Nucleon Wave Function from Lattice QCD

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Nucleon puzzle

What we really want:

Nucleon wave function

What we really get:

Lowest moments of the Nucleon Distribution Amplitude (NDA)

- ★ What are NDA's
- ★ How to calculate their moments on the lattice
- ★ Possible forms of the NDA's
- ★ From NDA to experiment

Simplifing the Problem

"What is the Nucleon Wave Function"

$$\Psi_{\mathrm{BS}}(x_i) = \langle 0 | T \left[q(x_1, k_{1,\perp}) q(x_2, k_{2,\perp}) q(x_3, k_{3,\perp}) \right] |p\rangle$$

x_i	–	longitudinal momentum fractions of partons
$k_{i,\perp}$	–	transverse momenta of quarks
$\|p\rangle$	–	baryon state with momentum p (e.g., Proton).

"Distribution Amplitude(s)"

$$\Phi(x_i, \mu) = Z(\mu) \int^{|k_\perp| \le \mu} [dk_{i,\perp}] \, \Psi_{\mathrm{BS}}(x_i, k_\perp)$$

Further simplifications:

- $\hookrightarrow$ Twist expansion
- $\hookrightarrow$ Expansion in local operators (moments of DA's)

Moments

"Expansion in **local** operators"

$$\langle 0|\epsilon^{abc}[i^l D^{\lambda_1}\dots D^{\lambda_l} q^a_\alpha(0)](C\gamma^\rho)_{\alpha\beta}[i^m D^{\mu_1}\dots D^{\mu_m} q^b_\beta(0)][i^n D^{\nu_1}\dots D^{\nu_n}(\gamma_5 q^c(0))_\gamma]|p\rangle$$
$$= - f_V V^{lmn} p^\rho p^{\lambda_1}\dots p^{\lambda_l} p^{\mu_1}\dots p^{\mu_m} p^{\nu_1}\dots p^{\nu_n} N_\gamma(p)$$

analogous for all Distribution Amplitudes

Moments of DA's:

$$V^{lmn} = \frac{1}{f_V}\int_0^1 [\mathrm{d}x]\; x_1^l x_2^m x_3^n V(x_1, x_2, x_3), \qquad [\mathrm{d}x] = \mathrm{d}x_1\; \mathrm{d}x_2\; \mathrm{d}x_3\; \delta(1-\sum_{i=1}^{3} x_i)$$

$\delta(1-\sum_{i=1}^{3} x_i)$	-	Momentum conservation
f_V	-	DA normalization constant

"Problem reduction for nucleon"

Two *up*-quarks, Isospin symmetry in the proton

$\hookrightarrow$ Only **one** Distribution amplitude:

$$\varphi^{lmn} = V^{lmn} - A^{lmn} \quad \text{or} \quad \phi^{lmn} = \frac{1}{3}(V^{lmn} - A^{lmn} + 2T^{lnm})$$

ϕ^{lmn} is preferable on the lattice. On the other hand

$$|p,\uparrow\rangle = \int_0^1 [\mathrm{d}x] \, \frac{\varphi(x_i)}{\sqrt{96 x_1 x_2 x_3}} |u^\uparrow(x_1) \left[u^\downarrow(x_2) d^\uparrow(x_3) - d^\downarrow(x_2) u^\uparrow(x_3)\right]\rangle$$

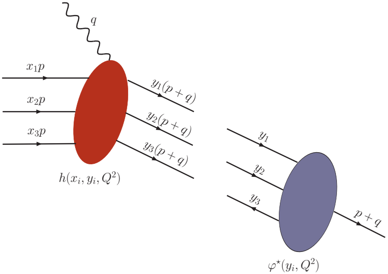
$\hookrightarrow$ φ^{100}, φ^{010} und φ^{001} - momentum fractions of quarks in nucleon.

Momentum conservation

$$\phi^{lmn} = \phi^{(l+1)mn} + \phi^{l(m+1)n} + \phi^{lm(n+1)} \qquad \phi^{000} \equiv 1$$

Form Factors and Distribution Amplitudes

Factorization for (very) large Q^2



exclusive processes, hard scattering

Generalization of Form Factors.

Different form factors are related to each other through (one) distribution amplitude.

$$\varphi(x_i, Q^2 \to \infty) = 120 x_1 x_2 x_3 f_N$$

$$G_M(Q^2) = \int_0^1 [\mathrm{d}x] \int_0^1 [\mathrm{d}y] \varphi^\star(y_i, Q^2) h(x_i, y_i, Q^2) \varphi(x_i, Q^2)$$

Operator Choice

Reduced symmetry due to discretization $\hookrightarrow$
☹ additional operator mixing

Construction of proper operators required

- $\hookrightarrow$ Irreducibly transforming operators have "minimal" mixing
- $\hookrightarrow$ Construct such operators from DA's operators

Isospin 1/2: $\mathcal{O}_{1,I=1/2} = \mathcal{O}_1(u_\alpha u_\beta d_\gamma) + i\gamma_4 \mathcal{O}_1(u_\alpha d_\beta u_\gamma)$

Correlator ($t \to \infty$)

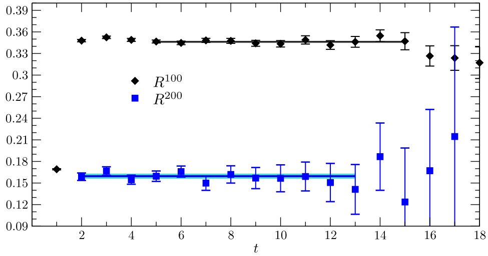
$$\langle \mathcal{O}_{1,I=1/2}(t)\bar{\mathcal{N}}(0)\rangle = f_N \phi^{lmn}(ip_1\gamma_1 - ip_2\gamma_2)(ip_3\gamma_3 - \gamma_4 E(\vec{p}))N(\vec{p})e^{-E(\vec{p})t}$$

$\hookrightarrow \vec{p} \neq 0 \not\leftrightarrow$ "good" Mixing

Constrained Analysis

DA correlator ratios

$$R^{lmn} = \frac{\phi^{lmn}}{S_{(l+m+n)}}$$



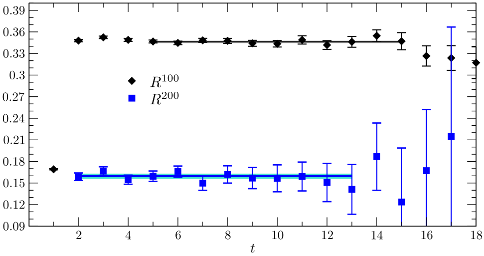
$$S_1 = \phi^{100} + \phi^{010} + \phi^{001}$$

$$S_2 = \phi^{200} + \phi^{020} + \phi^{002} + 2(\phi^{011} + \phi^{101} + \phi^{110})$$

Constrained Analysis

DA correlator ratios

$$R^{lmn} = \frac{\phi^{lmn}}{S_{(l+m+n)}}$$

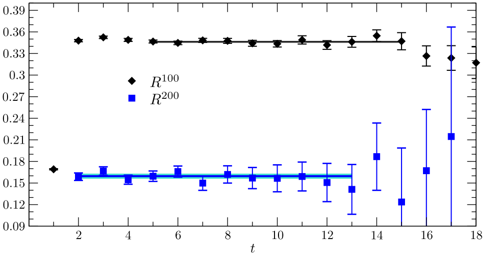


Problem: normalization of ϕ^{lmn} lost

Constrained Analysis

DA correlator ratios

$$R^{lmn} = \frac{\phi^{lmn}}{S_{(l+m+n)}}$$



Solution:

$$S_1 = \phi^{100} + \phi^{010} + \phi^{001} \equiv 1$$

Restore the normalization from the constraint

$$\sum_{l+m+n=1} \phi^{lmn} = \frac{1}{\sum_{l+m+n=1} Z_{lmn} R^{lmn}}$$

Z^{lmn}: Renormalization constants

Just Numbers

Results from QCDSF/DIK configurations at $\beta = 5.40$

Comparison with estimates at 1 GeV

	Asympt.	QCD-SR	BK	BLW	Latt. N	Latt. $N^\star(1535)$
φ^{100}	$\frac{1}{3} \approx 0.333$	0.560(60)	0.38	0.415	0.3999(13)(126)	0.4765(18)(92)
φ^{010}	$\frac{1}{3} \approx 0.333$	0.192(12)	0.31	0.285	0.2986(22)(111)	0.2523(28)(111)
φ^{001}	$\frac{1}{3} \approx 0.333$	0.229(29)	0.31	0.300	0.3015(9)(18)	0.2712(10)(29)
φ^{200}	$\frac{1}{7} \approx 0.143$	0.350(70)	$0.18^\star$	0.212	0.1792(26)(157)	0.2289(40)(71)
φ^{020}	$\frac{1}{7} \approx 0.143$	0.084(19)	$0.13^\star$	0.123	0.1459(66)(63)	0.0839(80)(264)
φ^{002}	$\frac{1}{7} \approx 0.143$	0.109(19)	$0.13^\star$	0.132	0.1354(42)(270)	0.1005(38)(176)
φ^{011}	$\frac{2}{21} \approx 0.095$	−0.030(30)	$0.08^\star$	0.053	0.0491(54)(351)	0.0457(58)(200)
φ^{101}	$\frac{2}{21} \approx 0.095$	0.102(12)	$0.10^\star$	0.097	0.1171(21)(66)	0.1250(29)(44)
φ^{110}	$\frac{2}{21} \approx 0.095$	0.090(10)	$0.10^\star$	0.093	0.1037(34)(266)	0.1227(50)(116)

From the moments to the DA

NDA is not completely determined by the (lowest) moments.
$\hookrightarrow$ Additional input required!

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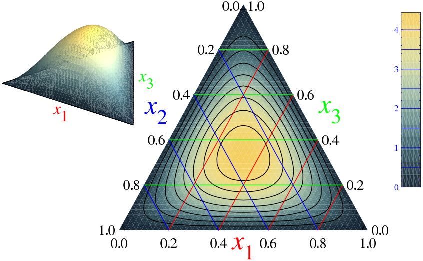
From renormalization group

$$\begin{aligned}\varphi(x_1,x_2,x_3,\mu) =& 120 x_1 x_2 x_3 f_N(\mu_0) L^{\frac{2}{3\beta_0}} \{1 + h_{10}(\mu_0)(x_1 - 2x_2 + x_3) L^{\frac{8}{3\beta_0}} \\ &+ h_{11}(\mu_0)(x_1 - x_3) L^{\frac{20}{9\beta_0}} \\ &+ h_{20}(\mu_0)\left[1 + 7(x_2 - 2x_1x_3 - 2x_2^2)\right] L^{\frac{14}{3\beta_0}} \\ &+ h_{21}(\mu_0)(1 - 4x_2)(x_1 - x_3) L^{\frac{40}{9\beta_0}} \\ &+ h_{22}(\mu_0)\left[3 - 9x_2 + 8x_2^2 - 12x_1x_3\right] L^{\frac{32}{9\beta_0}} \Big\}\end{aligned}$$

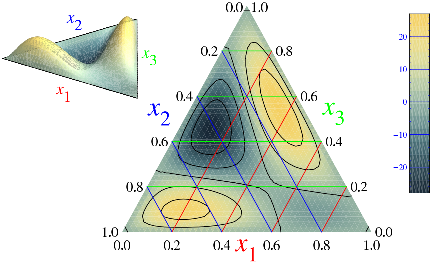
h_{ij} determined by ϕ^{lmn} $\qquad L = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}$ $\qquad \beta_0 = 11 - \frac{2}{3} n_f$

Next^2-to-leading conformal spin

Asymptotic form

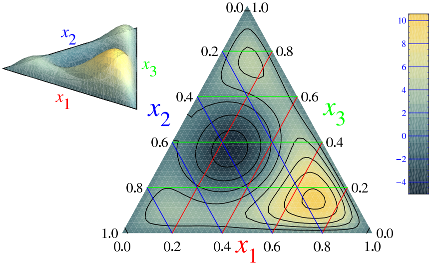


Nucleon model (ϕ^{101}, ϕ^{002}, ϕ^{200})



Next²-to-leading conformal spin

QCD-SR inspired form



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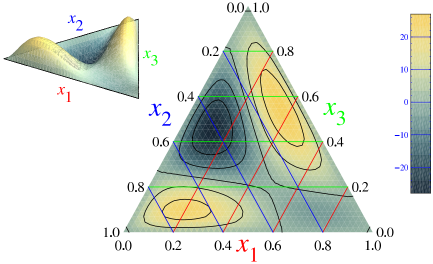
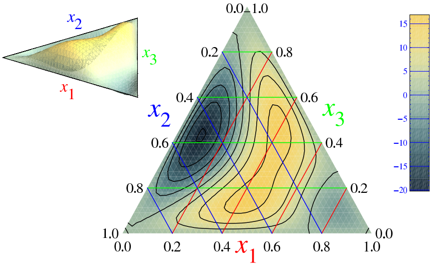
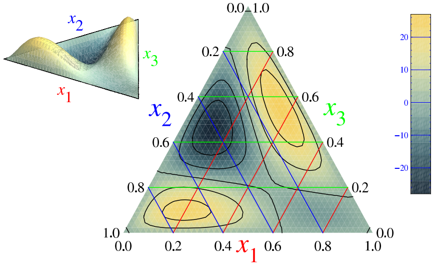


Next^2-to-leading conformal spin

$N^\star(1535)$ model (ϕ^{101}, ϕ^{002}, ϕ^{200})



Nucleon model (ϕ^{101}, ϕ^{002}, ϕ^{200})



Conclusions

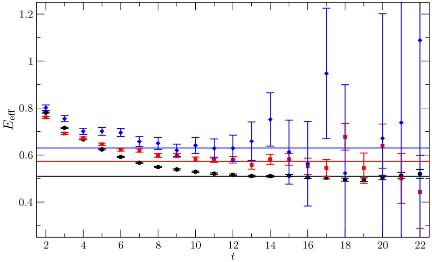
- Lattice QCD at the moment the method of choice for DA
- Accuracy improved by explicitly exploiting momentum conservation.
- Asymmetries are less pronounced compared to QCD-SR
- Symmetry in $u^{\downarrow}$ and $d^{\uparrow}$ (diquark)
- In good agreement with phenomenological values

The usual approach

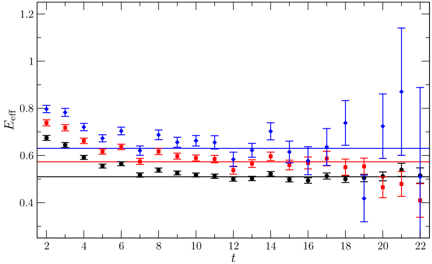
Effective energies for

required correlators

DA Correlators



Nucleon correlators



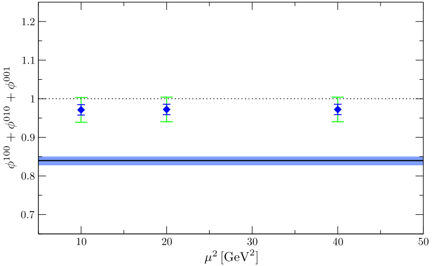
Nucleon correlator adds noise but does not really contain any additional information

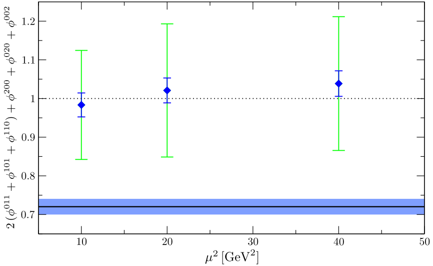
Quality of the results

Sum of the moments

$\phi^{100} + \phi^{010} + \phi^{001}$

$\phi^{200} + \phi^{020} + \phi^{002} + 2(\phi^{011} + \phi^{101} + \phi^{110})$





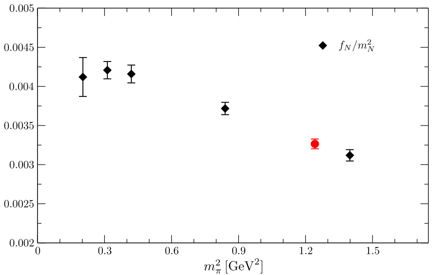
Errors are too large for the interesting quantities (asymmetries).

Chiral extrapolation I

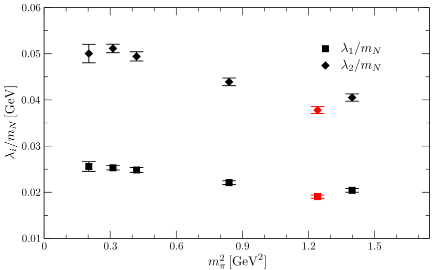
No results from χPT

Normalization constants

Leading twist



Next-to-leading twist



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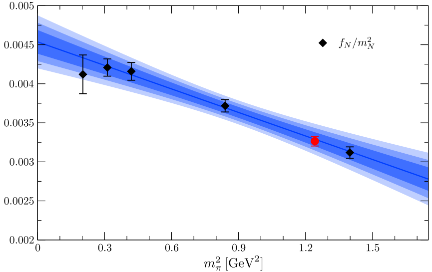

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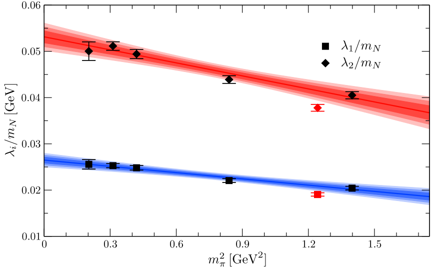
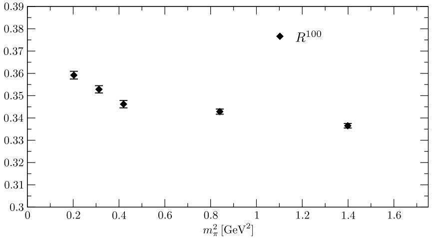


Chiral extrapolation II

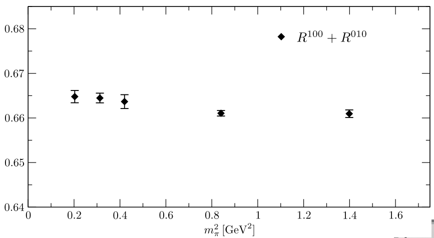
R^{100} and R^{010} not really linear
No strong deviation for other moments

First moments

R^{100}



$R^{100} + R^{010}$



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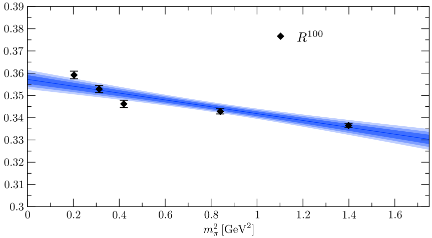
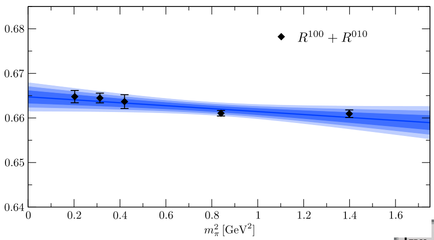
First moments

R^{100} (linear)



$R^{100} + R^{010}$ (linear)



Chiral extrapolation II

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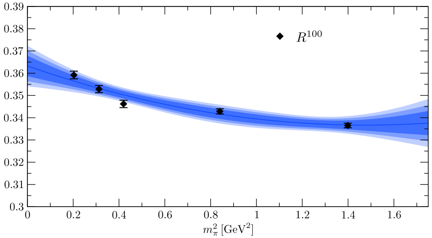
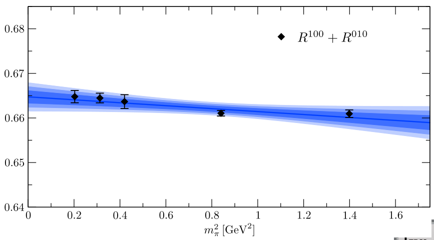
No strong deviation for other moments

First moments

R^{100} (quadratic)



$R^{100} + R^{010}$ (linear)

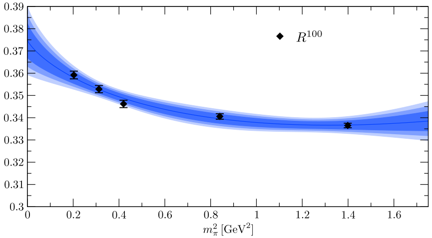


Chiral extrapolation II

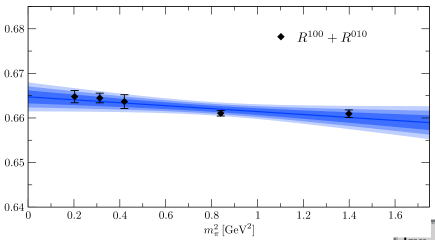
R^{100} and R^{010} not really linear
No strong deviation for other moments

First moments

R^{100} ("chiral log")



$R^{100} + R^{010}$ (linear)

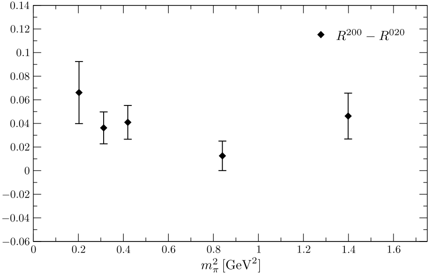


Chiral extrapolation III

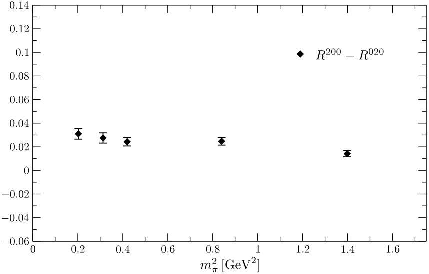
Asymmetries are the important quantities

For the second moments

Standard approach



Constrained analysis



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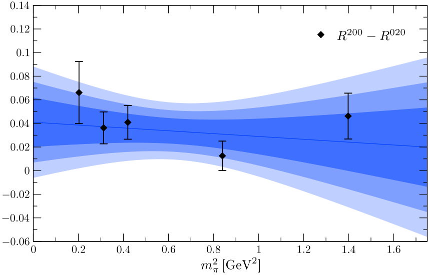
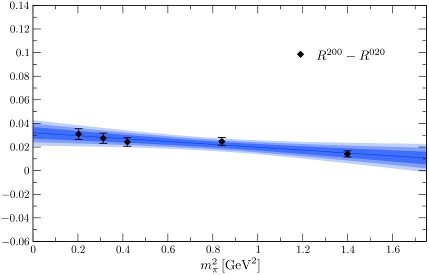


Constrained analysis



THANK YOU