Revisiting strong coupling QCD at finite baryon density and temperature

M.Fromm, Ph. de Forcrand
ETH, Zürich

Lattice'08, July 16th, 2008

Motivation

Strong coupling QCD (SCQCD) under investigation for > 25 years...

Analytical (Mean Field $1/d$)

- Mass spectrum: Kawamoto and Smit '81, Kluberg-Stern, Morel, Petersson '82
- Phase diagram, Damgaard, Kawamoto, Shigemoto '84
- Phase diagram with $1/g^2$ corrections: Faldt and Petersson '86, Bilić et al.'92
- Latest: Nishida '04, Kawamoto et al. '05, Miura and Ohnishi '08 (next talk)

Numerical

- Karsch and Mütter '89, MDP-approach ($T \approx 0$, $\mu \approx \mu_c$)
- Boyd et al. '92, MDP at $T \approx T_C, \mu = 0$
- Azcoiti et al. '99 (MDP under scrutiny)
- de Forcrand and Kim '06, HMC, mass spectrum

Strong coupling QCD

Some Definitions:

$$Z = Z(m, \mu, \beta) = \int \mathcal{D}U\mathcal{D}\bar{\chi}\mathcal{D}\chi\, \mathrm{e}^{-S_\mathrm{F} - \beta S_\mathrm{G}},$$

μ chemical potential, m staggered quark mass, $\beta = \frac{6}{g_0^2}$ inverse gauge coupling

$$\begin{aligned} S_\mathrm{G} &= \sum_P \left(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{tr}[U_P]\right) \\ S_\mathrm{F} &= \sum_{x,\nu} \bar{\chi}_x \left[\eta_{x\nu} U_{x\nu} \chi_{x+\nu} - \eta_{x\nu}^{-1} U_{x-\nu\nu}^\dagger \chi_{x-\nu}\right] + 2m \sum_x \bar{\chi}_x \chi_x \end{aligned}$$

$\eta_{x\nu} = \mathrm{e}^\mu$ $(\nu = 0)$ and $(-1)^{\sum_{\rho<\nu} x_\rho}$ otherwise.

Strong coupling QCD (SCQCD)

In Strong (infinite) coupling limit, $\beta = 0$ - can do integral in links $U_{x\nu}$ first [Rossi & Wolff]:

$$Z(m,\mu) = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \mathrm{e}^{2m\sum_x \bar{\chi}_x\chi_x} \prod_{\langle xy \rangle} F_{xy}$$

where $F_{xy} = \sum_{k=0}^{3} (-1)^k \alpha_k (M_x M_y)^k + \kappa/6 \left[\left(\bar{B}_x B_y\right)^3 + \left(\bar{B}_y B_x\right)^3 \right]$ and

$$\kappa = \begin{cases} 0, & \text{for } U(3) \\ 1, & \text{for } SU(3) \end{cases}$$

New degrees of freedom are color singlet

Monomers $M_x = \sum_{a,x} \bar{\chi}_{ax}\chi_{ax}$, $(\bullet)$, monomers per site $n_x = 0, \ldots, 3$

Dimers $D_{k,xy} = \frac{1}{k!}(M_x M_y)^k$ $(-, =, \equiv)$, bond number $n_b = 0, \ldots, 3$

(Anti-)Baryons $B_x = \chi_{1x}\chi_{2x}\chi_{3x}$, $\bar{B}_x = \bar{\chi}_{3x}\bar{\chi}_{2x}\bar{\chi}_{1x}$, $---$

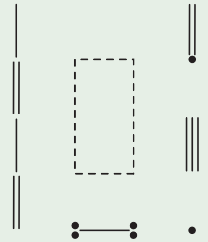
SCQCD loop gas

Self-avoiding loops C of $\bar{B}B_x$ pairs are formed, with *signed* weights $\rho(C)$,

$$Z(m,\mu) = \sum_{\{n_x, n_b, \Box\}} \prod_b \frac{(3-n_b)!}{3!\,n_b!} \prod_x \frac{3!}{n_x!}(2m)^{n_x} \prod_{\text{loops } C} \rho(C) \; ,$$

constraint $n_x + \sum_{b_x} n_{b_x} = 3$.

Example



- MDP description [Karsch & Mütter, 1989]: Signed baryonic loops are associated with polymer loops. Mapping the weight

$$\begin{aligned} \rho_B(C) &\rightarrow w_{polymer}(C) \\ \pm \cosh 3\mu/T &\rightarrow 1 \pm \cosh 3\mu/T \end{aligned}$$

softens the sign problem.

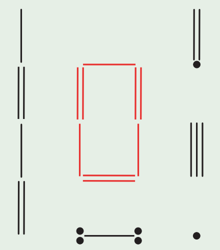
SCQCD loop gas

Self-avoiding loops C of $\bar{B}B_x$ pairs are formed, with *signed* weights $\rho(C)$,

$$Z(m,\mu) = \sum_{\{n_x, n_b, \Box\}} \prod_b \frac{(3-n_b)!}{3!\,n_b!} \prod_x \frac{3!}{n_x!}(2m)^{n_x} \prod_{\text{loops } C} \rho(C) \;,$$

constraint $n_x + \sum_{b_x} n_{b_x} = 3$.

Example



- MDP description [Karsch & Mütter, 1989]: Signed baryonic loops are associated with polymer loops. Mapping the weight

$$\begin{array}{rcl} \rho_B(C) & \rightarrow & w_{polymer}(C) \\ \pm\cosh 3\mu/T & \rightarrow & 1 \pm \cosh 3\mu/T \end{array}$$

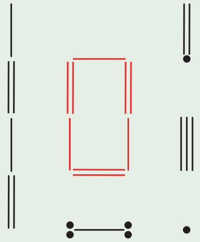
softens the sign problem.

The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields M_{bx_1}, M_{cx_2},i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi\, M_{bx_1} M_{cx_2} \mathrm{e}^{2m\sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$

Example



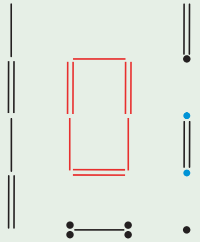
- Corresponds to introduction of additional monomers (”worm” head and tail)

The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields M_{bx_1}, M_{cx_2},i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \, M_{bx_1} M_{cx_2} \mathrm{e}^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy} .$$

Example



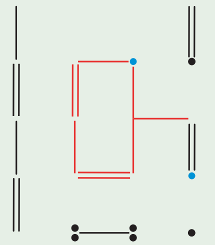
- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$

The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \dots$ introduce two additional meson fields M_{bx_1}, M_{cx_2},i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \, M_{bx_1} M_{cx_2} \mathrm{e}^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy} .$$

Example



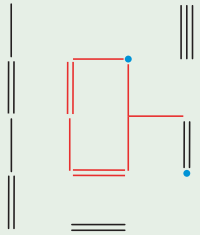
- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
- No restrictions in mass range (e.g. can take $m \to 0$) in contrast to local monomer changing Metropolis step $- \leftrightarrow \bullet\bullet$

The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields M_{bx_1}, M_{cx_2},i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi\, M_{bx_1} M_{cx_2} \mathrm{e}^{2m\sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$

Example



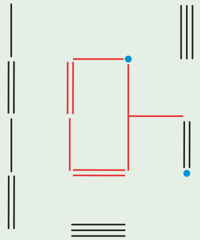
- Corresponds to introduction of additional monomers ("worm" head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
- No restrictions in mass range (e.g. can take $m \to 0$) in contrast to local monomer changing Metropolis step $- \leftrightarrow \bullet\bullet$
- See U(N) [Adams & Chandrasekharan '03]

The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields M_{bx_1}, M_{cx_2},i.e.

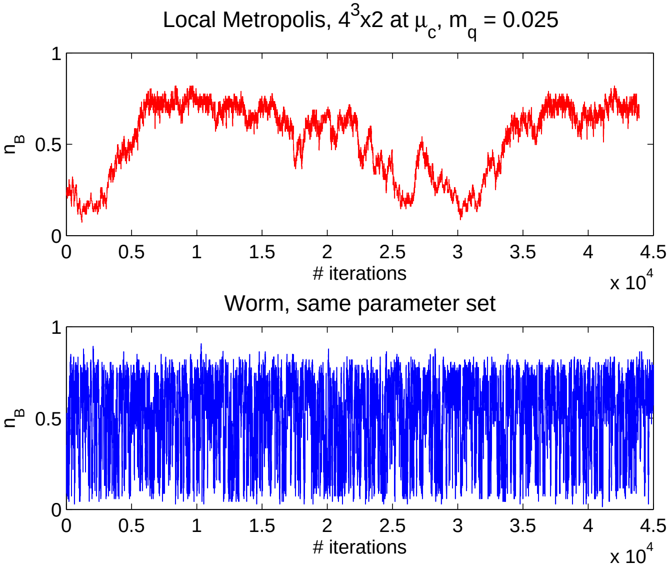
$$\langle M_{bx_1} M_{cx_2} \rangle = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \, M_{bx_1} M_{cx_2} \mathrm{e}^{2m\sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy} .$$

Example

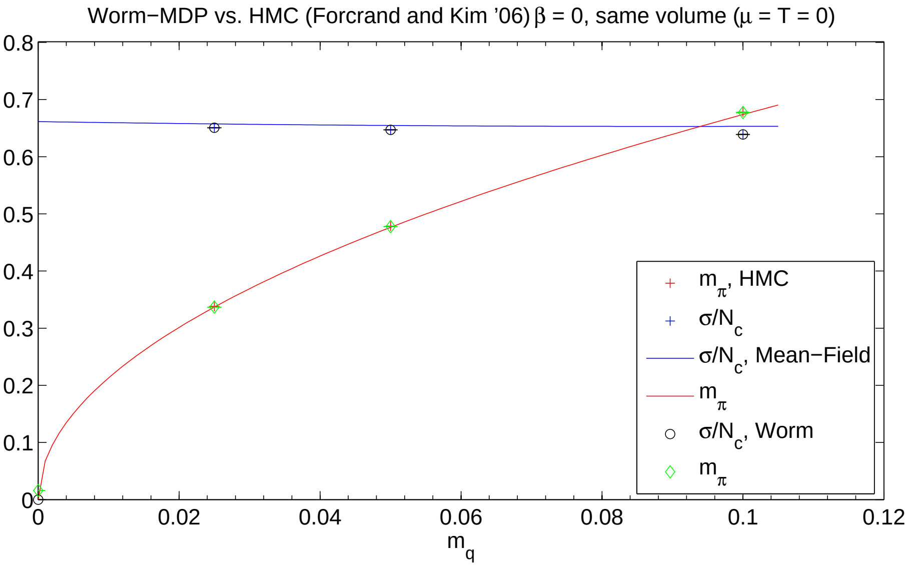


- Corresponds to introduction of additional monomers (”worm” head and tail)
- MC-update via heatbath shift of head, sample configs contributing to $\langle M_{bx_1} M_{cx_2} \rangle$
- No restrictions in mass range (e.g. can take $m \to 0$) in contrast to local monomer changing Metropolis step $- \leftrightarrow \bullet\bullet$
- See U(N) [Adams & Chandrasekharan ’03]

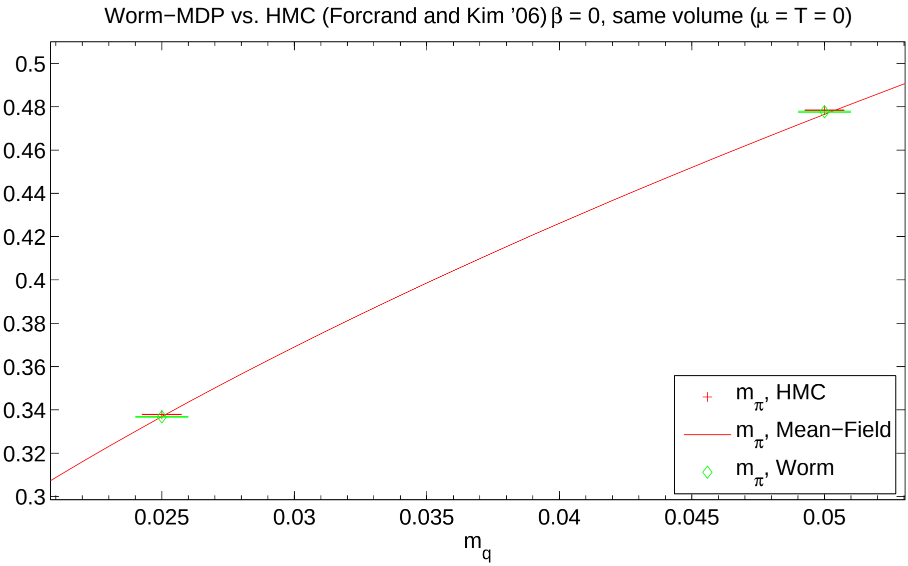
Simple efficiency test



Consistency check with HMC

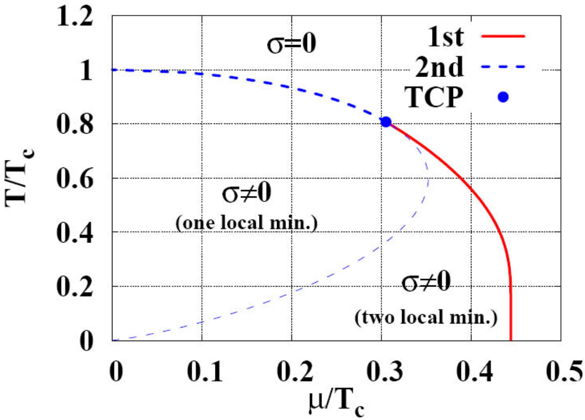


Consistency check with HMC



SCQCD chiral restoration transition

Phase diagram in the chiral limit as obtained by *mean-field* calculations [Damgaard et al. '85, Nishida '04, Kawamoto et al. '05]



Literature: $T_c = \frac{5}{3}$, $\mu_c(T=0) = 0.57 - 0.66$. $(\sigma \propto \langle \bar{\chi}_a \chi_a \rangle)$

SCQCD chiral restoration transition

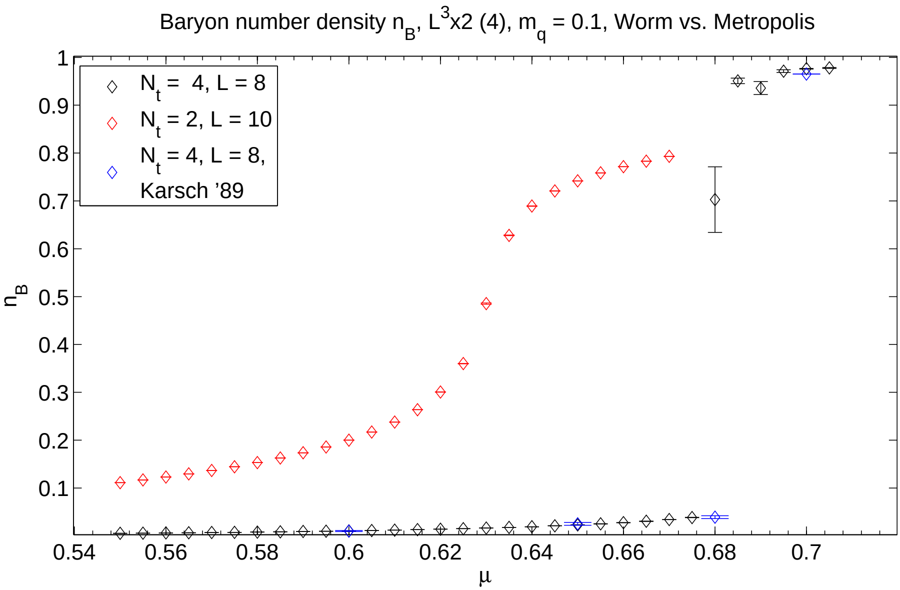
Puzzle

- Strong coupling MC-simulations [Karsch & Mütter 1989] at *finite* quark mass and $T = 1/4$ confirm 1st order finite μ transition and extrapolate to $\mu_c(T \approx 0, m = 0) = 0.63$ (in agreement with mean-field)
- However: Expect ($T = 0$)-phase transition when

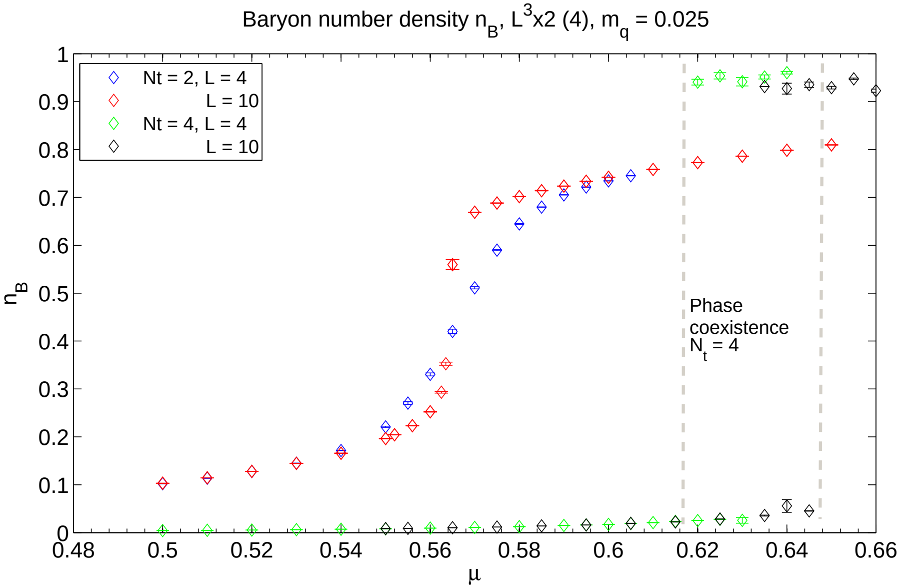
$$3\mu \geq F_B \approx M_{\mathrm{Nucleon}} \approx 3\,, \text{i.e.} \quad \mu_c \approx 1$$

- Nuclear attraction strong, $\mathcal{O}(300\,\mathrm{MeV})$?
- Or: Finite T effects (MC), extrapolation in m (MC) or mean field approach inaccurate ?
  $\rightarrow$ Check with worm-MC in the chiral limit, $T \approx 0$.
- Note: Mean field calculations with $1/g^2$ corrections [Bilić et al. 1992] show that $\mu_c \rightarrow M_N/3$

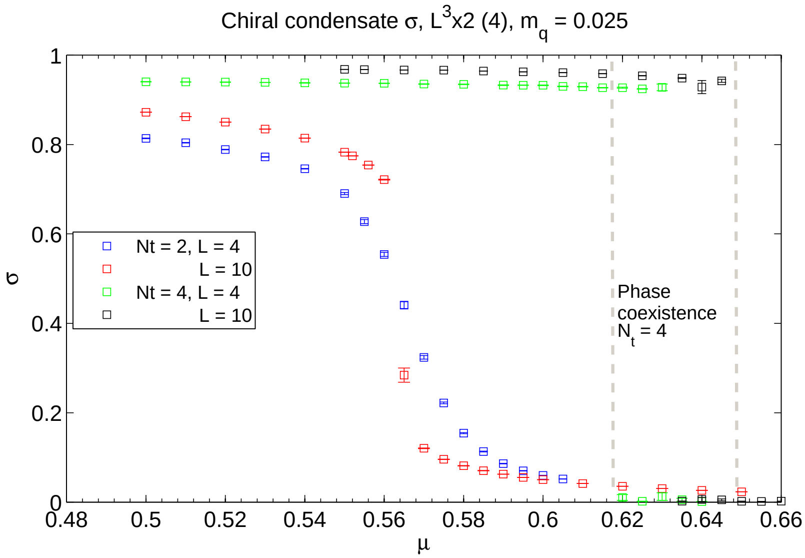
Preliminary Results, varying mass



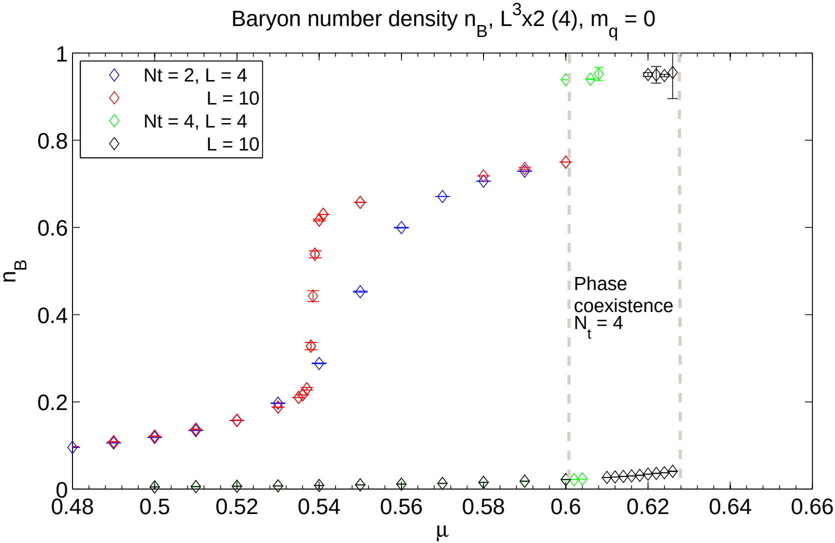
Preliminary Results, varying mass



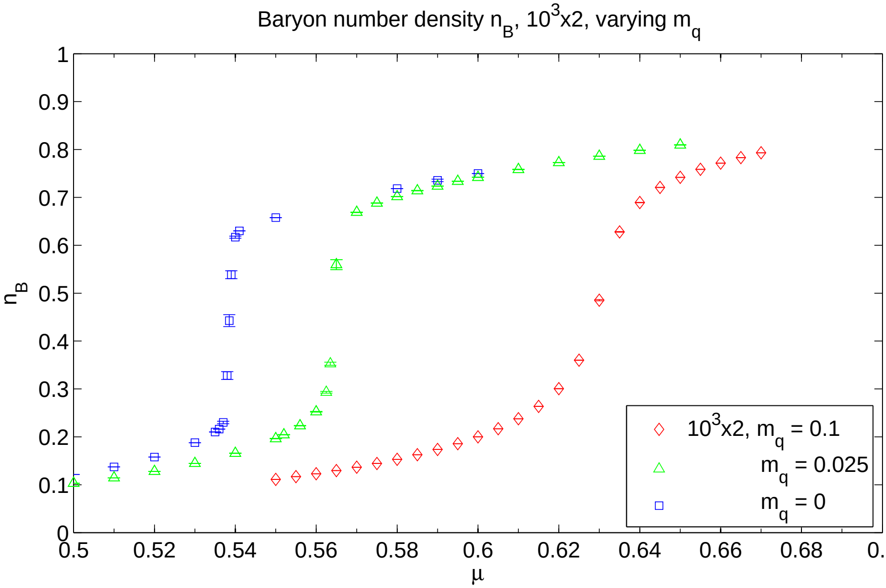
Preliminary Results, varying mass



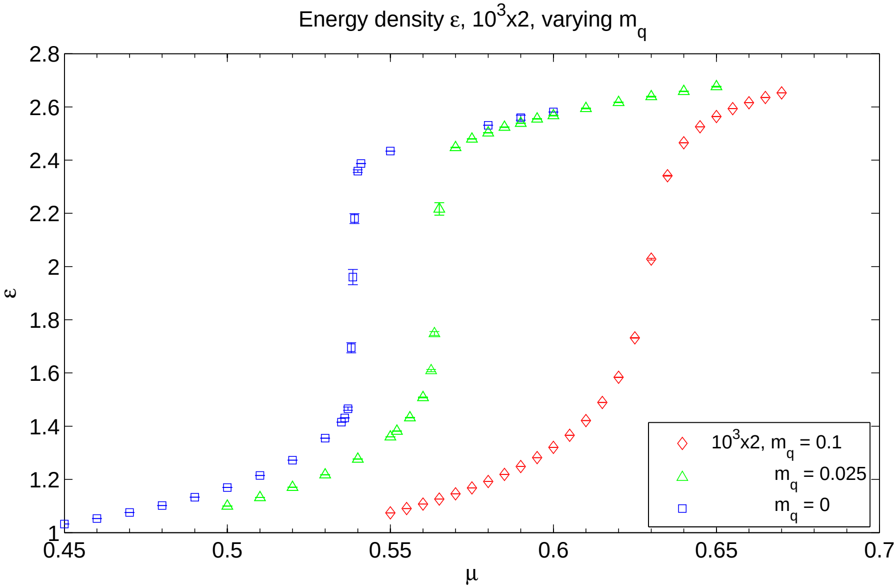
Preliminary Results, varying mass



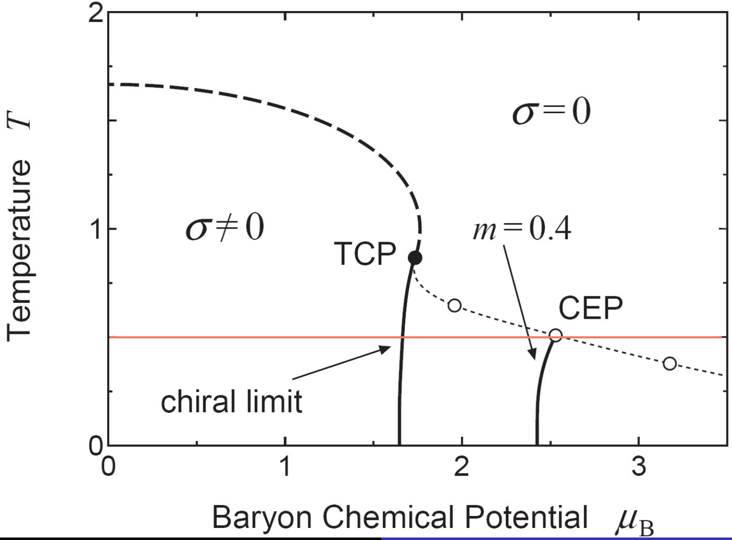
Preliminary Results, varying mass



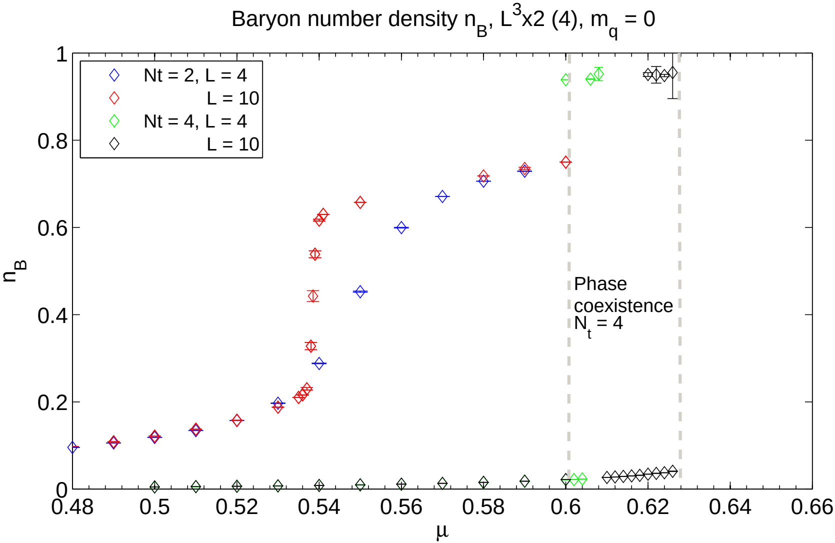
Preliminary Results, varying mass



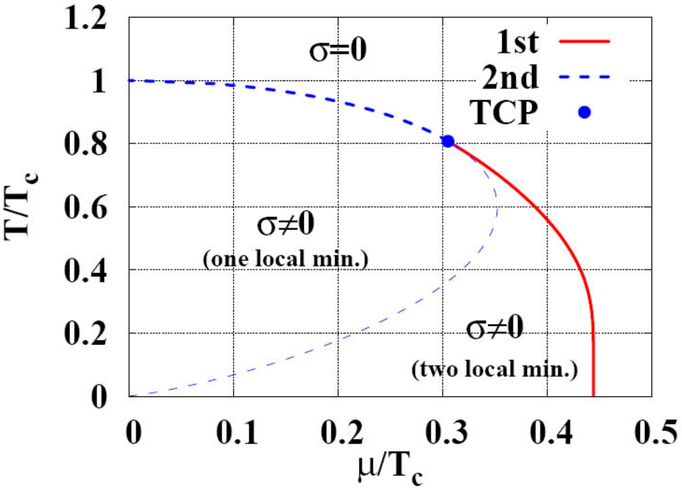
c.f. Mean-Field phase diagram, Nishida '04: qualitative agreement



Preliminary Results, $m = 0$

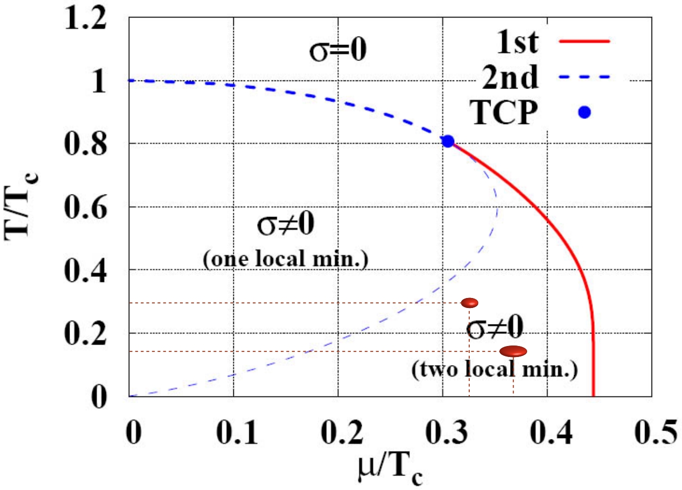


Preliminary Results, $m = 0$



Take $T_c = 5/3$, c.f. MC $T_c \approx 1.4$ [Boyd et al.'92]

Preliminary Results, $m = 0$



Take $T_c = 5/3$, c.f. MC $T_c \approx 1.4$ [Boyd et al.'92]

Summary & Outlook

- Can locate $T = 1/4$ transition with $\mu_c \approx 0.62$ in the chiral limit ($< m_B/3$), $T = 1/2$, $\mu_c \approx 0.54$
- Observe smoothening of finite T transition with increasing mass - in accord with mean-field

"Assignment"

- Extrapolation $T \to 0$ remains open,
  - Study 1st order PT with multicanonical algorithm
  - Include asymmetry γ to vary T continuously
  - Check mean-field predicted relation $T = \gamma^2/N_t$
- SCQCD Phase diagram
  - In the chiral limit: Locate TCP
  - CEP for varying quark mass
  - Flavor dependence of phase diagram
- Consistency check $\mu \to i\mu$

Average sign