Non-perturbative quark mass dependence in the heavy-light sector of two-flavour QCD

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Motivation

$$\mathcal{L}_{\rm HQET}(x) = \overline{\psi}_{\rm h}(x)\left[\ \underbrace{D_0 + m}_{\text{static limit}} - \frac{\omega_{\rm kin}}{2m}\,\mathbf{D}^2 - \frac{\omega_{\rm spin}}{2m}\,\sigma\mathbf{B}\ \right]\psi_{\rm h}(x) + \ldots\,,$$

m : heavy quark mass

- systematic expansion in $1/m$, accurate for $m \gg \Lambda_{\rm QCD}$, renormalizable & has a continuum limit
- matching $\{m, \omega_{\rm spin}, \cdots\} \Leftrightarrow \{\text{QCD parameters}\}$ required to make HQET an effective theory of QCD
- consider HQET as expansion of QCD in $1/z \equiv 1/(LM)$ and verify that its large-z behaviour complies with HQET
- tests may justify interpolations between the charm region (slightly above of it) and the static limit to the b-scale also in large-volume physics applications, e.g. to determine $F_{\rm B}$ [Alpha:JHEP02(2008)078]
- comparison to tests of quenched QCD [Heitger etal:JHEP11(2004)048]

Requirements

Finetuning

- **line of constant physics**; within our strategy to do a NP matching between QCD and HQET, we are working at

$$\bar{g}^2(L_1) \approx 4.484 \qquad L_1 m_\mathrm{l} \approx 0 \qquad z \equiv L_1 M \approx \textbf{const}$$

M : renormalization group invariant heavy quark mass

- *connection between bare & renormalized parameters of the theory* $\rightsquigarrow$ knowledge of improvement coeff. and renormalization constants crucial to invert

$$z(\kappa_\mathrm{h}) \equiv L_1 M = L_1 Z_\mathrm{M} \widetilde{m}_\mathrm{q,h} = L_1 Z_\mathrm{M} m_\mathrm{q,h}(1 + b_\mathrm{m} a m_\mathrm{q,h})$$

with $am_\mathrm{q,h} = (\kappa_\mathrm{h}^{-1} - \kappa_\mathrm{c}^{-1})/2$, $L_1/a \in \{20, 24, 32, 40\}$.
Quadratic equation with solutions

$$\kappa_\mathrm{h}(z) = \left[\frac{1}{\kappa_\mathrm{c}} - \frac{1}{b_\mathrm{m}}\left(1 - \sqrt{1 + z \cdot \frac{4b_\mathrm{m}}{(L_1/a)Z_\mathrm{M}}}\right)\right]^{-1}$$

restricts $z(L_1) < -(L_1/a)Z_\mathrm{M}/(4b_\mathrm{m})$.

Simulation parameters

- b_{m} and Z_{M} non-perturbatively computed by the Alpha-collaboration [Della Morte etal:PoS(LATTICE 2007)246]
  $\Rightarrow z < 22$ possible for $L_1 = 24, 32, 40$ and $z < 17$ for $L_1 = 20$
- we choose $\mathbf{z} \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$ to cover a wide range of masses $\leftrightarrow$ $M \sim (1.5, \ldots, 8.3)\mathrm{GeV}$
  (reference scale $L^* \approx 0.6\mathrm{fm}$ from [Alpha:JHEP07(2008)037])

L_1/a	β	κ_{c}	$L_1 m_{\mathrm{l}}$
20	6.1906	0.135997290	$+0.00055(13)$
24	6.3158	0.135772110	$-0.000145(66)$
32	6.5113	0.135421494	$+0.000143(36)$
40	6.6380	0.135192285	$+0.000024(24)$

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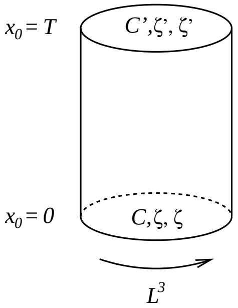
Framework

The Schrödinger functional as finite renorm. scheme

- *periodic* b.c. in space and Dirichlet in time
- fermion fields periodic in space up to a phase

$$\psi(x+\hat{k}L) = \mathrm{e}^{i\theta}\psi(x)$$
$$\overline{\psi}(x+\hat{k}L) = \mathrm{e}^{-i\theta}\overline{\psi}(x)$$

here we mainly use $\theta \in \{0, 0.5, 1\}$, where 0.5 is the angle in our dynamical configurations generated on apeNEXT @ DESY-Zeuthen



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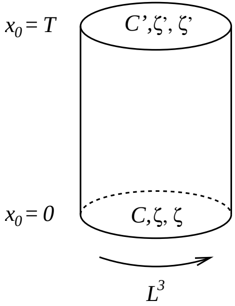
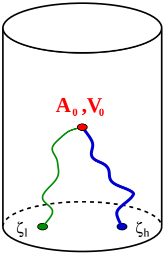


- multiplicative renormalization scheme where the kinematical parameters L, $T/L, \theta$ fixes the renormalization prescription
- mass independent renormalization scheme
- $N_\mathrm{f} = 2$ degenerate massless sea quarks ($m_\mathrm{l} \equiv m_\mathrm{light} = 0$)
- correlation functions are build from heavy-light valence quarks; light quark mass is set to the sea-quark mass (~ 0)

Finite volume observables

SF correlation functions ...



Boundary-to-bulk:

$$f_{\rm A}(x_0,\theta) = -\frac{a^6}{2L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z}} \left\langle \overline{\psi}_{\rm l}(x)\gamma_0\gamma_5\psi_{\rm h}(x)\,\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z})\right\rangle$$

$$k_{\rm V}(x_0,\theta) = -\frac{a^6}{6L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z},k} \left\langle \overline{\psi}_{\rm l}(x)\gamma_k\psi_{\rm h}(x)\,\zeta_{\rm h}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z})\right\rangle$$

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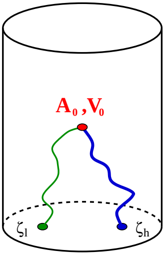

Boundary-to-boundary:

$$f_1(\theta) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}}\left\langle \overline{\zeta}_{\rm l}'(\mathbf{u})\gamma_5\zeta_{\rm h}'(\mathbf{v})\,\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z})\right\rangle$$

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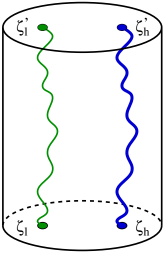

and additionally $f_{\rm P}$, $k_{\rm T}$ to improve $f_{\rm A}$, $k_{\rm V}$ respectively

Finite volume observables

... and derived quantities . . .

- provided that A_μ, V_μ denote *renormalized* currents,

$$Y_{\mathrm{PS}}(L,M) \equiv +\frac{f_{\mathrm{A}}(T/2)}{\sqrt{f_1}}\,, \qquad Y_{\mathrm{V}}(L,M) \equiv -\frac{k_{\mathrm{V}}(T/2)}{\sqrt{k_1}}\,,$$

$$R_{\mathrm{PS/V}}(L,M) \equiv -\frac{f_{\mathrm{A}}(T/2)}{k_{\mathrm{V}}(T/2)}\,, \quad R_{\mathrm{PS/P}}(L,M) \equiv -\frac{f_{\mathrm{A}}(T/2)}{f_{\mathrm{P}}(T/2)}\,,$$

  are finite quantities

- in the $O(a)$ improved lattice theory this amounts to replace e.g.

$$A_\mu \longrightarrow Z_{\mathrm{A}}[1+\frac{1}{2}b_{\mathrm{A}}(am_{\mathrm{q,l}}+am_{\mathrm{q,h}})] \times A_\mu$$

Finite volume observables

... and derived quantities

- for the same purpose **effective energies** are defined by

$$\Gamma_{\rm PS}(L,M) \equiv -\frac{\rm d}{{\rm d}x_0} \ln\left[\, f_{\rm A}(x_0)\,\right]\Big|_{x_0=T/2} = -\frac{f'_{\rm A}(T/2)}{f_{\rm A}(T/2)}\,,$$

$$\Gamma_{\rm V}(L,M) \equiv -\frac{\rm d}{{\rm d}x_0} \ln\left[\, k_{\rm V}(x_0)\,\right]\Big|_{x_0=T/2} = -\frac{k'_{\rm V}(T/2)}{k_{\rm V}(T/2)}\,,$$

$$\Gamma_{\rm av}(L,M) \equiv \tfrac{1}{4}\left[\Gamma_{\rm PS}(L,M) + 3\Gamma_{\rm V}(L,M)\right]$$

$$R_{\rm spin}(L,M) \equiv \ln(f_1/k_1)$$

- meaning of the observables from their large-volume behaviour (up to normalizations)

$$L \to \infty \;:\quad Y_{\rm PS}, Y_{\rm V} \to F_{\rm PS}, F_{\rm V} \qquad \text{: heavy-light decay constant,}$$

$$R_{\rm spin} \to m_{{\rm B}_0^*} - m_{{\rm B}_0} \qquad \text{: mass splitting}$$

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Effective theory predictions

at the classical level:

- current matrix elements expected to posses a power series expansion in $1/z \equiv 1/(LM)$
- leading term in expansion of CFs by replacing $\psi_\mathrm{b} \to \psi_\mathrm{h}$ & dropping O(1/m) terms $\rightsquigarrow$ static limit

$$f_\mathrm{A} \to f_\mathrm{A}^\mathrm{stat} \qquad \frac{f_\mathrm{A}^\mathrm{stat}(T/2)}{\sqrt{f_1^\mathrm{stat}}} \equiv X(L) = \lim_{z\to\infty} Y_\mathrm{PS}(L,M)$$

$$= \lim_{z\to\infty} Y_\mathrm{V}(L,M)$$

due to *heavy quark spin-symmetry* ($A_0^\mathrm{stat} \Leftrightarrow V_k^\mathrm{stat}$)

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Effective theory predictions

correspondence of HQET and QCD in quantum theory:

- scale dependent ren. of HQET implies logarithmic modifications

axial current renorm. $\qquad X_R(L) = Z_{\rm A}^{\rm stat}(\mu) X_{\rm bare}(L)$

depends logarithmically on the chosen renorm. scale μ

- no scheme dependence when going over to renormalization group invariants (RGI)

$$\lim_{\mu\to\infty} \left\{ [2b_0\bar{g}^2(\mu)]^{-\gamma_0/(2b_0)} X_R(L,\mu) \right\} = X_{\rm RGI} = Z_{\rm RGI} X_{\rm bare}(L)$$

$$\text{where} \qquad b_0 = \frac{11 - 2N_{\rm f}/3}{(4\pi)^2}\,, \qquad \gamma_0 = -\frac{1}{(4\pi)^2}\,,$$

are first order coeff.s of β and of the anomalous dimension of the axial current, respectively

- large-mass behaviour of the QCD observables:
(RGIs of the eff. theory)$\times$(logarithmically mass dependent functions C)

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Conversion to the matching scheme

translation to another renormalization scheme

Definition of the matching scheme: for arbitrary renormalized matrix elements Φ_{R} in QCD & the effective theory it should hold

$$\Phi_{\mathrm{R}}^{\mathrm{QCD}} = \Phi_{\mathrm{R}}^{\mathrm{HQET}}(\mu)\Big|_{\mu=m} + O(1/m)$$

- in perturbative QCD, m typically can either be the pole mass m_Q or the $\overline{\mathrm{MS}}$ mass $\overline{m}_*$

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example: static axial current; the conversion factor for $X_{\rm RGI}$ to Φ in this scheme is

$$\widehat{C}_{\rm PS}(\mu) = \left[2b_0\bar{g}^2(\mu)\right]^{\frac{\gamma_0}{2b_0}} \exp\left\{\int_0^{\bar{g}(\mu)} {\rm d}g \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}\Bigg|_{\mu=\overline{m}_*}$$

$$\mu\frac{\partial\Phi}{\partial\mu} = \gamma(g)\Phi\,, \qquad \gamma \equiv \gamma^{\rm match}$$: anomalous dim. in the matching scheme

$$\gamma(g) = \gamma^{\overline{\rm MS}}(g) + \rho(g)\,, \qquad \rho$$: matching of $\overline{\rm MS}$-renorm. HQET operators in QCD

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Matching coefficients $C_X(\Lambda_{\overline{MS}}/M)$

more convenient choice of the argument of the conversion functions $\widehat{C}_X$:

- change argument of $\widehat{C}_X$ to the ratio of RGIs, $M/\Lambda_{\overline{\rm MS}}$
  $\Rightarrow$ functions $C_X(M/\Lambda_{\overline{\rm MS}})$
- M = RGI quark mass, advantage: fixed in lattice calculations without perturbative uncertainties

one then expects the (heavy) quark mass dependence to obey

$$Y_X(L,M) \overset{M\to\infty}{\sim} C_{\rm X}\,(M/\Lambda_{\overline{\rm MS}})\; X_{\rm RGI}(L)\left(1+{\rm O}(1/z)\right), \qquad X = {\rm PS, V},\; z = ML\,,$$

$$R_{{\rm PS}/?}(L,M) \overset{M\to\infty}{\sim} C_{{\rm PS}/?}\,(M/\Lambda_{\overline{\rm MS}})\,[1]\left(1+{\rm O}(1/z)\right), \qquad ? = {\rm V, P},$$

$$R_{\rm spin}(L,M) \overset{M\to\infty}{\sim} C_{\rm spin}\,(M/\Lambda_{\overline{\rm MS}})\;\frac{X^{\rm spin}_{\rm RGI}(L)}{z}\left(1+{\rm O}(1/z)\right),$$

$$L\Gamma_{\rm av}(L,M) \overset{M\to\infty}{\sim} C_{\rm mass}\,(M/\Lambda_{\overline{\rm MS}}) \times z\; + {\rm O}(1)\,,$$

$$C_{\rm mass}\,(M/\Lambda_{\overline{\rm MS}}) \equiv \frac{m_Q}{M} = \frac{\overline{m}(\overline{m}_*)}{M}\,\frac{m_Q}{\overline{m}(\overline{m}_*)}$$

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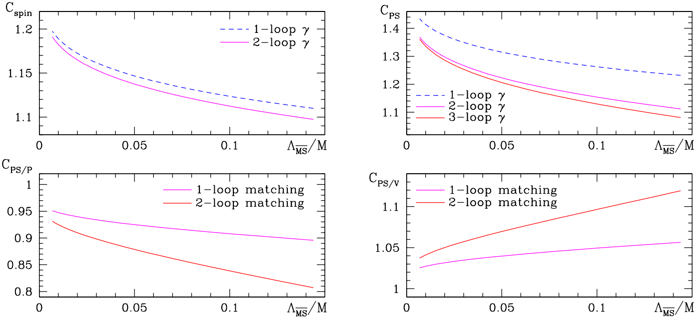
Matching coefficients $C_X(\Lambda_{\overline{MS}}/M)$

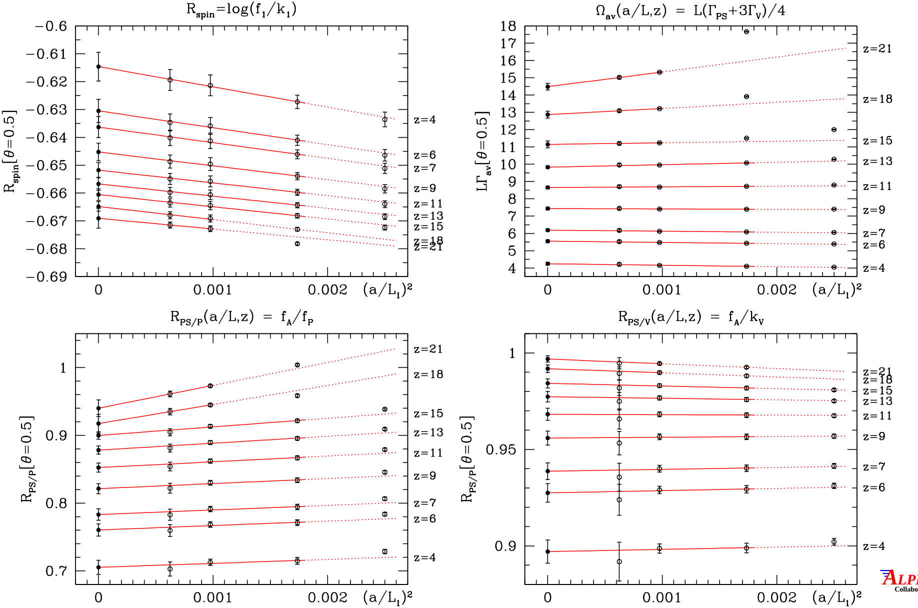
C_X : integrate perturbative RG equations (in the effective theory) in the matching scheme, using 4-loop $\beta(g)$, $\tau(g)$



- 3-loop $\gamma_2^{\overline{MS}}$ anomalous dimension (AD) from [Chetyrkin&Grozin,2003]
- C_{spin} constructed from the AD of $\overline{\psi}_h(x)\,\sigma\mathbf{B}\psi_h(x)$; 3-lp γ known

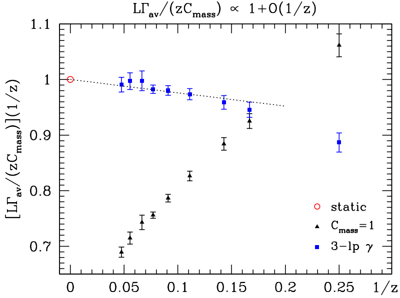
Results

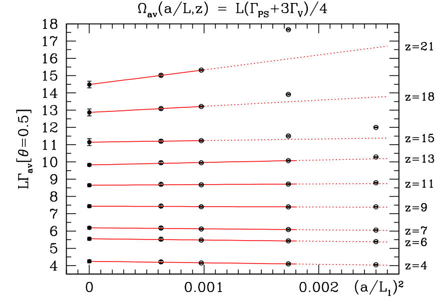
Continuum extrapolations . . .

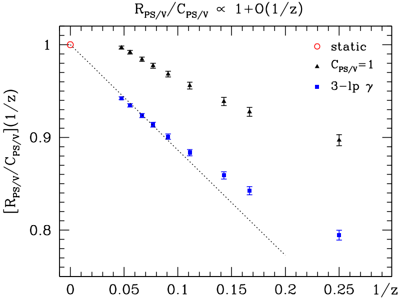


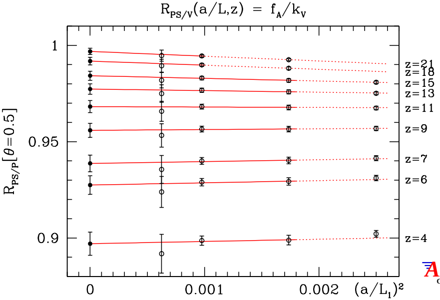
Results

Continuum extrapolations and asymtotics . . .









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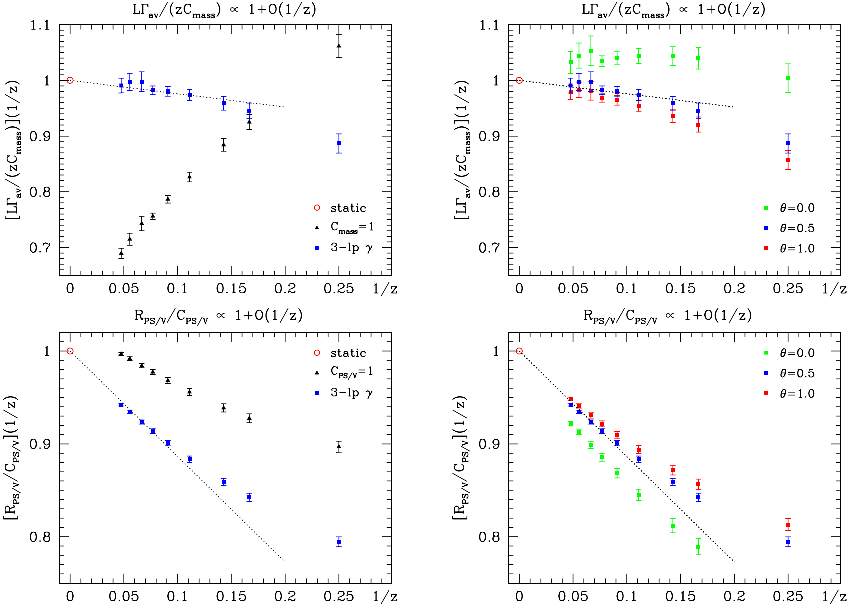
Results

Continuum extrapolations with asymptotics and universality



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Conclusions & perspectives

conclusions that can be drawn (maybe):

- nearly linear (1/z)-behaviour down to $1/z = 0.1 \leftrightarrow M \sim 4\mathrm{GeV}$ for all observables investigated so far
- $(1/z)^2$ corrections in spin splitting very small over the whole range of z covered
- slope in continuum extrapolations nearly equal for all z's in each observable seperately
- overall behaviour similar to quenched $\rightsquigarrow$ NP matching of QCD and HQET should also be as well behaved as in the quenched case

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what still need to be done:

- continuum limit and $1/z$-dependence of *heavy-light decay constant* (needs additional computations in HQET)
- *correlated fits* for a reliable error estimate – all z's at constant L computed on the same gauge background
- *3-loop γ^{spin} is available* $\rightsquigarrow$ implement it

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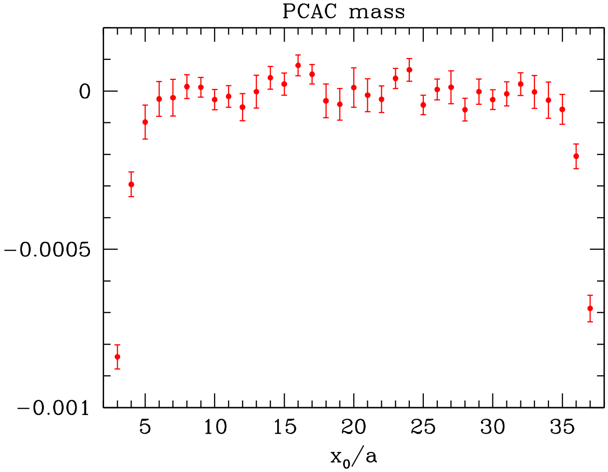
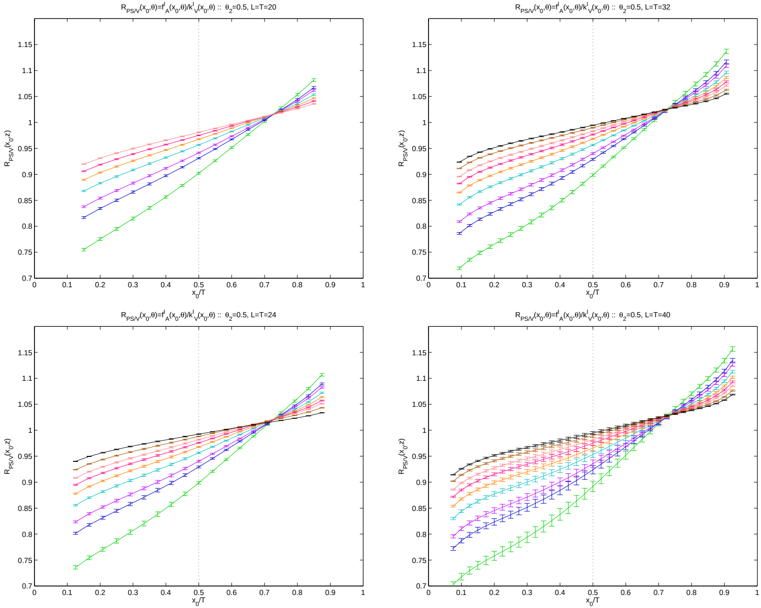
Thank you for your attention.

PCAC mass in the SF

at $L/a = 40$



recent observation in $R_{\mathrm{PS/V}}(x_0, z)$



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