Bottom spectroscopy on dynamical 2+1 flavor domain wall fermion lattices with a relativistic heavy quark action

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Heavy quarks

- Challenge:
  - Charm and Bottom quarks are too heavy for current lattice ensembles: m∼1/a
- Solutions:
  - Heavy quark effective theory (HQET)
  - Non-relativisitic QCD (NRQCD)
  - Relativisitic Heavy Quarks/ Fermilab (RHQ)
- RHQ action

$$S = \sum \bar{\psi}(m_0 a + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - \frac{1}{2} r_t (D^0)^2 - \frac{1}{2} r_s (\vec{D})^2 + \sum_{\mu,\nu} \frac{i}{4} c_P \sigma_{\mu\nu} F_{\mu\nu})\psi$$

(A. El-Khadra et al.(1997), S. Aoki et al.(2003), N. Christ et al.(2007))

  - Works for all lattice spacings and allows continuum limit.
  - Supports non-perturbative methods.
  - Only three paramters need to be tuned.
  - Errors of order $O((\vec{p}a)^2)$.

Calculations in full QCD

Instead of using perturbation theory or step scaling to calculate the spectrum from first principles. We match our calculation to experimental data to calibrate the RHQ action.

- Determine the RHQ parameters for heavy quark systems, with the lattice spacing from other methods. (at least 3 quantities needed)
- Predict other quantities of interest using the determined RHQ parameters.
- Determine the lattice scale together with the RHQ parameters. (at least 4 quantities needed)

What's been done on Charm

- RHQ paramteters are determined($\sim$1%) from charmonium spectrum and extrapolated to chiral limit.
  $m_0 a = 0.251(9)$ $c_P = 2.091(17)$ $\zeta = 1.242(10)$
- χ_{c0} and χ_{c1} masses are predicted in the chiral limit with less than 1% error.
  $m_{\chi_{c0}} = 3.424(11)$GeV exp. 3.415GeV
  $m_{\chi_{c1}} = 3.502(14)$GeV exp. 3.511GeV
- Lattice scale is determined from the charmonium and charm strange spectrum, also with errors $\sim$ 1%.
  $a^{-1} = 1.749(14)$GeV or $a^{-1} = 1.730(23)$GeV (diag corr. matrix) which are consistent with 1.73(2)GeV from Ω baryon.

(M. Li and H. Lin, arxiv:0710.0910 (hep-lat), lattice 07 proceeding)

Bottom in this work

- Explore the validity of this method in a regime with larger heavy quark momenta.
- Bottom-light has smaller discretization errors ($p \sim \Lambda_{QCD}$), thus is used to determine the RHQ paramters.
- Bottomonium states are predicted and compared to experimental numbers.
- Theoretical estimation of the errors is carried out to understand the $O((\vec{p}a)^2)$ systematic errors found in the numerical study.

Quantities calculated

- Spin-averaged (η_b,Υ,B_s,B_s^*)

$$m_{sa}^{hh} = \frac{1}{4}(m_{PS}^{hh} + 3m_{V}^{hh}) \quad m_{sa}^{hl} = \frac{1}{4}(m_{PS}^{hl} + 3m_{V}^{hl}) \tag{1}$$

- Hyperfine splitting

$$m_{hs}^{hh} = m_{V}^{hh} - m_{PS}^{hh} \quad m_{hs}^{hl} = m_{V}^{hl} - m_{PS}^{hl} \tag{2}$$

- Dispersion relation (mass ratio)

$$E^2 = m_1^2 + \frac{m_1}{m_2}p^2 \tag{3}$$

- Spin-orbit averaged and splitting (χ_{b0} and χ_{b1})

$$m_{sos}^{hh} = m_{AV}^{hh} - m_{S}^{hh} \tag{4}$$

$$m_{soa}^{hh} = \frac{1}{4}(m_{S}^{hh} + 3m_{AV}^{hh}) \tag{5}$$

- Heavy-heavy $^1P_1$ state(h_b)

Determination of the RHQ parameters

- Rough search on 16^3 lattices with initial parameters from tree level.
- Linear approximation in the appropriate region

$$Y(a) = \begin{pmatrix} m_1 a \\ m_2 a \\ m_3 a \\ m_4 a \\ 1 \end{pmatrix} = J \cdot \begin{pmatrix} m_0 a \\ c_P \\ \zeta \end{pmatrix} + A$$

Obtain parameters and a by minimizing the χ^2 defined as:

$$\chi^2 = (J \cdot X + A - Y(a))^T W^{-1} (J \cdot X + A - Y(a))$$

it is a quadratic function of

  - $X = (m_0 a, c_P, \zeta)^T$ if a is known,
  - $X^{New} = (m_0 a, c_P, \zeta, a)^T$ if a is unknown.

- J and A are determined from finite difference approximation to Y derivatives w.r.t. $\{m_0 a, c_P, \zeta\} \sim \{7.3(\pm 0.5), 4.0(\pm 1.0), 4.3(\pm 0.3)\}$.

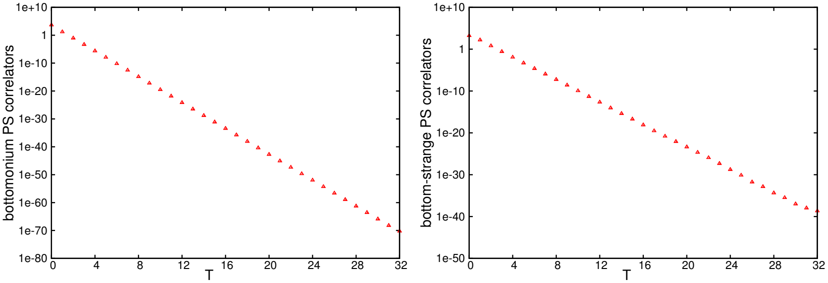
Dynamical DWF lattices

- IWASAKI β=2.13 lattices

volume	L_s	(m_{sea}, m_s)	Traj(step)	# of configs
$24^3 \times 64$	16	(0.005,0.04)	900-6880(20)	300x2
$24^3 \times 64$	16	(0.01,0.04)	1460-5060(40)	91x4
$24^3 \times 64$	16	(0.02,0.04)	1885-3605(20)	87x4

- For the m_{sea}=0.01 and 0.02 ensembles, we placed the sources at time 0 as well as 16, 32 and 48, so there are 4x statistics hidden for these two cases. For the m_{sea}=0.005 ensemble we place sources at 0 and 32 for each configuration.

Sample bottom correlators

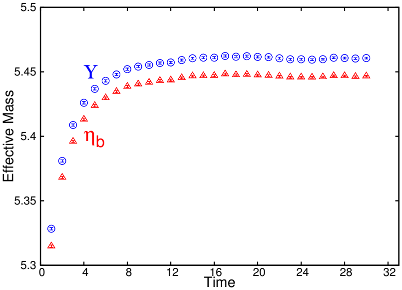


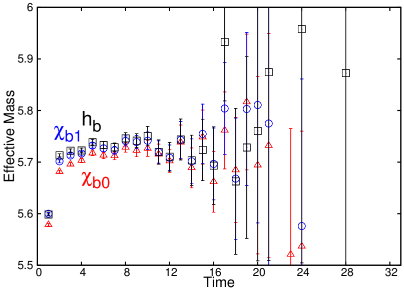
Here are some sample plots of the bottomonium and bottom strange pseudo-scalar correlators. One should notice the correltors are falling in orders of magnitude about 70 for bottomonium and 40 for bottom strange.

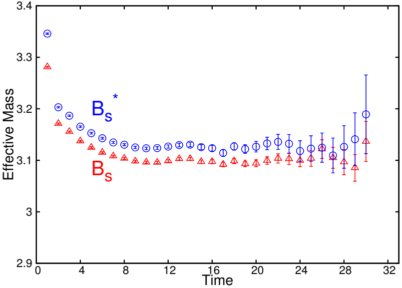
Numerical run details

- Box source with size 4
- Quark propagator precision.
  - Heavy propagator: Extreme CG stopping condition (10^{-60}) to ensure accuracy.
  - Light propagator: CG stopping condition 10^{-10}.
- Mass ratio m_1/m_2.
  - Obtained from Υ momentum dependence.
  - Only three smallest momentum are used and the fit is uncorrelated.
- Fitting ranges.
  - All fitting ranges for the correlators' time dependences are chosen from a close examination of the corresponding effective mass plot.

Effective masses







Sample effective mass plots for η_b,Υ,χ_{b0},χ_{b1},B_s and B_s^* at $m_{sea} = 0.005$. The fitting ranges are 14-30 for η_b and Υ; 5-12 for χ_{b0} and χ_{b1}; 10-25 for B_s and B_s^*. Masses for χ_{b0} and χ_{b1} might be subject to more systermatic errors as the plateaus are less manifest.

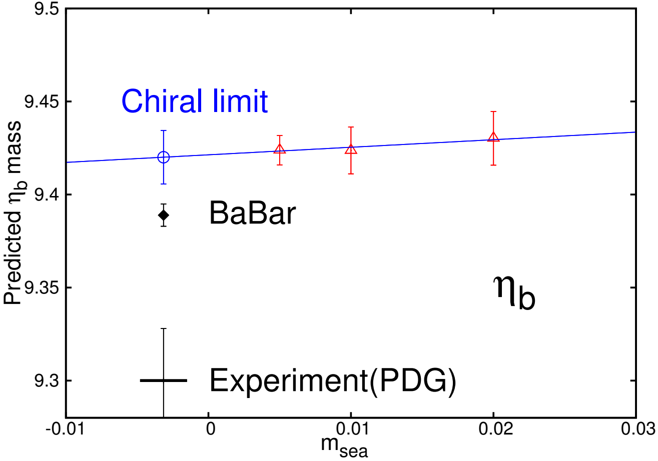
RHQ parameters

Determined RHQ parameters using quantities m_{B_s},$m_{B_s^*}$ and m_1/m_2 from Υ meson momentum dependence, and a^{-1}=1.73GeV is assumed from Ω baryon/charm study.

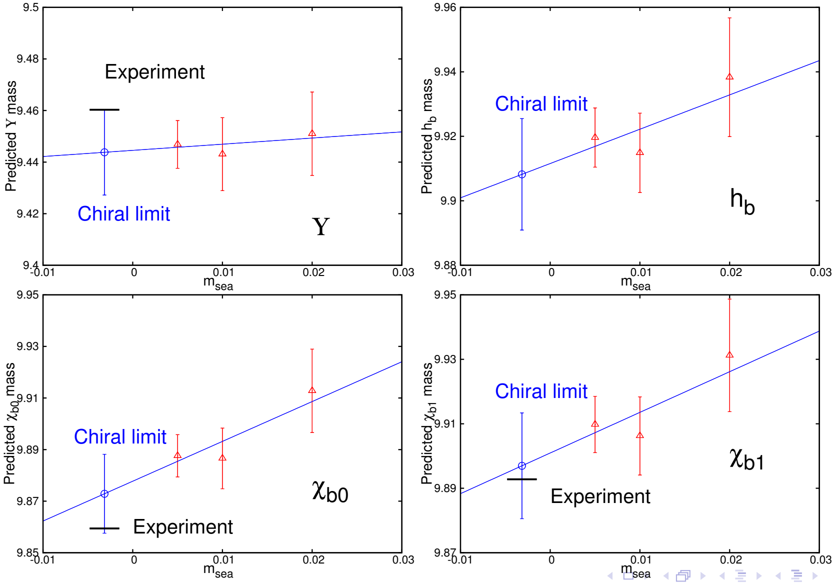
m_{sea}	$m_0 a$	c_P	ζ
0.005	7.37(7)	3.84(40)	4.21(3)
0.01	7.28(9)	3.28(40)	4.21(3)
0.02	7.30(11)	3.52(53)	4.24(4)
$-m_{res}$	7.38(12)	3.93(54)	4.19(4)

Note: m_0 is around 12.7GeV, which indicates serious distortion from the ma dependence in this regime.

Predictions



Predictions cont'd



Predictions cont'd

quantities	RHQ(MeV)	Exp.(MeV)	NRQCD(MeV)
m_{η_b}	9420(14)	9389(3)(3)†	
m_{Υ}	9444(17)	9460	
$m_{\chi_{b0}}$	9873(15)	9859	
$m_{\chi_{b1}}$	9897(16)	9893	
m_{h_b}	9908(17)	-	9900(3)(6)^{*1}
$m_{\Upsilon} - m_{\eta_b}$	23.7(3.7)	71(3)(3)†	61(14)*
$m_{\chi_{b1}} - m_{\chi_{b0}}$	24.0(3.5)	33.34	

Note:†: Numbers are from the new results from the BaBar collaboration(arxiv:0807.1086). The PDG number for η_b is 9300(28)MeV (from a single event) indicating a hyperfine splitting of 160(28)MeV.

Note: Our results only include the statistical errors.

* A Gray et al. Phys.Rev.D72:094507,2005 (hep-lat/0507013v2), errors include statistical, fitting and discretization errors, as well as radiative and relativistic corrections.

1 where 3 is the experimental error(PDG2004).

Theoretical estimation

Our errors for the bottomonium spectrum are typically on the order of 20-30MeV!

- Rough estimate of some a^2 operators (eg. $\hat{O} = \overline{\Psi}\vec{\gamma}\cdot\vec{D}\,\vec{D}^2\Psi$)
  $mv^2 \sim (\Upsilon(2S) - \Upsilon(1S)) \sim 500\text{MeV} \rightarrow \langle\hat{O}\rangle \sim \frac{p^4 a^2}{m_b} \sim 300\text{MeV}$

  Why such small errors in numerical results??
- Hydrogen atom Coulomb model

$$
\begin{aligned}
|\Upsilon, m_j\rangle &= \int \frac{d^3\vec{p}_1 d^3\vec{p}_2}{(2\pi)^{9/2}} \sum_{s_1 s_2} \delta^{(3)}(\vec{P}_0 - \vec{p}_1 - \vec{p}_2)\phi(\frac{\vec{p}_1 - \vec{p}_2}{2})_{1S} \\
&\times \langle 1 m_j | s_1 s_2\rangle a^\dagger(\vec{p}_1, s_1) b^\dagger(\vec{p}_2, s_2)|0\rangle
\end{aligned}
$$

Where a and b are free field quark and anti-quark annihination operators as in

$$
\Psi(\vec{x}) = \int \frac{d^3\vec{p}}{\sqrt{2E_p}} \frac{1}{(2\pi)^3} \sum_s \{u^s(p)e^{i\vec{p}\cdot\vec{x}} a(\vec{p}, s) + v^s(p)e^{-i\vec{p}\cdot\vec{x}} b^\dagger(\vec{p}, s)\}
$$

$$
\Longrightarrow \langle\hat{O}\rangle \sim \frac{5}{8} m_b^3 \alpha_s^4 a^2 = \begin{cases} \sim 40MeV & m_b = 4.0GeV, \alpha_s = 0.25 \\ \sim 146MeV & m_b = 4.0GeV, \alpha_s = 0.35 \end{cases}
$$

Outlook

What could we do next?

- Use some phenomenological models which reproduce the bottom spectrum to estimate the $O((\vec{p}a)^2)$ errors more accurately.
- Heavy-light spectrum on different sea quark masses for both charm and bottom system.
- Predictions/calculations of more states, like χ_{b2}, $\overline{b}c$ mesons and nucleons with one or more charm quarks etc.
- Move on to $32^3 \times 64$ lattices.
- Calculate matrix elements.

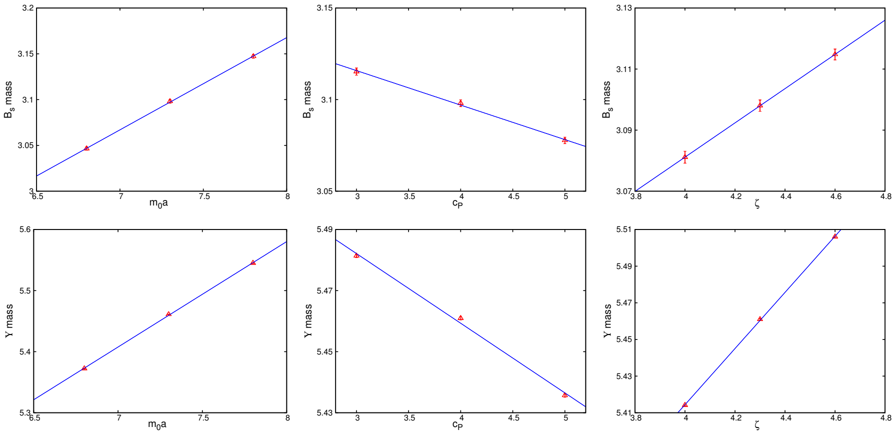
Summary

Conclusions:

- (m_0a, c_P, ζ) is determined by matching to physical quantities and extrapolated to chiral limit for the bottom system.
- Predictions of individual masses mostly agreed with the experiment and indicated small discretization errors (<30MeV). Theoretical estimation using Coulomb model needs accurate coupling constant α_s. Some phenomenological models might help to do a better estimate.
- Also achieved good precision of mass splittings (4MeV), but both of them deviate from the experimental values.
- Calculations for bottom-light system would double check the validity of the method in this regime and give more accurate results for both parameters and predictions.

Backup Slides

Linearity test



Lattice scale

Lattice scale and RHQ paramters determined from m_{B_s},$m_{B_s^*}$,m_{Υ} and m_1/m_2 and extrapolated to the chiral limit.

m_{sea}	$m_0 a$	c_P	ζ	a^{-1}GeV
0.005	7.72(59)	4.10(25)	4.33(23)	1.71(4)
0.01	7.72(57)	3.61(31)	4.36(21)	1.70(4)
0.02	7.46(62)	3.71(47)	4.30(24)	1.72(4)
-m_{res}	7.91(88)	4.27(45)	4.38(34)	1.70(6)