value of χ_t compared to large-N prediction



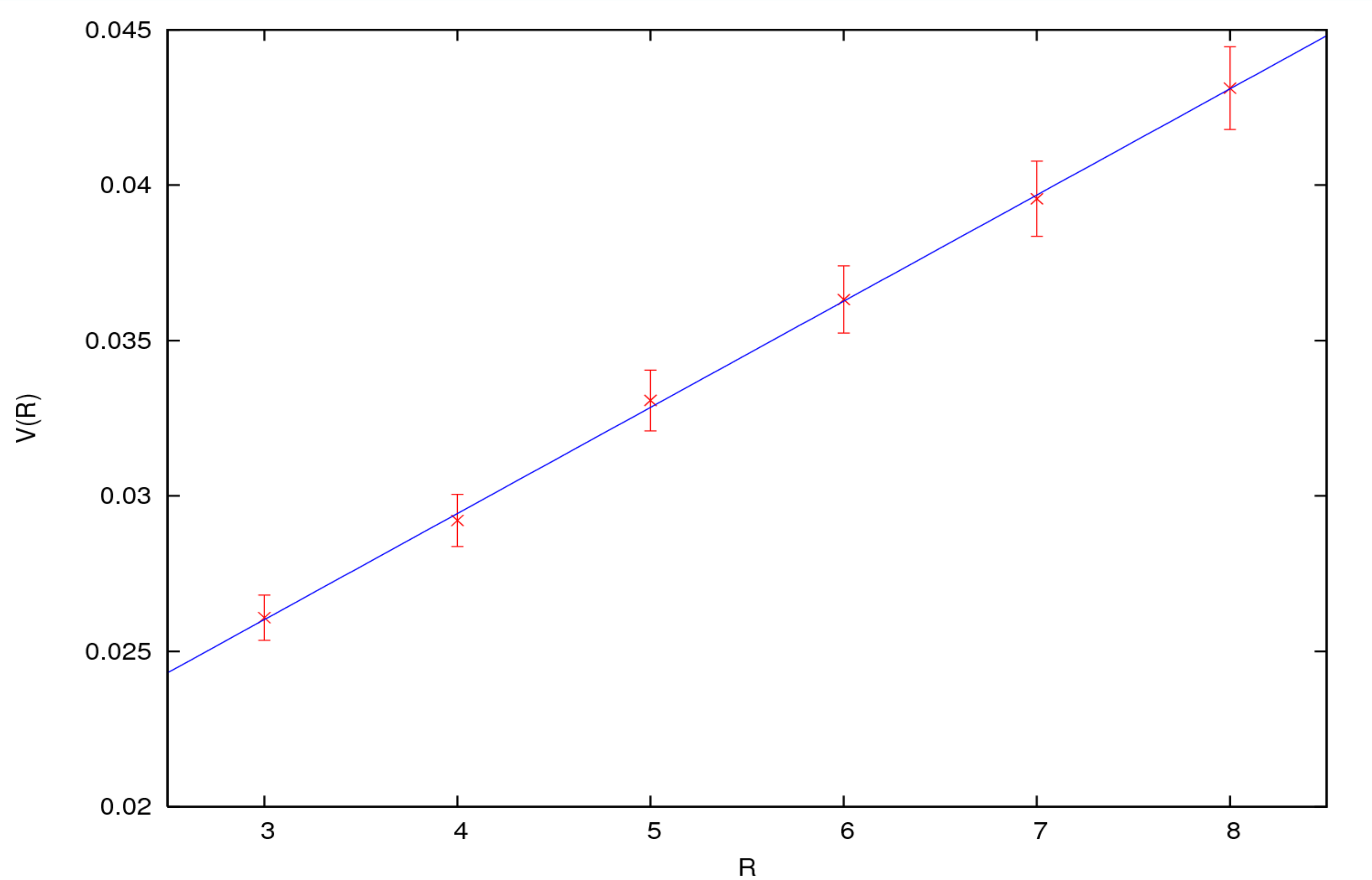
V(R) for CP^1, CP^5 and CP^9 for $R \in [3:8]$



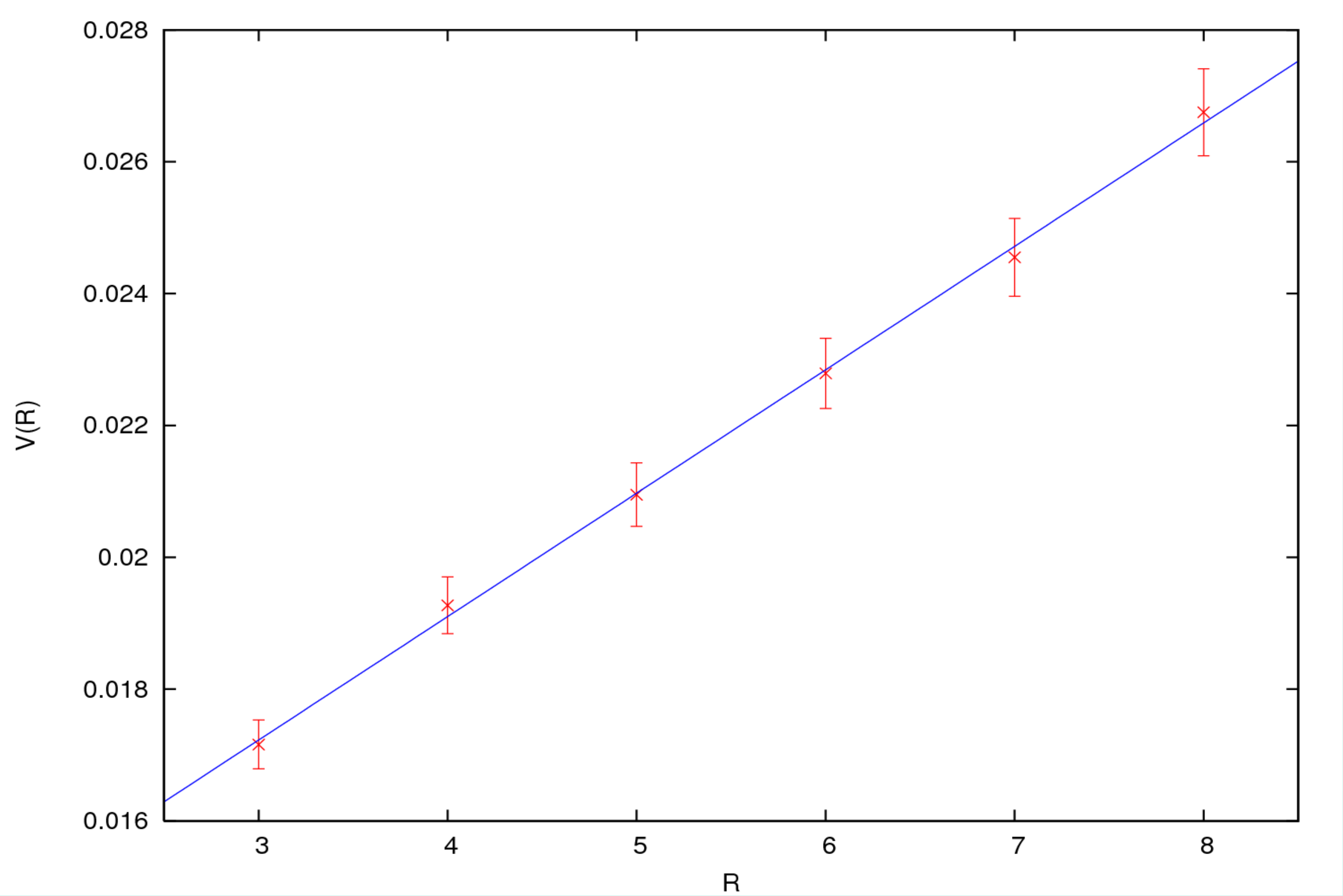
V(R) for CP^1 with q=0.3 for $R \in [3:8]$



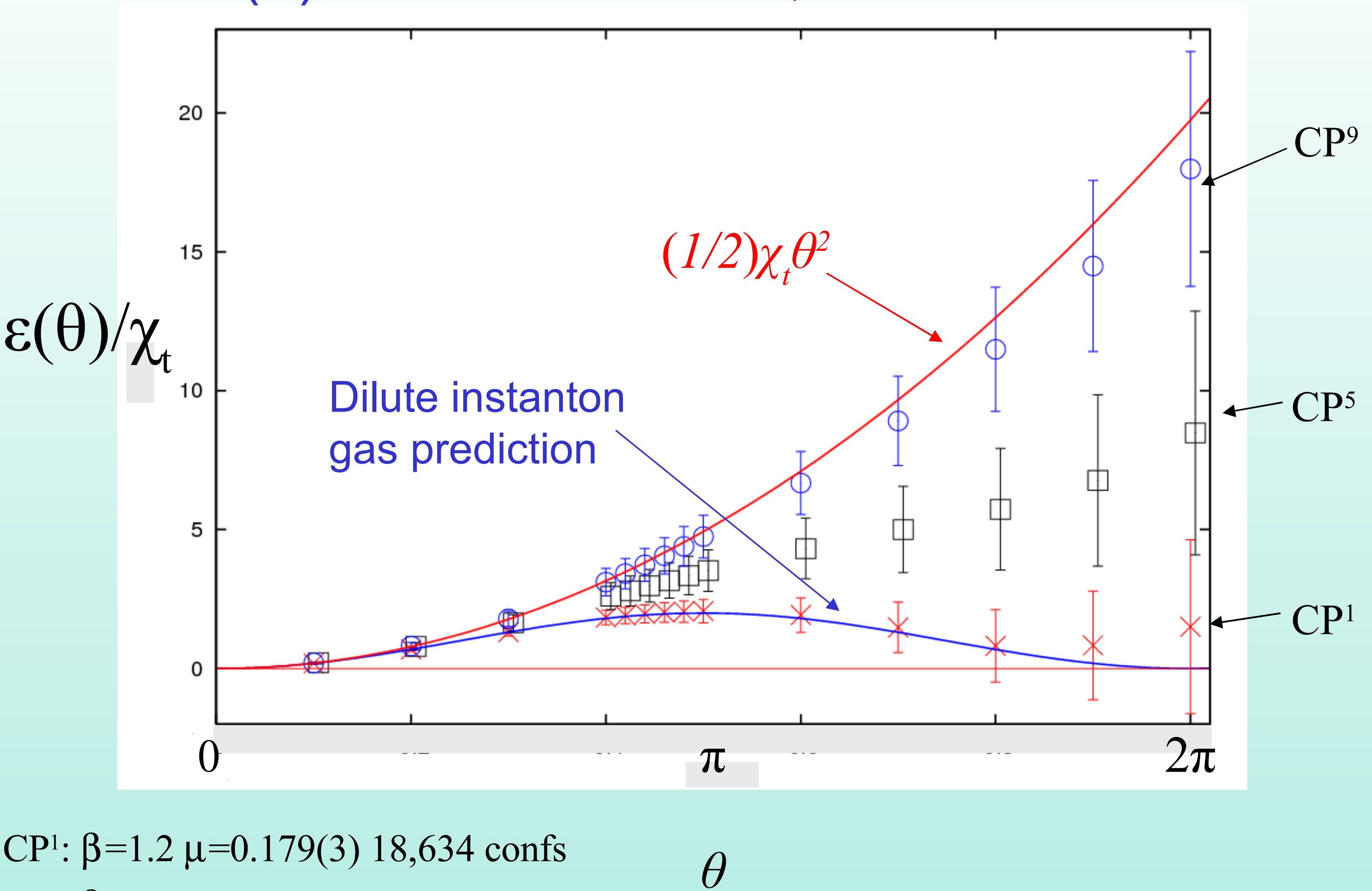
V(R) for CP^5 with q=0.3 for R∈[3:8]



V(R) for CP^9 with q=0.3 for R∈[3:8]



$\varepsilon(\theta)$ $0 \le \theta \le 2\pi$ for CP^1, CP^5 and CP^9

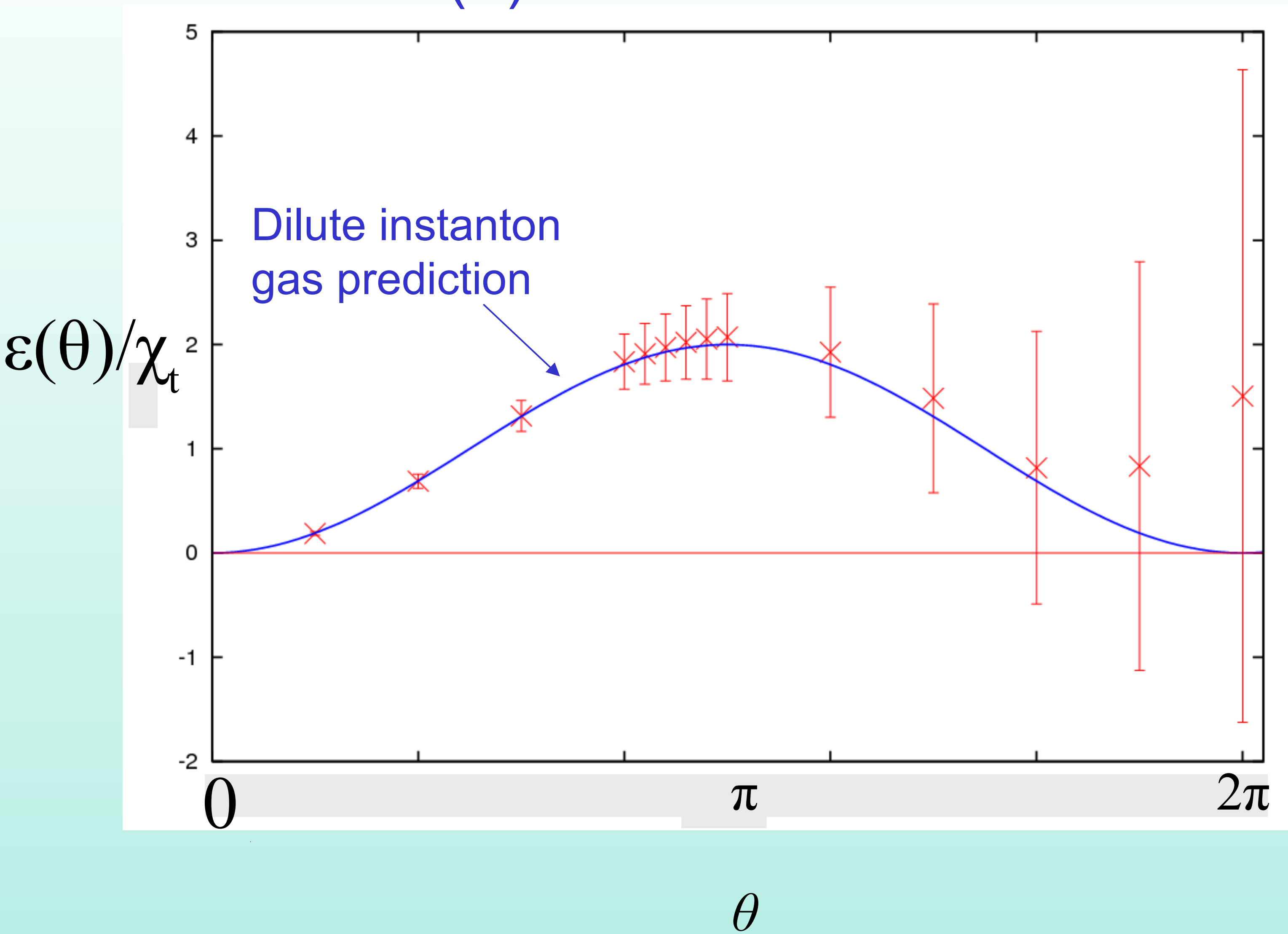


CP^1: β=1.2 μ=0.179(3) 18,634 confs

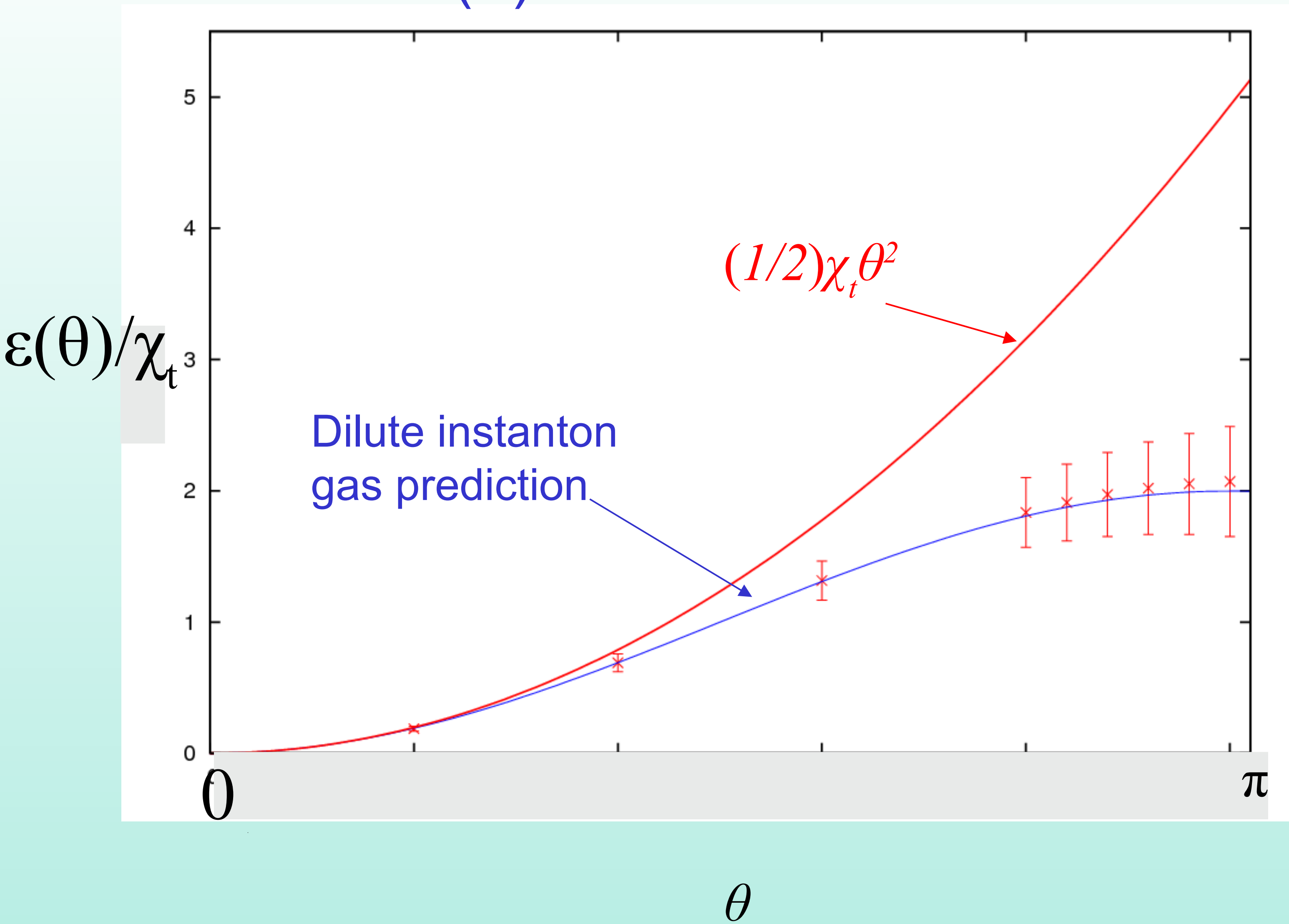
CP^5: β=0.9 μ=0.186(3) 9,312 confs

CP^9: β=0.8 μ=0.212(2) 15,059 confs

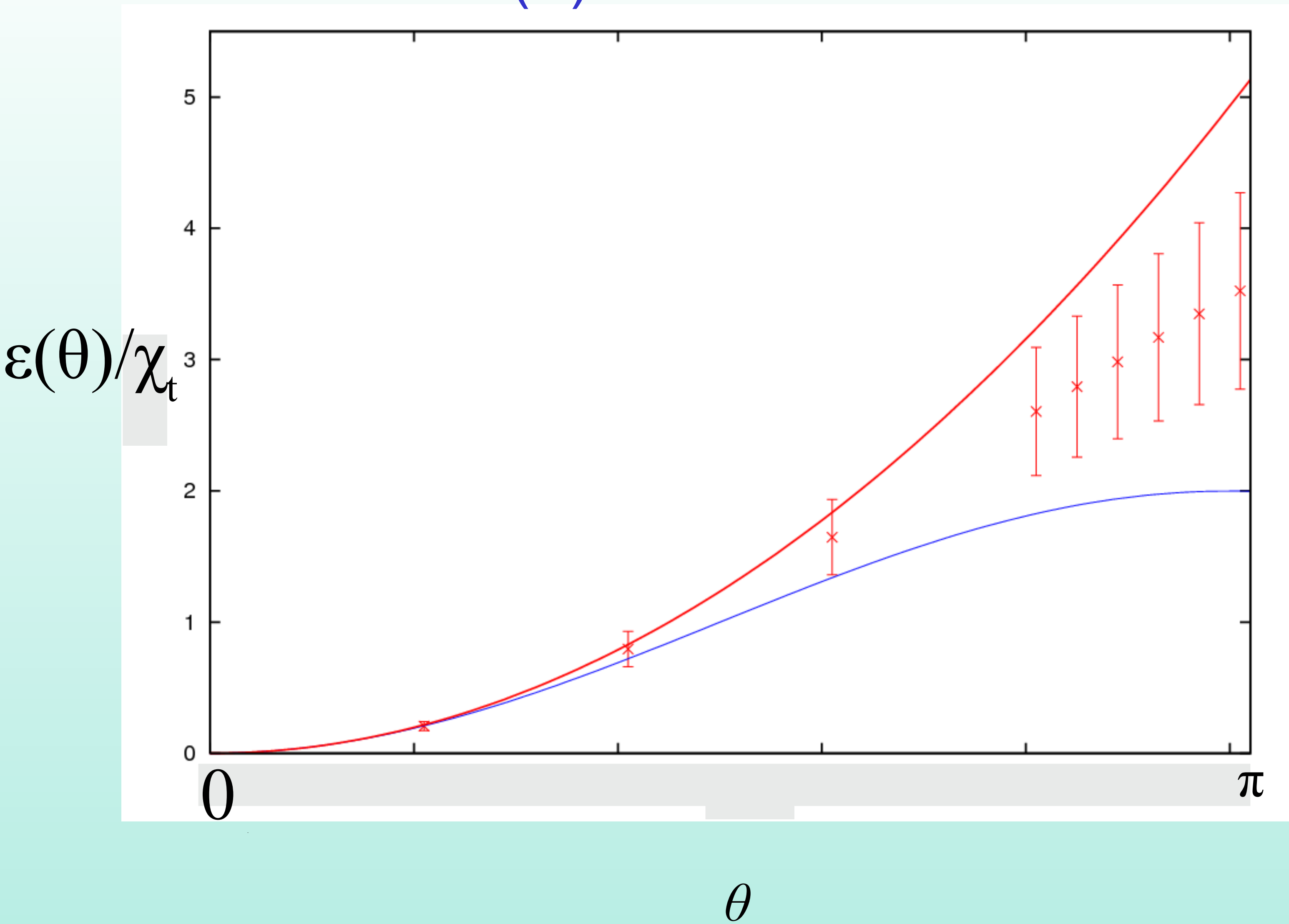
$\varepsilon(\theta)$ $0 \le \theta \le 2\pi$ for CP^1



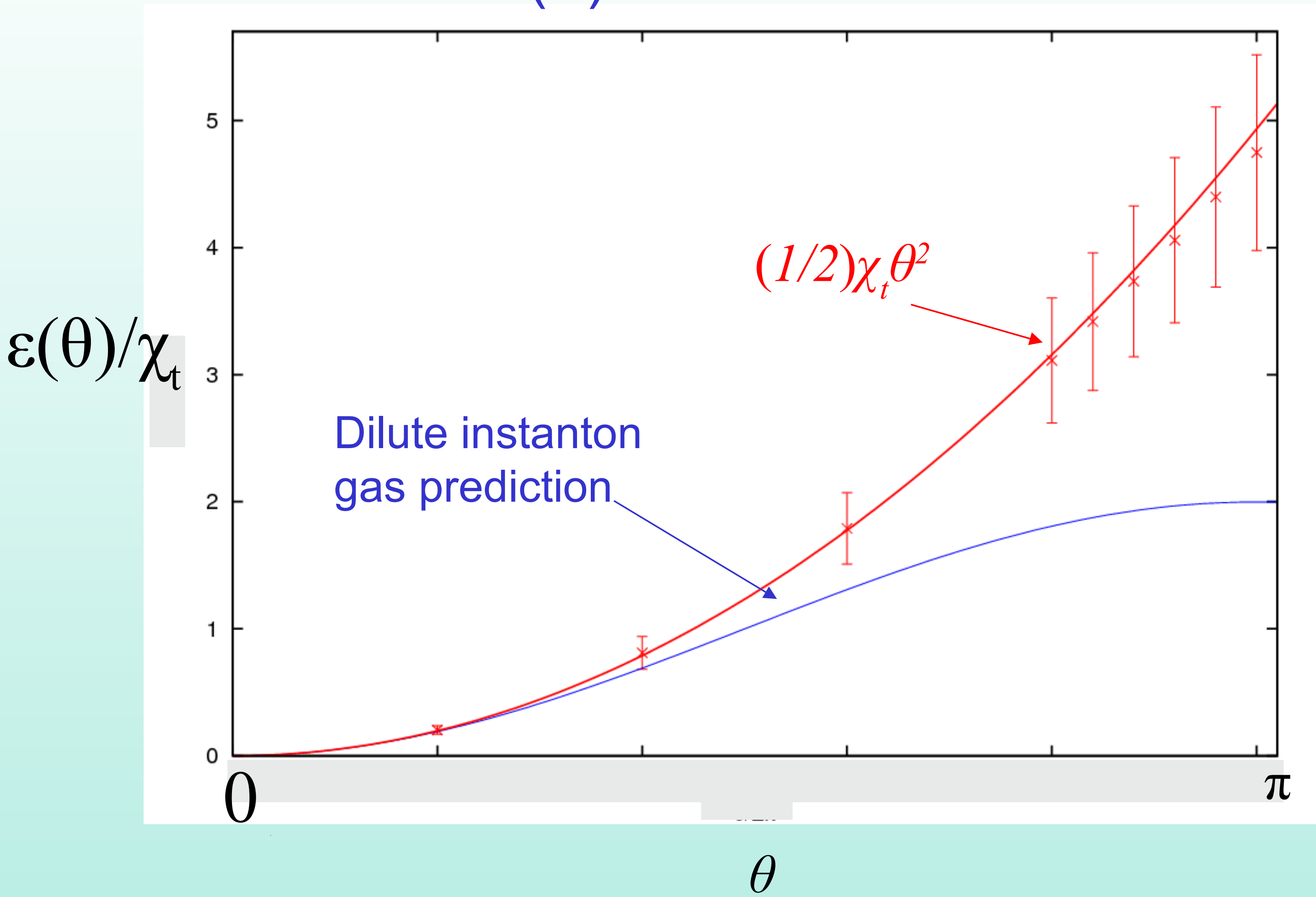
$\varepsilon(\theta)$ $0 \leq \theta \leq \pi$ for CP^1



$\varepsilon(\theta)$ $0 \leq \theta \leq \pi$ for CP^5



$\varepsilon(\theta)$ $0 \leq \theta \leq \pi$ for CP^9



Conclusions

- In $CP^{(N-1)}$ fractionally charged Wilson loops can serve as a useful probe of $\varepsilon(\theta)$ throughout the entire range $0 < \theta < 2\pi$
- For CP^1 $\varepsilon(\theta)$ is consistent with the dilute instanton gas approximation for $0 < \theta < 2\pi$
- For CP^9 $\varepsilon(\theta)$ closely follows the large N prediction for $0 < \theta < 2\pi$
- We see no evidence for string breaking in CP^5 or CP^9 in the loops we have studied (up to 10x10)