Chiral perturbation theory, $K \rightarrow \pi\pi$ decays and 2+1 flavor, domain wall QCD.

Lattice 2008

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for the RBC and UKQCD Collaborations

Outline

- $K \rightarrow \pi\,\pi$ from $K \rightarrow \pi$ and $K \rightarrow |0\rangle$
- Ensembles: 2+1 DWF
- Lattice matrix elements
- The chiral limit: *LEC's*
- Extrapolating to m_K = 495 MeV

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2+1 Flavor partially quenched chiral perturbation theory

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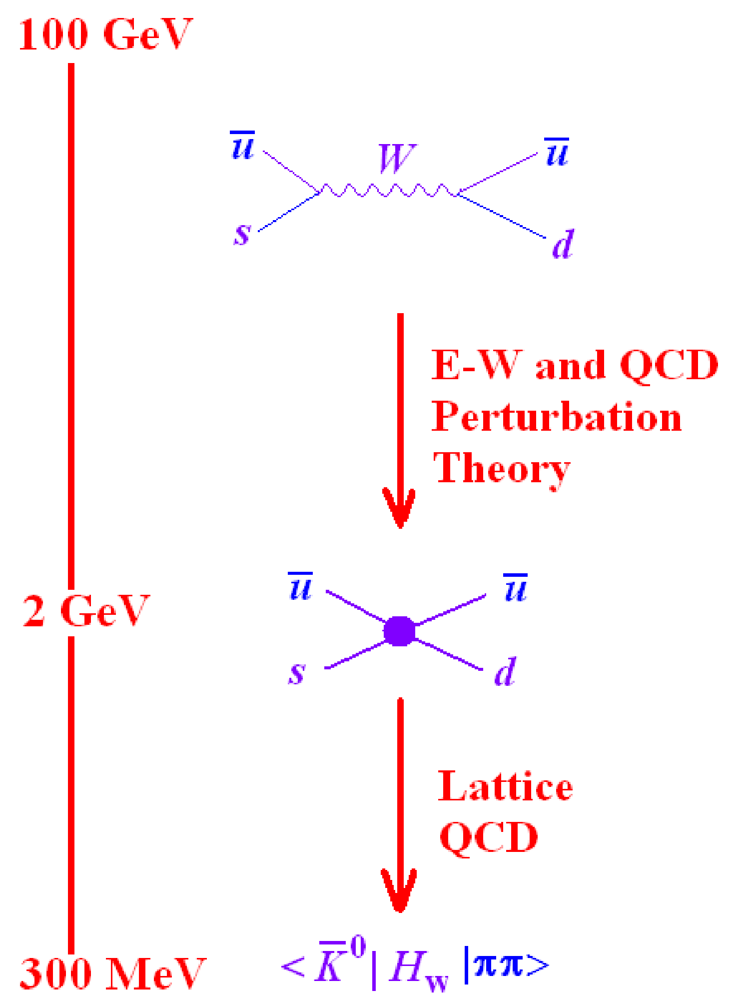
Physics Background

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{ts}^*}\frac{V_{us}^*}{V_{ud}} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Four quark operators

- **Current-current operators**

$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**

$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**

$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Chiral Perturbation Theory

- **Describe low energy QCD as an $SU(3)_L \times SU(3)_R$ covariant theory of *π*'s and *K*'s.**
- **In LO PQChPT the operators $Q_1 - Q_{10}$ can be expressed in terms of the four operators:**
- **In LO, all matrix elements of $Q_1 - Q_{10}$ are described by 8 LEC's:**

$$\mathcal{O}_{LO}^{(8,8)} = \mathrm{str}\left[\lambda_6 \Sigma Q \Sigma^\dagger\right]$$

$$\mathcal{O}_{LO,1}^{(8,1)} = \mathrm{str}\left[\lambda_6 \partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right]$$

$$\mathcal{O}_{LO,2}^{(8,1)} = 2B_0 \mathrm{str}\left[\lambda_6 \left(\Sigma \mathcal{M} + \mathcal{M}^\dagger \Sigma^\dagger\right)\right]$$

$$\mathcal{O}_{LO}^{(27,1)} = t_{kl}^{ij} \left(\Sigma \partial_\mu \Sigma^\dagger\right)_i^k \left(\Sigma \partial^\mu \Sigma^\dagger\right)_j^l$$

$\Delta I = 3/2$:

α_{27}, α_{88} α_{88m}

$\Delta I = 1/2$:

$\alpha_{3\bar{3}}$, α_{81A}, α_{81S}, $\alpha_{81\text{-}5}$, $\alpha_{81\text{-}6}$

Chiral Perturbation Theory (con't)

- At LO the needed LEC's can be determined from

  $< K \mid Q_i \mid 0 >$ and $< K \mid Q_i \mid \pi >$
- Avoids dealing with $\mid \pi \pi >$ final states
- Method of Bernard, *et al.*, Phys. Rev. D32, 2343 (1985).
- Present work is an extension of the RBC quenched calculation: Blum, *et al.*, Phys.Rev.D68:114506 (2003).
- Exploit both $m_{val} = m_{sea}$ and $m_{val} \neq m_{sea}$:
  - Partially quenched ChPT
  - Simplify penguin operators using only partially quenched singlets.

Matrix elements

Matrix Elements

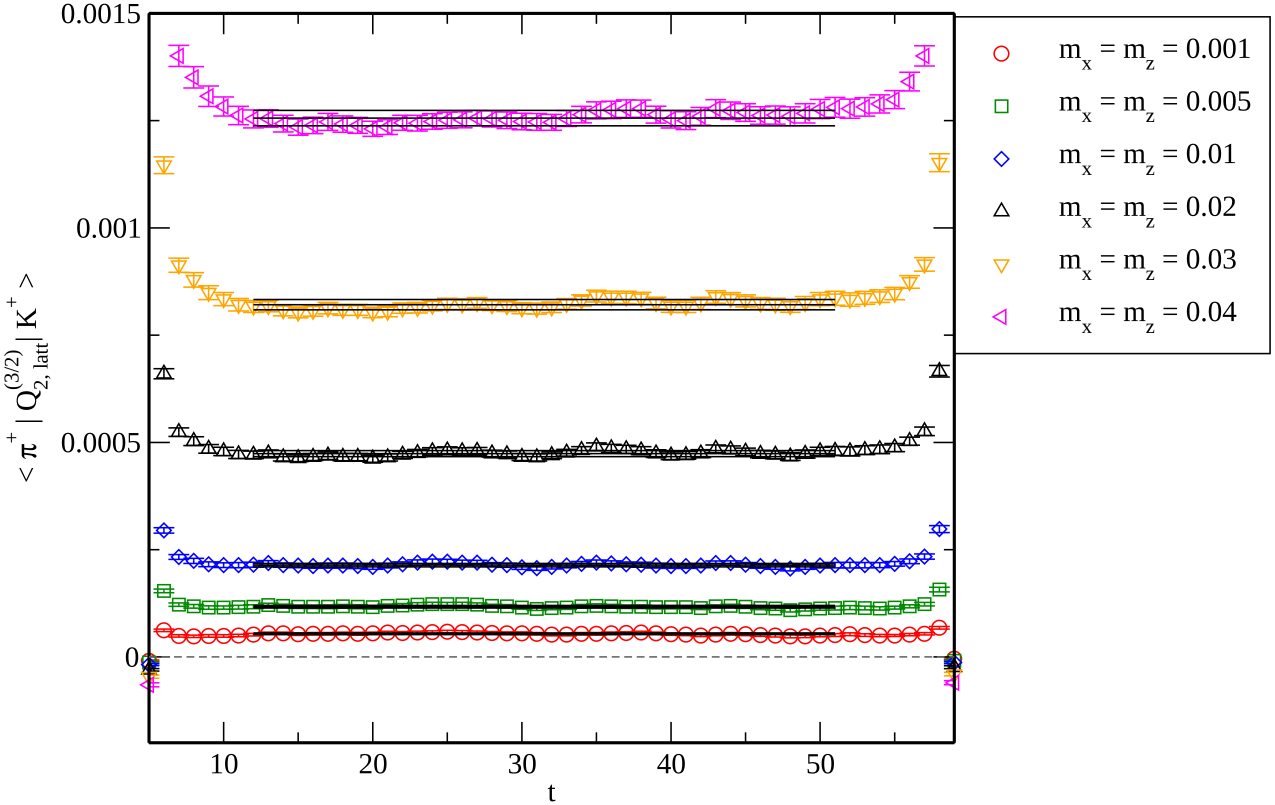
- Use 24^3 x 64, RBC/UKQCD 2+1 flavor configurations:
  - ml = 0.005 (m_π = 331 MeV) 76 configs, 80 mdt separation
  - ml = 0.01 (m_π = 419 MeV) 74 configs, 80 mdt separation
- Use 0.001, 0.005, 0.01, 0.02, 0.03 and 0.04 valence quark masses giving pion masses (MeV):

	0.001	0.005	0.01	0.02	0.03	0.04
0.001	241	294	349	438	512	576
0.005	294	**338**	387	469	539	600
0.01	349	387	**430**	505	570	629
0.02	438	469	505	570	629	682
0.03	512	539	570	629	682	732
0.04	576	600	629	682	732	779

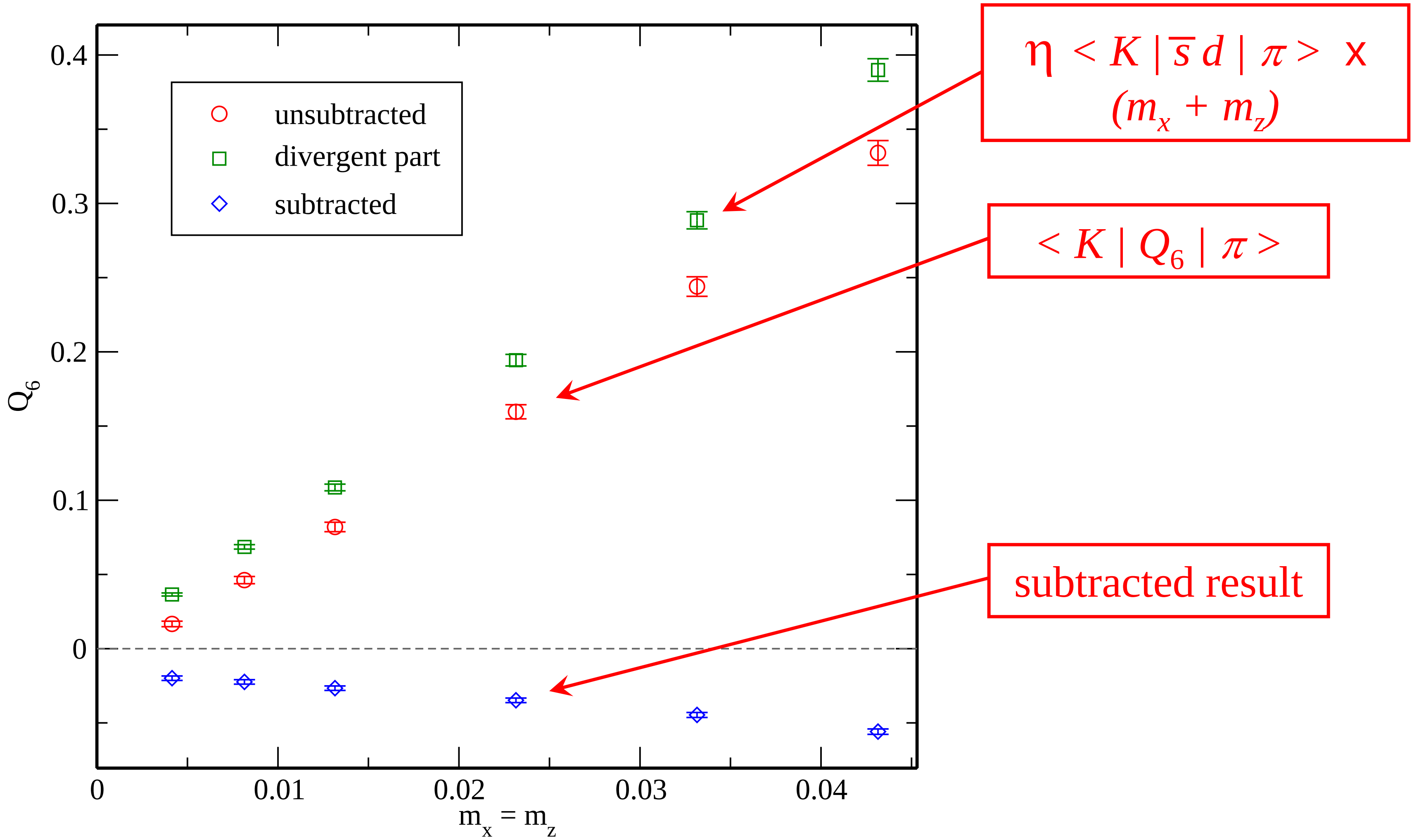
- Use strange quark mass m_s = 0.04 (15% too large)
- Residual mass m_{res} = 0.00315.

Example Q_2 matrix element

$m_{sea} = 0.005 \qquad m_x = m_z$



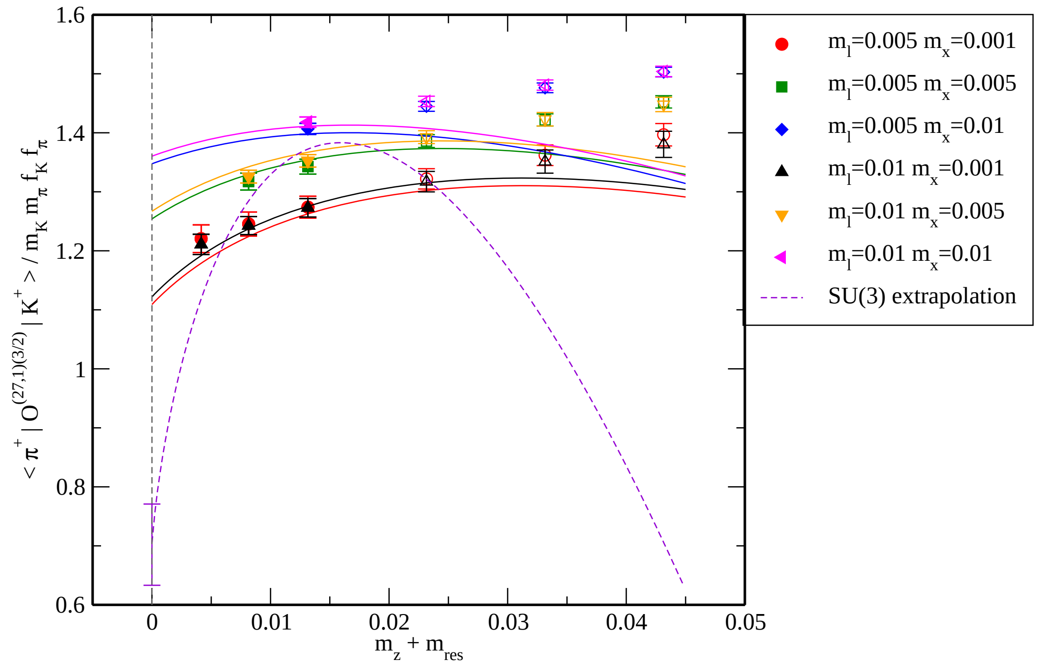
Subtraction for Q_6 Matrix Element



Chiral Extrapolation

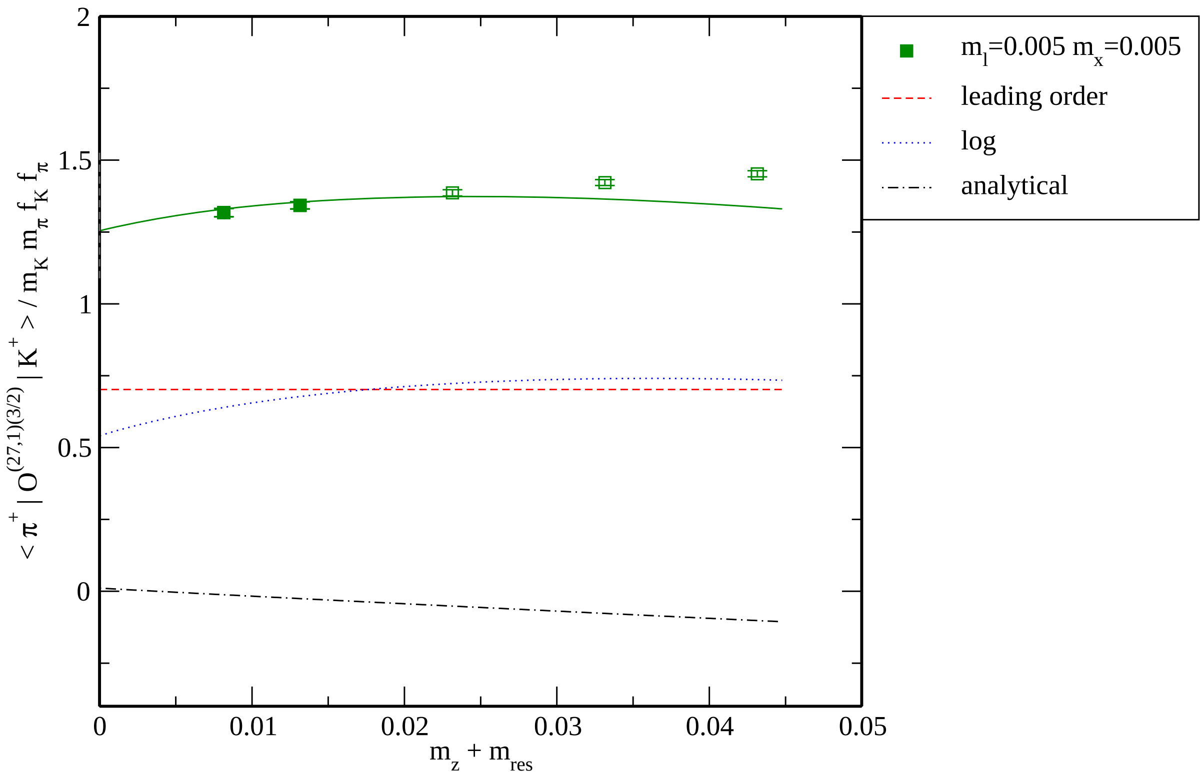
Determination of α_{27}

- Fit to points with $(m_{val} + m_{res})_{avg} \leq 0.013$
- PQChPT describes this data
- Large, ~50% correction!?
- Same large ChPT corrections as RBC/UQKCD, arXiv:0804.0473 (see talks of Enno Scholz and Chris Kelly)
- Fit does not work without $m_K m_\pi f_K f_\pi$ division.



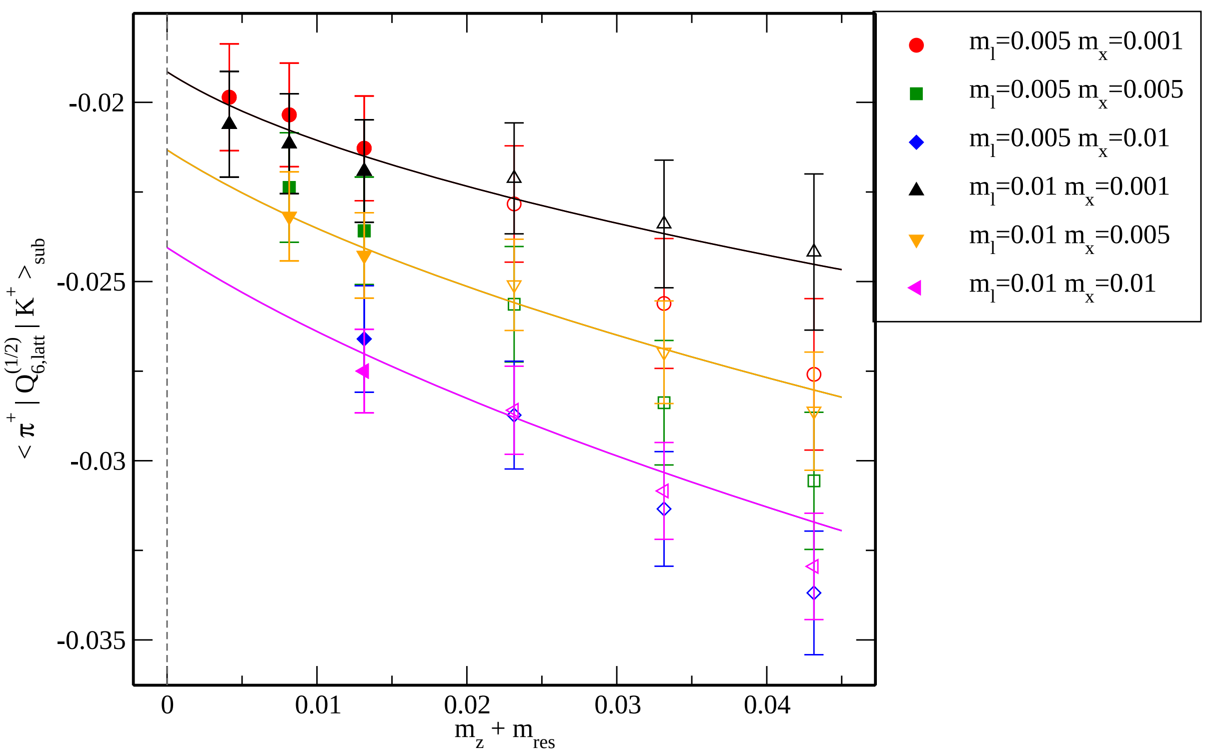
Relative size of LO and NLO terms

- LO and NLO log terms are the same size.
- Consistent results if we divide by $m_K m_\pi (f_K f_\pi)^2$
- Double the difference between two fits to estimate systematic error.



Determination of α_6

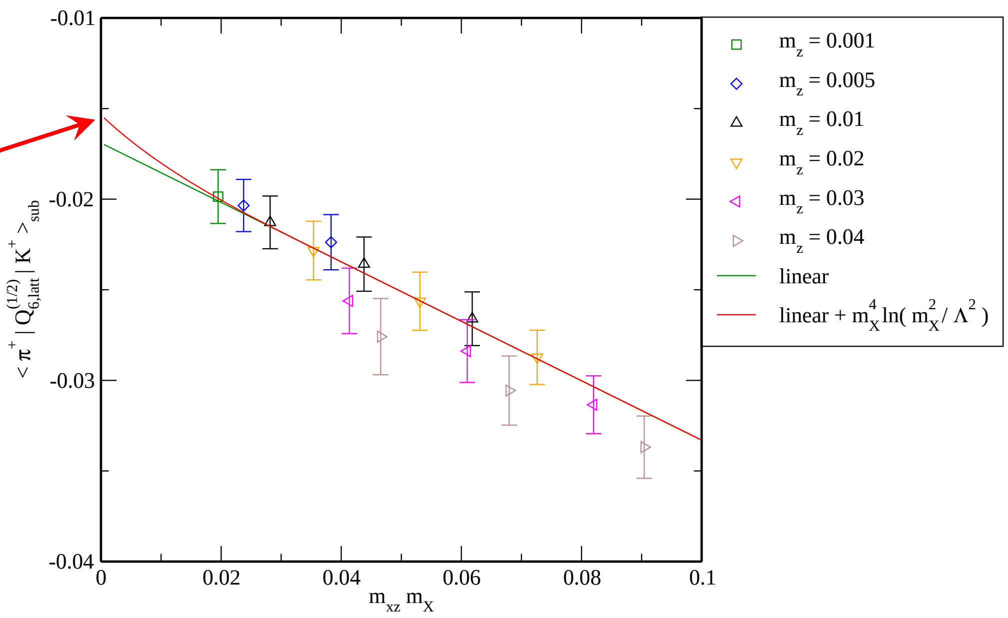
- NLO fit not possible, insufficient data to determine 8 LEC's.
- LO fit works well for large mass range.
- Omitted NLO logs are important!



Effect of NLO logs on α_6

- Chose $m_{max} = 0.005$.
- Use linear fit for $m_{max} \leq m$
- Use chiral log for $m \leq m_{max}$
- Match value, slope and curvature at $m = m_{max}$

Slope doubled by including NLO chiral log



Results for LEC's

Q_i	$\alpha^{(1/2)}_{i,\mathrm{ren}}$	$\alpha^{(3/2)}_{i,\mathrm{ren}}$
1	$-6.6(15)(66)\times 10^{-5}$	$-2.48(24)(39)\times 10^{-6}$
2	$9.9(21)(99)\times 10^{-5}$	$-2.47(24)(39)\times 10^{-6}$
3	$-0.8(31)(21)\times 10^{-5}$	**0.0**
4	$1.62(44)(162)\times 10^{-4}$	**0.0**
5	$-1.52(29)(152)\times 10^{-4}$	**0.0**
6	$-4.1(7)(41)\times 10^{-4}$	**0.0**
7	$-1.11(17)(18)\times 10^{-5}$	$-5.53(85)(91)\times 10^{-6}$
8	$-4.92(72)(75)\times 10^{-5}$	$-2.46(37)(37)\times 10^{-5}$
9	$-9.8(20)(98)\times 10^{-5}$	$-3.72(37)(59)\times 10^{-6}$
10	$6.8(15)(68)\times 10^{-5}$	$-3.69(37)(59)\times 10^{-6}$

- Q_1– Q_6, Q_9, Q_{10} in $(\mathrm{GeV})^4$ Q_7, Q_8 in $(\mathrm{GeV})^6$
- Heroic 7-operator NPR performed!

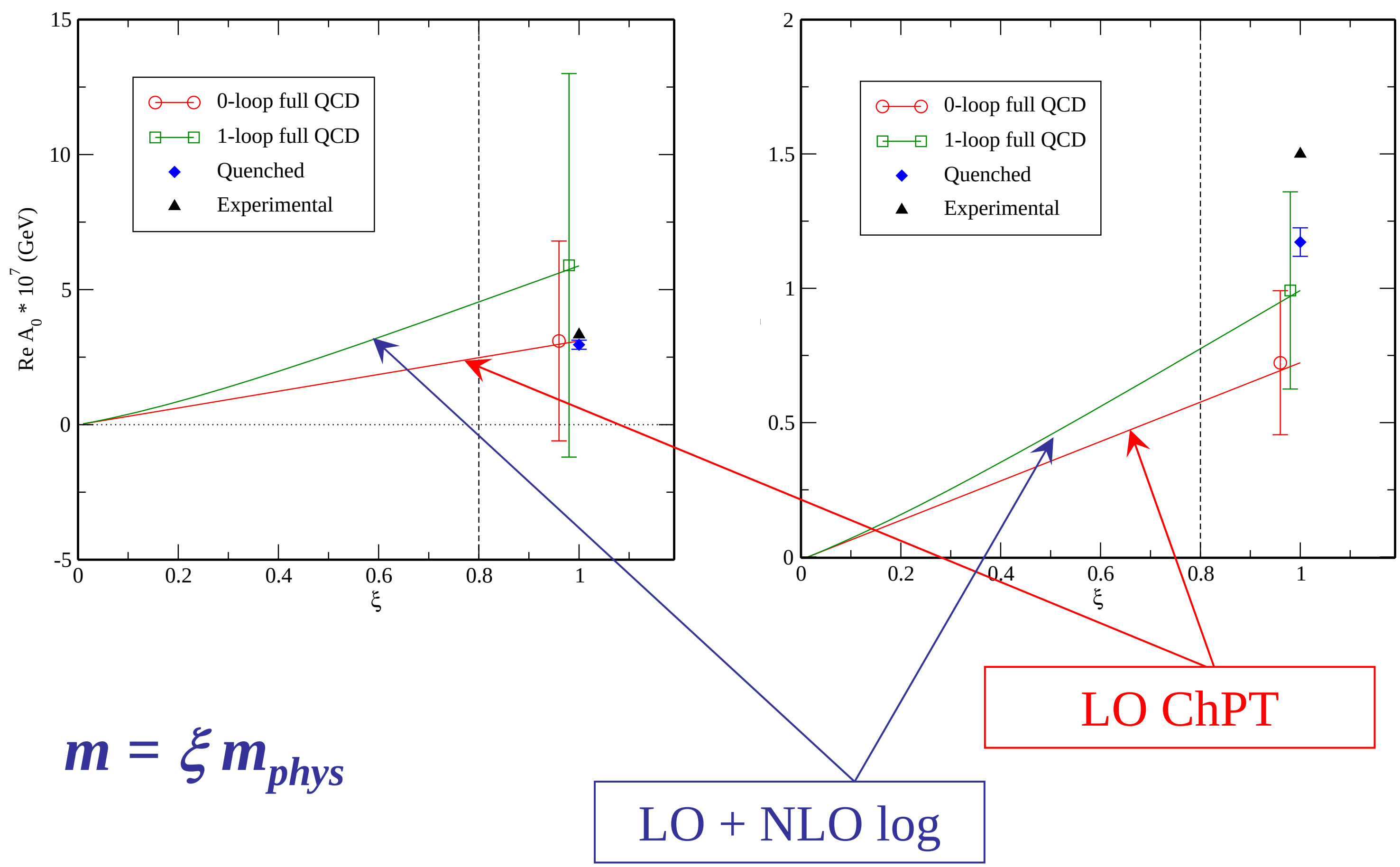
$K \rightarrow \pi\pi$ decay

Estimate $K \rightarrow \pi\,\pi$ decay amplitudes

- Made difficult by 100% errors on important LEC's.
- Conventional NLO extrapolation impeded by:
  - 2+1 flavor ChPT formula not available
  - Not all LEC's have been determined
- ChPT likely does not apply to the physical kaon:
  - Find 100% NLO corrections at m_{PS} = 430 MeV
  - $(m_K/m_{PS})^2 = (495/430)^2 = 1.3$
  - Corroboration of RBC/UKQC arXiv:0804.0473 (see talks of Chris Kelly, Bob Mawhinney and Enno Scholz)
- Attempt rough estimates using two extrapolations:
  - LO ChPT
  - LO+ only NLO logs with Λ_{chiral} = 1 GeV (no analytic terms)

Estimate $K \rightarrow \pi\pi$ amplitudes (con't)

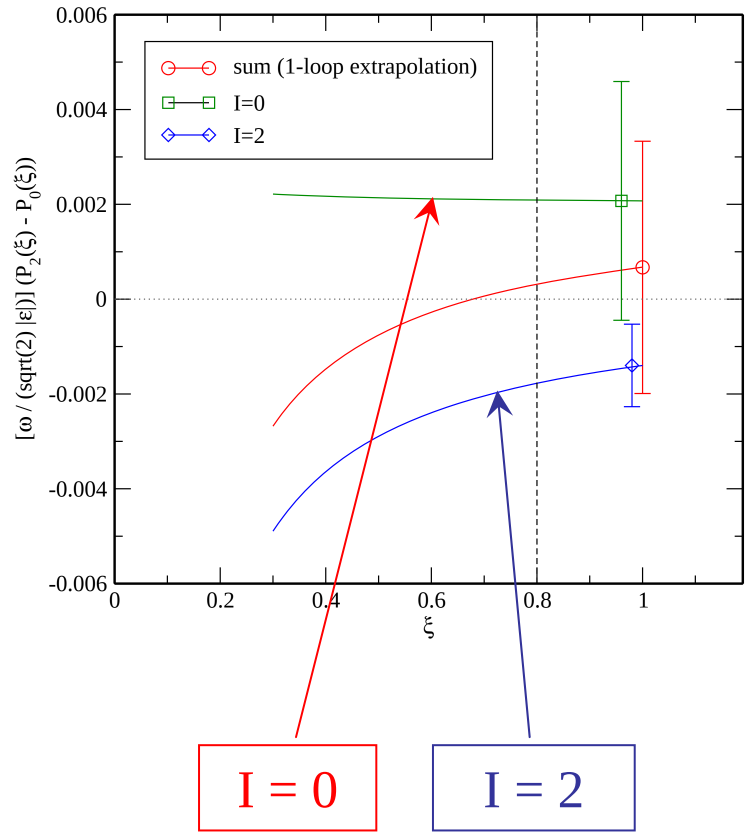
Re A_0 **Re A_2**



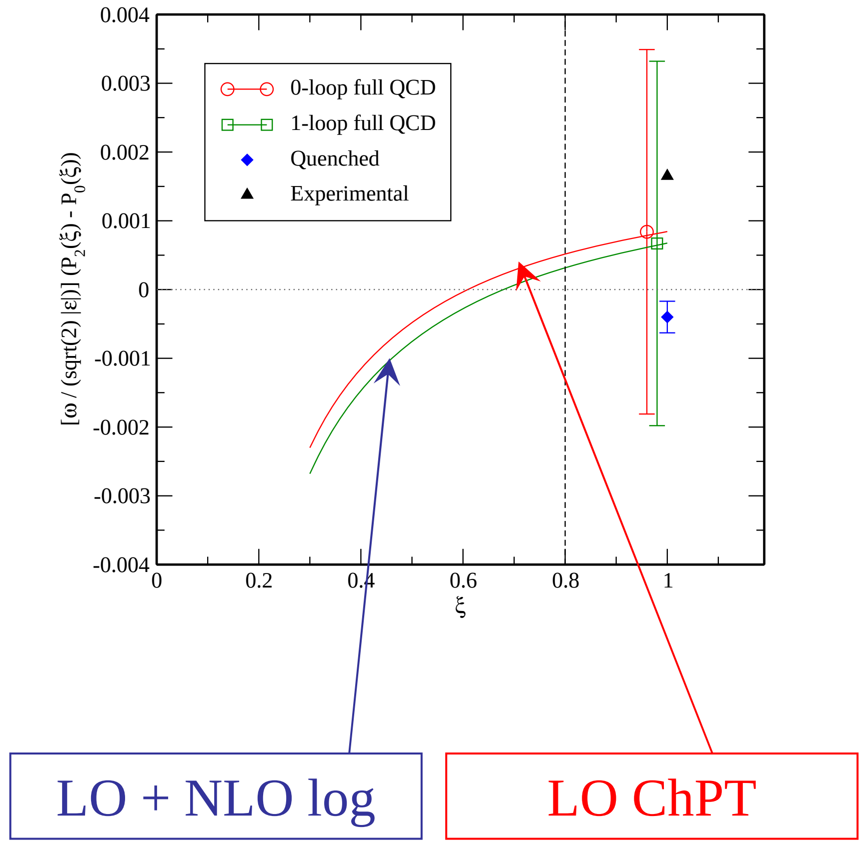
$m = \xi\, m_{phys}$

Estimate $K \rightarrow \pi\,\pi$ amplitudes (con't)

Re ε′/ε



Re ε′/ε



$m = \xi\, m_{phys}$

Conclusion

Quantity	This analysis	Quenched	Experiment
$\mathrm{Re}A_0$ **(GeV)**	$4.5(11)(53) \times 10^{-7}$	$2.96(17) \times 10^{-7}$	3.33×10^{-7}
$\mathrm{Re}A_2$ **(GeV)**	$8.57(99)(300) \times 10^{-9}$	$1.172(53) \times 10^{-8}$	1.50×10^{-8}
$\mathrm{Im}A_0$ **(GeV)**	$-6.5(18)(77) \times 10^{-11}$	$-2.35(40) \times 10^{-11}$	
$\mathrm{Im}A_2$ **(GeV)**	$-7.9(16)(39) \times 10^{-13}$	$-1.264(72) \times 10^{-12}$	
$1/\omega$	$50(13)(62)$	$25.3(1.8)$	**22.2**
$\mathrm{Re}(\epsilon'/\epsilon)$	$7.6(68)(256) \times 10^{-4}$	$-4.0(2.3) \times 10^{-4}$	1.65×10^{-3}

- ChPT approach to $K \rightarrow \pi\pi$ faces severe difficulties.
- RBC/UKQCD studying physical $\pi\pi$ final states.
- DWF on coarse lattices and large volumes: 4 → 5 fm?
- Vranas auxiliary determinant (talk of Dwight Renfrew)