Determining bare quark masses for $N_f = 2 + 1$ dynamical simulations

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Overview

- Physics overview
- Lattice set-up: $N_f = 2 + 1$ dynamical QCD on anistropic lattices.
- How to match simulation data to the physical quark mass point.
- "Newport News" co-ordinates
- Mass ratio extrapolations: m_H / m_Ω
- **Preliminary** $N_f = 2 + 1$ spectrum
- r_0 determination
- Summary

Physics overview

- The first dynamical, $N_f = 2 + 1$, anisotropic lattice simulations.
- The anisotropic lattice is particularly well-suited to spectroscopy calculations
- The spectrum collaboration aims to investigate the spectrum and decays of light mesons and baryons, including isoscalar mesons.
- **First step:** how should the strange quark mass be set in a Wilson-like $N_f = 2 + 1$ setting?
- How should contact with the physical light and strange quark masses be made?

Anisotropic lattice action

$\mathcal{O}(a^2)$ tree-level improved gauge action

$$S_G^{\xi}[U] = \frac{\beta}{N_c}\left\{\frac{1}{\xi_0}\sum_{x,s>s'}\left[\frac{5}{3u_s^4}\mathcal{P}_{ss'}(x) - \frac{1}{12u_s^6}\mathcal{R}_{ss'}(x)\right] + \xi_0\sum_{x,s}\left[\frac{4}{3u_s^2u_t^2}\mathcal{P}_{st}(x) - \frac{1}{12u_s^4u_t^2}\mathcal{R}_{st}(x)\right]\right\},$$

- $\mathcal{P}$ is the 1×1 plaquette, $\mathcal{R}$ is the 2×1 rectangle
- $u_t = 1$, $u_s = \langle\Box\rangle^{1/4}$
- Configurations generated using RHMC.
- Action parameters needed to restore rotational symmetries are tuned non-perturbatively (Robert Edwards; previous talk).
- Perturbative determinations underway (Justin Foley's talk)

Anisotropic lattice action

$\mathcal{O}(a)$ Sheikholeslami-Wohlert improved quark action

$$S_F^{\xi}[U,\overline{\psi},\psi] = a_s^3 a_t \sum_x \overline{\psi}(x) Q \psi(x)$$

$$Q = \left[m_0 + \nu_t W_t + \nu_s W_s - \frac{a_s}{2} \left(c_{\rm t} \sigma_{st} F^{st} + \sum_{s<s'} c_{\rm s} \sigma_{ss'} F^{ss'} \right) \right]$$

- All links are spatially stoutened; $n_\rho = 2, \rho = 0.14$
- $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$, $F_{\mu\nu}(x) = \frac{1}{4}\mathrm{Im}(\mathcal{P}_{\mu\nu}(x))$
- $W_\mu = \nabla_\mu - \frac{a_\mu}{2}\gamma_\mu \Delta_\mu$
- $\nabla_\mu f(x) = \frac{1}{2a_\mu}\left[U_\mu(x) f(x+\mu) - U_\mu^\dagger(x-\mu) f(x-\mu) \right]$
- $\Delta_\mu f(x) = \frac{1}{a_\mu^2}\left[U_\mu(x) f(x+\mu) + U_\mu^\dagger(x-\mu) f(x-\mu) - 2f(x) \right]$

Simulation parameters

Volume	$a_t m_s^0$	$a_t m_l^0$	m_π / m_ρ
$12^3 \times 96$	-0.0539	-0.0539	0.833(7)
$12^3 \times 96$	-0.0539	-0.0698	0.742(9)
$12^3 \times 96$	-0.0539	-0.0793	0.69(2)
$12^3 \times 96$	-0.0539	-0.0825	0.59(2)
$16^3 \times 96$	-0.0539	-0.0825	0.61(2)
$12^3 \times 96$	-0.0617	-0.0617	0.812(12)
$16^3 \times 128$	-0.0742	-0.0742	0.6880(18)
$16^3 \times 128$	-0.0742	-0.0808	0.571(5)
$16^3 \times 128$	-0.0742	-0.0830	0.490(6)
$16^3 \times 128$	-0.0742	-0.0840	0.444(7)
$24^3 \times 128$	-0.0742	-0.0840	0.447(4)

- Between 175-770 configurations have been analysed on these ensembles.

How were the bare quark masses chosen?

- Quark action breaks chiral symmetry: the quarks have an additive mass renormalisation.
- This complicates quark mass setting: how can we vary the bare parameters in the lattice action to approach the physical theory?
- "Partially quenched" critical mass varies with $a_t m_s, a_t m_l$ and $m_l^{\rm crit}$ varies with bare parameters ...
- ... as does r_0/a_s.
- Our solution: track hadron mass ratios, and follow "well-chosen" lines of constant bare $a_t m_s^0$.
- Avoid all reference to the lattice spacing in observables.
- Extrapolate/interpolate hadron mass ratios to physical point. We use m_Ω as a reference scale. Ω is QCD-stable, has mild light-quark dependence in χ-PT and mild finite-volume dependence.

Choosing co-ordinates for the $N_f = 2 + 1$ theory space

Always use physical, dimensionless observables; use m_Ω to set the scale everywhere.

"Newport News" parameterisation

Parameterise the strange and light quark masses using:

$$l_\omega = \frac{9m_\pi^2}{4m_\Omega^2}$$

$$s_\omega = \frac{9(2m_K^2 - m_\pi^2)}{4m_\Omega^2}$$

The numerators are proportional to the quark masses at leading-order in χ-PT.

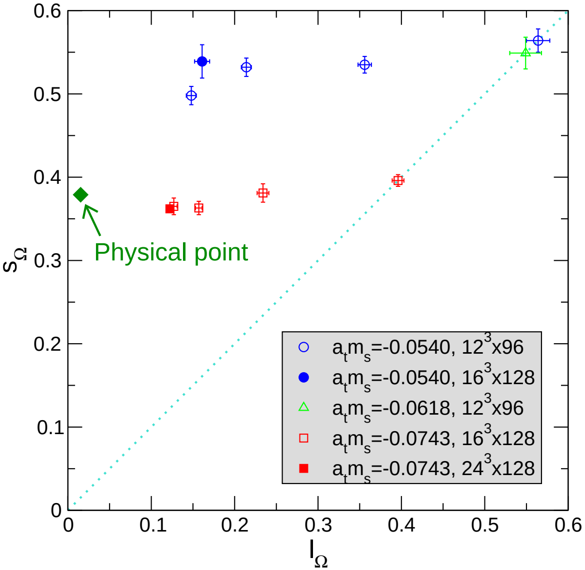
Choosing co-ordinates for the $N_f = 2 + 1$ theory space

"Newport News" parameterisation

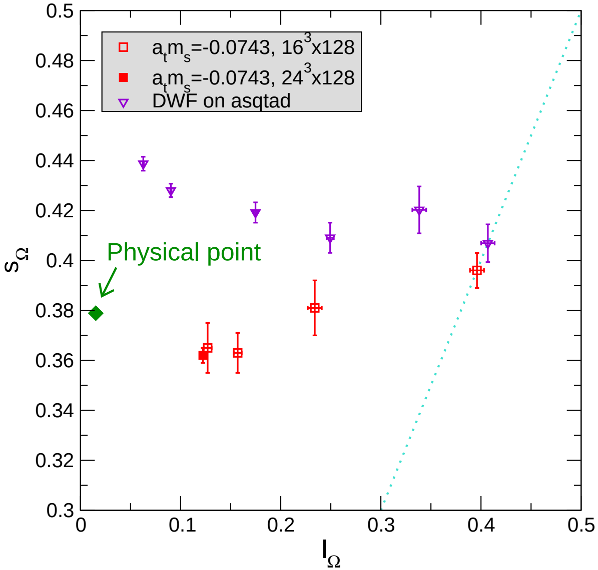
$$l_\omega = \frac{9m_\pi^2}{4m_\Omega^2}, s_\omega = \frac{9(2m_K^2 - m_\pi^2)}{4m_\Omega^2}$$

- In the $N_f = 3$ theory, $l_\Omega = s_\Omega$ and as $m_q \longrightarrow \infty, l_\Omega \longrightarrow 1$
- The "real world" is at $(l_\Omega^*, s_\Omega^*) = (0.0153, 0.3789)$
- We kept the bare strange quark mass parameter in the lattice lagrangian fixed in three separate runs.
- The third "main branch" uses a best guess strange quark.

Approaching the physical point



Approaching the physical point



Simulation parameters

Volume	$a_t m_s^0$	$a_t m_l^0$	l_Ω	s_Ω
$12^3 \times 96$	-0.0539	-0.0539	0.564(14)	0.564(14)
$12^3 \times 96$	-0.0539	-0.0698	0.356(8)	0.535(10)
$12^3 \times 96$	-0.0539	-0.0793	0.214(6)	0.532(11)
$12^3 \times 96$	-0.0539	-0.0825	0.148(6)	0.498(11)
$16^3 \times 96$	-0.0539	-0.0825	0.161(9)	0.539(20)
$12^3 \times 96$	-0.0617	-0.0617	0.549(19)	0.549(19)
$16^3 \times 128$	-0.0742	-0.0742	0.396(7)	0.396(7)
$16^3 \times 128$	-0.0742	-0.0808	0.234(7)	0.381(11)
$16^3 \times 128$	-0.0742	-0.0830	0.157(4)	0.363(8)
$16^3 \times 128$	-0.0742	-0.0840	0.127(4)	0.365(10)
$24^3 \times 128$	-0.0742	-0.0840	0.1223(16)	0.362(3)

Approaching the physical point

- For this lattice action, lines of constant bare $a_t m_s$ are close to horizontal.
- Different actions will have different trajectories in (l_Ω, s_ω).
- Indication is the $m_s = -0.0743$ simulations are close to the physical point, but undershoot slightly.
- Corrected quark mass can be interpolated.
- With current data-set, physics can be interpolated in strange quark mass. Interpolation should be reasonable, since last set of runs come close to physical point.

$N_f = 2 + 1$ simple spectroscopy

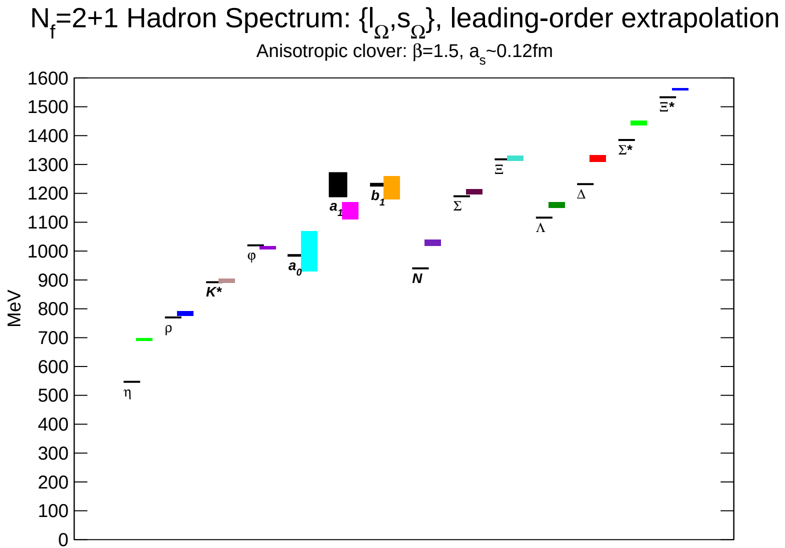
- At each simulation point, once (l_Ω, s_Ω) are determined, they represent quark masses in extrapolations (based on χ-PT).
- Mass ratios (using m_Ω as a common scale) are extrapolated
- At this stage, only the most naive chiral fits are attempted:

extrapolation: m_Ω ratios, (l_Ω, s_Ω) parameterisation

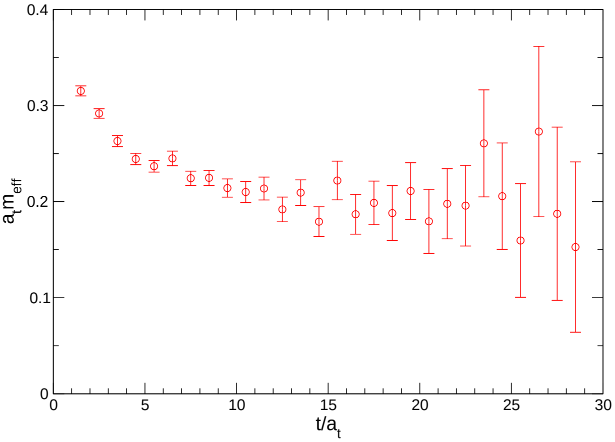
$$\frac{m_H}{m_\Omega} = a_H + b_H l_\Omega + c_H s_\Omega$$

- Parameterisation gives a good representation of the data in the range of simulation parameters attempted here; all fits are good.
- Fits using *e.g.* $a_t m_H$ were more problematic.

$N_f = 2 + 1$ PRELIMINARY simple spectroscopy



a_0 effective mass

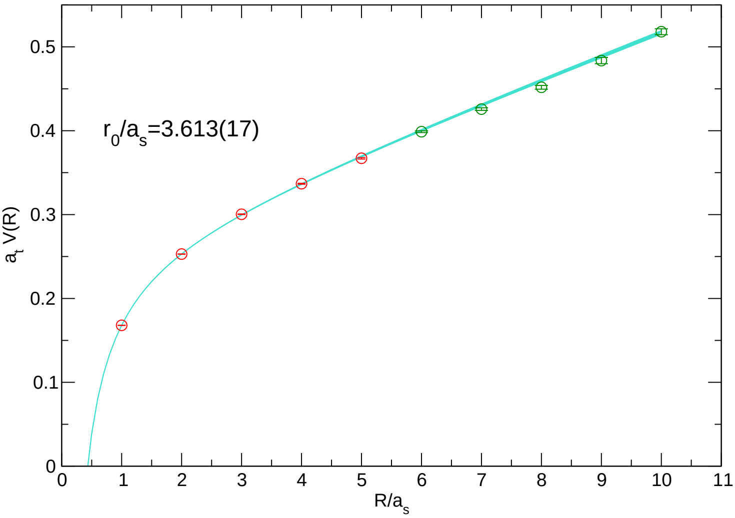


- Smeared-smeared correlator measurement.
- Simplest $\bar{\psi}\psi$ same-site operator construction.
- No quenched artefacts. Mass is consistent with $1 GeV$

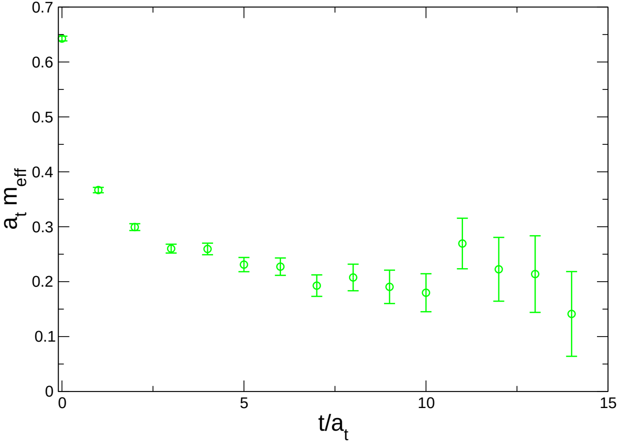
Measuring r_0

- Following the same philosophy, a determination of $r_0 m_\Omega$ at the physical point can be used to compute r_0 in QCD.
- Use a small 5×5 basis of operators, built from stout-smeared links
- Raw temporal links are used
- No evidence of string-breaking in these measurements
- The systematic uncertainties are yet to be controlled; I won't present a result today.

$N_f = 2 + 1$ simple spectroscopy



Effective mass from smeared spatial Wilson loops



- Smeared Wilson-loop basis measurement.
- Difficult to get a good plateau from the Wilson loop.

Conclusions

- First simulations of $N_f = 2 + 1$ QCD with dynamical quarks on anisotropic lattices are underway.
- The problem of setting the input strange quark mass has a simple solution: track movement of simulations in (l_Ω, s_Ω) plane.
- Access to χ-PT extrapolations is helped by fitting mass ratios (using m_Ω) with l_Ω and s_Ω representing the quark masses.
- Physical units only appear once data is extrapolated to (l^*_Ω, s^*_Ω)
- These extrapolations give an encouraging first look at the low-lying spectrum on these ensembles; still more to do!
- The anisotropic lattice still gives good resolution of correlation functions that fall rapidly into noise; more interesting physics to come from these ensembles!