Physical matrix elements for $\Delta I = 3/2$ channel $K \to \pi\pi$ decays Using 2+1 Flavor Domain Wall Fermions

Lattice 2008

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Introduction

- $K \to \pi\pi$ decays on the lattice are interesting because the typical energies involved are less than Λ_{QCD} so that QCD effects are important in this decay.
- The direct CP violating parameter ϵ'/ϵ can be found from $K \to \pi\pi$ calculations.
- Need domain wall fermions (DWF) and a large lattice size to get reasonable uncertainties. The RBC/UKQCD $32^3 \times 64$, $L_s = 16$ 2+1 flavor lattices are used.
- Chiral extrapolations from unphysical masses are problematic, however with this lattice size we can approach physical pion masses.

Effective Hamiltonian

The weak interactions are included in an effective Hamiltonian

$$\mathcal{H}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} c_i(\mu) Q_i$$

where $c_i(\mu)$ are the Wilson coefficients and Q_i are four quark operators, for example

$$\begin{aligned}
Q_1 &= \bar{s}_a \gamma_\mu (1-\gamma^5) d_a \bar{u}_b \gamma^\mu (1-\gamma^5) u_b \\
Q_2 &= \bar{s}_a \gamma_\mu (1-\gamma^5) d_b \bar{u}_b \gamma^\mu (1-\gamma^5) u_a \\
Q_9 &= \frac{3}{2} \bar{s}_a \gamma_\mu (1-\gamma^5) d_a \sum_q e_q \bar{q}_b \gamma^\mu (1-\gamma^5) q_b \\
Q_{10} &= \frac{3}{2} \bar{s}_a \gamma_\mu (1-\gamma^5) d_b \sum_q e_q \bar{q}_b \gamma^\mu (1-\gamma^5) q_a
\end{aligned} \tag{1}$$

χPT

- For masses near the chiral limit, chiral perturbation theory (χPT) can be used to make predictions for the forms of matrix elements.
- The leading order chiral Lagrangian is written in terms of

$$\Sigma = \exp\left[\frac{2i\phi^a\lambda^a}{f}\right]$$

where the ϕ^a are the real pseudo-scalar meson fields, and is given by

$$\mathcal{L}_{LO} = \frac{f^2}{8} Tr[\partial_\mu \Sigma \partial^\mu \Sigma] + \frac{f^2 B_0}{4} Tr[\chi^\dagger \Sigma + \Sigma^\dagger \chi]$$

where $\chi = diag(m_u, m_d, m_s)$ and
$B_0 = \frac{m^2_{\pi^+}}{m_u+m_d} = \frac{m^2_{K^+}}{m_u+m_s} = \frac{m^2_{K^0}}{m_d+m_s}$

χPT

- Matrix elements of the parts of the weak operators that transform in a definite way under SU(3) and isospin are calculated by forming out of the Σ field all possible operators that transform in the given way at a given order. A linear combination of these with arbitrary coefficients, called low energy constants (LECs) is taken to represent the weak operator in question.
- At next to leading order and with 0 momentum in the initial and final states, the matrix elements in χPT will depend on meson masses squared and to the fourth power, the LECs, and the other parameters in the Lagrangian.

Extraction of LECs

- Since physical pions would require a larger box size than is currently available, the strategy to calculate matrix elements at unphysical kinematics in order to extract LECs from χPT, and then to use these LECs and χPT to calculate matrix elements at physical kinematics.
- In order to extract the necessary LECs for physical $K \to \pi\pi$ matrix elements given a limited number of ensembles, it is necessary either to resort to partial quenching, in which the masses of the quarks in the fermion determinant are different than the masses of the propagating quarks, or to considering pions with non-zero momentum.
- Laiho and Soni (hep-lat 0306035) have treated the case of partially quenched χPT at NLO with sea quarks of equal mass.

Phenomenological Fits

- Since χPT has been problematic in extracting $K \to \pi\pi$ matrix elements from $K \to vac$ and $K \to \pi$ matrix elements (see talk by Norman Christ), and since it is only available for the unquenched case at the moment, a potentially useful alternative when meson masses are already near their physical values would be to do a simple phenomenological extrapolation of the matrix element to physical masses.
- For example, we can just include all linear and quadratic terms (or even just linear terms) in the masses which are varied:

$$\begin{aligned} \langle \pi\pi | Q_i | K \rangle = A_i + B_i^{(1)} m_l + B_i^{(2)} m_s + B_i^{(3)} m_{sea} + C_i^{(1)} m_l^2 \\ + C_i^{(2)} m_s^2 + C_i^{(3)} m_{sea}^2 + C_i^{(4)} m_l m_s + C_i^{(5)} m_l m_{sea} + C_i^{(6)} m_s m_{sea} \end{aligned} \tag{2}$$

where $m_l = m_u^{val} = m_d^{val}$, $m_s = m_s^{val}$, and $m_{sea} = m_l^{sea}$. (The strange sea quark mass m_s^{sea} is not varied).

Non-Zero Momenta

- χPT for $K \rightarrow \pi\pi$ matrix elements with pions having non-zero momentum has been worked out by Sachrajda et. al. (hep-lat 0208007) and Laiho and Soni (hep-lat 0203106).
- In practice data with non-zero momenta can be very noisy.
- There are some methods for dealing with this, such as antiperiodic, and in general twisted boundary conditions.
- Non-zero momentum will be used as a consistency/way to obtain more statistics later on since it requires the calculation of additional propagators.

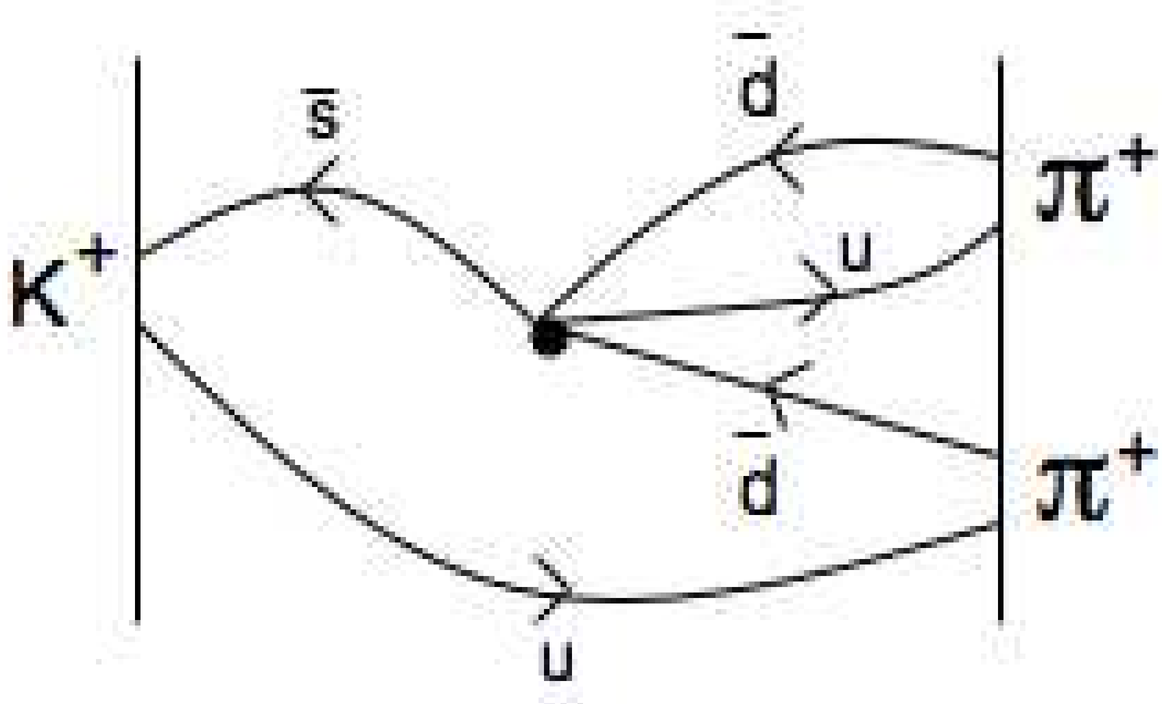
Calculating Lattice Matrix Elements

- Focusing on the weak operator that transforms like (27,1) under SU(3) and changes isospin by $\Delta I = 3/2$, we can relate the physical matrix element $\langle \pi^+\pi^0|\mathcal{O}^{(27,1),3/2}|K^+\rangle$ to an unphysical, easier to compute, matrix element via the Wigner Eckhart theorem:

$$\langle \pi^+\pi^+|\mathcal{O}'^{(27,1),3/2}|K^+\rangle = \frac{2\sqrt{2}}{\sqrt{3}}\langle \pi^+\pi^0|\mathcal{O}^{(27,1),3/2}|K^+\rangle \qquad (3)$$

- The unphysical matrix element on the left hand side has only one diagram that contributes to it:

Diagram for $K^+ \to \pi^+\pi^+$



Calculating Lattice Matrix Elements

- On the lattice we can calculate

$$C_{\mathcal{O}} = \langle 0|T\left(O_{\pi\pi}(t_{sink})\mathcal{O}_W(t)O_K^{\dagger}(t_{source})\right)|0\rangle \quad (4)$$

where

$$O_{\pi\pi}(t) = \bar{d}(t)\gamma^5 u(t)\bar{d}(t)\gamma^5 u(t) \quad (5)$$

$$O_K^{\dagger}(t) = \bar{u}(t))\gamma^5 s(t) \quad (6)$$

$$\mathcal{O}_W(t) = \mathcal{O}'^{(27,1),3/2}(t) \quad (7)$$

$$= \bar{s}(t)\gamma_\mu(1-\gamma^5)d(t)\bar{u}(t)\gamma^\mu(1-\gamma^5)d(t) \quad (8)$$

Calculating Lattice Matrix Elements

- Inserting a complete set of states we find that the leading exponential behavior far from the source and the sink is

$$C_{\mathcal{O}} \sim Z_{\pi\pi} \mathcal{M} Z_K^* \exp[-E_{\pi\pi}|t - t_{sink}|] \exp[-m_K|t - t_{source}|] \quad (9)$$

where $\mathcal{M}$ is the desired matrix element $\langle \pi^+\pi^+ | \mathcal{O}_W | K^+ \rangle$ and $Z_{\pi\pi}$ and Z_K are normalization factors from the K and $\pi\pi$ correlators.

$$C_K = \langle 0 | T \left(O_K(t) O_K^\dagger(t_{source}) \right) | 0 \rangle \quad (10)$$

$$\sim Z_K Z_K^* \exp[-m_K |t - t_{source}|] \quad (11)$$

$$C_{\pi\pi} = \langle 0 | T \left(O_{\pi\pi}(t) O_{\pi\pi}^\dagger(t_{sink}) \right) | 0 \rangle \quad (12)$$

$$\sim Z_{\pi\pi} Z_{\pi\pi}^* \exp[-E_{\pi\pi} |t - t_{sink}|] \quad (13)$$

Calculating Weak Matrix Elements

- We can also divide

$$\frac{C_{\mathcal{O}}}{C_K C_{\pi\pi}} = \frac{\mathcal{M}}{Z^*_{\pi\pi} Z_K} \tag{14}$$

to get a quantity whose leading exponential behavior far from the source and sink is a constant.

Details of the Lattice Calculation

- Carried out on RBC/UKQCD $32^3 \times 64$, $L_s = 16$ 2+1 flavor domain wall fermion lattices.
- The inverse lattice spacing is a^{-1} =2.42(4) GeV (see talk by Enno Scholz)
- We add and subtract propagators with periodic and antiperiodic boundary condtions from each other. The resultant periodic plus antiperiodic (P+A) propagator has a source at t=0 and the resultant periodic minus antiperiodic (P-A) propagator effectively has a source at t=64. These provide the left and right walls for the Kaon and the two Pions respectively.
- Wall source propagators (zero momentum) quark propagators are used. The time t at which the weak operator is located is varied.

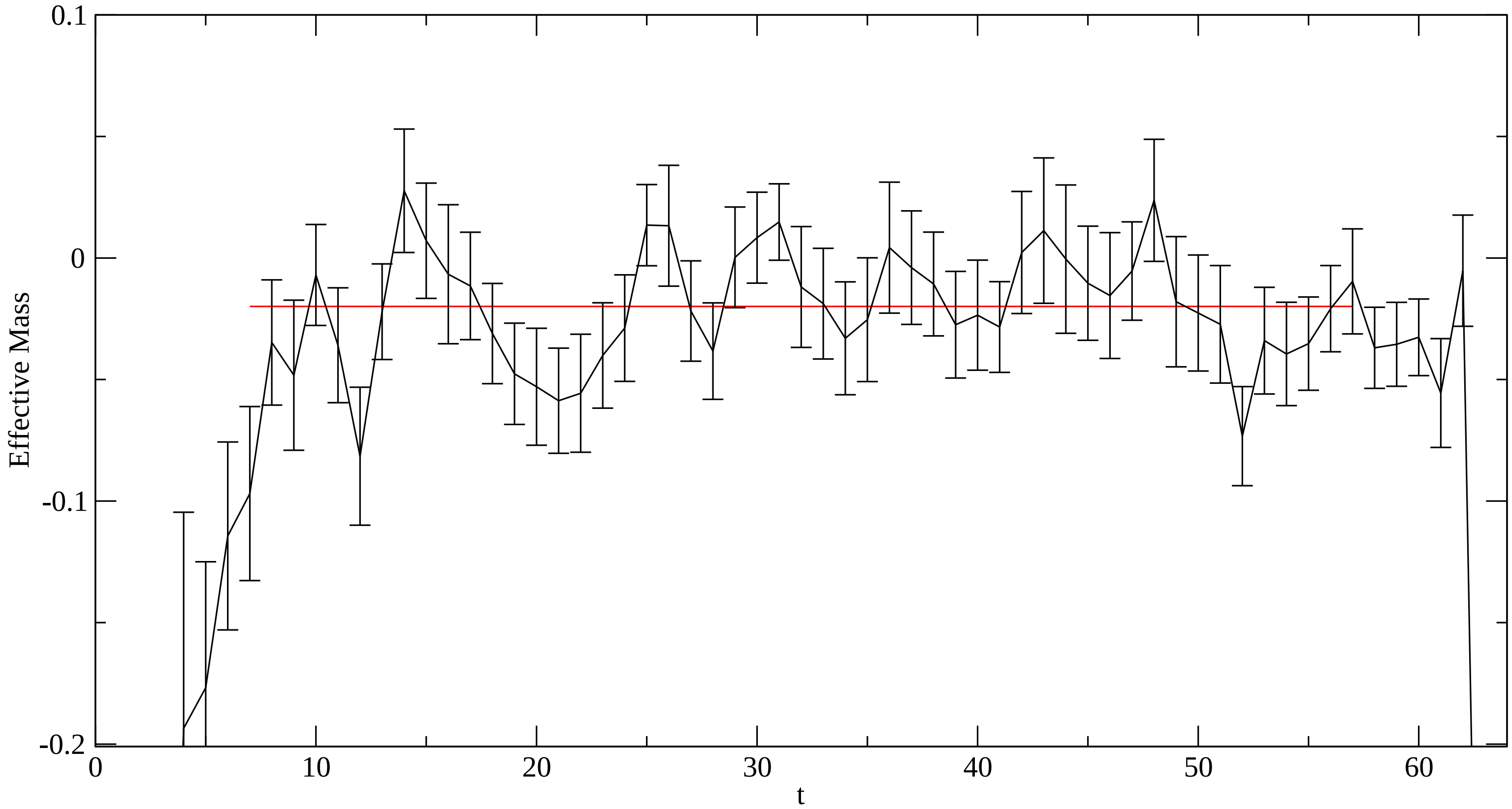
Results

- Calculations were performed on the $m_{sea} = 0.004$ (37 configurations) and $m_{sea} = 0.008$ (101 configurations) ensembles, with valence quark masses 0.002, 0.004, 0.006, 0.008, 0.025, 0.030.
- All possible valence mass combinations such that $m_s \geq m_l$. This gives 21 different mass combinations per sea quark mass.
- If $C_{\mathcal{O}}$ has the expected exponential behavior then an effective mass plot of this quantity

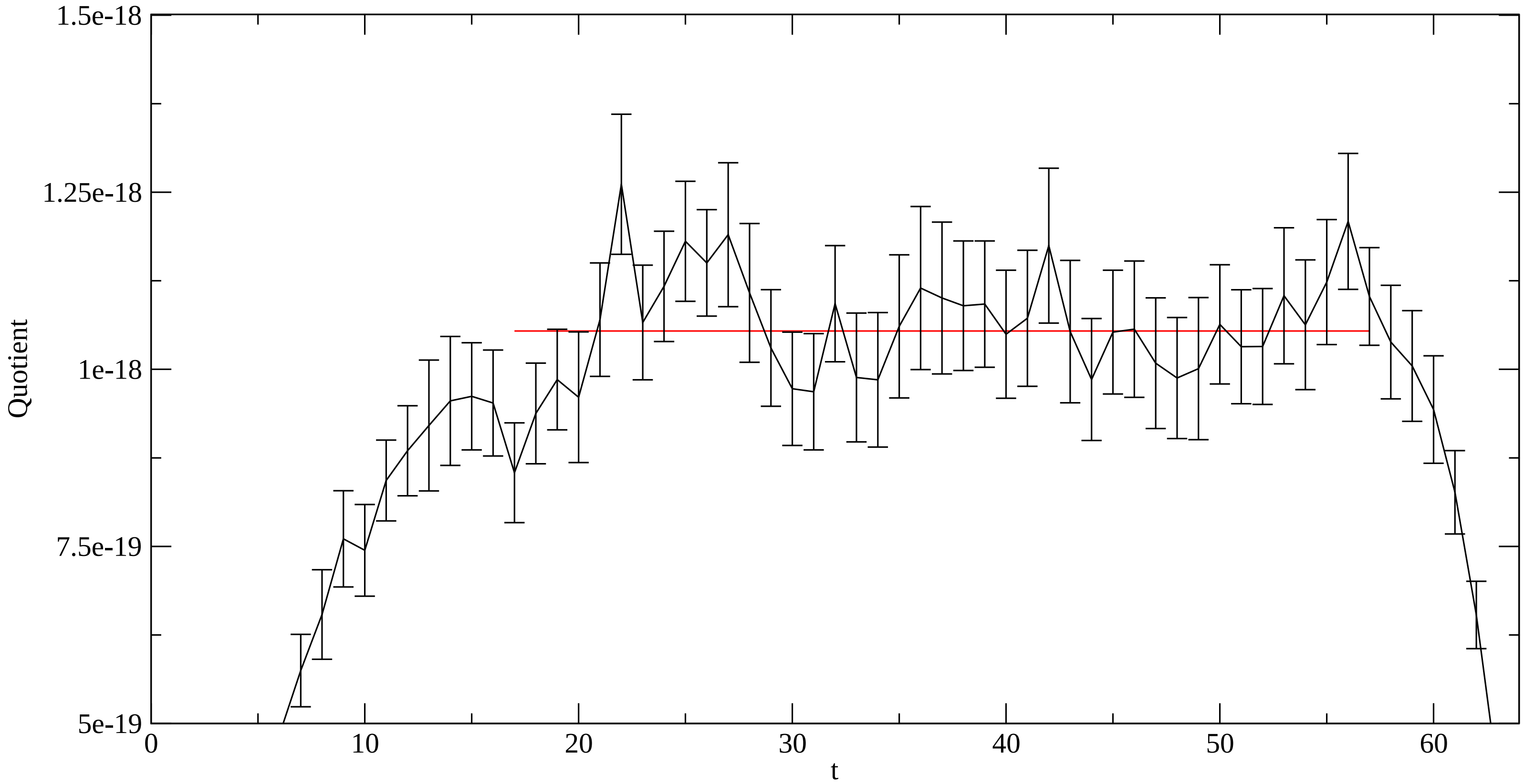
$$m_{eff} = -\ln\left(\frac{C_{\mathcal{O}}(t)}{C_{\mathcal{O}}(t-1)}\right) \qquad (15)$$

  should have a plateau of value $m_K - E_{\pi\pi}$.
- The quotient $\frac{C_{\mathcal{O}}}{C_K C_{\pi\pi}}$ should also be constant in this region.

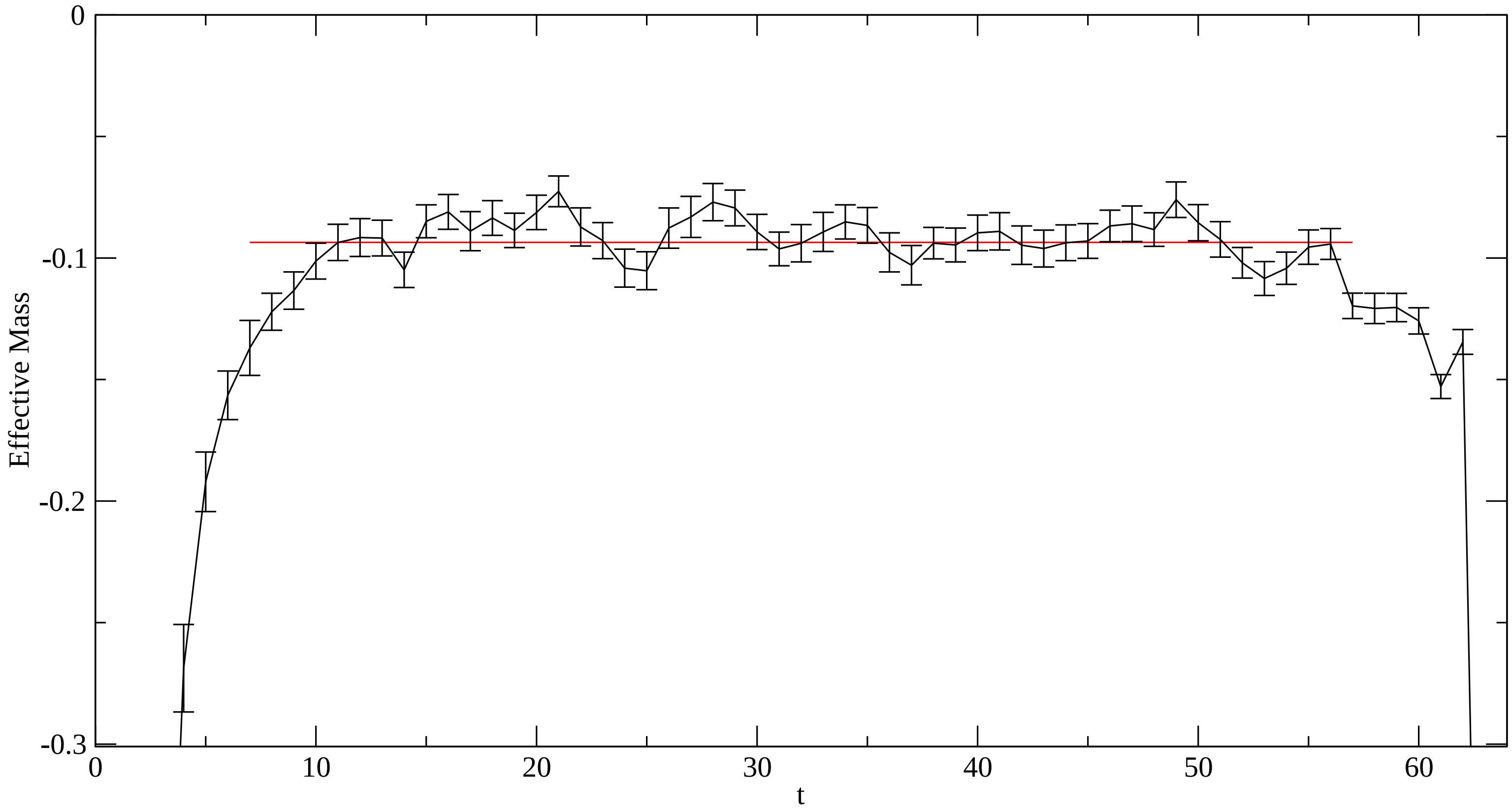
Results - Effective Mass Plot $m_{sea} = 0.004$, $m_s = 0.03$, $m_l = 0.004$ (Unitary Point)



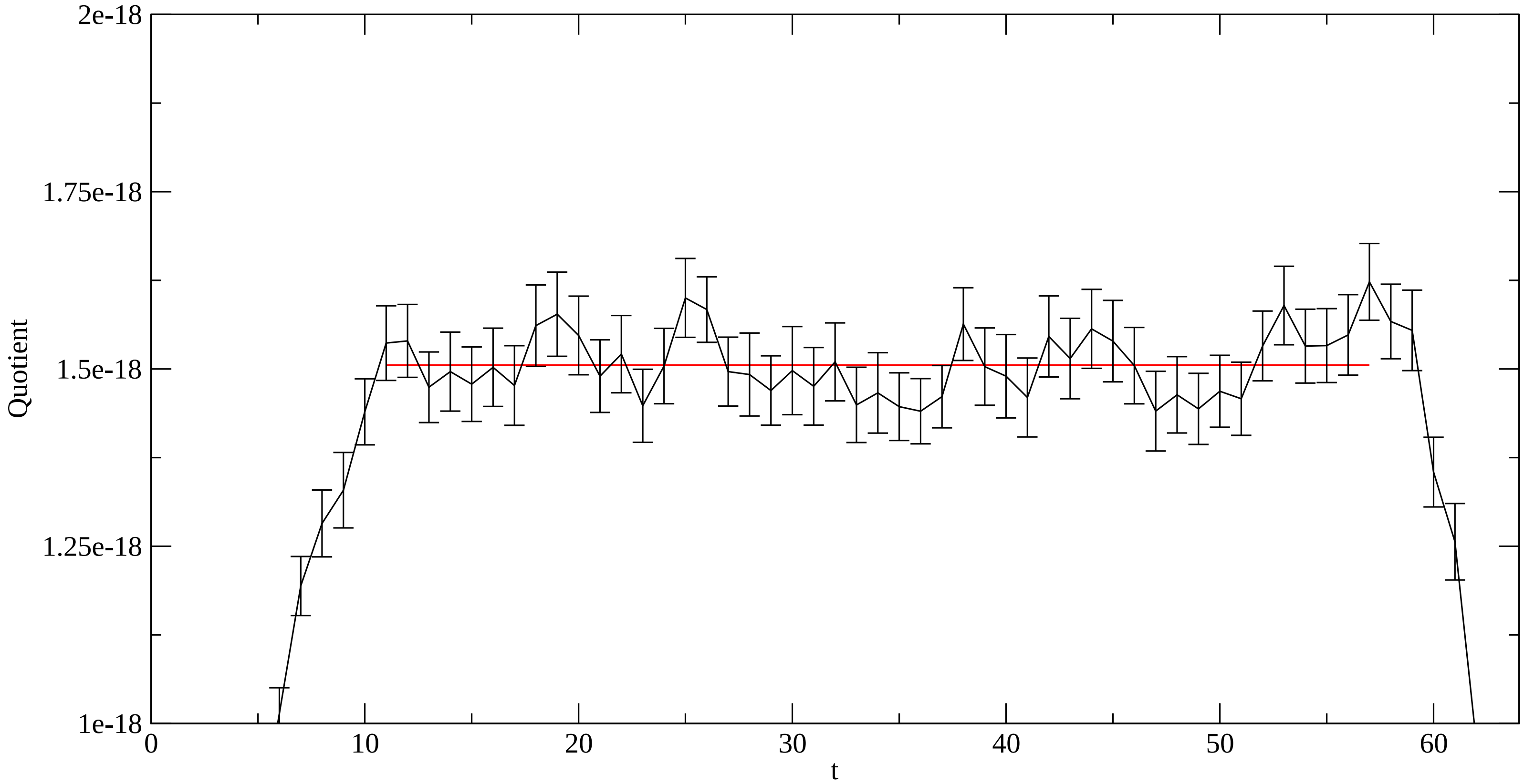
Results - Quotient $m_{sea} = 0.004$, $m_s = 0.03$, $m_l = 0.004$ (Unitary Point)



Results - Effective Mass Plot $m_{sea} = 0.008$, $m_s = 0.03$, $m_l = 0.008$ (Unitary Point)



Results - Quotient $m_{sea} = 0.008$, $m_s = 0.03$, $m_l = 0.008$ (Unitary Point)



Results - $m_{sea} = 0.004$ Table of Values (Lattice Units)

m_s	m_l	m_K	$E_{\pi\pi}$	$\lvert\mathcal{M}\rvert$
0.03	0.03	0.3215(5)	0.645(1)	0.00347(8)
0.03	0.025	0.3076(6)	0.588(1)	0.00313(7)
0.03	0.008	0.2558(8)	0.343(2)	0.00197(6)
0.03	0.006	0.2489(8)	0.303(2)	0.00183(7)
0.03	0.004	0.242(1)	0.255(2)	0.00169(8)
0.03	0.002	0.234(1)	0.196(2)	0.0015(1)
0.025	0.025	0.2932(6)	0.588(1)	0.00312(7)
0.025	0.008	0.2388(7)	0.343(2)	0.00194(6)
0.025	0.006	0.2316(8)	0.303(2)	0.00179(7)
0.025	0.004	0.2241(9)	0.255(2)	0.00164(8)
0.025	0.002	0.216(1)	0.196(2)	0.0015(1)

Results - $m_{sea} = 0.004$ Table of Values (Lattice Units) cont'd

m_s	m_l	m_K	$E_{\pi\pi}$	$\vert\mathcal{M}\vert$
0.008	0.008	0.1703(7)	0.343(2)	0.00187(6)
0.008	0.006	0.1605(7)	0.303(2)	0.00168(6)
0.008	0.004	0.1499(7)	0.255(2)	0.00148(6)
0.008	0.002	0.1386(8)	0.196(2)	0.00122(7)
0.006	0.006	0.1500(7)	0.303(2)	0.00168(6)
0.006	0.004	0.1387(7)	0.255(2)	0.00146(6)
0.006	0.002	0.1264(8)	0.196(2)	0.00118(6)
0.004	0.004	0.1264(8)	0.255(2)	0.00145(6)
0.004	0.002	0.1128(8)	0.196(2)	0.00115(5)
0.002	0.002	0.0972(8)	0.196(2)	0.00111(5)

Results - $m_{sea} = 0.008$ Table of Values (Lattice Units)

m_s	m_l	m_K	$E_{\pi\pi}$	$\|\mathcal{M}\|$
0.03	0.03	0.3225(3)	0.6471(7)	0.00348(5)
0.03	0.025	0.3087(3)	0.5908(7)	0.00315(5)
0.03	0.008	0.2581(4)	0.3482(8)	0.00194(3)
0.03	0.006	0.2519(5)	0.3079(9)	0.00179(3)
0.03	0.004	0.2456(5)	0.2612(9)	0.00163(4)
0.03	0.002	0.2394(6)	0.202(1)	0.00148(6)
0.025	0.025	0.2944(3)	0.5908(7)	0.00314(5)
0.025	0.008	0.2412(4)	0.3482(8)	0.00191(3)
0.025	0.006	0.2344(4)	0.3079(9)	0.00176(3)
0.025	0.004	0.2276(5)	0.2613(9)	0.00159(4)
0.025	0.002	0.2208(6)	0.202(1)	0.00141(5)

Results - $m_{sea} = 0.008$ Table of Values (Lattice Units) cont'd

m_s	m_l	m_K	$E_{\pi\pi}$	$\lvert\mathcal{M}\rvert$
0.008	0.008	0.1728(4)	0.3482(8)	0.00183(3)
0.008	0.006	0.1631(4)	0.3079(9)	0.00165(3)
0.008	0.004	0.1528(4)	0.2613(9)	0.00146(3)
0.008	0.002	0.1419(4)	0.202(1)	0.00123(3)
0.006	0.006	0.1526(4)	0.3079(9)	0.00165(3)
0.006	0.004	0.1415(4)	0.2612(9)	0.00145(3)
0.006	0.002	0.1295(4)	0.202(1)	0.00121(3)
0.004	0.004	0.1292(4)	0.2613(9)	0.00144(3)
0.004	0.002	0.1157(4)	0.202(1)	0.00119(3)
0.002	0.002	0.1000(4)	0.202(1)	0.00118(3)

Outlook

- Accumulate more statistics on the $m_{sea} = 0.004$ ensemble. (This is happening as we speak).
- Need to multiply by the Wilson coefficient and perform NPR to get a physical value.
- With a pion mass of ~ 240 MeV for the lowest valence quark mass 0.002, we are approaching physics.
- There are $48^3 \times 64$ lattices in preparation on which we might hope to have physical pions.