Search for Chiral Fermion Actions on Non-Orthogonal Lattices

Presented by Michael I. Buchoff

In collaboration with Paulo Bedaque, Brian Tiburzi, and Andre Walker-Loud

P.F. Bedaque, M.I.B, B.C. Tiburzi, and A. Walker-Loud (Phys. Lett. B 662, 449-455), 2008

P.F. Bedaque, M.I.B, B.C. Tiburzi, and A. Walker-Loud (Phys. Rev. D 78, 017502), 2008

Introduction

- Why use non-orthogonal lattices ("graphene", "hyperdiamond", etc.)?

- Complications with proposed "graphene" lattice actions

- Advantages and disadvantages of other non-orthogonal lattice actions

Brief Intro on Graphene

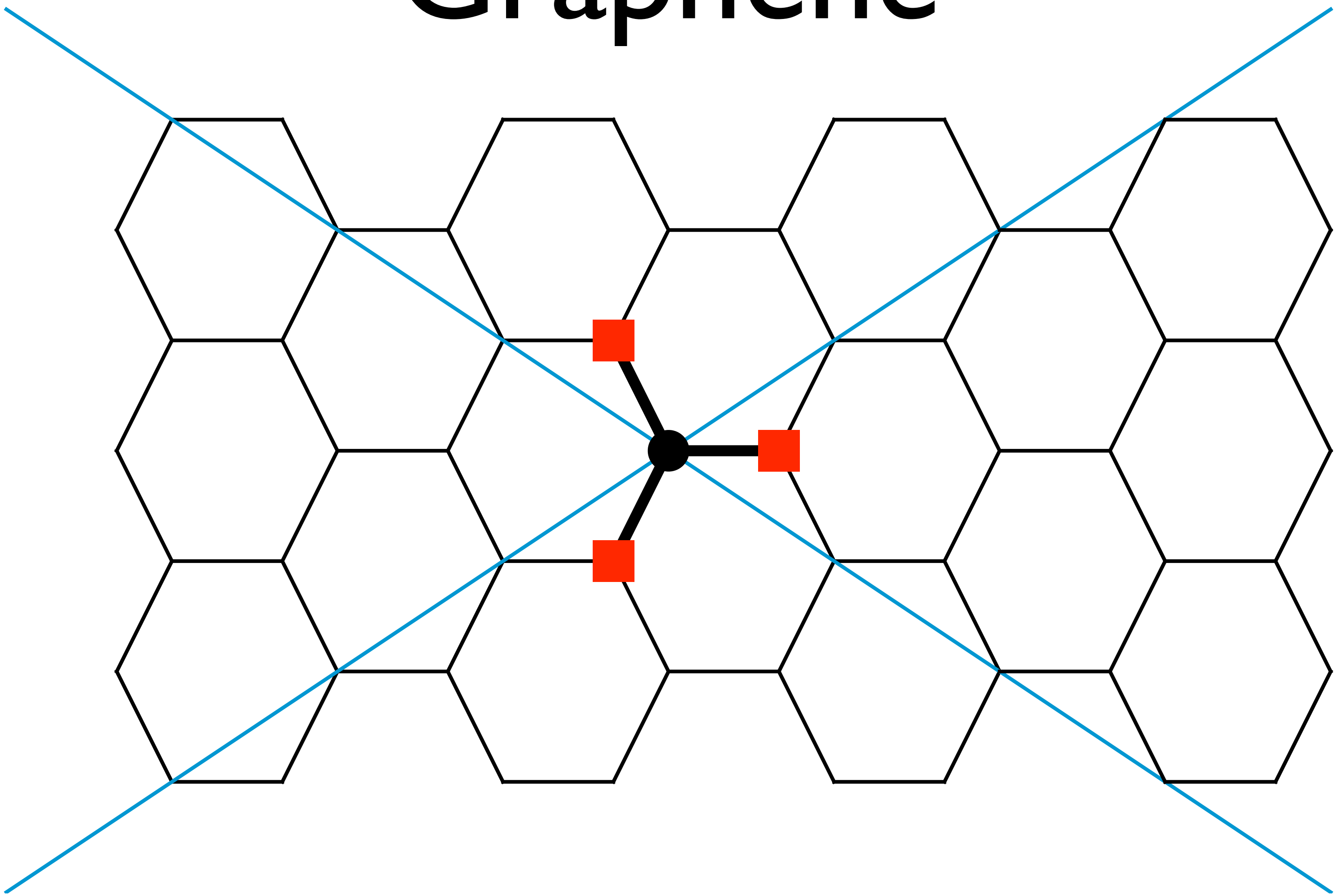
- Two dimensional honeycomb (hexagonal) lattice

- Leads to Dirac fermions in the massless limit

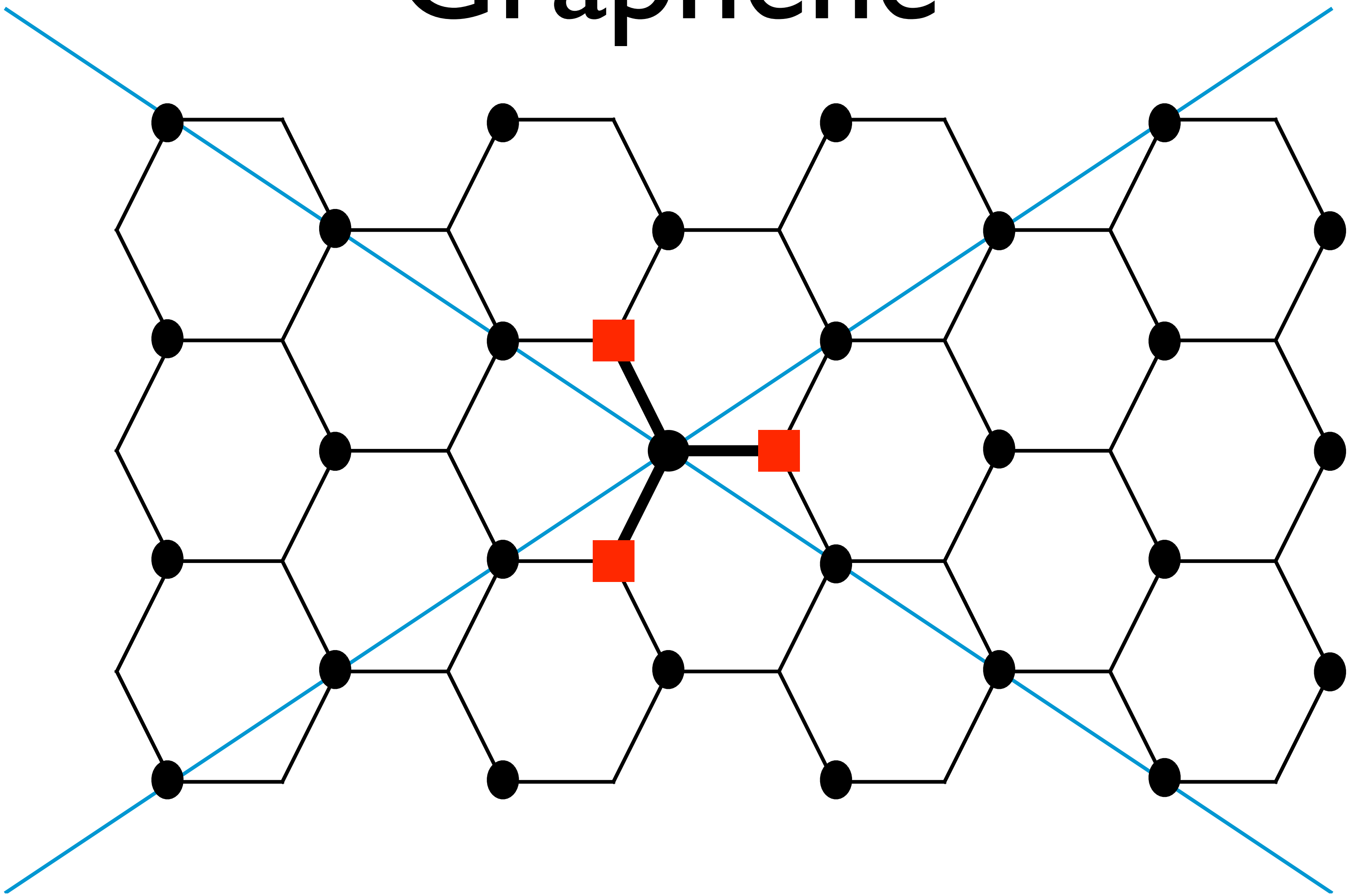
-This effect lead to great interest
throughout condensed matter
community

Graphene

Graphene



Graphene



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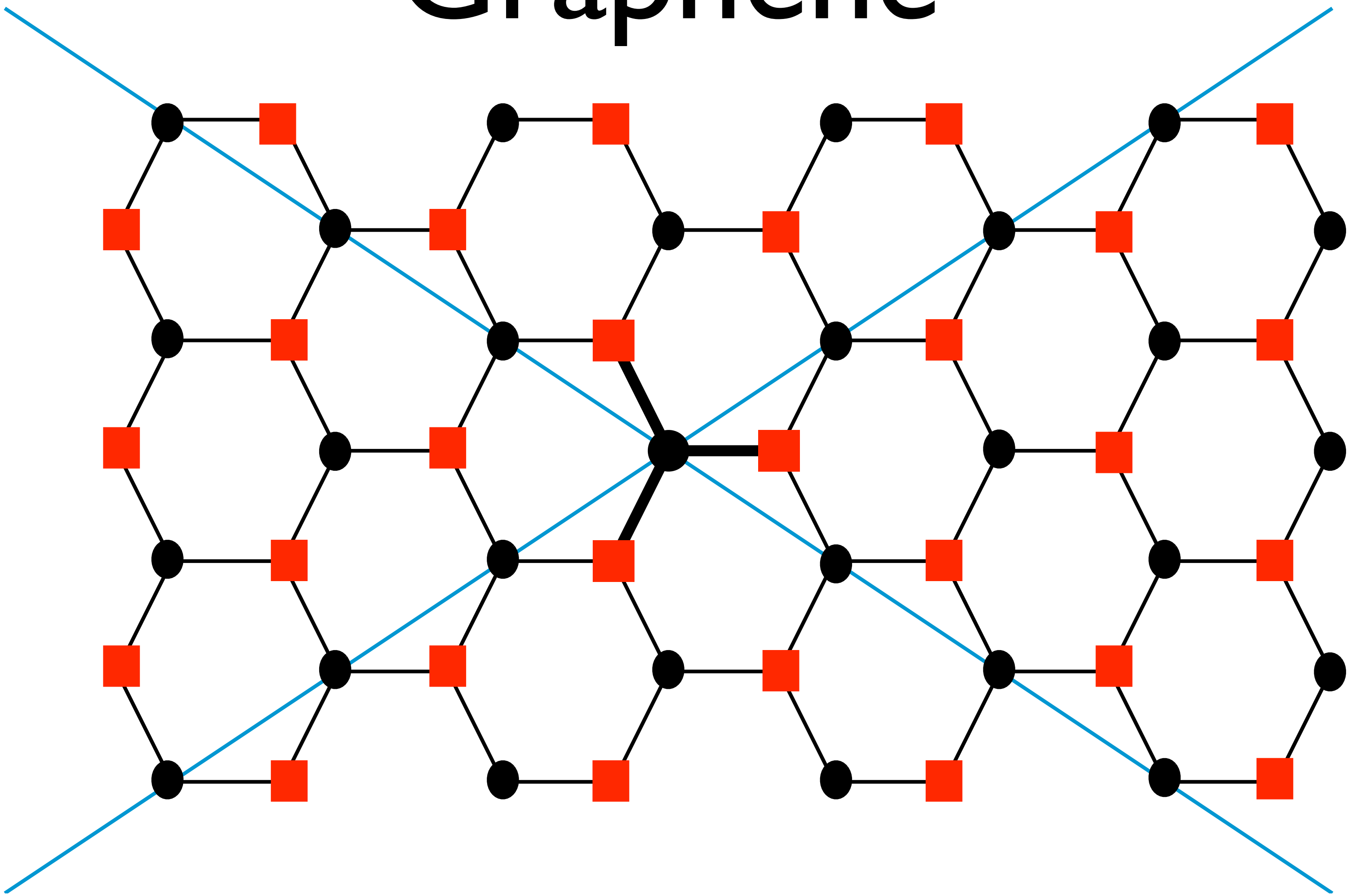

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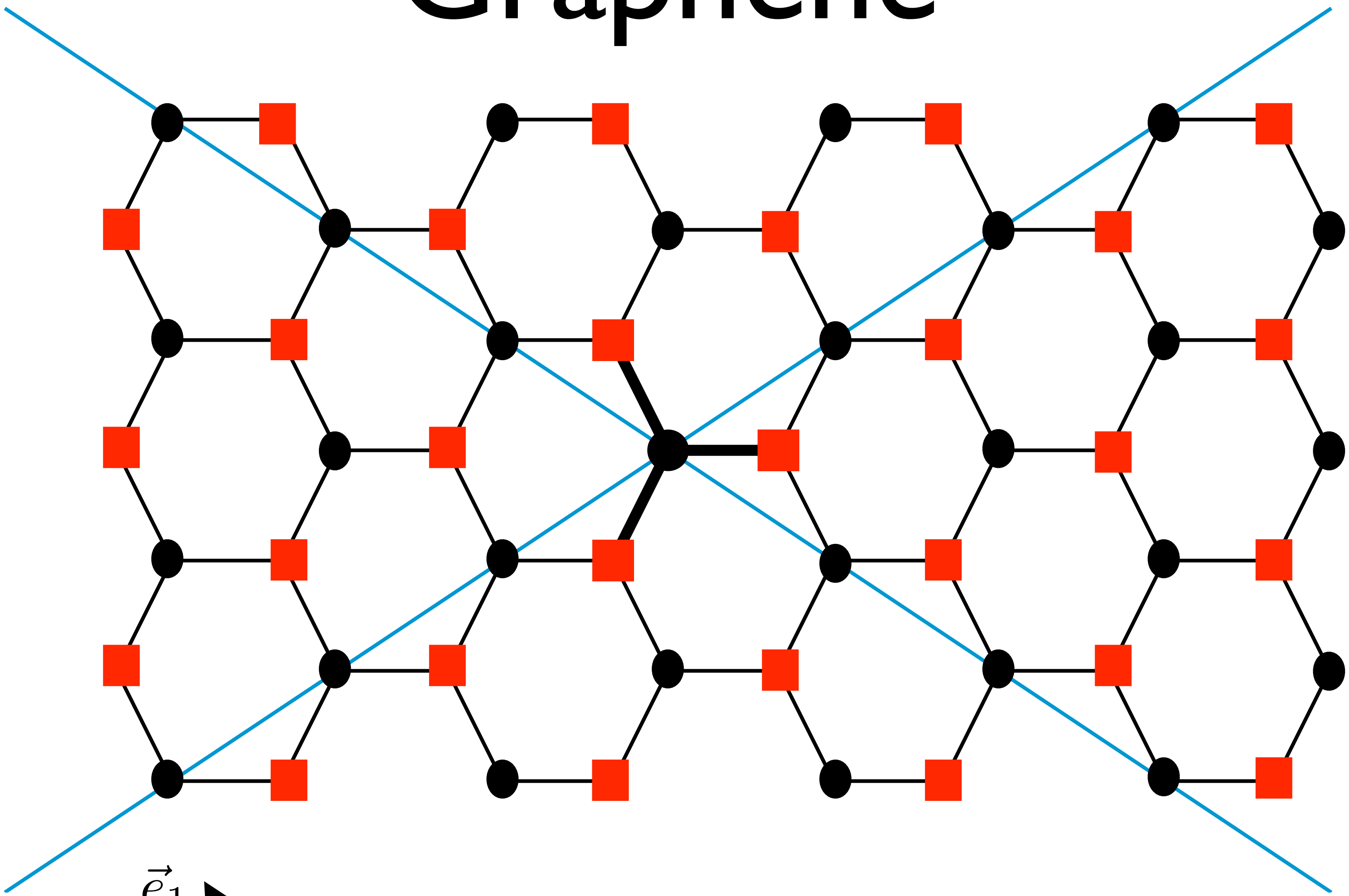
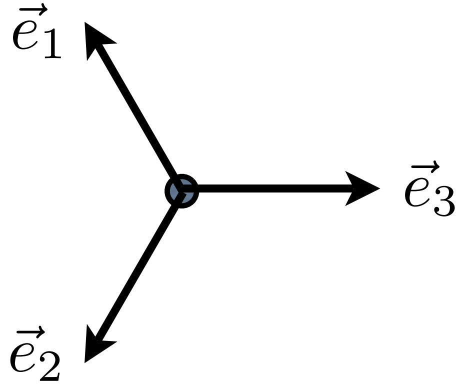
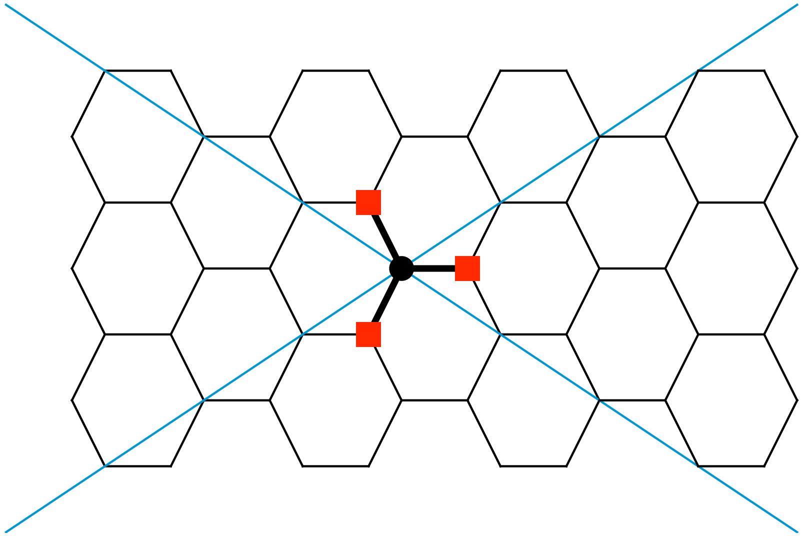




$$\sum_\alpha \mathbf{e}^\alpha = 0 \quad \text{for } \alpha = 1, 2, 3$$

Graphene (cont.)



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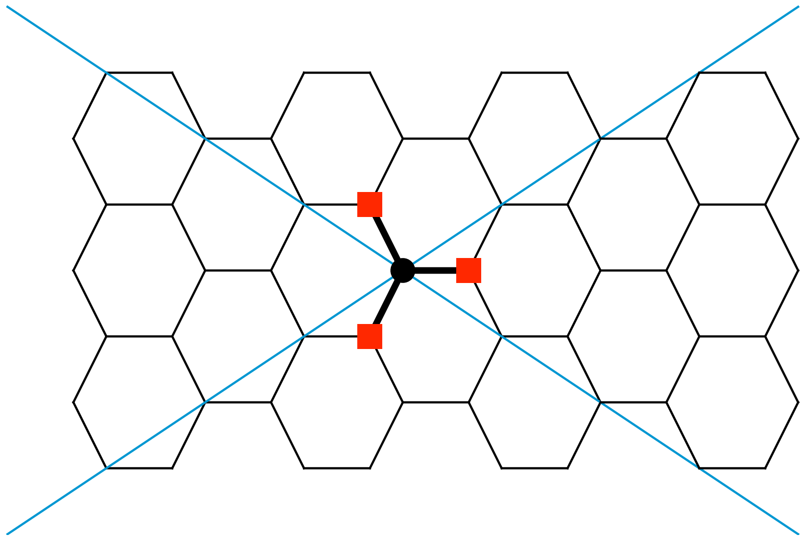

- 1+1 dimensional graphene lattices in the massless limit lead to chiral Dirac fermions.

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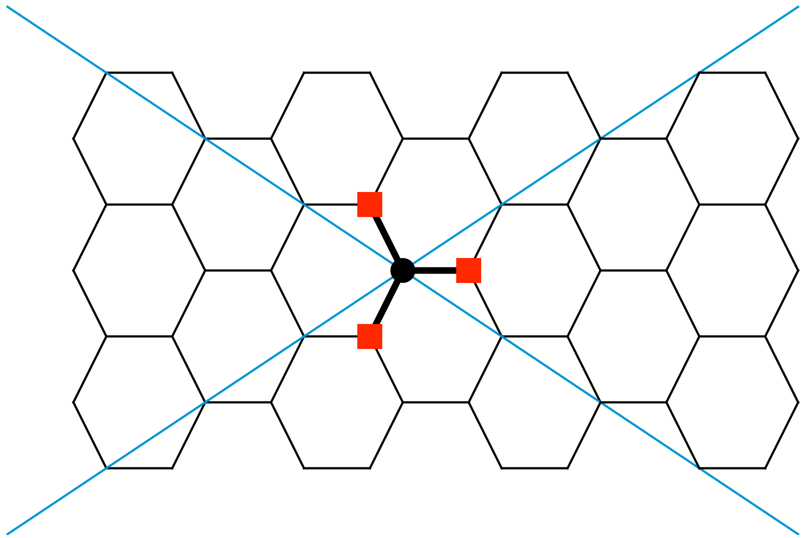

- 1+1 dimensional graphene lattices in the massless limit lead to chiral Dirac fermions.

- Question: Can this construction be extended to a four dimensional lattice gauge theory?

Graphene-Inspired Lattice

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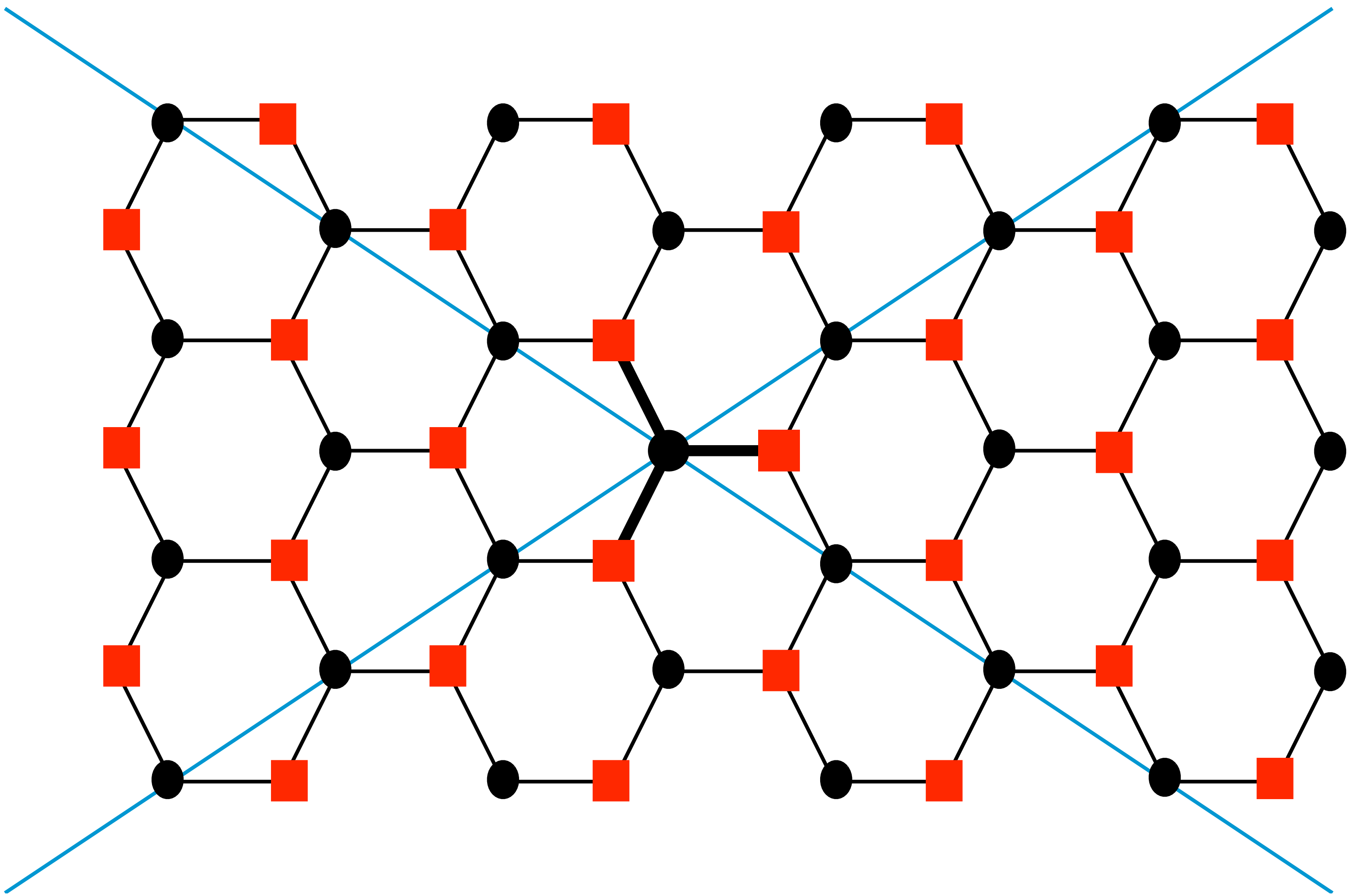

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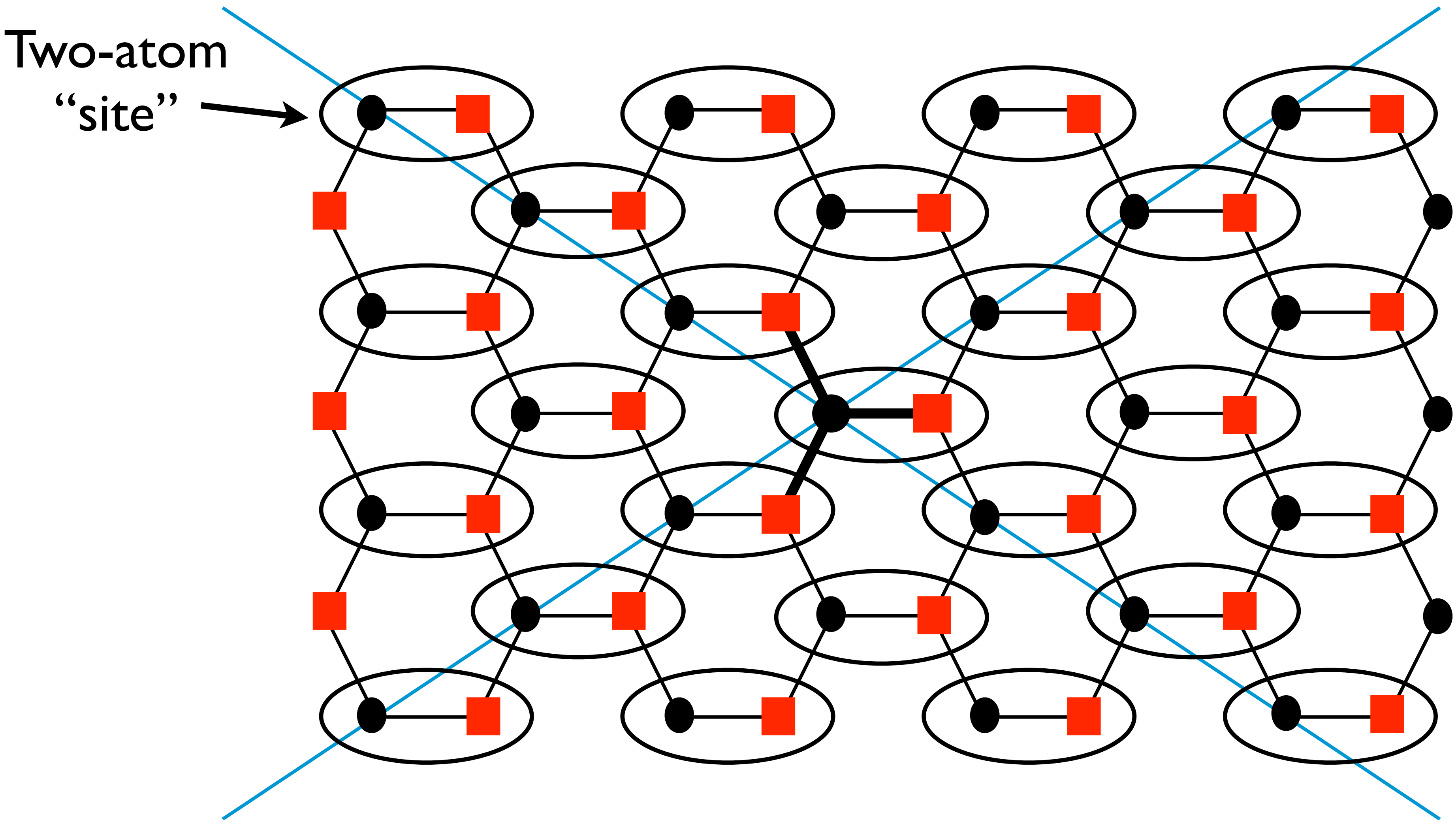

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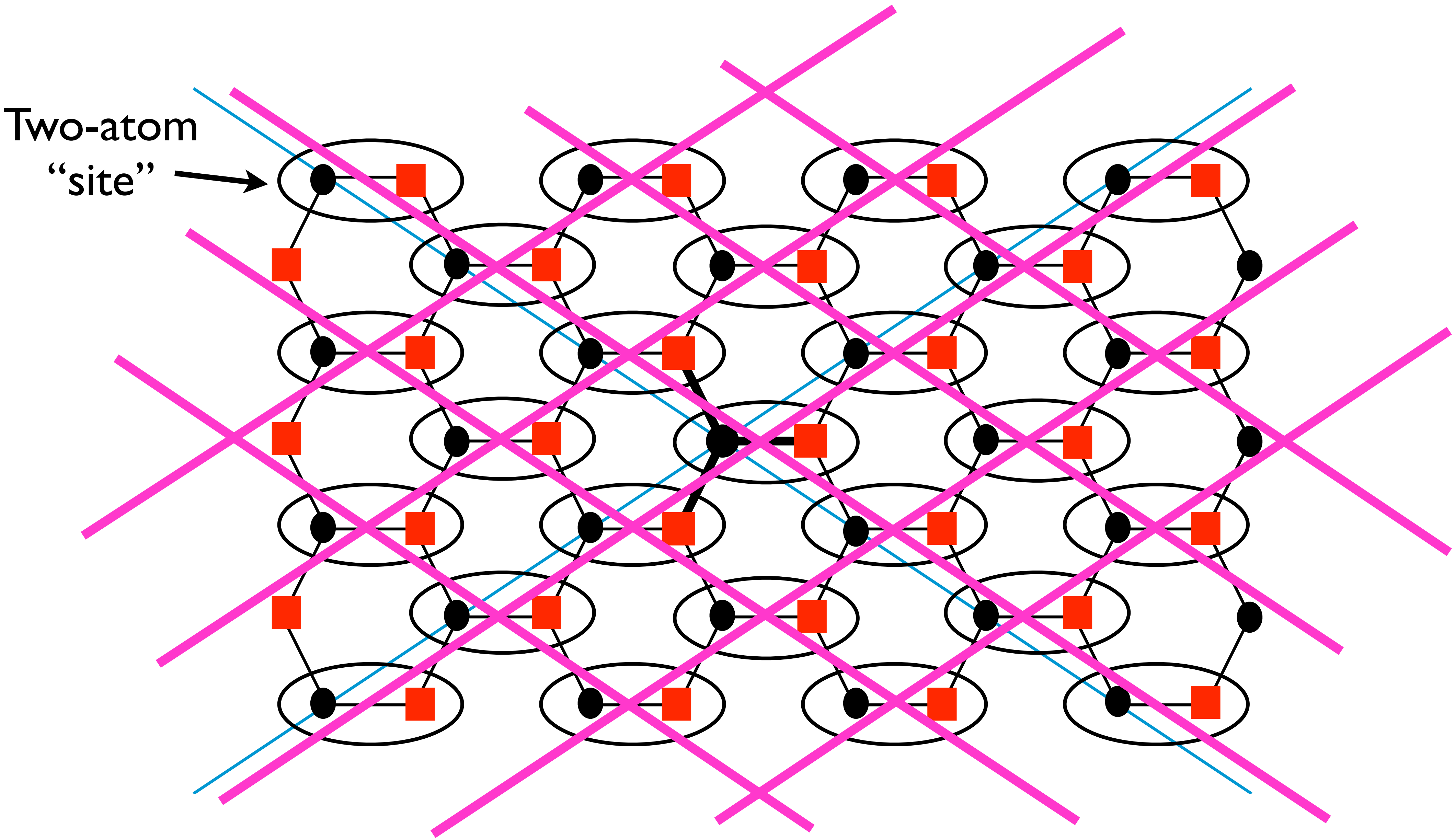

Borici-Creutz Action

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$$S_{BC} = \frac{1}{2}\sum_x \Bigg[\sum_\mu \left(\overline{\psi}_{x-\mu}\,\mathbf{e}^\mu\cdot\Gamma\,\psi_x - \overline{\psi}_{x+\mu}\,\mathbf{e}^\mu\cdot\Gamma^\dagger\,\psi_x\right) + \overline{\psi}_x\,\mathbf{e}^5\cdot\Gamma\,\psi_x - \overline{\psi}_x\,\mathbf{e}^5\cdot\Gamma^\dagger\,\psi_x \Bigg]$$

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$$\Gamma_\mu = (\vec{\gamma}, i\gamma_4)$$

$$\mu = 1,2,3,4$$

$$\begin{aligned}
\mathbf{e}^1 &= (\ \ 1,\ \ 1,\ \ 1,\ B\ ) \\
\mathbf{e}^2 &= (\ \ 1,-1,-1,\ B\ ) \\
\mathbf{e}^3 &= (-1,-1,\ \ 1,\ B\ ) \\
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\mathbf{e}^5 &= -(\ \ 0,\ \ 0,\ \ 0, 4BC)
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Momentum Space:

$$S_{BC} = \int_p \overline{\psi}_p \left[i \sum_\mu \left(\sin(p_\mu)\vec{\mathbf{e}}^\mu \cdot \vec{\gamma} + B\gamma_4(\cos(p_\mu) - C) \right) \right] \psi_p$$

Borici-Creutz (cont.)

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Chiral Symmetry

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Good News:

Chiral Symmetry

Minimal Doubling (two Poles): If $B \neq 0$ and $0 < C < 1$

$$p_\mu^{(1)} = \tilde{p}(\ 1, \ 1, \ 1, \ 1 \ ) \qquad p_\mu^{(2)} = -\tilde{p}(\ 1, \ 1, \ 1, \ 1 \ ) \qquad \cos(\tilde{p}) = C$$

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Bad News:

Broken Discrete Symmetries:

Parity: $\psi(\vec{p}, p_4) \rightarrow \gamma_4 \psi(-\vec{p}, p_4)$

Charge Conjugation: $\psi(\vec{p}, p_4) \rightarrow C\bar{\psi}^T(\vec{p}, p_4)$

Time Reversal: $\psi(\vec{p}, p_4) \rightarrow \gamma_5\gamma_4\psi(\vec{p}, -p_4)$

Radiative Corrections

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Relevant Operators: $$\frac{1}{a}\sum_j c_3^{(j)}\mathcal{O}_3^{(j)}$$

$$\mathcal{O}_3^{(1)} = 4iB\overline{\psi}\gamma_4\psi = i\sum_\mu \overline{\psi}(\mathbf{e}^\mu\cdot\gamma)\psi$$

$$\mathcal{O}_3^{(2)} = 4iB\overline{\psi}\gamma_4\gamma_5\psi = i\sum_\mu \overline{\psi}(\mathbf{e}^\mu\cdot\gamma)\gamma_5\psi$$

$\mu = 1, 2, 3, 4$

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- Terms will be generated unless additional symmetry of the action prevents them.

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4D: If $\mathbb{Z}_5$ (or A_5 or S_5) symmetric

$$\sum_\alpha \mathbf{e}^\alpha = 0 \quad \text{for } \alpha = 1,2,3,4,5 \;\Rightarrow\; \mathcal{O}_3^{(1)} = i \sum_\alpha \overline{\psi}(\mathbf{e}^\alpha \cdot \gamma)\psi = 0$$

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- Does Borici-Creutz have the minimal $\mathbb{Z}_5$ symmetry?

Permutation Symmetry?

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$$\psi_x = \begin{pmatrix}\phi_x \\ \chi_x\end{pmatrix}$$

$$\overline{\psi}_p = (\overline{\phi}_p, \overline{\chi}_p)$$

$$\Sigma = (\vec{\sigma}, -1)$$

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Type I (first two lines)

For $B = 1/\sqrt{5}$
and $C = 1$

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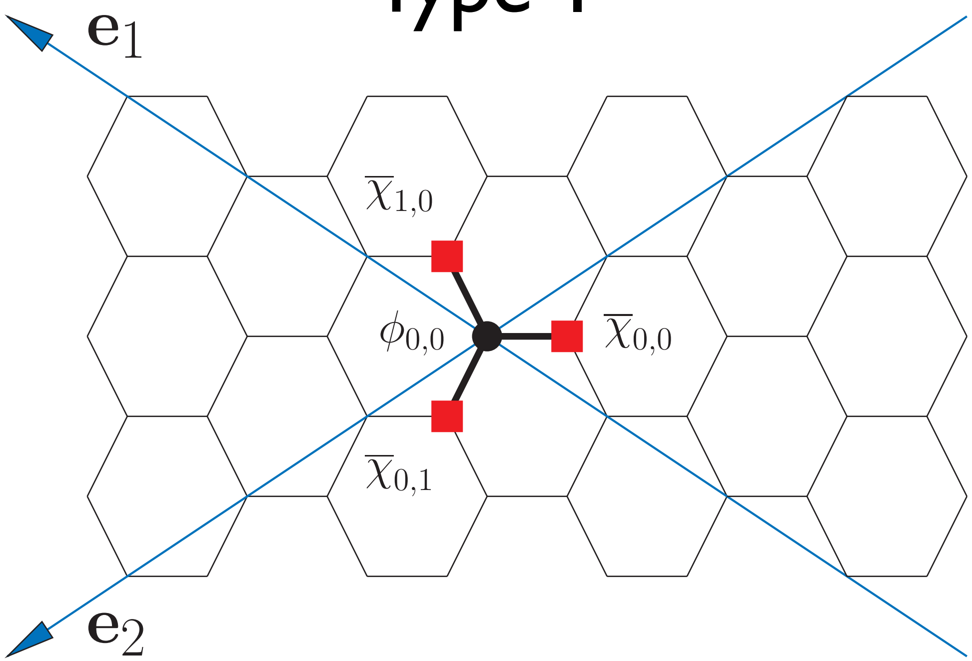

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Type 1 (first two lines)

Type 2 (last two lines)

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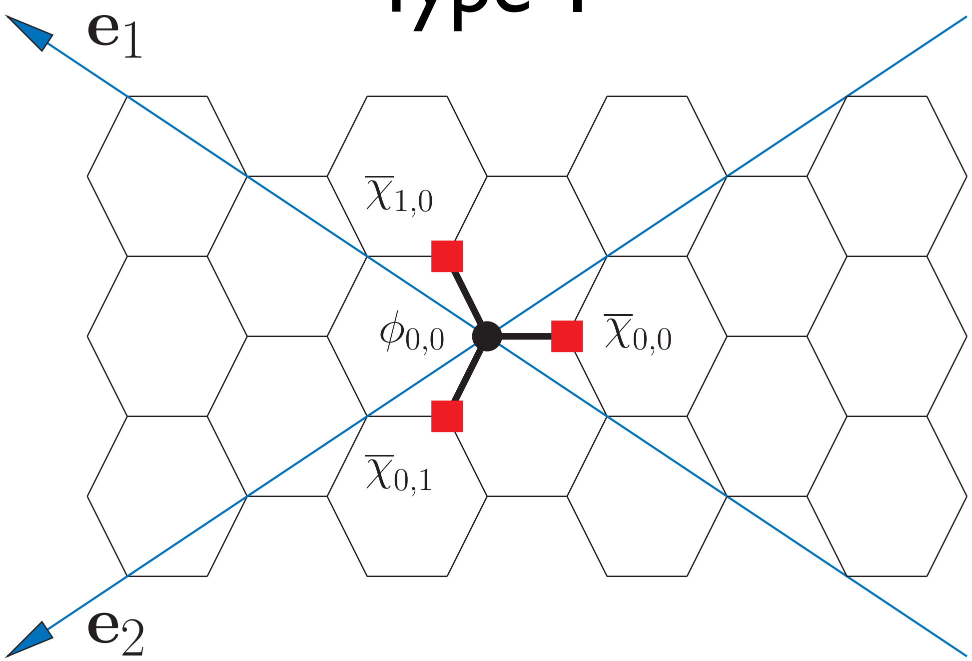
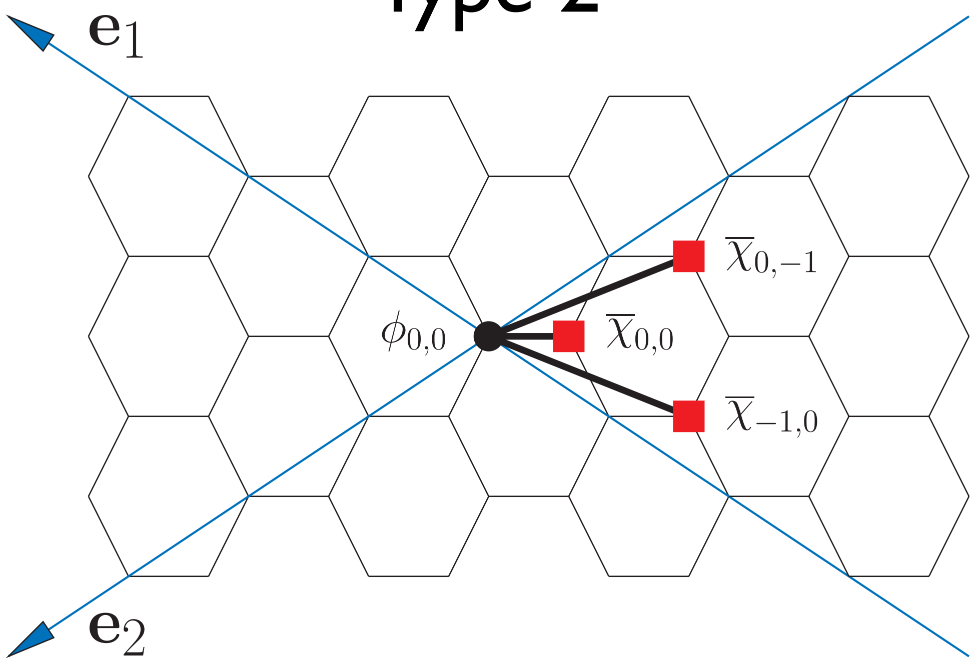


Type 2



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Action around pole $p_\mu \simeq 0$: $\overline{\psi}(i\vec{\gamma}\cdot\vec{k} + \gamma_5\gamma_4 k_4)\psi$

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“Mutilated” Fermions

“Hyperdiamond”

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- Find action with enough permutation symmetry

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$$S = \sum_x \Bigg[\sum_{\mu=1}^{4} \left(\overline{\phi}_{x-\mu}\, \sigma \cdot \mathbf{e}^\mu \, \chi_x - \overline{\chi}_{x+\mu}\, \overline{\sigma} \cdot \mathbf{e}^\mu \, \phi_x \right)$$

$$+ \overline{\phi}_x \, \sigma \cdot \mathbf{e}^5 \, \chi_x - \overline{\chi}_x \, \overline{\sigma} \cdot \mathbf{e}^5 \, \phi_x \Bigg]$$

"Hyperdiamond"

- Find action with enough permutation symmetry

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$$\begin{aligned}
\mathbf{e}^1 &= \tfrac{1}{4}(\ \ \sqrt{5},\ \ \sqrt{5},\ \ \sqrt{5},\ 1) \\
\mathbf{e}^2 &= \tfrac{1}{4}(\ \ \sqrt{5}, -\sqrt{5}, -\sqrt{5},\ 1) \\
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$$\sum_\alpha \mathbf{e}^\alpha = 0$$

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Has A_5 symmetry → No relevant operators

Hyperdiamond (cont.)

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Ex:

$$p_1 = -p_2 = -p_3 = p_4 = \cos^{-1}(-2/3)$$

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  - To this point, no non-orthogonal action has accomplished this balance.