S-parameter & pseudo-NG boson mass from lattice QCD

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Lattice Gauge Theory in the LHC era

- Lattice Gauge Theory (LGT) has been successfully applied to a wide range of physics.
- What can we do using LGT in the LHC era?

Technicolor (TC) [Weinberg(1979), Susskind(1979)]

- Strongly interacting gauge theory

- "χ-symmetry" of TC is dynamically broken at Λ_{TC} (as in QCD).
  ➡Triggers EW symmetry breaking
  ➡Weak bosons acquire their masses.

- Typically, $m_{W^\pm} = g_2 F_{TC}/2 \Leftrightarrow F_{TC} \sim 250$ GeV
  (F_{TC}:technipion decay constant)
  ➡$\Lambda_{TC} \sim (F_{TC} / f_\pi) \times \Lambda_{QCD} \sim 2600 \times \Lambda_{QCD}$

- Elementary scalar is not necessary.
  ➡No "hierarchy problem"

- Attractive candidate for the Higgs sector in the SM

Two key observables in TC

- *S-parameter* [Peskin, Takeuchi (1990,1992)]
  - tends to be sizably affected in TC.
- *Light pseudo-NG bosons*
  - often appear with a mass detectable in LHC (sometimes appear in the excluded region).

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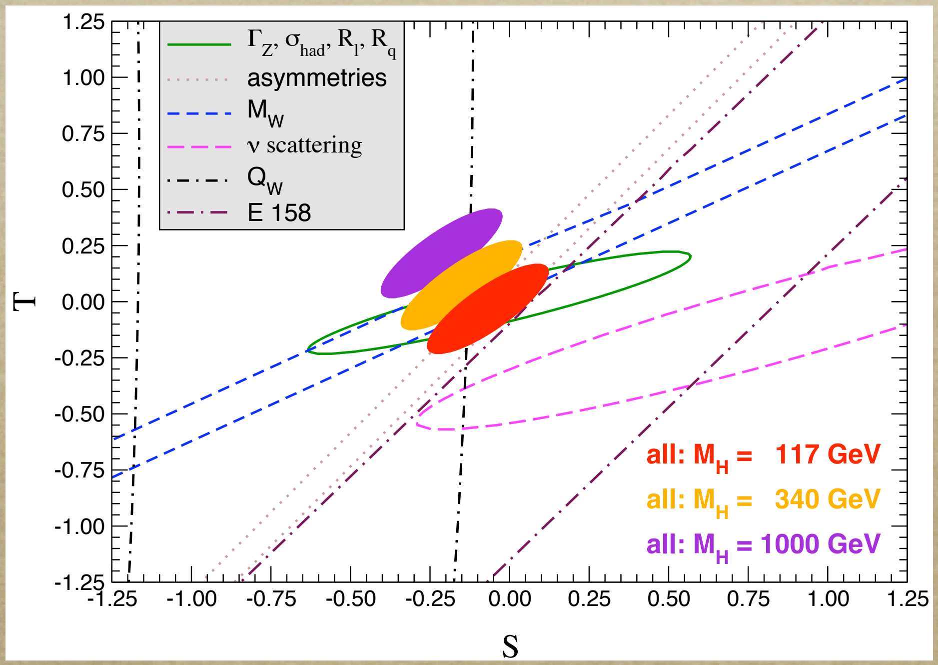
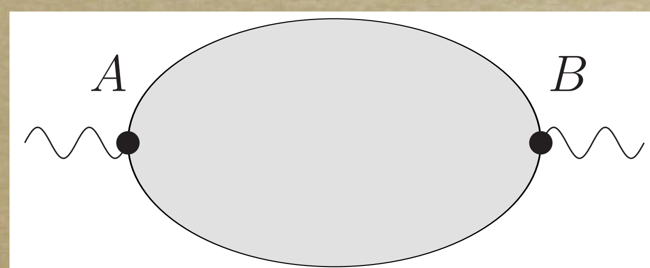


S-parameter [Peskin, Takeuchi (1990,1992)]

- Parameterizes "potential new physics contributions" to the EW gauge bosons' self-energy. "Oblique correction"
- Useful for New Physics search using the EW precision data

$$S = 16\pi \left[\frac{\partial \left[q^2 \left(\Pi^{(1)}_{VV} - \Pi^{(1)}_{AA} \right) \right]}{\partial q^2} \right]_{q^2=0}$$

$$i \int d^4x \, e^{iq\cdot x} \langle 0 \,|\, T\left[J_{A,\mu}(x) J_{B,\nu}(0) \right] |\, 0 \rangle = \left(g_{\mu\nu}\, q^2 - q_\mu q_\nu \right) \Pi^{(1)}_{AB}(q^2) - q_\mu q_\nu \, \Pi^{(0)}_{AB}(q^2)$$



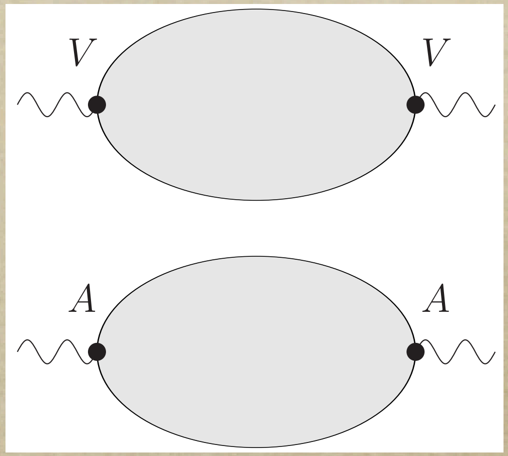
<*VV-AA*> ⇒ *S*-parameter

S-parameter and L_{10}

Interesting scale ~ Λ_{TC} ⇔ Low energy TC⇒ ChPT in TC

- In ordinary QCD ChPT [Gasser and Leutwyler (1984,1985)]

$$\Pi^{(1)}_{V-A}(q^2) = \Pi^{(1)}_{VV}(q^2) - \Pi^{(1)}_{AA}(q^2)$$
$$= -\frac{f_\pi^2}{q^2} - 8\, L^r_{10}(\mu) - \frac{\ln\left(\frac{m_\pi^2}{\mu^2}\right) + \frac{1}{3} - H(4m_\pi^2/q^2)}{24\pi^2}$$



$$H(x) = (1+x)\left[\sqrt{1+x}\,\ln\left(\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right) + 2\right]$$

L_{10}: one of LEC's in ChPT

- Reinterpret QCD→TC, and substitute the result

$$S = -16\pi\left[L^r_{10}(\mu) - \frac{1}{192\pi^2}\left\{\ln\left(\frac{\mu^2}{m_H^2}\right) - \frac{1}{6}\right\}\right]$$

Therefore determining L_{10} is equivalent to determining S-parameter.

Pseudo NG Boson Mass [Peskin(1980), Preskill(1981)]

- TC models ⇒ too many NG bosons.
- One standard wayout : introduce extra gauge symmetry which explicitly breaks χ-symmetry.
- Then NG bosons acquire the mass, and become pseudo-NG.

$$m_{\rm PNG}^2 = G \int_0^\infty dq^2 q^2 \left[\Pi_T^{(1)}(q^2) - \Pi_X^{(1)}(q^2) \right]$$

G : model dependent coefficient
Π_T :VP of currents corresponding to unbroken generators
Π_X :VP of currents corresponding to broken generators

Once the underlying TC theory is specified, the NP part is independent of further details.

Pseudo NG Boson Mass [Peskin(1980), Preskill(1981)]

- A well known example is the charged pion in QED+QCD theory.
- QED interaction explicitly breaks chiral symmetry of QCD.

DGMLY sum rule in the chiral limit [Das,Guralnik,Mathur,Low,Young (1967)]

$$m_{\pi^\pm}^2 = -\frac{3\alpha}{4\pi}\int_0^\infty dq^2\,\frac{q^2\,\Pi^{(1)}_{V-A}(q^2)|_{m_q=0}}{f^2}$$

<*VV-AA*> comes in again.

With different method,
Duncan, Eichten ,Thacker(1998), Blum, Doi, Hayakawa, Izubuchi, Yamada(2007), Namekawa, Kikukawa(2006)

In this work

- Consider two-flavor QCD as TC, and calculate $\Pi_{\mathrm{V-A}}(q^2)$ on the lattice.
- Evaluate
  - ✓ L_{10} (or S-parameter) through

$$\Pi^{(1)}_{V-A}(q^2) = -\frac{f_\pi^2}{q^2} - 8\,L^r_{10}(\mu) - \frac{\ln\left(\dfrac{m_\pi^2}{\mu^2}\right) + \dfrac{1}{3} - H(4m_\pi^2/q^2)}{24\pi^2}$$

  - ✓ $m_{\pi^\pm}{}^2$ (or pseudo-NG boson mass) from

$$m^2_{\pi^\pm} = -\frac{3\alpha}{4\pi}\int_0^\infty dq^2\,\frac{q^2\,\Pi^{(1)}_{V-A}(q^2)|_{m_q=0}}{f^2}$$

- Compare with their experimental values.

Demonstrate the feasibility of the lattice technique for these quantities.

$\langle VV-AA \rangle$ on the lattice

- In continuum, WT Identity guarantees that $\langle VV-AA \rangle$ vanishes if there is no spontaneous nor explicit χ-sym breaking.
- If the lattice formulation explicitly breaks χ-sym, it is difficult to disentangle the effect of the SχSB from the explicit breaking due to the lattice artifact.
- Exact χ-sym is required in this calculation to extract the physic from $\langle VV-AA \rangle$. [Sharpe(2007)]

⬇

Overlap fermion formalism

Simulation Parameters

β	2.30			
# of site	$16^3 \times 32$			
Gauge	Iwasaki + extra Wilson quarks + ghosts ($m_0 = 1.6$)			
dynamical and valence quarks	overlap ($m_0 = 1.6$)			
m_{sea}	0.015	0.025	0.035	0.050
# of traj.	10,000	10,000	10,000	10,000

r_0=0.49 fm ⇒ a=0.1184(12)(11) fm (1/a=1.67(2)(2) GeV)

$(L/a)^3$ x $(T/a)=16^3 \times 32 \Rightarrow V \approx (1.9\ fm)^3$

- lightest pion ⇒ $m_\pi \approx 290$ *MeV*, $m_\pi L \approx 2.8$.
- *Calculation is done in a fixed topological sector $Q_{top}=0$.*

Current correlator in continuum

$$i\int d^4x\, e^{iq\cdot x}\langle\, 0\,|\, T\left[J_\mu(x)J_\nu^\dagger(0)\right]|\, 0\,\rangle = \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right)\Pi_J^{(1)}(q^2) - q_\mu q_\nu \Pi_J^{(0)}(q^2),$$

$$J_\mu(x) = \begin{cases} V_\mu(x) = \bar{q}_1(x)\gamma_\mu q_2(x), \\ A_\mu(x) = \bar{q}_1(x)\gamma_\mu\gamma_5\, q_2(x), \end{cases}$$

Current correlator on the lattice

$$\Pi_{J\mu\nu}(\hat{q}) = \sum_x e^{i\hat{q}\cdot x}\langle 0|\, T\left[J_\mu^{(21)}(x) J_\nu^{(12)}(0)\right]|0\rangle$$
$$= \sum_{n=0}^{\infty} B_J^{(n)}(\hat{q}_\mu)^{2n}\delta_{\mu\nu} + \sum_{n,m=1}^{\infty} C_J^{(n,m)}(\hat{q}_\mu)^{2n-1}(\hat{q}_\nu)^{2m-1}$$

$V_\mu^{(12)} = Z\,\bar{q}_1\gamma_\mu(1 - aD/2m_0)q_2$ and similarly defined $A_\mu^{(12)}$

Z=1.3842(3) is common, and determined nonperturbatively.

- The currents are not conserved ones. c.f. [Kikukawa, A. Yamada (1999)]
- Many terms representing lattice artifacts show up.
  (only $B_J^{(0)}$ & $C_J^{(1,1)}$ are physically relevant.)
- But the exact symmetry between V_μ and A_μ simplifies the analysis!

Cancellation of the artifacts in $\Pi_{V\text{-}A}$

With our V_μ and A_μ, $\langle VV{-}AA\rangle$ exactly vanishes in the absence of both explicit and spontaneous breakings as in continuum.

The artifacts arising in short distance vanishes in $\langle VV{-}AA\rangle$.

The artifacts coupling to long distance physics are numerically investigated, and found to be negligibly small in $\langle VV{-}AA\rangle$.

Therefore, we write $\langle VV{-}AA\rangle$ as

$$\Pi_{V\,\mu\nu} - \Pi_{A\,\mu\nu} = \left(\hat{q}^2\delta_{\mu\nu} - \hat{q}_\mu\hat{q}_\nu\right)\Pi^{(1)}_{V-A} - \hat{q}_\mu\hat{q}_\nu\Pi^{(0)}_{V-A}$$

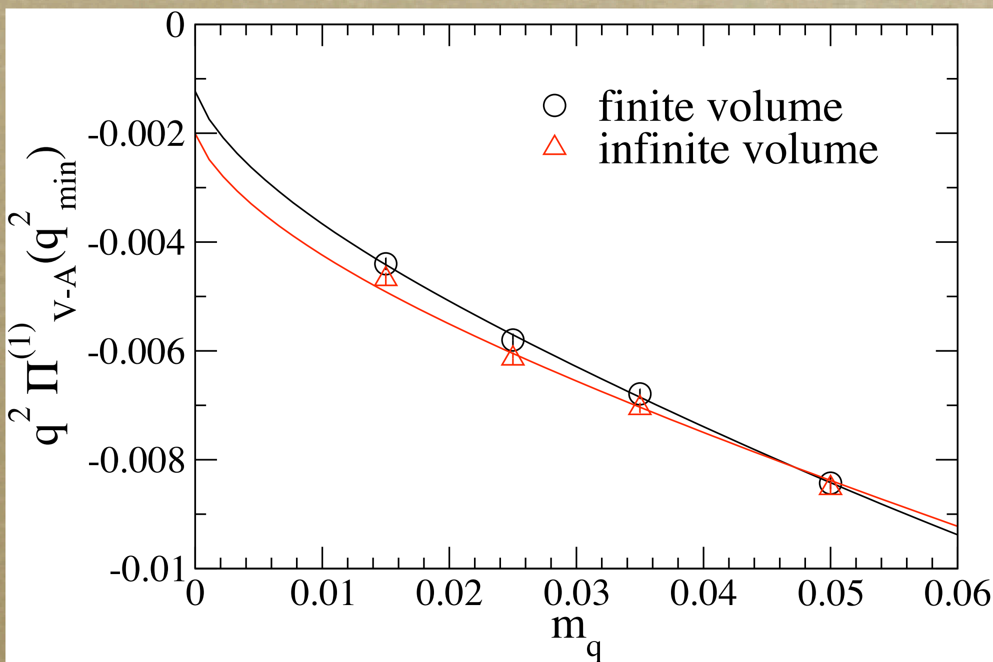
By considering $\mu=\nu$ and $\mu\neq\nu$, we extract $\Pi^{(0)}_{V-A}(q^2)$, $\Pi^{(1)}_{V-A}(q^2)$

L_{10} from $\Pi_{V\text{-}A}^{(1)}(q^2)$

- ChPT predicts [Gasser & Leutwyler (1984)]

$$\Pi^{(1)}_{V-A}(q^2) = -\frac{f_\pi^2}{q^2} - 8\,L^r_{10}(\mu_\chi) - \frac{\ln\left(\frac{m_\pi^2}{\mu_\chi^2}\right) + \frac{1}{3} - H(x)}{24\pi^2}$$

($x=4m_\pi^2/q^2$, $H(x)$ is known function.)



Fit the data to the ChPT prediction using the measured f_π and m_π.

$L_{10}(m_\rho) = -5.2(2)(^{+0}_{-3})(^{+5}_{-0})\times10^{-3}$
($\chi^2/dof=0.5,\ 2.3$)

$L_{10}(Exp) = -5.09(47)\times10^{-3}$

Pseudo-NG boson mass

$$m_{\pi^\pm}^2 = -\frac{3\alpha}{4\pi}\int_0^\infty dq^2\, \frac{q^2\,\Pi_{V-A}^{(1)}(q^2)|_{m_q=0}}{f^2}$$

Integral region is separated at q^2=2 to avoid discretization effects.

‣ Small q^2 region: Integrate fit func.

Functional forms adopted :

$$\hat{q}^2\Pi_{V-A}^{(1),\mathrm{fit}}(\hat{q}^2) \;=\; -\hat{f}_\pi^2 + \frac{\hat{q}^2\hat{f}_V^2}{\hat{q}^2+\hat{m}_V^2} - \frac{\hat{q}^2\hat{f}_A^2}{\hat{q}^2+\hat{m}_A^2} - \frac{\hat{q}^2}{24\pi^2}\,\frac{X(\hat{q}^2)}{1+x_5\,(Q_\rho^2)^4},$$

where $Q_\rho^2 = \hat{q}^2/(a^2 m_\rho^2)$

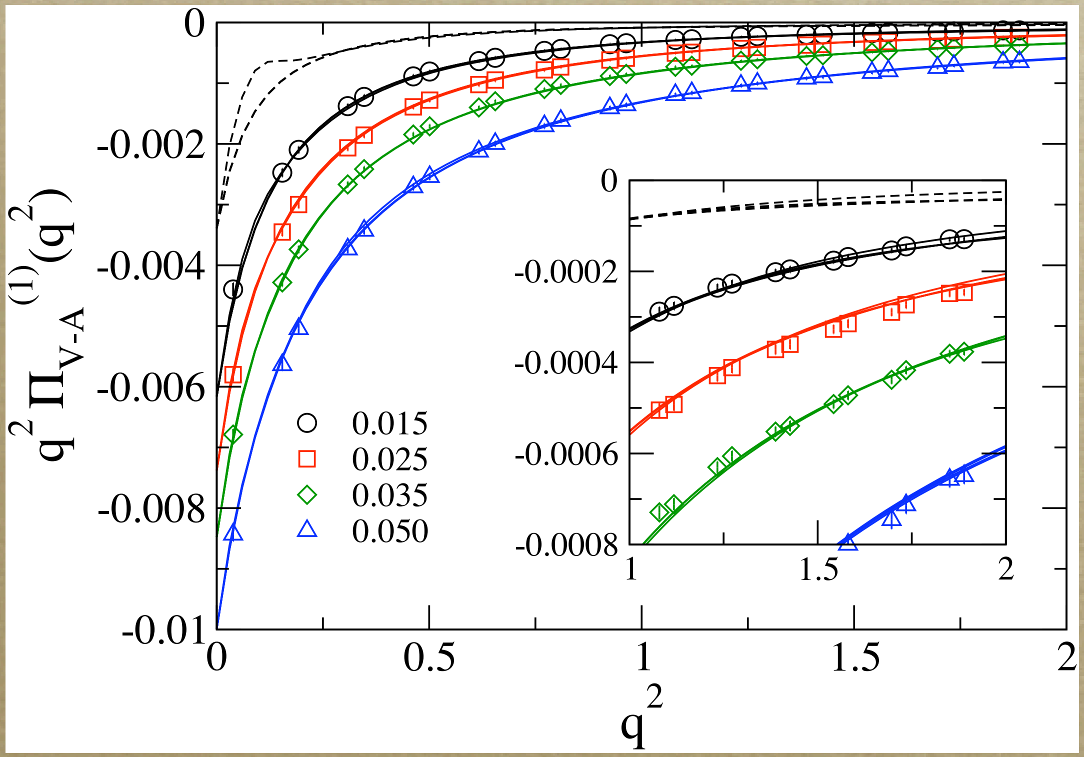
- 1st and 2nd Weinberg sum rules are imposed.

$$\hat{f}_\pi^2 = \hat{f}_V^2 - \hat{f}_A^2, \quad \hat{f}_A\hat{m}_A = \hat{f}_V\hat{m}_V,$$
$$\hat{f}_V = x_1 + x_3\,\hat{m}_\pi^2, \quad \hat{m}_V = x_2 + x_4\,\hat{m}_\pi^2$$

- $X(q^2)$ are chosen to be consistent with the ChPT (OPE) prediction in small (large) q^2.

$$X(q^2) = \begin{cases} \ln\left(\dfrac{\hat{m}_\pi^2}{\hat{m}_\rho^2}\right) + \dfrac{1}{3} - H(4\hat{m}_\pi^2/\hat{q}^2) + x_6\,Q_\rho^2 \\ x_6\,Q_\rho^2\ln(Q_\rho^2). \end{cases}$$

Pseudo-NG boson mass



- Small q^2 region: Integrate fit func.

$$\Delta m_\pi^2|_{\hat{q}^2\leq 2.0} = 676(50) \text{ and } 811(12)\,\text{MeV}^2$$

- Large q^2 region: OPE predicts

$$\Pi^{(1)}_{V-A}(q^2) \sim a_6/(q^2)^3$$

a_6 = [–0.001, –0.01] GeV^6

$$\Delta m_\pi^2|_{q^2\geq 2.0} = 182(149)\ \text{MeV}^2$$

$$\Delta m_\pi^2 = 993(12)(^{+\ \ 0}_{-135})(149)\ \text{MeV}^2$$

Errors: *(statistical)(chiral extrapolation)(large q^2)*

Exp: $\Delta m_\pi^2 = 1261.2\ \text{MeV}^2$

Summary

- We used overlap fermion to calculate the S-parameter and pNG boson mass in 2-flavor QCD. Chiral symmetry on the lattice is essential in this calculation.
- Both the calculations reasonably reproduced the experimental values. Thus the feasibility of the LGT to calculate these quantities is demonstrated.
- The study of more realistic TC models is an interesting extension.
- LGT may be able to directly investigate physical quantities relevant for the LHC phenomenology.

Cancellation of the artifacts in *VV-AA*

- Define a measure of artifacts

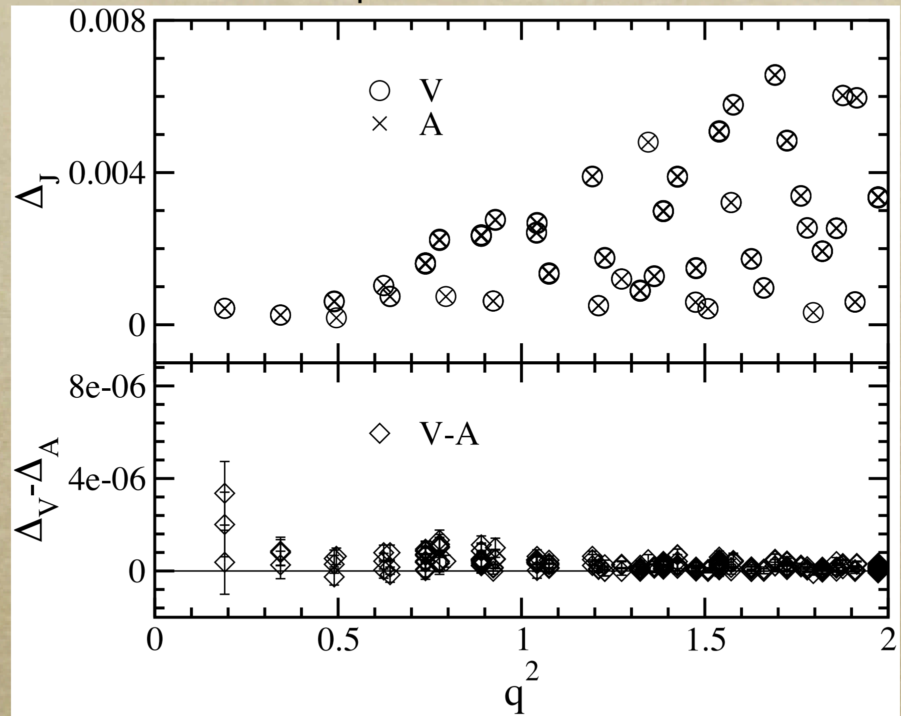
$$\Delta_J = \sum_{\mu,\nu} \hat{q}_\mu \hat{q}_\nu \left(\frac{1}{\hat{q}^2} - \frac{\hat{q}_\nu}{\sum_\lambda (\hat{q}_\lambda)^3} \right) \Pi_{J\,\mu\nu}$$

$$\Delta_J = \sum_{n=1}^{\infty} B_J^{(n)} \left(\frac{Q^{(2n+2)}}{q^2} - \frac{Q^{(2n+3)}}{Q^{(3)}} \right) + \sum_{n,m=1}^{\infty} C_J^{(n,m)} Q^{(2n)} \left(\frac{Q^{(2m)}}{q^2} - \frac{Q^{(2m+1)}}{Q^{(3)}} \right)$$

$(n = m = 1 \text{ is not included})$

Δ_J entirely consists of lattice artifacts!

Results at m_q=0.015 is shown.



In the difference〈VV−AA〉, irrelevant terms cancel!

$\Pi_{\text{V-A}}{}^{(0)}(q^2)$

In the spectral representation,

$$q^2\Pi^{(0)}_{V-A}(q^2) = \frac{f_\pi^2 m_\pi^2}{q^2 + m_\pi^2} + (\text{excited states} \sim O(m_q^2))$$

- The obtained $\Pi_{V\text{-}A}{}^{(0)}(q^2)$ is compared to the spectral rep.
- For f_π and m_π, the measured values are used.