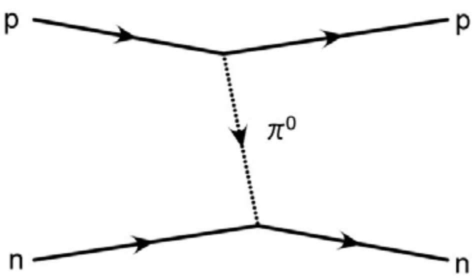
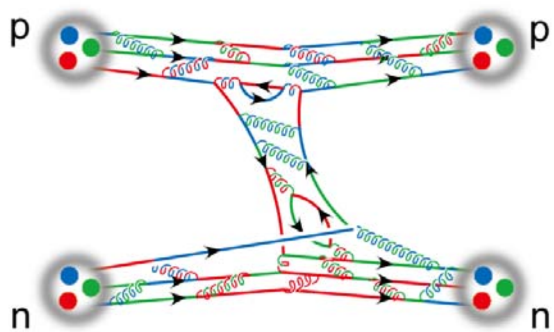
Nuclear forces from quenched and NF=2+1 full lattice QCD using the PACS-CS gauge configurations



N.Ishii (Univ. Tsukuba)
S.Aoki (Univ. Tsukuba)
T.Hatsuda (Univ. Tokyo)
for the PACS-CS Collab.



Plan of talk

1. Background
2. Tensor force from quenched lattice QCD
3. Nuclear force from NF=2+1 full lattice QCD using the PACS-CS gauge configurations
4. Summary

1.background

Nuclear force serves as one of the most important building blocks in nuclear physics.

✓ long distance (r > 2fm)

OPEP（one pion exchange） [H.Yukawa (1935)]

✓ medium distance (1fm < r < 2fm) ∼2π

multi pion and heavier meson exchanges(“σ”, ρ, ω,..)

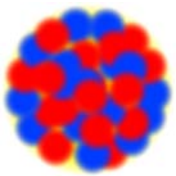
The attractive pocket is responsible for bound nuclei.

✓ short distance (r < 1fm)

Strong repulsive core [R.Jastrow (1951)]

The repulsive core plays an important role for

(a) stability of nuclei
(b) super nova explosion of type II
(c) maximum mass of neutron stars



The origin of the repulsive core remains an open problem.

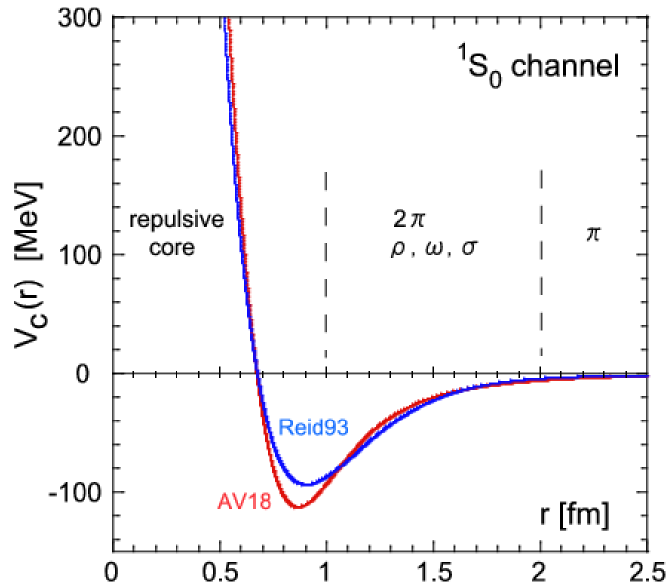
(1) vector meson exchange model
(2) constituent quark model
Pauli forbidden states and color magnetic interaction
(3) etc.

Since nucleons overlap with each other in this region, the short distance property should reflect

internal structure of nucleon in terms of quark and gluon degrees of freedom

⇒ Lattice QCD first principle approach to the nuclear force

The nuclear force



Reid93 is from
V.G.J.Stoks et al., PRC**49**, 2950 (1994).

AV16 is from
R.B.Wiringa et al., PRC**51**, 38 (1995).

Approaches to NN potential from lattice QCD

S.Aoki et al.(CP-PACS Collab.), Phys. Rev. D71,094504(2005).

extention

1. Method, which uses static quarks
2. Method, which uses Bethe-Salpeter wave function(We use this)

N.Ishii, S.Aoki, T.Hatsuda, Phys.Rev.Lett.99,022001('07).

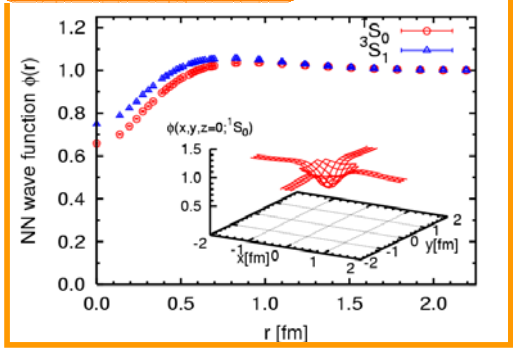
Advantage :
Since our potential is constructed from wave function,
it is expected to become faithful to the NN scattering data in the near future.

① NN wave function(BS wave function) is constructed from lattice QCD

② NN potential is reconstructed from NN wave function by using Schrodinger equation.

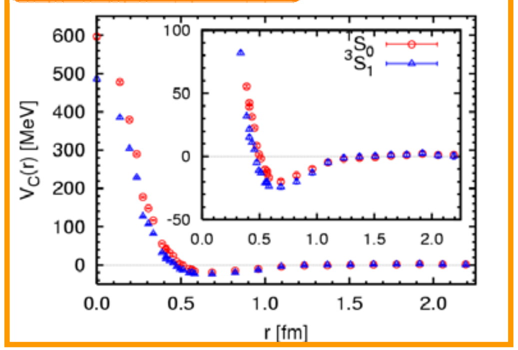
lattice QCD

wave function



Schrodinger eq.

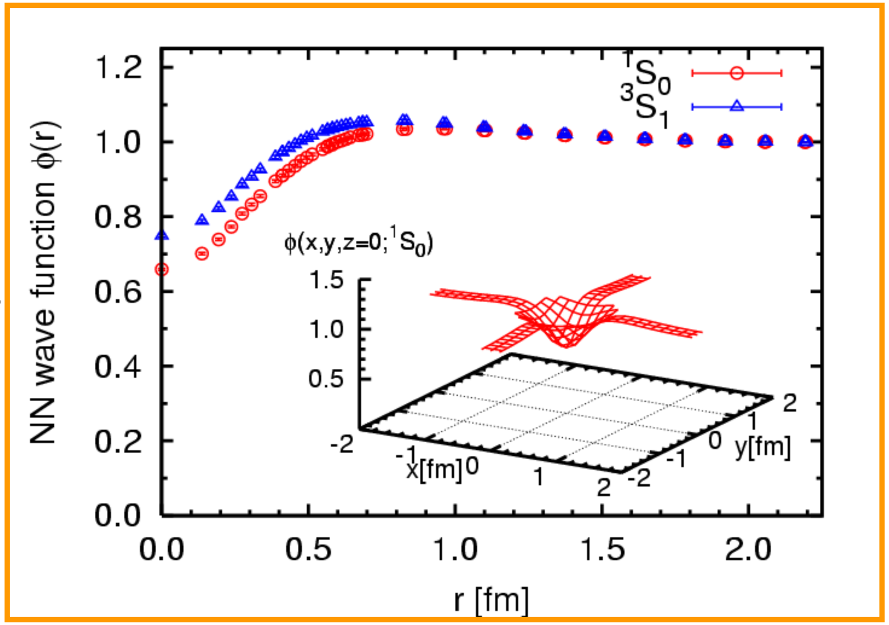
NN potential



$$V(r) = \frac{(E - H_0)\,\psi(\vec{x})}{\psi(\vec{x})}$$

BS wave function

- Quantum mechanical NN wave function is an approximate concept in QCD.
- The object, which provides the closest concept is equal-time Bethe-Salpeter(BS) wave function



$$\boxed{\psi_{\alpha\beta}(\vec{x}-\vec{y}) \equiv \lim_{t\to+0} \langle 0 | \mathrm{T}[p_\alpha(\vec{x},t)\ n_\beta(\vec{y},0)] | NN \rangle}$$

$$p(x) \equiv \varepsilon_{abc}\left(u_a{}^T C\gamma_5 d_b\right)u_c(x)$$
$$n(y) \equiv \varepsilon_{abc}\left(u_a{}^T C\gamma_5 d_b\right)d_c(y)$$

✓ amplitude to find proton-like three quarks at x and neutron-like three quarks at y.

✓ asymptotic behavior in large |x-y|

$$\boxed{\psi(r) \approx e^{i\delta_0(k)} \frac{\sin(kr+\delta_0(k))}{kr} + \cdots \quad \text{(s-wave)}}$$

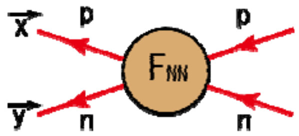
✓ it satisfies Schrodinger-like equation.

$$\boxed{\left(\vec{\nabla}^2 + k^2\right)\psi_E(\vec{r}) = m_N \int d^3r' U(\vec{r},\vec{r}\,')\psi_E(\vec{r}\,')}$$

For derivation, see
C.-J.D.Lin et al., NPB619,467 (2001).
S.Aoki et al., CP-PACS Collab., PRD71,094504(2005).
S.Aoki, T.Hatsuda, N.Ishii, arXiv:0805.2462[hep-ph].

- equal-time BS wave function is obtained from 4 point correlator of nucleons at large t region.

$$F_{NN}(\vec{x},\vec{y},t) \equiv \langle 0 | \mathrm{T}[p(\vec{x},t+0)\,n(\vec{y},t)\ \ \overline{p}(0)\,\overline{n}(0)] | 0 \rangle$$
$$= \sum_m \langle 0 | p(\vec{x})\,n(\vec{y}) | m \rangle e^{-E_m t} \langle m | \overline{p}(\vec{0})\,\overline{n}(\vec{0}) | 0 \rangle$$
$$= \boxed{A_0\ \psi_{E_0}(\vec{x}-\vec{y})\ e^{-tE_0} + \cdots}$$



(Contributions from excited states are suppressed exponentially.)

Potentials from BS wave function ($J^P=0^+$)

Derivative expansion to the potential term U(r,r')
after imposing constraints from various symmetry:

$$U(\vec{r},\vec{r}') = \left(V_C(r) + V_T(r)S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S} + O(\vec{\nabla}^2)\right)\delta(\vec{r}-\vec{r}').$$

$$S_{12} \equiv 3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})/r^2 - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

$V_C(r)$, $V_T(r)$, $V_{LS}(r)$ are called as central, tensor, and LS potentials.
These three potentials play an impotant role in convensional nuclear physics.

- **$J^P=0^+$:**
  Contributions from $V_T(r)$ and $V_{LS}(r)$ vanish.
  Only $V_C(r)$ survives.

$$V(\vec{r},\vec{r}') = V_C(r) + \cancel{V_T(r)}S_{12} + \cancel{V_{LS}(r)}\vec{L}\cdot\vec{S} + \cancel{O(\vec{\nabla}^2)}$$

Schrodinger eq. is arranged as

$$V_C(r;{}^1S_0) = \frac{(E-H_0)\psi(\vec{x};{}^1S_0)}{\psi(\vec{x};{}^1S_0)}$$

Potentials from BS wave function ($J^P=1^+$)

Derivative expansion to the potential term U(r,r')
after imposing constraints from various symmetry:

$$U(\vec{r},\vec{r}')=\left(V_C(r)+V_T(r)S_{12}+V_{LS}(r)\vec{L}\cdot\vec{S}+O(\vec{\nabla}^2)\right)\delta(\vec{r}-\vec{r}').$$

$$S_{12}\equiv 3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})/\vec{r}^2-\vec{\sigma}_1\cdot\vec{\sigma}_2$$

- **$J^P=1^+$:**

Contribution from $V_T(r)$ and $V_{LS}(r)$ survive

$$V(\vec{r},\vec{r}')=V_C(r)+V_T(r)S_{12}+V_{LS}(r)\vec{L}\cdot\vec{S}+\cancel{O(\vec{\nabla}^2)}$$

$V_T(r)$ generates a coupling
between s-wave component and d-wave component.

Approach 1

As the first step, we repeat the same procedure as $J^P=0^+$ by neglecting V_T and V_{LS}.

$$V(\vec{r},\vec{r}')=V_C(r)+\cancel{V_T(r)S_{12}}+\cancel{V_{LS}(r)\vec{L}\cdot\vec{S}}+\cancel{O(\vec{\nabla}^2)}$$

What is obtained in this procedure is
"central potential, which reproduce exact $^3S_1$ WF"
= "effective central potential".

$$V_C^{eff}(r;{}^3S_1)=\frac{(E-H_0)\psi(\vec{x};{}^3S_1)}{\psi(\vec{x};{}^3S_1)}$$

Approach 2

We restrict ourselves to the strictly local contribution.

$$V(\vec{r},\vec{r}')=V_C(r)+V_T(r)S_{12}+\cancel{V_{LS}(r)\vec{L}\cdot\vec{S}}+\cancel{O(\vec{\nabla}^2)}$$

Schrodinger eq for $J^P=1^+$ becomes.

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2+V_C(\vec{r})+V_T(\vec{r})S_{12}\right]\phi(\vec{r})=E\phi(\vec{r})$$

s-wave

"d-wave"

Projection op's

$$P\phi(\vec{r})\equiv\frac{1}{24}\sum_{R\in O}\phi(R\vec{r});\quad Q\equiv 1-P$$

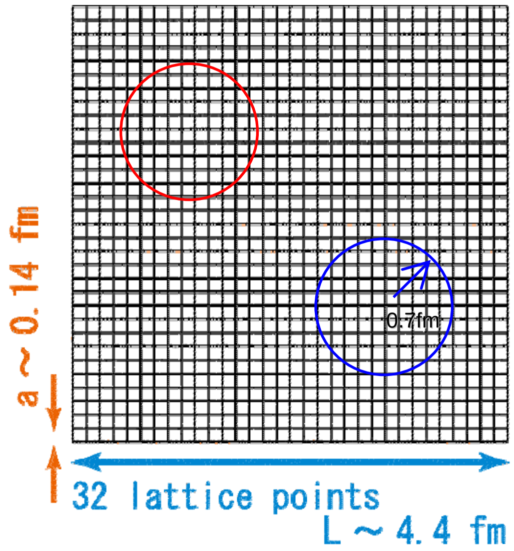
$$\left[-\frac{1}{2\mu}\vec{\nabla}^2+V_C(\vec{r})\right][P\phi](\vec{r})+V_T(\vec{r})[PS_{12}\phi](\vec{r})=E\cdot[P\phi](\vec{r})\qquad(1)$$

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2+V_C(\vec{r})\right][Q\phi](\vec{r})+V_T(\vec{r})[QS_{12}\phi](\vec{r})=E\cdot[Q\phi](\vec{r})\qquad(2)$$

By solving this coupled equation, we obtain V_C and V_T.

2.Tensor force from quenched lattice QCD

Lattice QCD set up for tensor force



- Quenched QCD
- standard plaquette gauge action
  - ✓ $\beta = 5.7$
  - ✓ $1/a = 1.44(2)$ GeV ($a \sim 0.14$ fm)
  - ✓ 32^4 lattice (4.4^4 fm^4)
  - ✓ 1947 gauge configs are used.
- standard Wilson quark action
  - ✓ $\kappa = 0.1665$
  - ✓ $m_{pi} \sim 0.53$ GeV, $m_N \sim 1.34$ GeV
  - ✓ Dirichlet(periodic) BC along temporal(spatial) direction on time-slice t=0
  - ✓ wall source on time-slice t=5 to avoid possible boundary artifacts.
- Numerical calculation is performed with Blue Gene/L at KEK

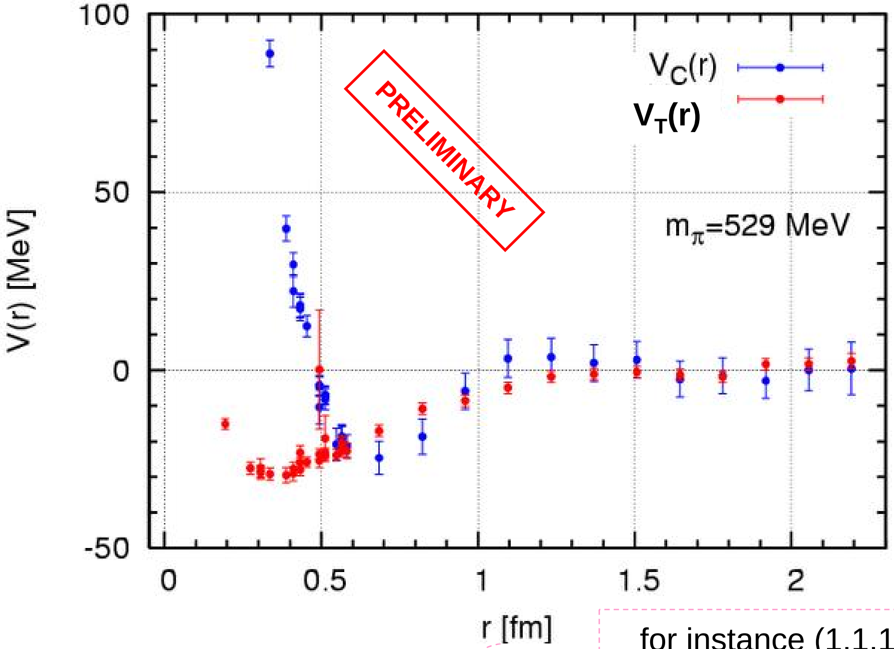
Tensor force from quenched lattice QCD

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V_C(\vec{r})\right][P\phi](\vec{r}) + V_T(\vec{r})[PS_{12}\phi](\vec{r}) = E\cdot[P\phi](\vec{r}) \qquad (1)$$

$$\left[-\frac{1}{2\mu}\vec{\nabla}^2 + V_C(\vec{r})\right][Q\phi](\vec{r}) + V_T(\vec{r})[QS_{12}\phi](\vec{r}) = E\cdot[Q\phi](\vec{r}) \qquad (2)$$

In this talk, we use |J=1,M=0>
V_T and V_C are obtained
by using spin $(\alpha,\beta)=(0,1)$ component of these equations.

NN potentials



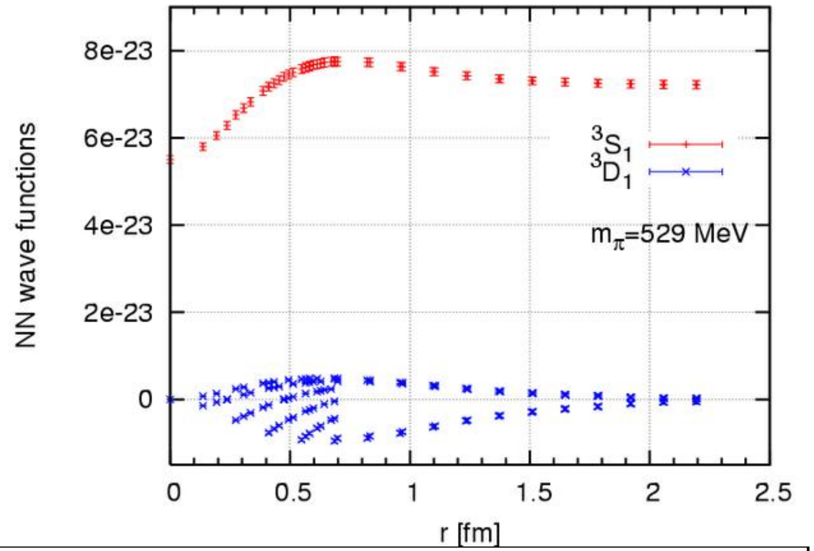
for instance (1,1,1)

(0,1) component of d-wave wave function is proportional to

$$Y_{2,0}(\theta,\phi) \propto 3\cos^2\theta - 1$$

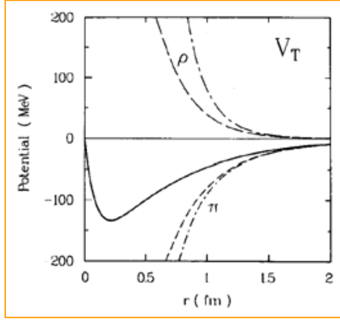
For points, which satisfies $3\cos^2\theta=1$, these two equations are not independent,
leading to large error bar near these points.

BS wave function



Comments:

- Tensor force is important for the stability of heavy nuclei together with the repulsive core.
- Tensor force plays an important role in nuclear structures.
- Empirical determination of tensor force is known to involve large uncertainties especially at short distance due to the centrifugal barrier.
- This shape of tensor force is expected from cancellation of contributions from π and ρ.



from
R.Machleidt,
Adv.Nucl.Phys.**19**

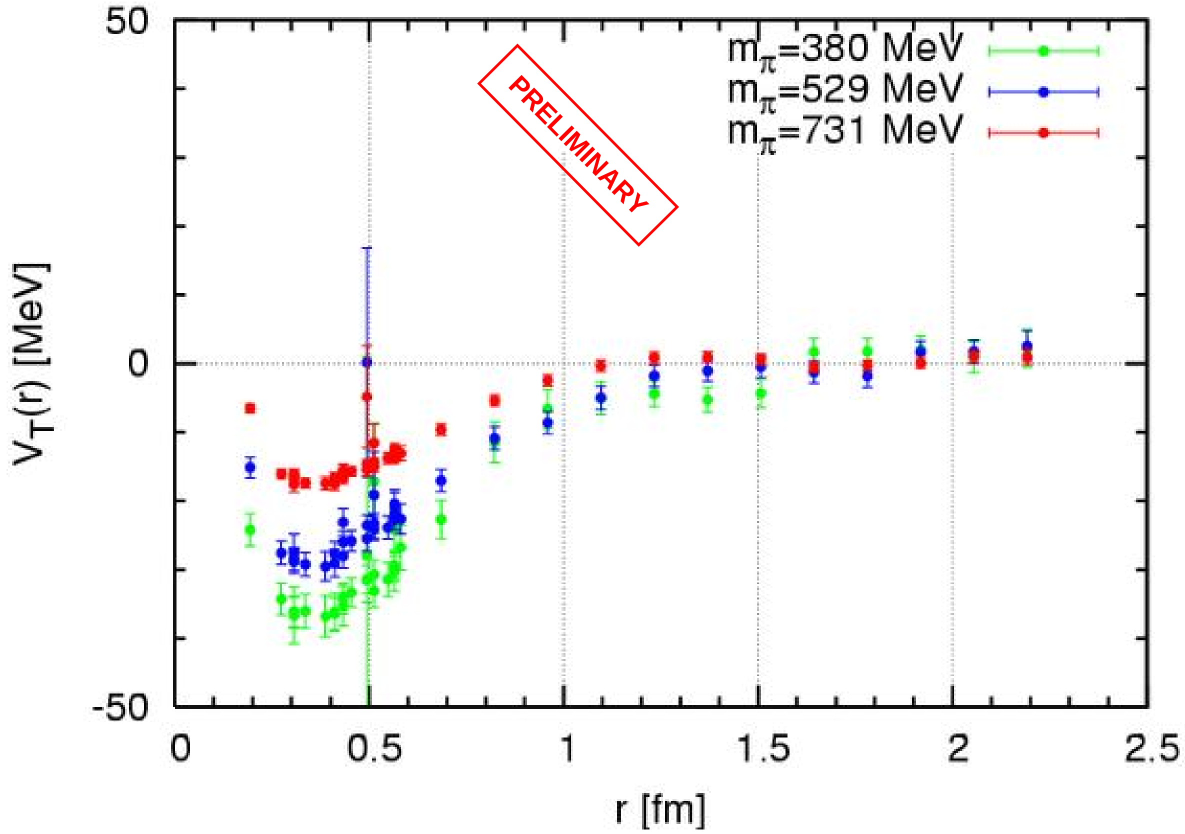
Fig. 3.7. The contributions from π and ρ (dashed) to the T = 0 tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

- This method can be straightforwardly extended to LS force.

Quark mass dependence

Tensor force is calculated by changing quark mass.

Tensor force becomes enhanced in the light quark mass region.



(1) m_π=380MeV: Nconf=2020

[28 exceptional configurations have been removed]

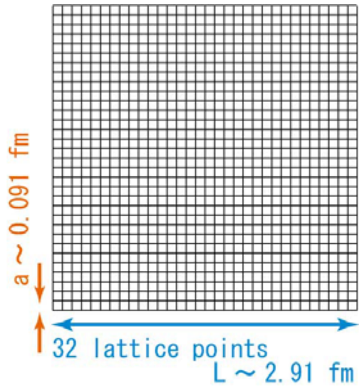
(2) m_π=529MeV: Nconf=1947

(3) m_π=731MeV: Nconf=1000

3. Nuclear force from 2+1 flavor full QCD using PACS-CS gauge configurations

PACS-CS collaboration is generating 2+1 flavor gauge configurations in significantly light quark mass region on a large spatial volume

- 2+1 flavor full QCD
  S.Aoki et al., PACSCS Collab., arXiv:0807.1661[hep-lat]
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark (clover) action
  with a non-perturbatively improved coefficient c_{SW}=1.715
- 1/a=2.17 GeV (a ~ 0.091 fm). L=32a ~ 2.91 fm



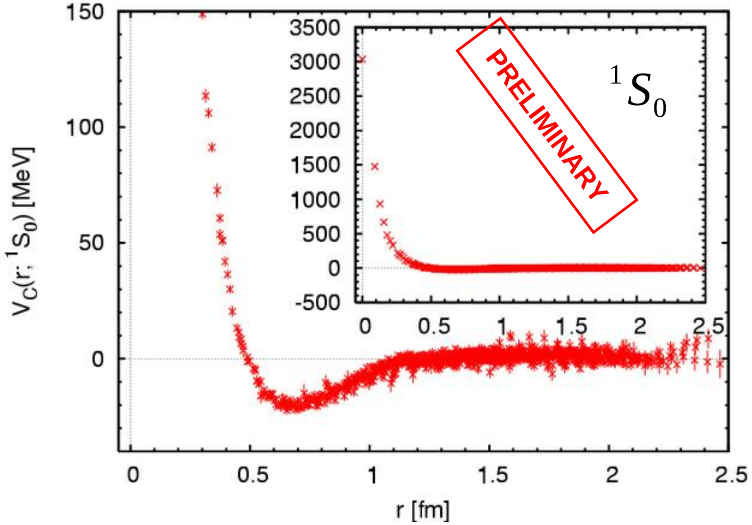
We use some of these gauge configurations to calculate nuclear force from full lattice QCD.

In this talk, we present preliminary results of

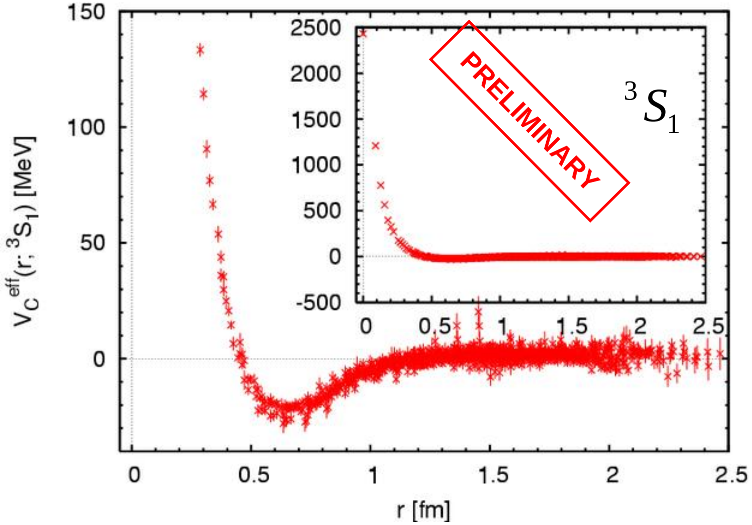
- κ_{ud}=0.13700, κ_s=0.13640(m_{pi} ~ 730 MeV)
  - ✓ Nconf=122 (gauge config's are picked up every 50 traj.)
  - ✓ Each config is used four times by changing the position of the wall source on t=0,16,32,48 planes.
  - ✓ number of data is doubled by using charge conjugation and time reversal symmetry.
- κ_{ud}=0.13770, κ_s=0.13640(m_{pi} ~ 300 MeV)
  - ✓ Nconf=422 (gauge config's are picked up every 20 traj.)
  - ✓ Each config is used once. (single position of the wall source on t=0 plane)
  - ✓ Number of data is doubled by using charge conjugation and time reversal symmetry.

(Effective) central potentials (m_{pi} ~ 730 MeV) from NF=2+1 full QCD

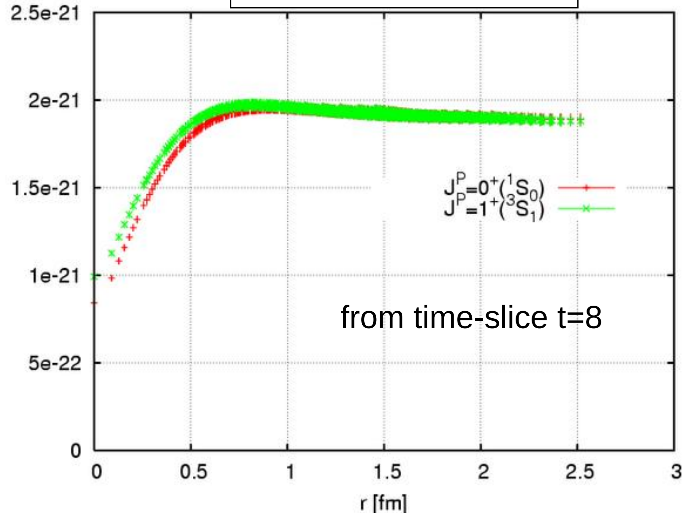
$$V_C(r;{}^1S_0) = \frac{(E-H_0)\psi(\vec{x};{}^1S_0)}{\psi(\vec{x};{}^1S_0)}$$



$$V_C^{eff}(r;{}^3S_1) = \frac{(E-H_0)\psi(\vec{x};{}^3S_1)}{\psi(\vec{x};{}^3S_1)}$$

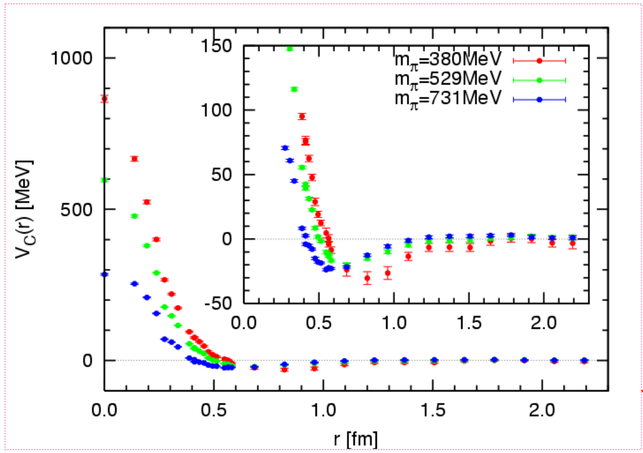


BS wave functions



Comments:

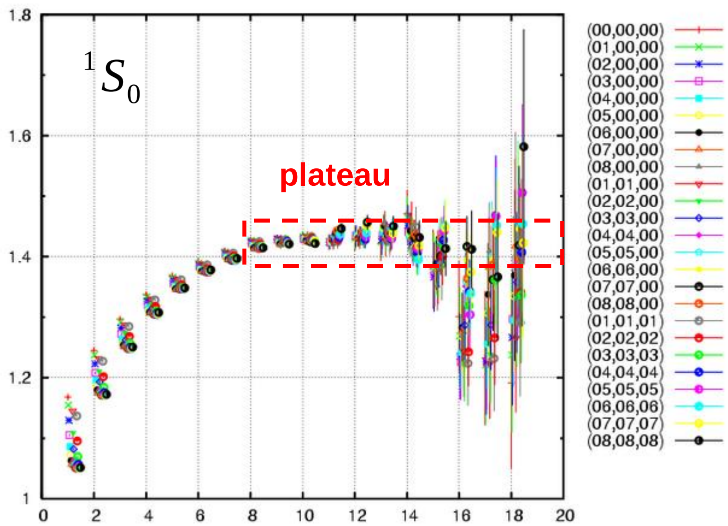
- m_{pi} ~ 730 MeV
- results from time-slice t=8 for $^1S_0$ and t=9 for $^3S_1$, where ground state saturation is expected.
- A remarkable difference is the strength of the repulsive core:



$^1S_0$ central potential from quenched QCD

Reason is currently under investigation.

Ground state saturation (m_{pi} ~ 730 MeV case)



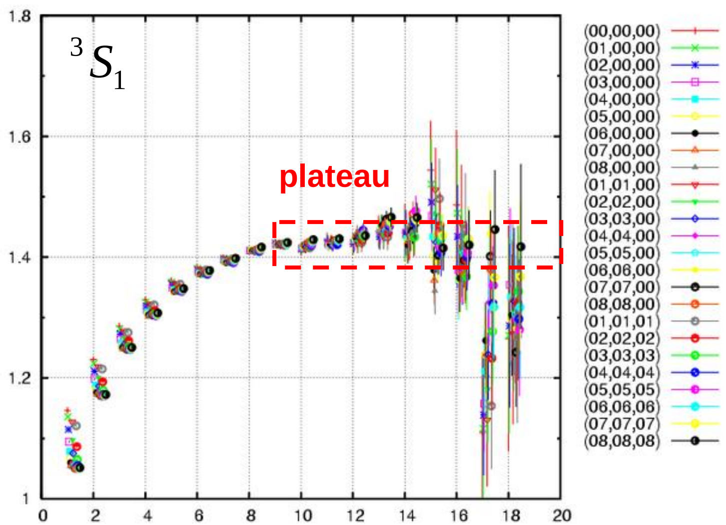
For fixed x, effective mass plot of t-dependence of nucleon four point correlator is ploted.

$$m_{eff}(t;\vec{x}) \equiv \log\left(\frac{F_{NN}(\vec{x},t)}{F_{NN}(\vec{x},t+1)}\right)$$

Since

$$F_{NN}(\vec{x},t) = A_0\,\psi_0(\vec{x})\,e^{-tE_0} + (\text{excited states})$$

These effective mass have plateaux at $E=E_0$ for all x. (Each plateau may start at different t.)

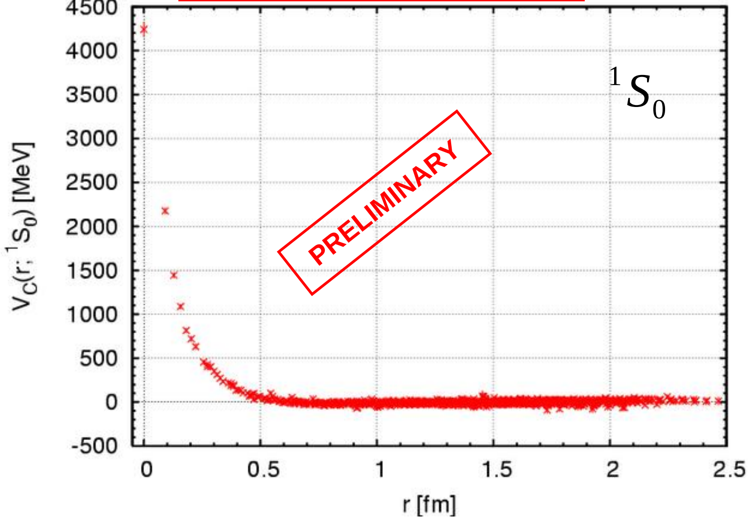


For $^1S_0$, plateaux appear at t=8, beyond which the ground state saturation is expected.

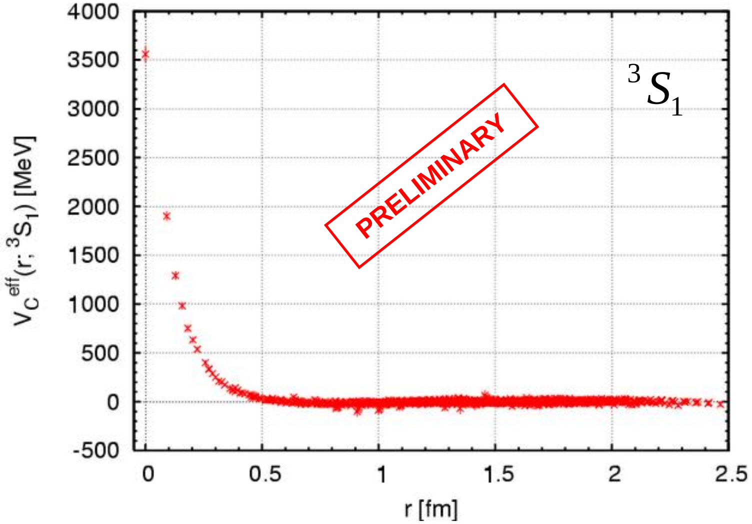
For $^3S_1$, plateaux appear at t=9, beyond which the ground state saturation is expected.

(Effective) central potentials ($m_{pi} \sim 300$ MeV) from NF=2+1 full QCD

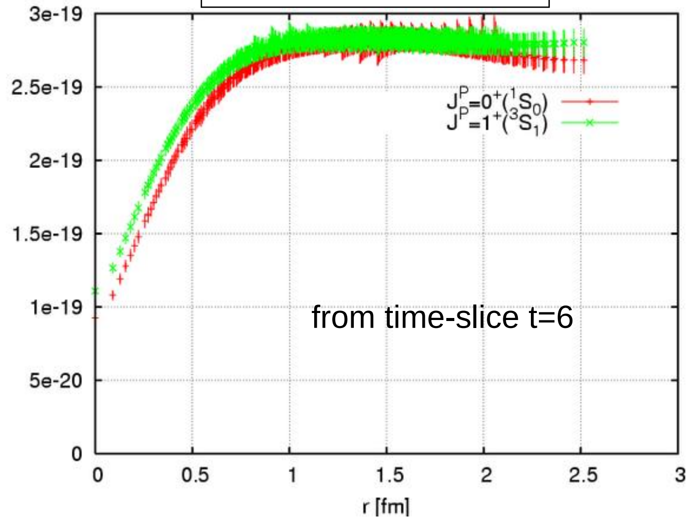
$$V_C(r; {}^1S_0) = \frac{(E - H_0)\psi(\vec{x}; {}^1S_0)}{\psi(\vec{x}; {}^1S_0)}$$



$$V_C^{eff}(r; {}^3S_1) = \frac{(E - H_0)\psi(\vec{x}; {}^3S_1)}{\psi(\vec{x}; {}^3S_1)}$$

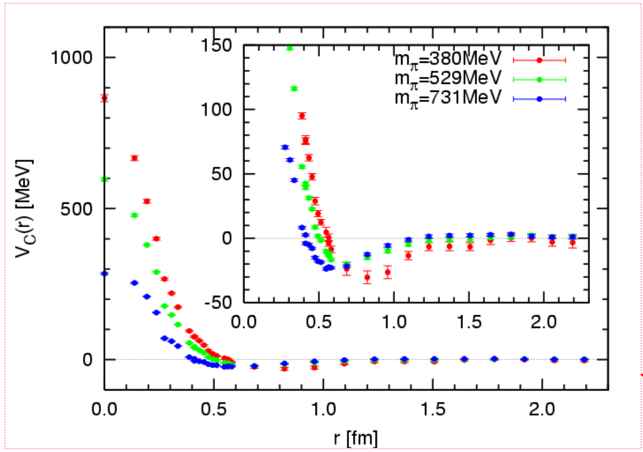


BS wave functions



Comments:

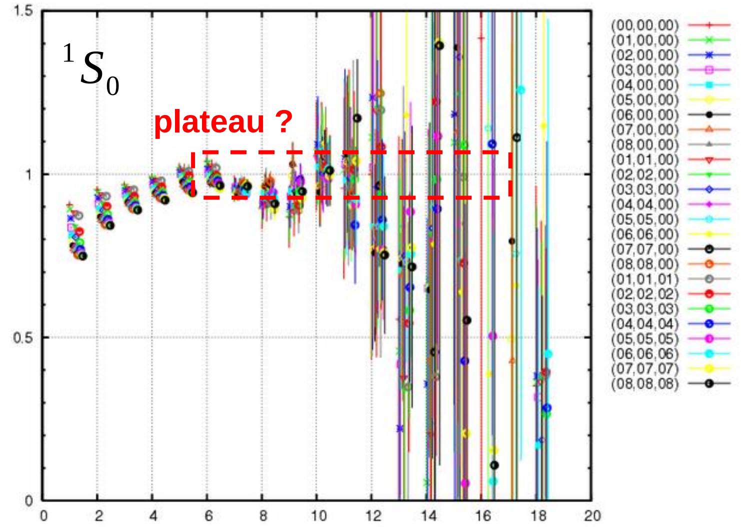
- $m_{pi} \sim 300$ MeV
- results from time-slice t=6 both for $^1S_0$ and $^3S_1$,
- We need much more statistics (to see the attractive pocket).
- A remarkable difference is the strength of the repulsive core:



Reason is currently under investigation.

$^1S_0$ central potential from quenched QCD

Ground state saturation (m_{pi} ~ 300 MeV case)



For fixed x, effective mass plot of t-dependence of nucleon four point correlator is ploted.

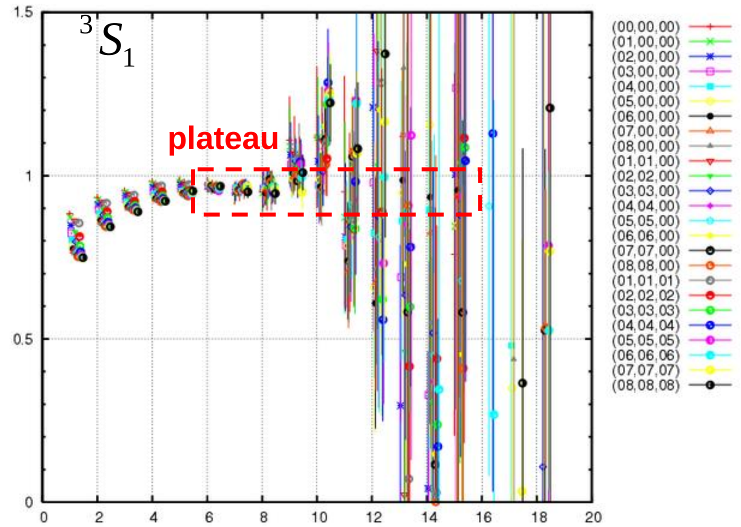
$$m_{eff}(t;\vec{x}) \equiv \log\left(\frac{F_{NN}(\vec{x},t)}{F_{NN}(\vec{x},t+1)}\right)$$

Since

$$F_{NN}(\vec{x},t) = A_0\,\psi_0(\vec{x})\,e^{-tE_0} + (\text{excited states})$$

These effective mass have plateaux at $E=E_0$ for all x. (Each plateau may start at different t.)

For $^1S_0$, we need more statistics to determine the stable plateaux.



For $^3S_1$, plateaux appear at t=6, beyond which the ground state saturation is expected.

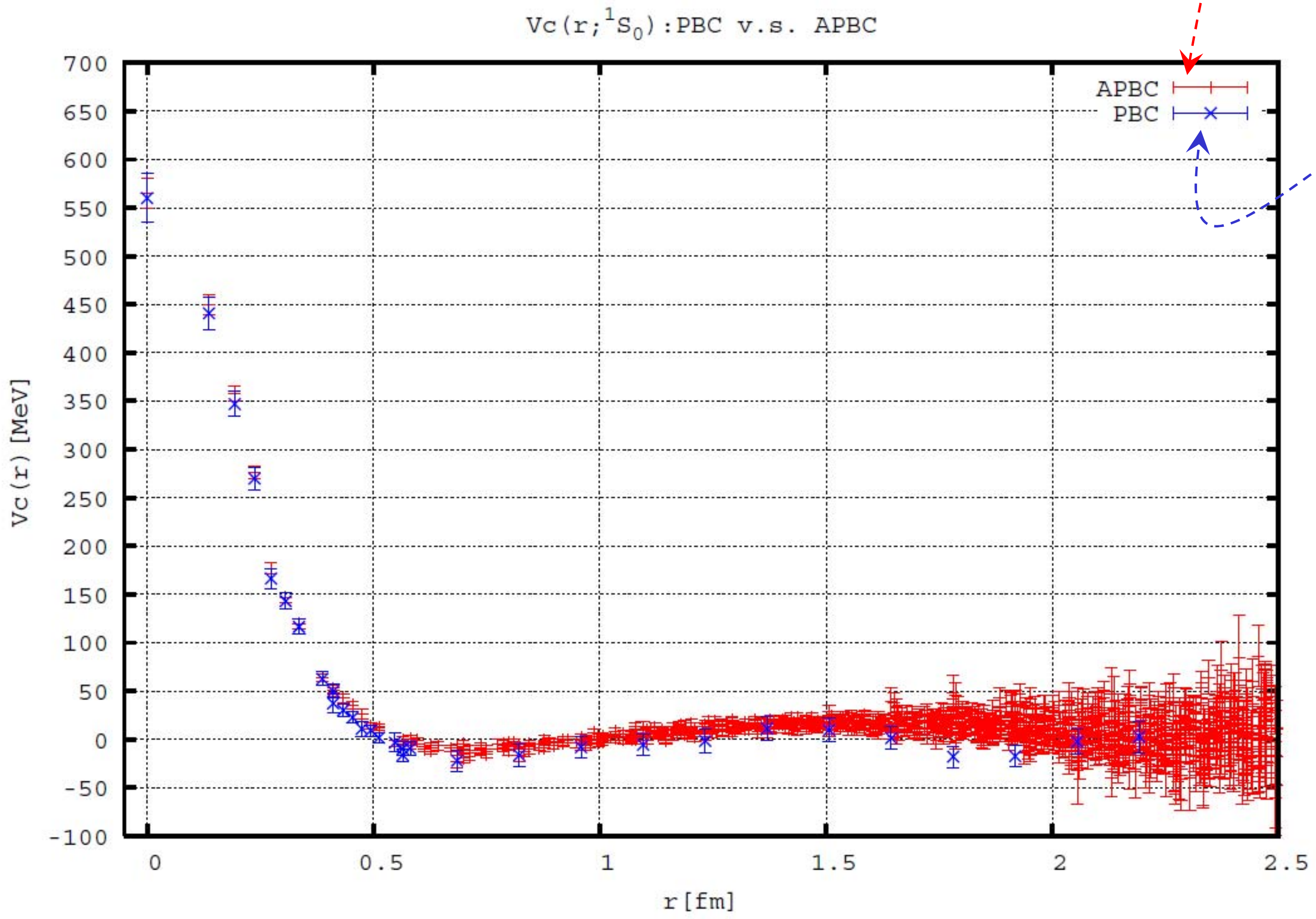
4.Summary

- We have extended our method of calculating (effective) central NN potential to tensor force.
    - Preliminary results are presented.
    - The strength of tensor force is enhanced in the light quark mass region, which suggests the importance of lattice QCD calculation with light quark mass.
- We have presented preliminary results of (effective) central NN potentials from full QCD by using 2+1 flavor gauge configurations generated by PACS-CS collaboration.
    - A remarkable difference was found that the strength of repulsive core is considerably larger than the quenched calculations.
      The reason for this is currently under investigation.

END

BACKUP SLIDES

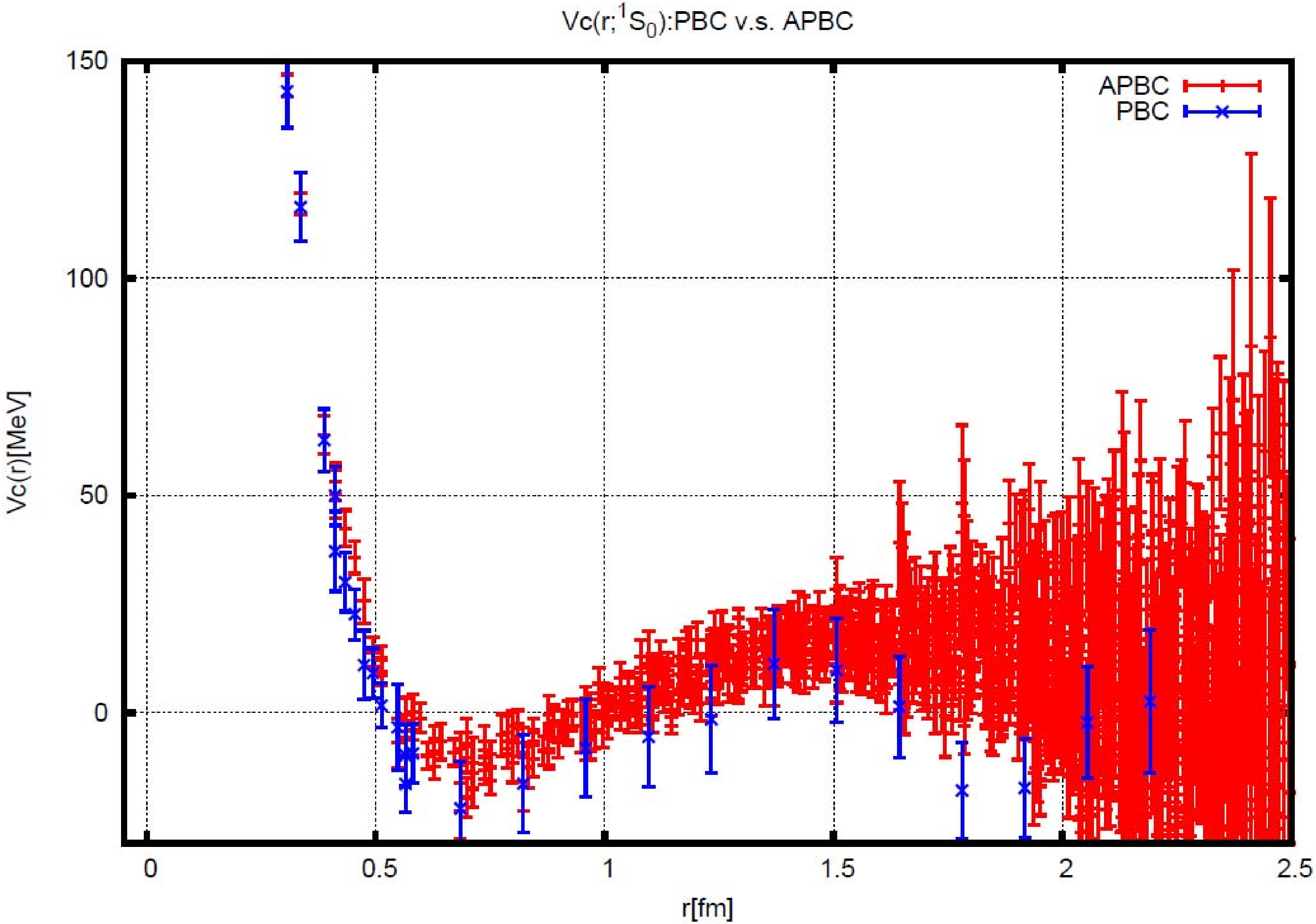
Energy dependence of NN potential (from Aoki's talk)



Potential generated from wave function at E ~ 50 MeV

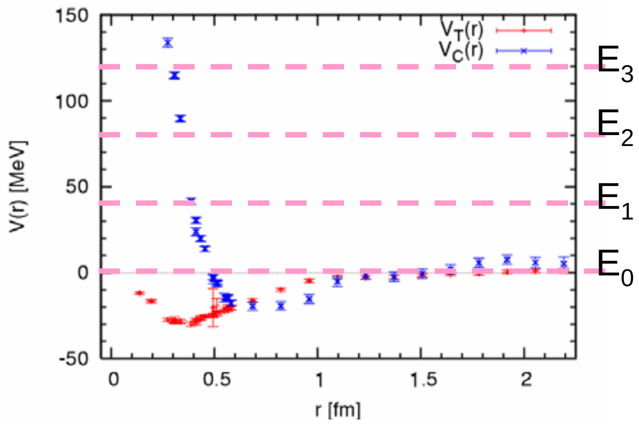
Potential generated from wave function at E ~ 0 MeV

Energy dependence of NN potential (from Aoki's talk)



Toward a potential, which is more faithful to the NN scattering data

By using BS wave function at multiple energies
to construct energy independent potential
⇒
potential, which is more faithful to NN scattering data
(⇔ more correct scattering phase shift)



- asympototic form of BS wave function:

$$\phi(r) \approx e^{i\delta_0(k)} \frac{\sin(kr + \delta_0(k))}{kr} + \cdots \quad \text{(s-wave)}$$

- Our potential is so constructed as to reproduce the used wave functions simultaneously.

⇒

It can reproduce the phase shift exactly at the energies,which we use to construct the potential.

This topic is closely related to the following important problems of our method through the inverse scattering theory:

- Expected distortion of nucleon at short distance
  (In terms of QCD, it corresponds to the uncertainties arising from chose of nucleon interpolating fields.)
- Orthogonality of the Bethe-Salpeter wave functions
- Non-locality of the potential
- Energy dependence of the potential.

It is quite important to consider these problems for establishment of our method in the near future.

General form of NN potential

★ By imposing following constraints:

- Probability (Hermiticity):
- Energy-momentum conservation:
- Galilei invariance:
- Spatial rotation:
- Spatial reflection:
- Time reversal:
- Quantum statistics:
- Isospin invariance:

The most general (off-shell) form of NN potential is given as follows:
[see S.Okubo, R.E.Marshak,Ann.Phys.4,166(1958)]

$$V = V^0 + V^\tau \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

$$V^i = V_0^i + V_\sigma^i \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{LS}^i \cdot (\vec{L} \cdot \vec{S}) + \{V_T^i, S_{12}\} + \frac{1}{2}\{V_{\sigma p}^i, (\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p})\} + \frac{1}{2}\{V_Q^i, Q_{12}\}$$

$$Q_{12} \equiv \frac{1}{2}\left[(\vec{\sigma}_1 \cdot \vec{L})(\vec{\sigma}_2 \cdot \vec{L}) + (\vec{\sigma}_2 \cdot \vec{L})(\vec{\sigma}_1 \cdot \vec{L})\right]$$

where $V_j^i = V_j^i(\vec{r}^{\,2}, \vec{p}^{\,2}, \vec{L}^2), \quad \vec{p} \equiv i\vec{\nabla}$

★ If we keep the terms up to O(p), we are left with the convensional form of the potential in nuclear physics:

$$\boxed{V = \underbrace{V_0(r) + V_\sigma(r)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)}_{V_C(r)} + V_{LS}(r)\,\vec{L} \cdot \vec{S} + V_T(r)\,S_{12} + O(\vec{\nabla}^2).}$$