Nucleon Electromagnetic Form Factors with Domain Wall Fermions on an Asqtad Sea

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Introduction

Nucleon electromagnetic form factors:

$$\langle N(P')|J^{\mu}_{EM}(x)|N(P)\rangle = e^{i(P'-P)\cdot x}\bar{u}(P')\left[\gamma^{\mu}F_1(q^2) + i\sigma^{\mu\nu}\frac{q^{\nu}}{2M_N}F_2(q^2)\right]u(P)$$

Consider only isovector case:

$$J^{\mu}_{EM}(x) = \bar{u}(x)\gamma^{\mu}u(x) - \bar{d}(x)\gamma^{\mu}d(x)$$

Disconnected diagram contributions cancel in the isospin limit.

Outline

Why DWF on an Asqtad Sea?

- Domain Wall Fermions (DWF)
  - Pros: Good chiral symmetry, only broken with a controllable small amount, parametrized by m_{res}.
  - Cons: 5-dimensional fermions, costly to generate gauge configurations (and do measurements)
- $O(a^2)$ Tadpole-improved Staggered Fermions (Asqtad)
  - Pros: inexpensive, MILC 2+1 flavor dynamical gauge configurations with light pion masses publicly available
  - Cons: break flavor symmetry, have only remnant $U(1)$ chiral symmetry
- DWF+Asqtad gives an economical way to achieve both chiral symmetry and flavor symmetry in the valence sector, and provides an early access to important physical results using chiral fermions

Why DWF on an Asqtad Sea?

Generalization of continuum chiral perturbation theory straightforward.

- DWF on DWF: continuum-like chiral perturbation theory

  RBC-UKQCD, arXiv:0804.0473

  - To NLO, residual chiral symmetry breaking only introduces a constant shift (m_{res}) to the input quark masses
  - To NLO, no additional parameters. Discretization errors contained in the low energy constants.

- DWF on Asqtad: need only small corrections to the unitary theory

  Chen, O'Connell and Walker-Loud, arXiv:0706.0035

  - Expressed in terms of m_π measured on the lattice
  - Existing continuum ChPT can be modified in a universal way

- Can make use of the available continuum chiral extrapolation formulae, with slight modifications.

Parameters

- MILC coarse ensembles: $a \approx 0.125$ fm

$L^3 \times T$	$(am_l)/(am_s)^{\text{asqtad}}$	# confs	m_π^{DWF} [MeV]
$20^3 \times 64$	0.007/0.05	464	293
$20^3 \times 64$	0.01/0.05	628	356
$20^3 \times 64$	0.02/0.05	477	495
$20^3 \times 64$	0.03/0.05	561	597
$20^3 \times 64$	0.04/0.05	348	688
$20^3 \times 64$	0.05/0.05	423	758
$28^3 \times 64$	0.01/0.05	274	353

- Parameters for valence DWF:
  - Valence quark masses tuned to match the asqtad Goldstone pion masses
  - Domain wall height $M_5 = 1.7$, tuned to minimize m_{res}
  - $L_s = 16 \Rightarrow am_{\text{res}} \approx 0.001$.

LHPC Hadron Structure Projects

- Recent Publications on Mixed-Action Calculation:
  - Nucleon axial charge: *Phys.Rev.Lett.96:052001,2006*
  - Generalized parton distributions: *Phys.Rev.D77:094502,2008*
  - Hadron spectroscopy: *arXiv:0806.4549*

- Other talks on hadron structure:
  - Generalized form factors:
    John Negele, Friday @ Hadron Structure

  - Preliminary DWF on DWF results:
    Sergey Syritsyn, Friday @ Hadron Structure

So What's New ...

On the two $m_\pi \approx 355$ MeV ensembles...

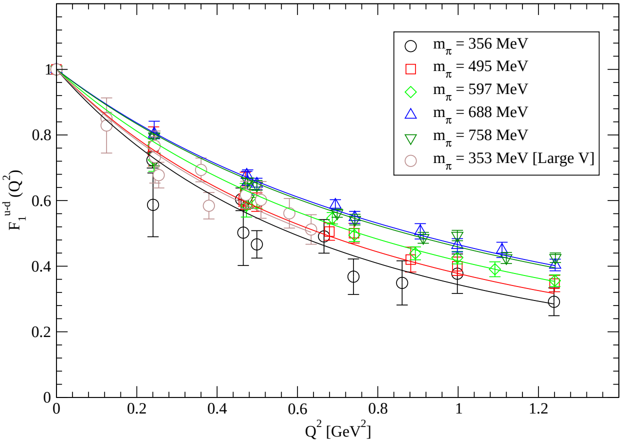
	Previous Calculation	Current Calculation
B.C.	Dirichlet at $t = T/2$	anti-periodic at $t = T$
$\tau_{snk} - \tau_{src}$	10	9
# sources	1 proton	4 proton + 4 anti-proton
sink	1 sink	4 sinks calculated at once

The improvements:

- 4 coherent sinks calculated at once; saved computational time by a factor of 4
  Gauge averaging cancels out contaminations from other sinks
- $8\mathrm{X}$ more measurements reduced the statiscal errors by roughly $\sqrt{8} \approx 3\mathrm{X}$.
- Shorter source-sink separation further reduced statistical noise
  $\Longrightarrow$ Overall error reduction: $4\mathrm{X}$
- One new lighter mass at $m_\pi = 293$ MeV.

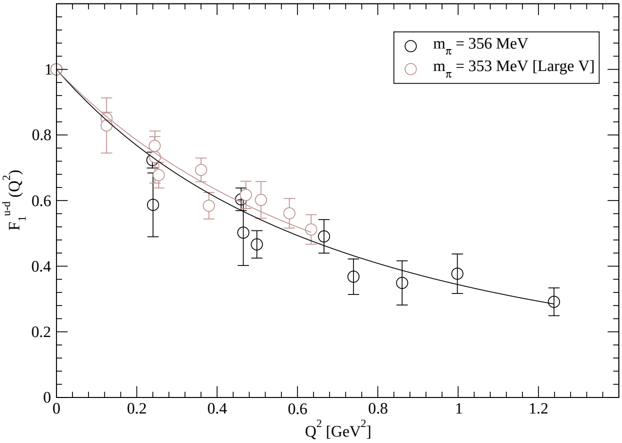
Isovector Dirac Form Factor

Previous results at different pion masses:



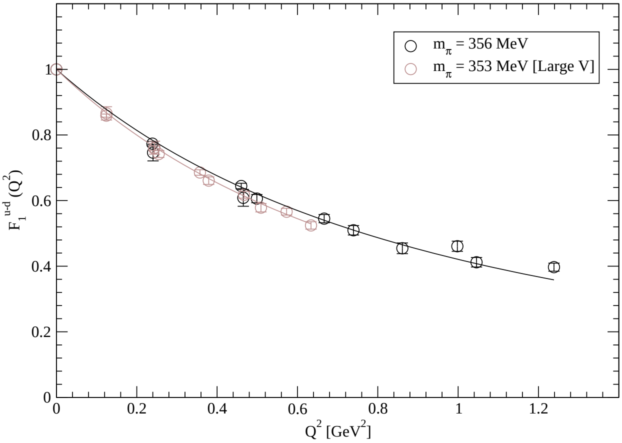
Isovector Dirac Form Factor

Previous results at $m_\pi \approx 355$ MeV for two different volumes:



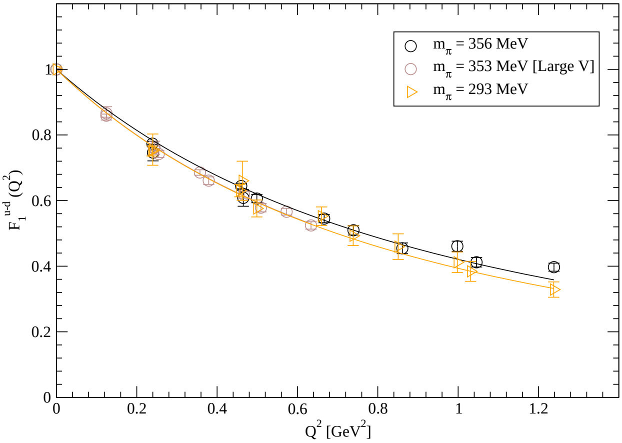
Isovector Dirac Form Factor

Improved results at $m_\pi \approx 355$ MeV for two different volumes:



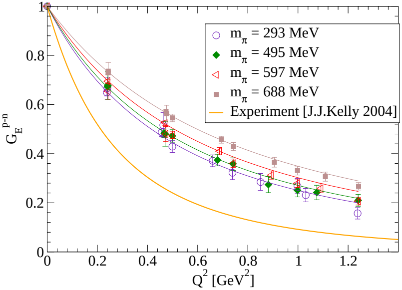
Isovector Dirac Form Factor

New results at $m_\pi \approx 293$ MeV:



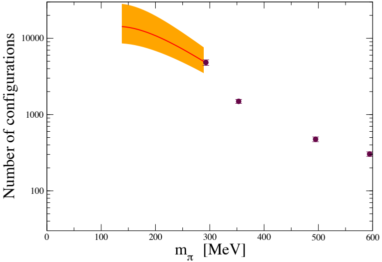
Towards the Physical World

- Isovector Sachs form factor for four different pion masses, compared with Kelly's parametrization of the experimental data
- Mild pion mass dependence
- Still a long way to go towards the physical world



- Going to the physical limit, cost grows exponentially to achieve the same precision (3%)

What it takes



Based on signal to noise ratio 30 (at T=10)

K. Orginos

The Calculation

- Nucleon 2pt and 3pt correlation function:

$$C^{\text{2pt}}(\tau, P) = \sum_{j,k} \left(\Gamma_{\text{unpol}}\right)_{jk} \langle N_k(\tau, P)\overline{N}_j(\tau_{\text{src}}, P)\rangle ,$$

$$C^{\text{3pt}}_{\mathcal{O}}(\tau, P', P) = \sum_{j,k} \left(\Gamma_{\text{pol}}\right)_{jk} \langle N_k(\tau_{\text{snk}}, P')\mathcal{O}(\tau)\overline{N}_j(\tau_{\text{src}}, P)\rangle ,$$

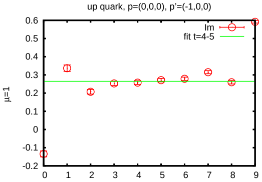
- Source: Gaussian smeared, constructed from HYP-smeared gauge links
- Lattice momenta

$$\vec{P} = \frac{2\pi}{La}\vec{p}, \quad p_i \in -L, -L+1, ..., L-1, L$$

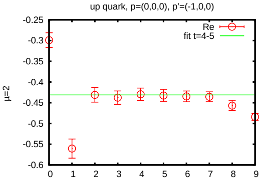
  - Two sink momenta: $\vec{p'} = (1,0,0), (-1,0,0)$
  - Various momentum transfer: $\vec{q} = p' - p$, restricted to $\vec{q}^2 < 10$

Plateaus

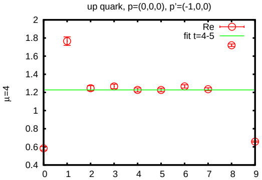
$$R_{\mathcal{O}}(\tau, P', P) = \frac{C^{\text{3pt}}_{\mathcal{O}}(\tau, P', P)}{C^{\text{2pt}}(\tau_{\text{snk}}, P')} \times \left[\frac{C^{\text{2pt}}(\tau_{\text{snk}} - \tau + \tau_{\text{src}}, P)\; C^{\text{2pt}}(\tau, P')\; C^{\text{2pt}}(\tau_{\text{snk}}, P')}{C^{\text{2pt}}(\tau_{\text{snk}} - \tau + \tau_{\text{src}}, P')\; C^{\text{2pt}}(\tau, P)\; C^{\text{2pt}}(\tau_{\text{snk}}, P)} \right]^{1/2}$$



$$\text{Im}\,\mu = 1 \quad \propto \quad F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)$$



$$\text{Re}\,\mu = 2 \quad \propto \quad F_1(Q^2) + F_2(Q^2)$$

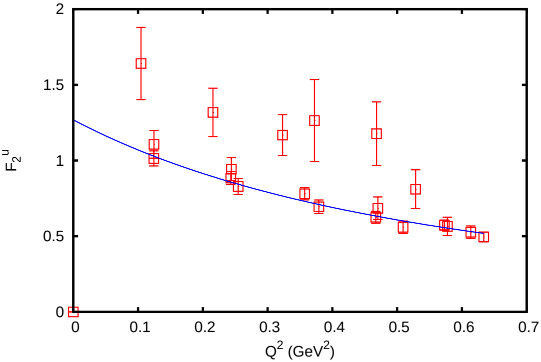


$$\text{Re}\,\mu = 4 \quad \propto \quad c \left[F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2) \right]$$

Overdetermined analysis:

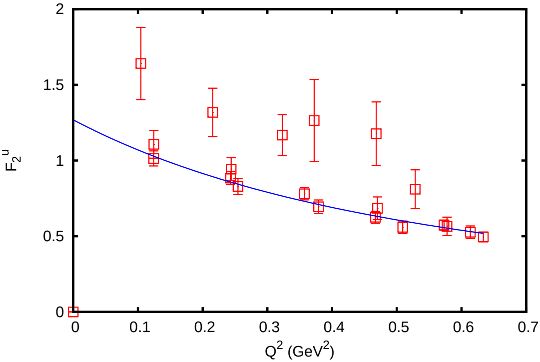
- singlular value decomposition to obtain optimal values for F_1 and F_2

Recognizing Correlations



- Several “outliers”... Systematics?

Recognizing Correlations



- Several "outliers"... Systematics?
- Data highly correlated; Deviations from the universal curve quantified by

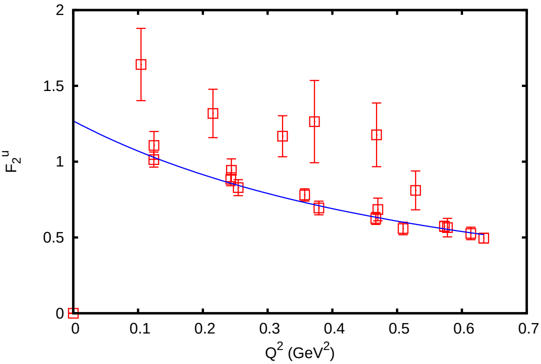
$$\text{correlated } \chi^2/N = \sum_{n=0}^{N-1} (F_i^{(n)} - \hat{F}_i) C_{ij}^{-1} (F_j^{(n)} - \hat{F}_j) = 1.9(1.1)$$

Recipes for the High-momentum Data

- The offenders share the same characteristics:
  - $\vec{p'} = (-1, 0, 0)$
  - $\vec{p}$ has at least one component of 2 or 3.
  - Much noisier
- They usually do not affect the final analysis due to large errrors
- Removed from the analysis to avoid unnecessary bias
- Same rule applied to all ensembles

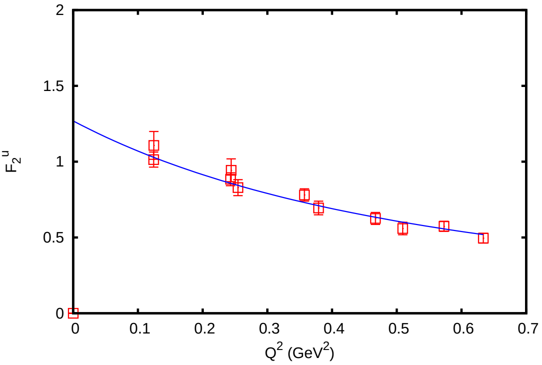
Recipes for the High-momentum Data

Before



Recipes for the High-momentum Data

After



Chiral Extrapolations for Nucleon Form Factors

- We use heavy baryon chiral perturbation theory including the Δ resonance, third order in small scale expansion (SSE).

  Bernard, Fearing, Hermert and Meissner (1998)

- Low-energy constants:
  g_A, F_π, M_N, Δ, c_A, c_V + counter terms

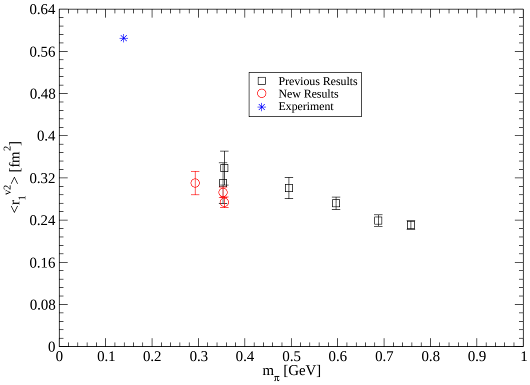
- Ideally would like to determine all the constants from lattice.
- Reality: Not enough data to constrain all the parameters; use phenomenological input

g_A	F_π [MeV]	M_N [MeV]	Δ [MeV]	c_A
1.2	92.4	939	293	1.5

Dirac Radius

- One-parameter dipole fits to $F_1(Q^2) = \frac{1}{(1+Q^2/M_1^2)^2}$ with $Q^2 \leq 0.4$ GeV^2

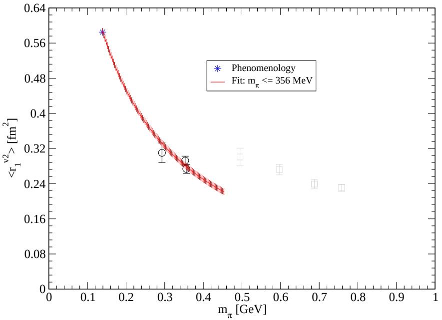
$$\langle r_1^2 \rangle = 12/M_1^2$$



Dirac Radius

$$
\begin{aligned}
(r_1^v)^2 = &-\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7g_A^2 + \left(10g_A^2 + 2\right)\log\left[\frac{m_\pi}{\lambda}\right]\right\} \\
&-\frac{12B_{10}^{(r)}(\lambda)}{(4\pi F_\pi)^2} + \frac{c_A^2}{54\pi^2 F_\pi^2}\left\{26 + 30\log\left[\frac{m_\pi}{\lambda}\right]\right. \\
&\left.+30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right]\right\}.
\end{aligned}
$$

- One free parameter $B_{10}^{(r)}(\lambda)$
- Fit reproduces phenomenological value
- But...



Note: Phenomenological value for isovector $\langle r_1^2 \rangle$ taken from Mergell, Meissner and Drechsel, hep-ph/9506375

Dirac Radius

$$(r_1^v)^2 = -\frac{1}{(4\pi F_\pi)^2}\left\{1 + 7g_A^2 + \left(10g_A^2 + 2\right)\log\left[\frac{m_\pi}{\lambda}\right]\right\}$$
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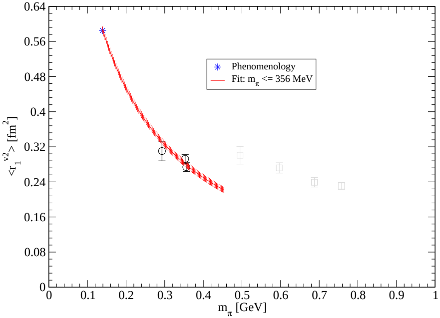
$$\left. +30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right]\right\}.$$

- One free parameter $B_{10}^{(r)}(\lambda)$
- Fit reproduces phenomenological value
- But...
  - Fit misses heavy points badly
  - New and old results employed different techniques; could potentially have different systematics
  - Further investigations needed.

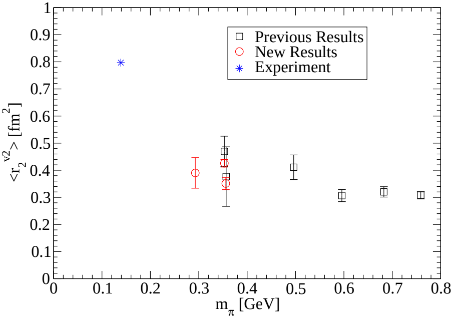


Note: Phenomenological value for isovector $\langle r_1^2 \rangle$ taken from Mergell, Meissner and Drechsel, hep-ph/9506375

Pauli Radius

- Two-parameter dipole fits:
  $F_2(Q^2) = \frac{F_2(0)}{(1+Q^2/M_2^2)^2}$
  with $Q^2 \leq 0.8$ GeV^2
  $$\langle r_2^2 \rangle = 12/M_2^2$$

$$\begin{aligned}(r_2^v)^2 &= \frac{g_A^2 M_N}{8F_\pi^2 \kappa_v(m_\pi)\pi m_\pi} \\ &+ \frac{c_A^2 M_N}{9F_\pi^2 \kappa_v(m_\pi)\pi^2\sqrt{\Delta^2 - m_\pi^2}} \\ &\times \log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right]\end{aligned}$$



- Decouple $\kappa_v(m_\pi)$ and $(r_2^v)^2$ by doing fits to
  $$\kappa_v(m_\pi)\,(r_2^v)^2$$
- No free parameter;

Pauli Radius

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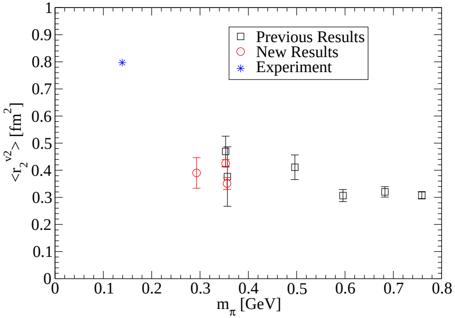
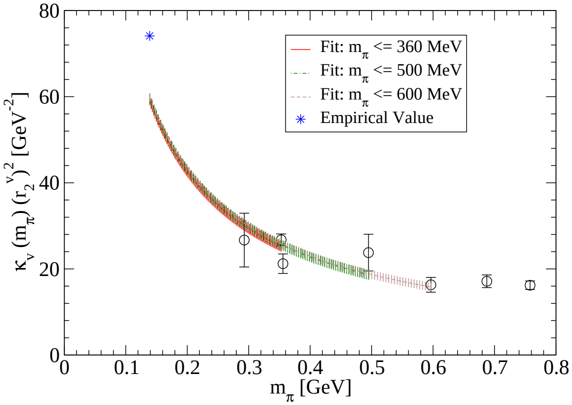
  $F_2(Q^2) = \frac{F_2(0)}{(1+Q^2/M_2^2)^2}$
  with $Q^2 \leq 0.8$ GeV^2
  $$\langle r_2^2 \rangle = 12/M_2^2$$

$$
\begin{aligned}
(r_2^v)^2 &= \frac{g_A^2 M_N}{8F_\pi^2 \kappa_v(m_\pi)\pi m_\pi} \\
&+ \frac{c_A^2 M_N}{9F_\pi^2 \kappa_v(m_\pi)\pi^2\sqrt{\Delta^2 - m_\pi^2}} \\
&\times \log\left[\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1}\right] \\
&+ \frac{24 M_N}{\kappa_v(m_\pi)} B_{c2}
\end{aligned}
$$



- Decouple $\kappa_v(m_\pi)$ and $(r_2^v)^2$ by doing fits to
  $$\kappa_v(m_\pi)\,(r_2^v)^2$$
- No free parameter; add a “core” term to the formula [QCDSF, PRD 71, 034508(2005)]

Pauli Radius



- B_{c2} found to be consistent with 0 with different fit ranges

Anomalous Magnetic Moment

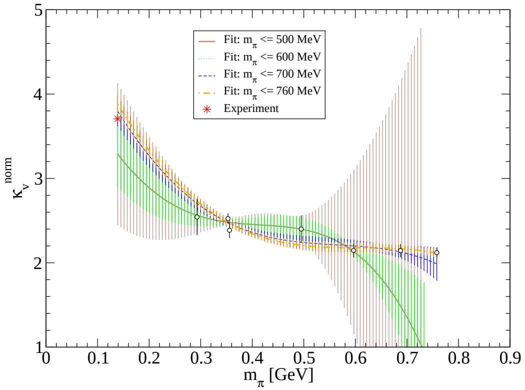
- From two-parameter dipole fits

$$\kappa_v^{lat} = F_2(0), \quad \kappa_v^{\text{norm}} = \kappa_v^{\text{lat}} \frac{M_N^{\text{phys}}}{M_N^{\text{lat}}}$$

- $\kappa_v(m_\pi)$ involves three free parameters:

$$\kappa_v^0, \; E_1^r(\lambda), \; \text{and} \; c_V$$

- Fits surprisingly stable
- But not a claim that HBChPT should work well to that heavy region

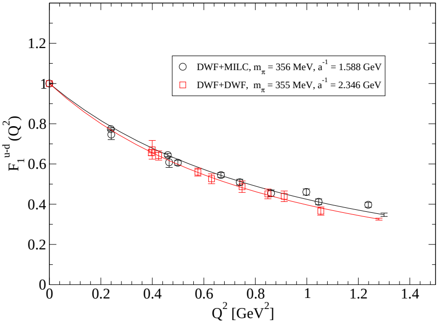


Fit Range	κ_V^0	c_V [GeV^{-1}]	E_1^r (0.6 GeV)	χ^2/d.o.f
$\leq$ 500 MeV	4.9(1.1)	3.5(4.5)	-3.4(7)	1.2
$\leq$ 600 MeV	4.9(5)	3.4(1.4)	-3.4(4)	0.6
$\leq$ 700 MeV	5.43(21)	1.8(4)	-3.00(16)	0.9
$\leq$ 760 MeV	5.55(14)	1.51(21)	-2.90(10)	0.8

Comparison with DWF Results

- LHPC now joins RBC and UKQCD in doing full domain wall fermion calculations.
- Available now (hadron structure):
  - $24^3 \times 64$, $a \approx 0.11$ fm, $m_\pi \approx 330$ MeV
  - $32^3 \times 64$, $a \approx 0.09$ fm, $m_\pi \approx 300 \,\&\, 350$ MeV
- Comparing coarse and fine DWF ensembles reveals little finite-a dependence.

(see S.Syristyn's talk on Friday)



- No significant discretization errors in the mixed-action calculations compared with full DWF.

Summary and Outlook

- New calculation improved the statistics for the three light-mass ensembles by a factor of 4.
- $\langle (r_1^v)^2 \rangle$, $\langle (r_2^v)^2 \rangle$ and κ_v show qualitative agreement with experiment.
- Mixed-action results compare well with full domain wall calculations.

- May improve statistics on the heavy-mass ensembles using the same techniques as the light-mass ensembles to have comparable systematics.
- New era: together with RBC and UKQCD collaborations, moving on to the full domain wall calculations towards the physical pion mass.