High Temperature Confinement in SU(N) Gauge Theories

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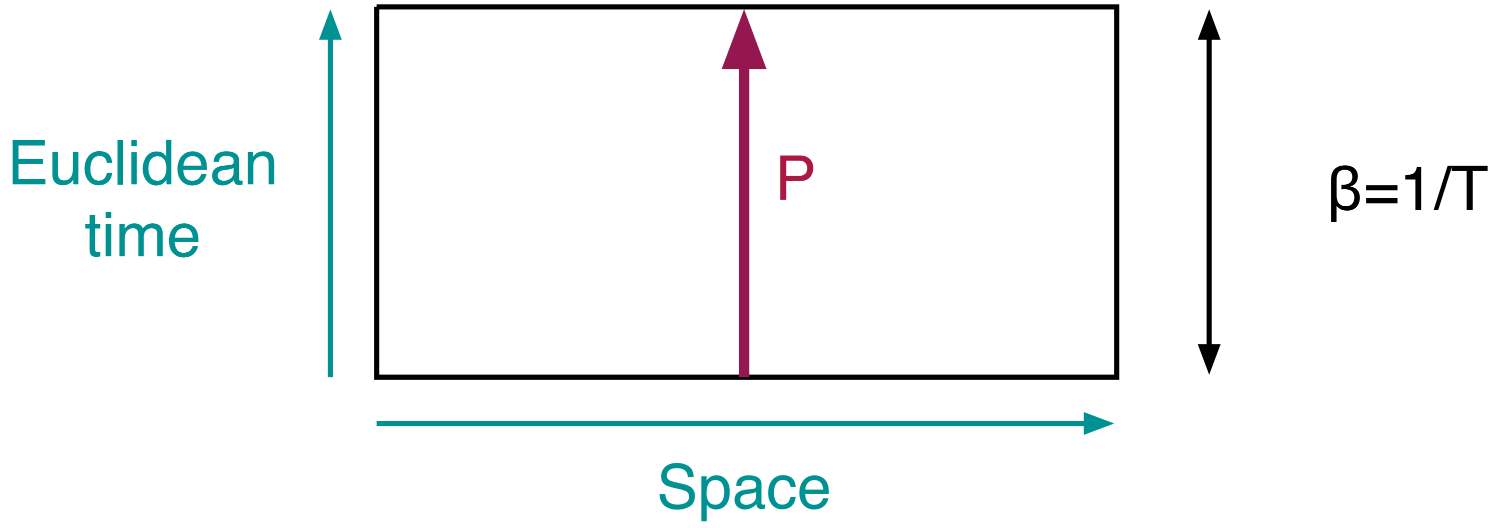
Lattice 2008

Summary of Claims

- SU(N) gauge theories can be extended in such a way that confinement occurs at high temperature.
- This high-temperature confining region is continuously connected to the low-T confined phase of pure gauge theories.
- Timelike string tensions are perturbatively calculable, and are of order $(gT)^2$.
- Spacelike string tensions are semiclassically calculable, and realize the dual superconductivity picture of quark confinement ('t Hooft 1976; Mandelstam 1976).

Role of Polyakov Loop



The operator P represents the insertion of a static quark

$$P\left(\vec{x}\right)=\mathcal{P}\exp\left[i\int_0^\beta dtA_4\left(\vec{x},t\right)\right]$$

Pure gauge theories have a global Z(N) symmetry

$$P\to zP \qquad z=e^{\frac{2\pi i}{N}}$$

Order parameter for deconfinement transition in pure gauge theories

Confined $\langle Tr_F P\rangle=0$

Deconfined $\langle Tr_F P\rangle\neq 0$

1 loop effective potential

Free energy density of a boson in a representation R with spin degeneracy s moving in a Polyakov loop background P at non-zero temperature and density. The fermion expression is similar.

$$f_b = sT \int \frac{d^d k}{(2\pi)^d} Tr_R \left[\ln\left(1 - P e^{\beta\mu - \beta\omega_k}\right) + \ln\left(1 - P^+ e^{-\beta\mu - \beta\omega_k}\right) \right]$$

With standard boundary conditions (periodic for bosons, antiperiodic for fermions), 1-loop effects always favor the deconfined phase.

$$f_b = -sT \int \frac{d^d k}{(2\pi)^d} \sum_{n=1}^{\infty} \frac{1}{n} \left[e^{n\beta\mu - n\beta\omega_k} Tr_R P^n + e^{-n\beta\mu - n\beta\omega_k} Tr_R P^{+n} \right]$$

High Temperature Confinement (HTC)

- SU(N) gauge theory with fermions in the adjoint representation, with periodic boundary conditions.
- Adjoint fermions respect Z(N) symmetry. With antiperiodic boundary conditions, they would lower the deconfinement temperature.
- Periodic boundary conditions are “wrong.”
- Ensemble partition function is Witten index for SuSy models

$$Z = Tr\left[(-1)^F\, e^{-\beta H}\right] \qquad \beta\mu \to i\pi$$

The high-temperature effective potential

$$V_{1-loop}\left(P,\beta,m,N_f\right)=\frac{1}{\pi^2\beta^4}\sum_{n=1}^{\infty}\frac{Tr_A P^n}{n^2}\left[\underset{\substack{\uparrow\\ \text{Fermion}}}{2N_f\beta^2m^2K_2\left(n\beta m\right)}-\underset{\substack{\uparrow\\ \text{Gauge}}}{\frac{2}{n^2}}\right]$$

N_f = number of Dirac flavors

- Fermion contribution would favor deconfined phase with normal anti-periodic boundary conditions.
- Periodic boundary conditions alter the phase structure radically.
- Unsal and Yaffe (2008) propose adding m=0 limit as a deformation of the pure gauge theory.

How High Temperature Confinement Works

Simplify

$$V_{1-loop}\left(P,\beta,m,N_f\right)=\frac{1}{\pi^2\beta^4}\sum_{n=1}^{\infty}\frac{Tr_A P^n}{n^2}\left[2N_f\beta^2 m^2 K_2\left(n\beta m\right)-\frac{2}{n^2}\right]$$

to

$$-h_A Tr_A P=-h_A\left[\left|Tr_F P\right|^2-1\right]$$

If $h_A>0$, the deconfined phase is favored, with $Tr_F(P)$ non zero.

If $h_A<0$, the confined phase is favored, with $Tr_F(P)$ zero.

Complete story a little more complicated!

Z(N) Symmetry

There is a unique set of eigenvalues of an SU(N) matrix that is invariant under Z(N)

$$P_0 = w \cdot diag\left[1, z, z^2, .., z^{N-1}\right]$$

$$z = e^{2\pi i/N}$$

z is the generator of Z(N)

$$zP_0 = gP_0g^+$$

z permutes the eigenvalues

$$Tr_F\left[P_0^k\right] = 0 \quad k = 1, 2, .., N-1$$

Meisinger, Miller, and Ogilvie 2002

P_0 is the global minimum of the effective potential in the high-temperature confined phase.

Small fermion mass means confinement

T^4 term dominates, and has Z(N) symmetric minimum

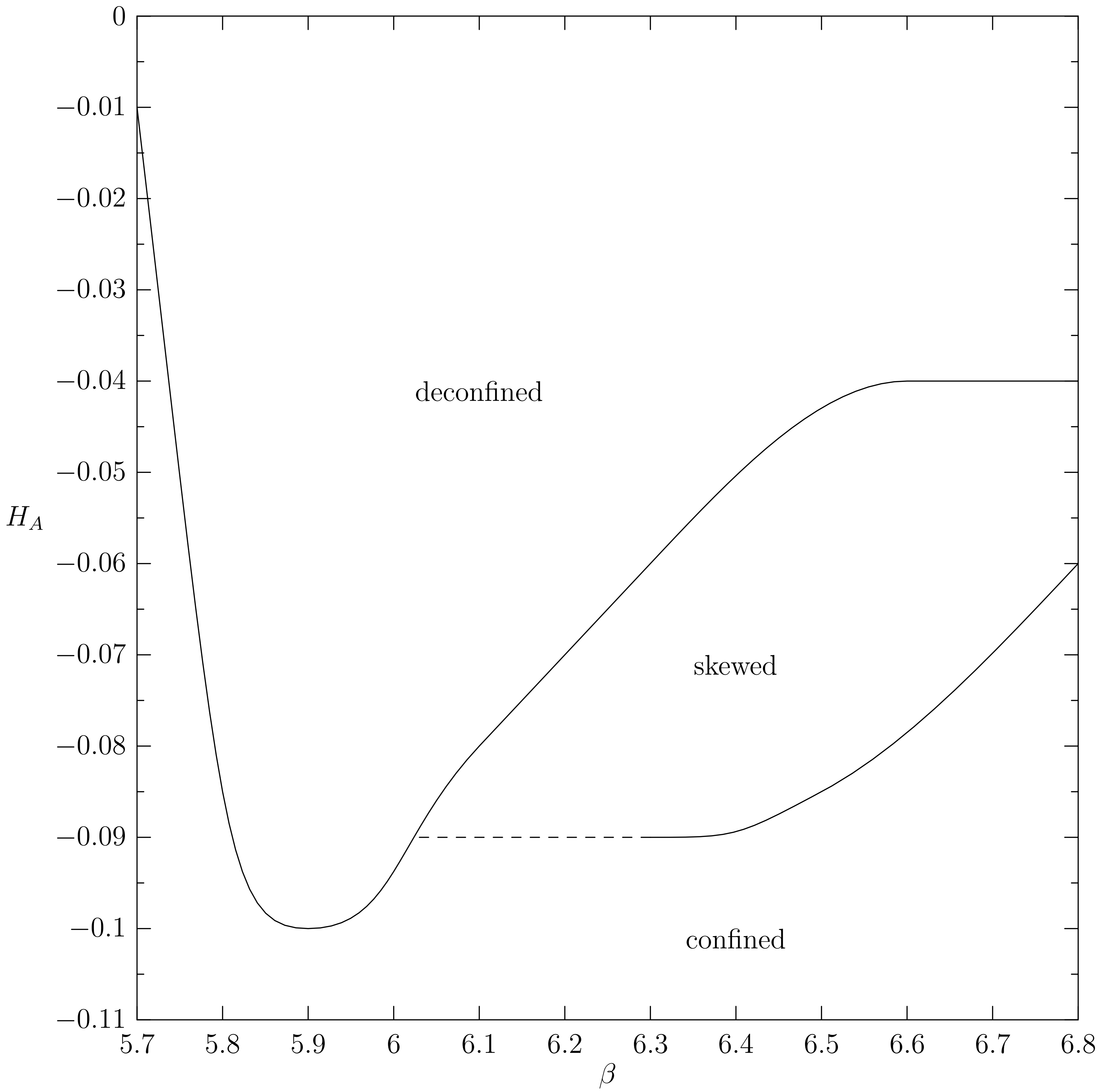
$$m/T \ll 1 \qquad\qquad P_{jk} = \delta_{jk} e^{i\phi_j}$$

$$V_{1-loop} \approx \sum_{j,k=1}^{N} (1 - \frac{1}{N}\delta_{jk}) \frac{2\,(2N_f - 1)\,T^4}{\pi^2} \left[\frac{\pi^4}{90} - \frac{1}{48\pi^2} \left(\phi_j - \phi_k\right)^2 \left(\phi_j - \phi_k - 2\pi\right)^2 \right]$$
$$- \sum_{j,k=1}^{N} (1 - \frac{1}{N}\delta_{jk}) \frac{N_f m^2 T^2}{\pi^2} \left[\frac{\pi^2}{6} + \frac{1}{4} \left(\phi_j - \phi_k\right) \left(\phi_j - \phi_k - 2\pi\right) \right]$$

Meisinger and Ogilvie, 2002

Can m be small enough? It certainly can be if the scale of chiral symmetry breaking is gT or smaller.

Phase Diagram for SU(3)



H_A is an external coupling to the adjoint Polyakov loop, representing the effect of heavy adjoint quarks. Negative values of H_A correspond to periodic boundary conditions.

HTC region is continuously connected to usual confined phase.

Myers and Ogilvie, 2008

String Tension Scaling

Timelike string tension between k quarks and k antiquarks is measured by

$$\left\langle Tr_F P^k\left(\vec{x}\right) Tr_F P^{+k}\left(\vec{y}\right)\right\rangle \simeq \exp\left[-\frac{\sigma_k^{(t)}}{T}\left|\vec{x}-\vec{y}\right|\right]$$

Two proposed scaling behaviors of pure gauge theory:

Casimir: $$\sigma_k = \sigma_1 \frac{k(N-k)}{N-1}$$

sine-law: $$\sigma_k = \sigma_1 \sin\left[\frac{\pi k}{N}\right]$$

Perturbative Confinement for Polyakov Loops

$$\left\langle Tr_F P^k\left(\vec{x}\right) Tr_F P^{+k}\left(\vec{y}\right)\right\rangle \simeq \exp\left[-\frac{\sigma_k^{(t)}}{T}\left|\vec{x}-\vec{y}\right|\right]$$

String tensions are calculable perturbatively in the high-temperature confining region from small fluctuations about the confining minimum of the effective potential. The scale is naturally O(gT).

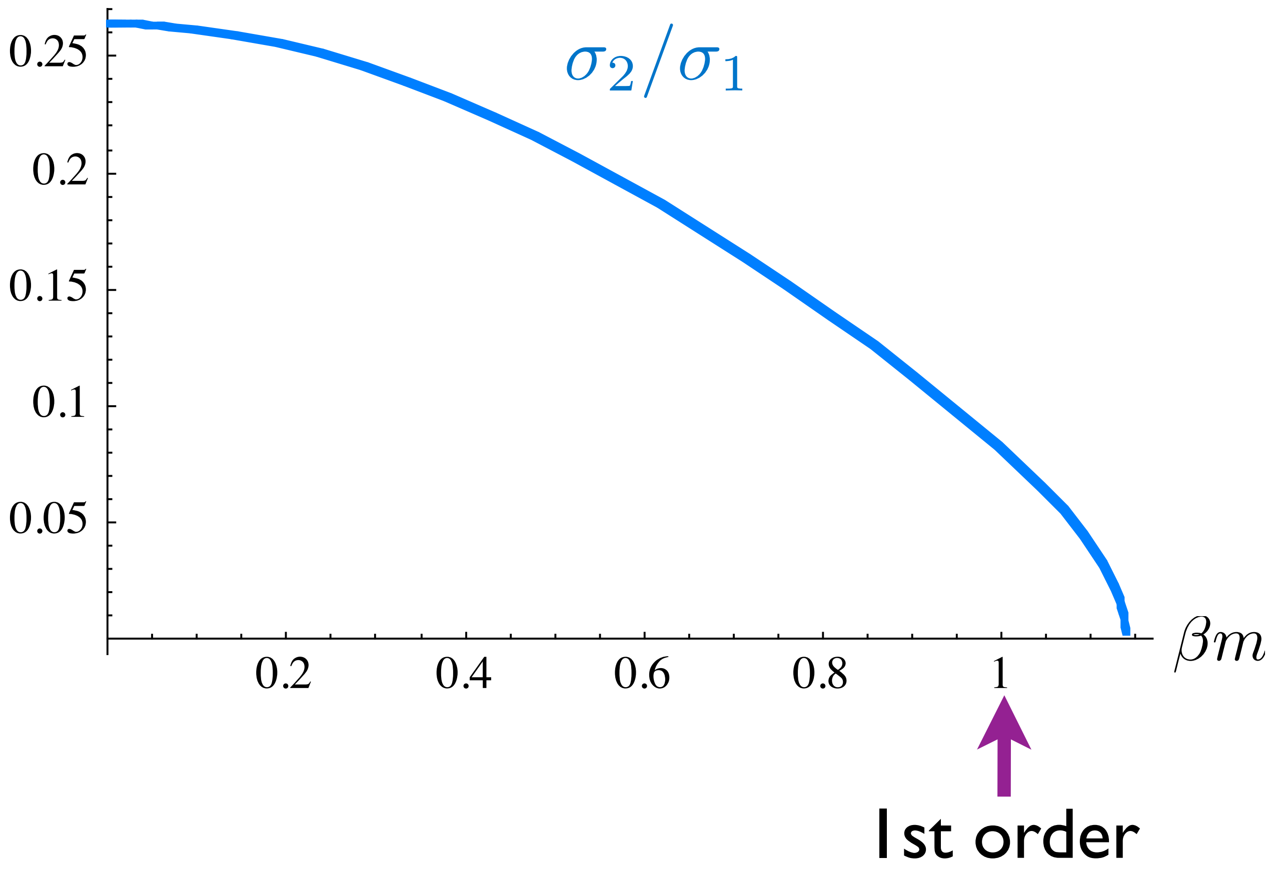
$$\begin{aligned}\left(\frac{\sigma_k^{(t)}}{T}\right)^2 &= g^2 N\frac{2N_f m^2}{2\pi^2}\sum_{j=0}^{\infty}\left[K_2\left((k+jN)\beta m\right)+K_2\left((N-k+jN)\beta m\right)-2K_2\left((j+1)N\beta m\right)\right]\\ &\quad -g^2 N\frac{T^2}{3N^2}\left[3\csc^2\left(\frac{\pi k}{N}\right)-1\right]\end{aligned}$$

The m=0 limit is simple:

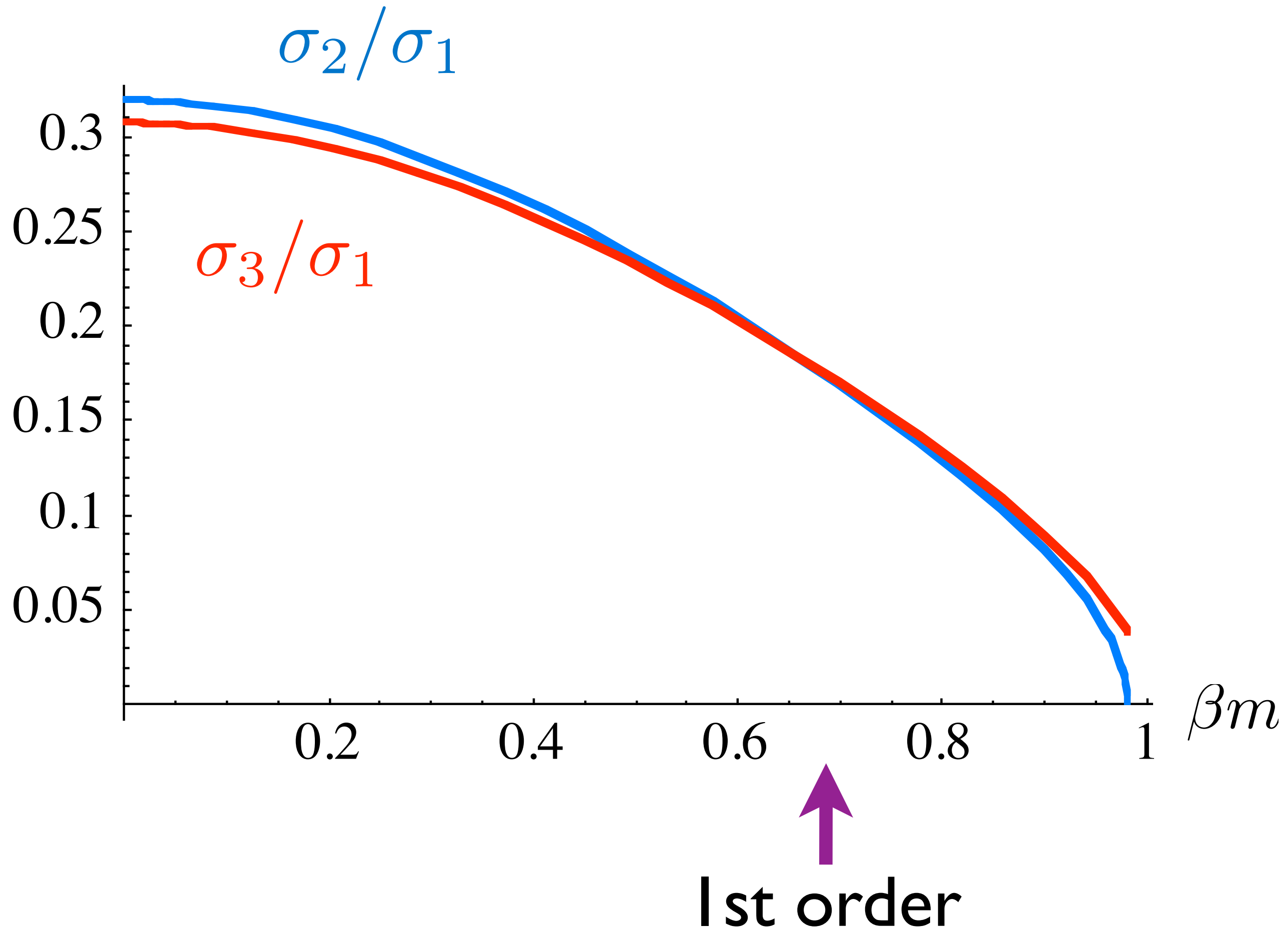
$$\left(\frac{\sigma_k^{(t)}}{T}\right)^2 = \frac{\left(2N_f-1\right)g^2T^2}{3N}\left[3\csc^2\left(\frac{\pi k}{N}\right)-1\right]$$

Polyakov loop string tensions

SU(4)



SU(6)



σ=0 marks spinodal point, the limit of metastability.

Magnetic Monopoles in HTC region

Confining minimum of effective potential breaks SU(N) to U(1)$^{N-1}$. P (or A$_4$) plays role similar to adjoint Higgs field.

N fundamental monopoles with charge proportional to affine roots of SU(N).

$$\frac{2\pi}{g}\alpha_j$$

$$\alpha_j = \hat{e}_j - \hat{e}_{j+1} \qquad \alpha_N = \hat{e}_N - \hat{e}_1$$

A finite-temperature instanton (caloron) is composed of N different monopoles Kraan & van Ball 1998; Lee & Lu 1998.

Effective Action for Monopoles

Generalized sine-Gordon model represents
monopole/anti-monopole gas

$$S_{mag} = \int d^3x \left[\frac{T}{2} (\partial \rho)^2 - 2\xi \sum_{j=1}^{N} \cos \left(\frac{2\pi}{g} \alpha_j \cdot \rho \right) \right]$$

Polyakov 1977; Unsal and Yaffe 2008

ρ = dual field to U(1)$^{N-1}$ magnetic field

ξ = monopole fugacity $\quad \xi = \frac{\det_{mono}}{T} e^{-S_{mono}}$

N degenerate inequivalent minima: $\quad \rho_{0k} = g\mu_k \qquad k = 1, .., N-1$

μ_k = simple fundamental weights $\quad \alpha_j \cdot \mu_k = \delta_{jk} \qquad e^{2\pi i \mu_k} \in Z(N)$

Spatial Wilson Loops and dual kinks

A spatial Wilson loop in the x-y plane introduces a discontinuity in the z direction in the field dual to B.

$$W\left[\mathcal{C}\right] = \mathcal{P}\exp\left[i\oint_{\mathcal{C}} dx_j \cdot A_j\right]$$

We can move this discontinuity out to spatial infinity; then the string tension of the spatial wilson loop is the interfacial energy of a kink interpolating between vacua.

Straight line ansatz in Lie algebra gives:

$$\rho(z) = g\mu_k q(z)$$

Giovannangeli and Korthals Altes 2002

$$\sigma_k^{(s)} = \frac{8}{\pi}\left[\frac{g^2 T\xi}{N}k\left(N-k\right)\right]^{1/2}$$

Exact for N=2,3; may be only upper bound for N>3.

Behavior should be easily distinguishable from Casimir and sine-law.

Predictions for lattice to confirm

- Phase structure and thermodynamics, especially for higher N, but also existence of a tricritical point for SU(2).
- Temporal string tension scaling, measured by Polyakov loops.
- Spatial string tension scaling, measured by spatial Wilson loops.
- Spatial string tensions proportional to monopole density.

Questions simulations can answer

(that we don’t yet have theory for)

- How does the crossover to the conventional low-temperature confined region behave?
- Can we understand chiral symmetry breaking in the high-temperature confining region? Unsal (2007) has proposed a detailed picture.
- Can we get close to the BPS limit for larger masses in HTC region?

Confinement in 3+1

- Hard problem, many ideas, not much success outside lattice simulations.
- Finite temperature gauge theories are easier, because the Polyakov loop is easier to work with than the Wilson loop.
- Finite temperature also gives us a rich phase structure to explore- “Make the problem harder” or “More knobs to turn.”
- Surprising recent results: semiclassical confinement at high T continuously connected to low-T phase of pure gauge theory.

How we got here

- Dual Superconductivity picture of quark confinement ‘t Hooft 1976; Mandelstam 1976
- KvBLL Calorons Kraan & van Ball 1998; Lee & Lu 1998
- $\mathcal{N}=1$ Supersymmetry at finite “T”: Davies *et al.* 1999
- Caloron determinants: Diakonov *et al.* 2004, 2005
- Confinement at high T: Unsal 08; Myers and Ogilvie 08; Unsal and Yaffe 08

Spacelike Confinement in 3 regions

We would like to understand the spatial string tension in 3 regions:

- HTCQCD phase: within our grasp
- low-temperature, confining phase?
- high-temperature, deconfined phase?

Recall that sine-law behavior is one possibility.

$\mathcal{PT}$, Affine Toda, & Confinement

The affine Toda model is non-Hermitian but $\mathcal{PT}$-symmetric, and has kink solutions with a sine-law mass spectrum.

$$S_{Toda} = \int d^3x \left[\frac{T}{2} (\partial\rho)^2 - \xi \sum_{j=1}^{N} \exp\left(i \frac{2\pi}{g} \alpha_j \cdot \rho \right) \right]$$

This is a effective field theory for a gas of monopoles, but no anti-monopoles.

$$\sigma_k^{(s)} = \frac{2N}{\pi} \left[g^2 T \xi \right]^{1/2} \sin\left(\frac{\pi k}{N} \right)$$

Toda- Hollowood 1992; SU(N)- Diakonov & Petrov 2007

Three Possible Behaviors

Affine sine-Gordon
(HTC region)

$$\sigma_k^{(s)} = \frac{8}{\pi}\left[\frac{g^2 T\xi}{N} k\,(N-k)\right]^{1/2}$$

Affine Toda
(sine law)

$$\sigma_k^{(s)} = \frac{2N}{\pi}\left[g^2 T\xi\right]^{1/2} \sin\left(\frac{\pi k}{N}\right)$$

All roots
(Casimir)

$$\sigma_k^{(s)} = \frac{8}{\pi}\left[\frac{g^2 T\xi}{N}\right]^{1/2} k\,(N-k)$$

We‘re not done yet!