

# Quark Propagators at the confinement and deconfinement phases

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# Main Results

- \* Quark propagators have negative norm contributions at confinement phase.
- \* This feature remains at deconfinement phase.

# Introduction

Quark propagator

- Quark confinement
- Chiral symmetry breaking

Quark propagators relate to two phase transitions in QCD.

Quark confinement

Pole mass, Asymptotic state

Chiral symmetry breaking

Order parameter:  $\langle \psi(0) \bar{\psi}(x) \rangle_{x \rightarrow 0}$

# Introduction2

At finite temperature

low

Quark : confined

Chiral symmetry : breaking

high

Quark : deconfined

Chiral symmetry : restored

How do behaviors of quark propagators change?

Do quark propagators have no pole at confinement phase  
and one or some pole(s) at deconfinement?

# Formulation

## Quark propagator

$$G(x, y) = \langle \psi(x) \bar{\psi}(y) \rangle = \langle W^{-1}(x, y; U) \rangle$$

$$S_f = \sum_{x, y} \bar{\psi}(x) W(x, y; U) \psi(y)$$

in our calculations, **Clover fermion**

## Time-time correlation function

$$G(x_4 - y_4) = \langle \psi(x_4) \bar{\psi}(y_4) \rangle = \sum_{\vec{x}, \vec{y}} \langle W^{-1}(x, y; U) \rangle$$

when  $\vec{p} = 0$

$$G(t) = G_4(t) \gamma_4 + G_s(t)$$



# Formulation2

## Time-time correlation function(One pole case)

$$G(t) = \frac{Z_1}{2V \cosh(m\beta/2)} [\cosh(m(t - \beta/2))\gamma_4 - \sinh(m(t - \beta/2))]$$

## Effective mass

$$\frac{G_4(t)}{G_4(t+1)} = \frac{\cosh(m(t - \beta/2))}{\cosh(m(t+1 - \beta/2))}$$

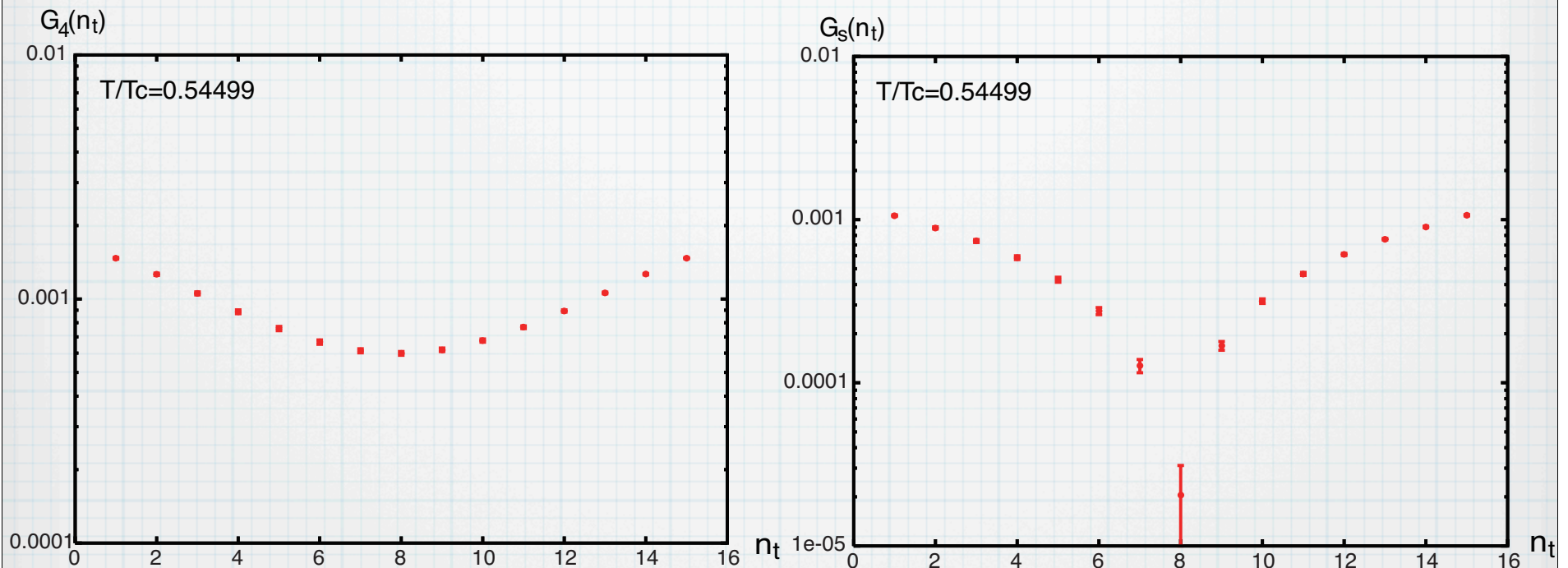
$$\frac{G_s(t)}{G_s(t+1)} = \frac{\sinh(m(t - \beta/2))}{\sinh(m(t+1 - \beta/2))}$$

# Numerical Conditions

- \* Quenched approximation
- \* Pla. gauge action + Wilson fermions with Clover
- \* Gauge Fixing : Landau Gauge
- \* Thermalization : 1000
- \* Sweeps between measurements : 1000
- \* # of Configuration : 20 - 50
- \*  $\beta = 6.10$ ,  $\kappa = 0.1345559, 0.1353591$
- \*  $\beta = 6.25$ ,  $\kappa = 0.1346226, 0.1352633$
- \* Confinement phase :  $N_t = 16, N_s = 24, 32$
- \* Deconfinement phase :  $N_t = 8, N_s = 24, 32$

# Numerical Result 1

Time-time correlation function at confinement phase



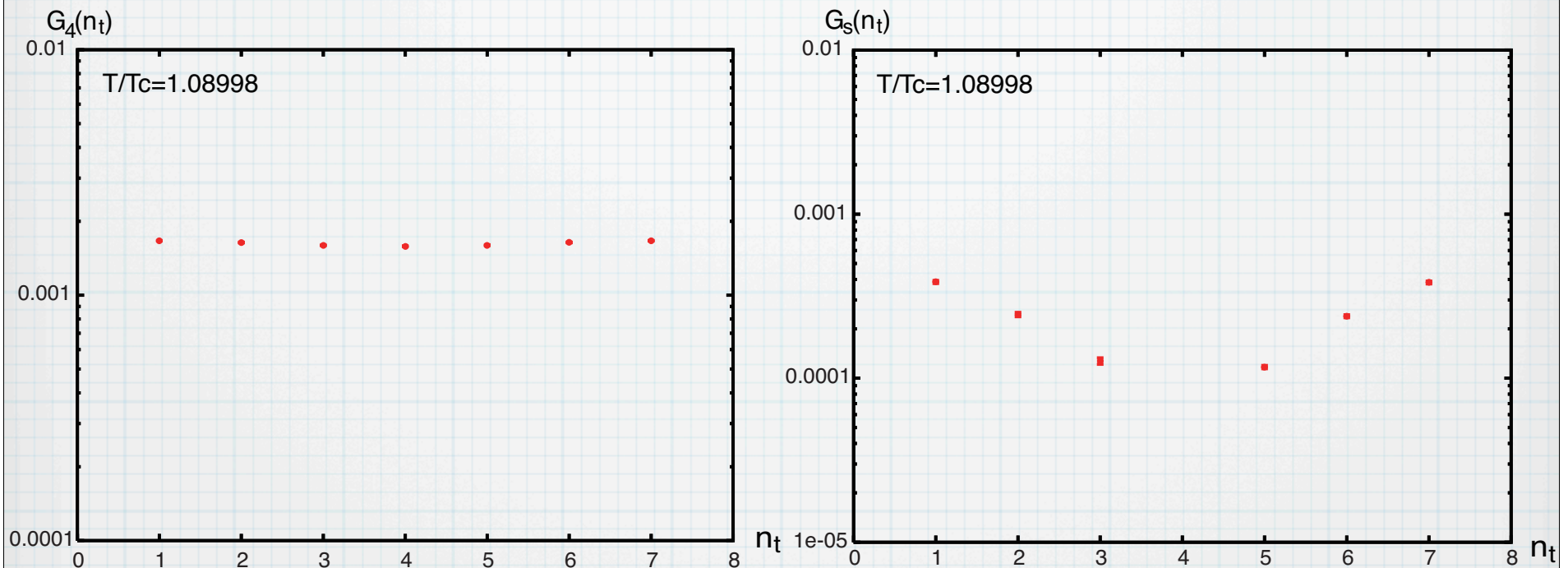
\*  $\beta = 6.10$ ,  $\kappa = 0.1345559$

\*  $N_t = 16$ ,  $N_s = 24$



# Numerical Result 2

Time-time correlation function at **deconfinement phase**

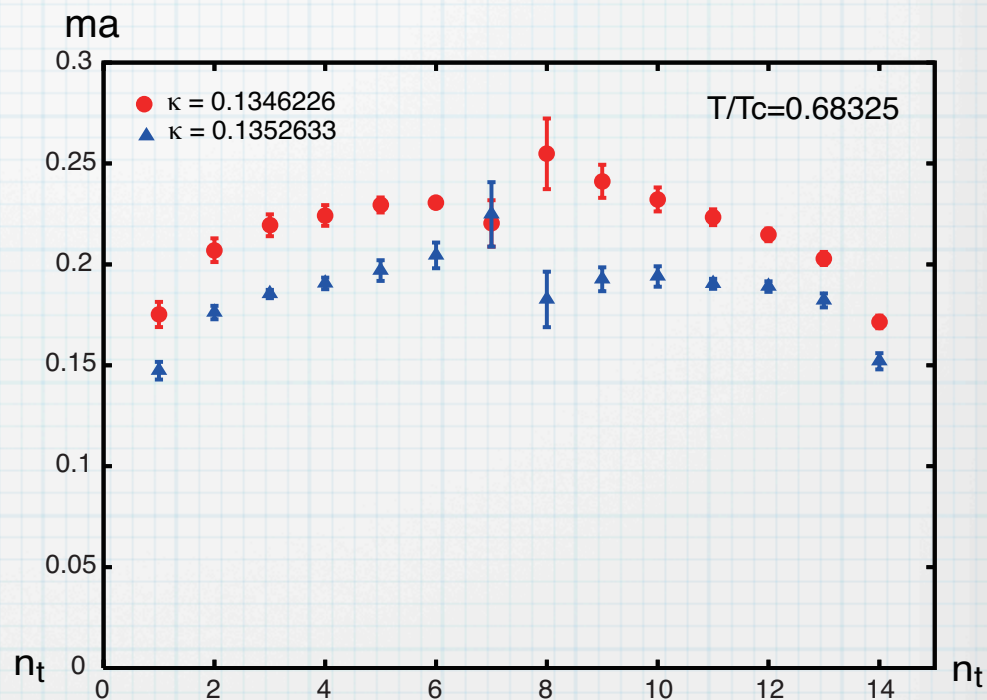
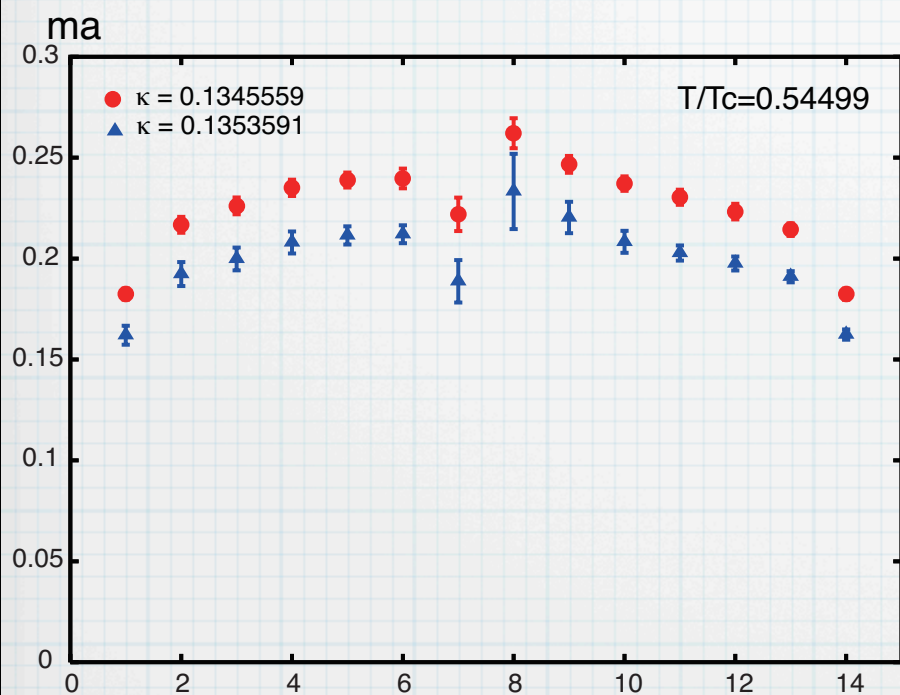


\*  $\beta = 6.10$ ,  $\kappa = 0.1345559$

\*  $N_t = 8$ ,  $N_s = 24$

# Numerical Result 3

Effective mass at confinement phase



\*  $\beta = 6.10$

\*  $N_t = 16, N_s = 24$

\*  $\beta = 6.25$

\*  $N_t = 16, N_s = 24$

# For details

$$G(t) = \rho_1 e^{-m_1 t} + \rho_2 e^{-m_2 t} \quad (m_1 < m_2)$$

$$m_{\text{eff}} = \ln \frac{G(t)}{G(t+1)} = \ln \frac{\rho_1 e^{-m_1 t}}{\rho_1 e^{-m_1(t+1)}} = m_1 \quad (t \rightarrow \infty)$$

$$m'_{\text{eff}} = \ln \frac{G(t)}{G(t+1)} = \ln \frac{\rho_1 e^{-m_1 t} + \rho_2 e^{-m_2 t}}{\rho_1 e^{-m_1(t+1)} + \rho_2 e^{-m_2(t+1)}}$$

$$= m_1 + \ln \frac{1 + \frac{\rho_2}{\rho_1} e^{-(m_2 - m_1)t}}{1 + \frac{\rho_2}{\rho_1} e^{-(m_2 - m_1)(t+1)}} \quad (t \rightarrow 0)$$

$$\frac{\rho_2}{\rho_1} > 0 \quad \Rightarrow \quad \frac{1 + \frac{\rho_2}{\rho_1} e^{-(m_2 - m_1)t}}{1 + \frac{\rho_2}{\rho_1} e^{-(m_2 - m_1)(t+1)}} > 1 \quad \Rightarrow \quad m_{\text{eff}} < m'_{\text{eff}}$$

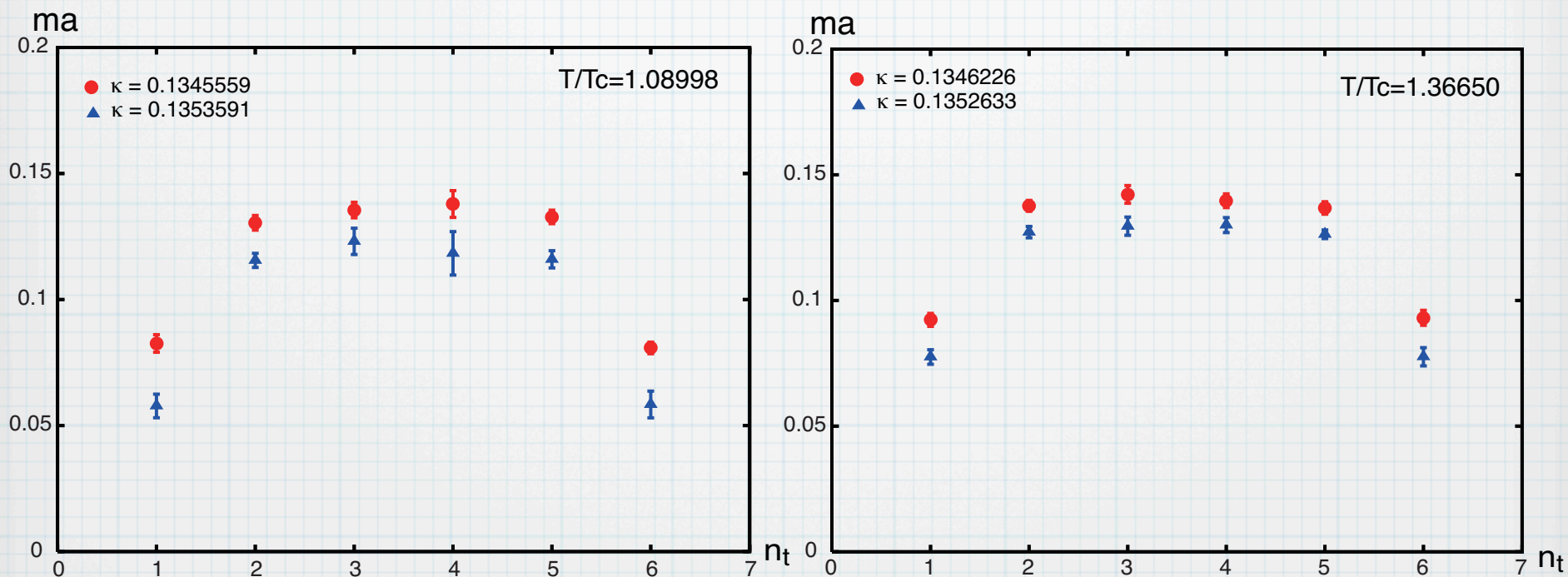
$$\frac{\rho_2}{\rho_1} < 0 \quad \Rightarrow \quad \frac{1 + \frac{\rho_2}{\rho_1} e^{-(m_2 - m_1)t}}{1 + \frac{\rho_2}{\rho_1} e^{-(m_2 - m_1)(t+1)}} < 1 \quad \Rightarrow \quad m_{\text{eff}} > m'_{\text{eff}}$$

namely *violation of positivity*. If a certain degree of freedom has negative norm contributions in its propagator, it cannot describe a physical asymptotic state, *i.e.* there is no Källén–Lehmann spectral representation for its propagator.

R. Alkofer, W. Detmold, C. S. Fischer, P. Maris,  
Phys.Rev. D70 (2004) 014014.

# Numerical Result 4

Effective mass at **denfinement phase**



\*  $\beta = 6.10$

\*  $N_t = 8, N_s = 24$

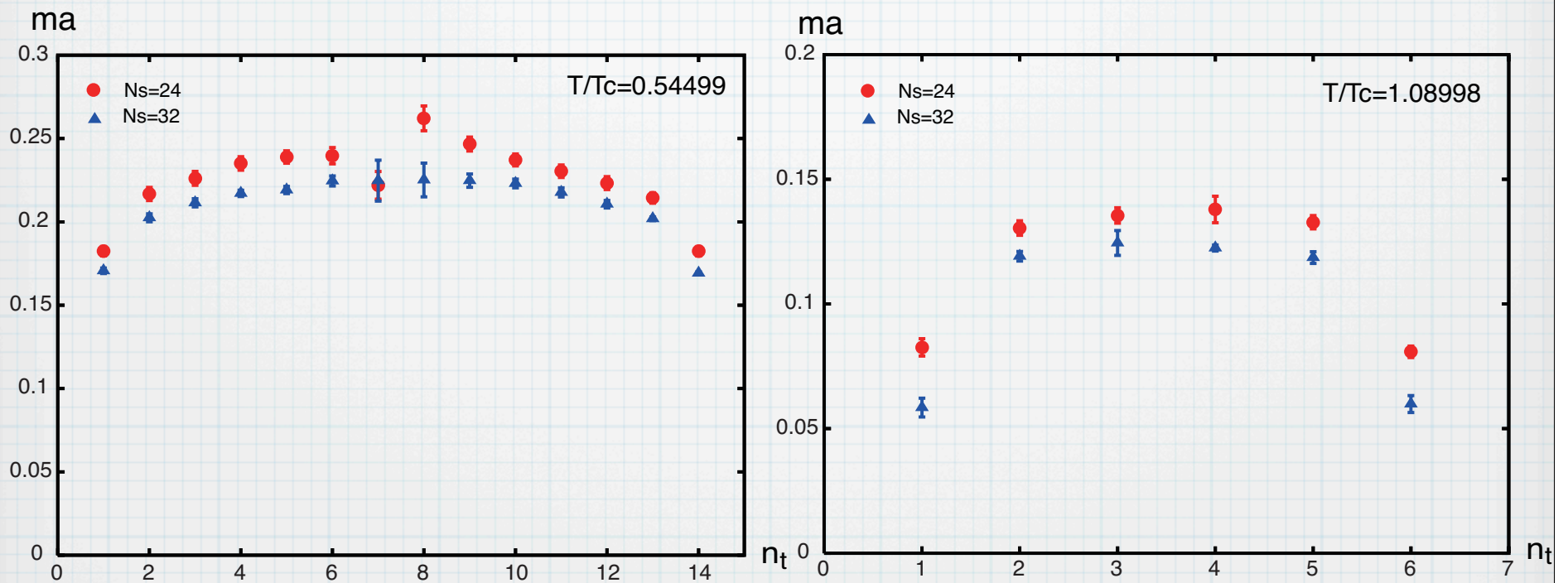
\*  $\beta = 6.25$

\*  $N_t = 8, N_s = 24$



# Numerical Result 5

Volume dependence of effective mass

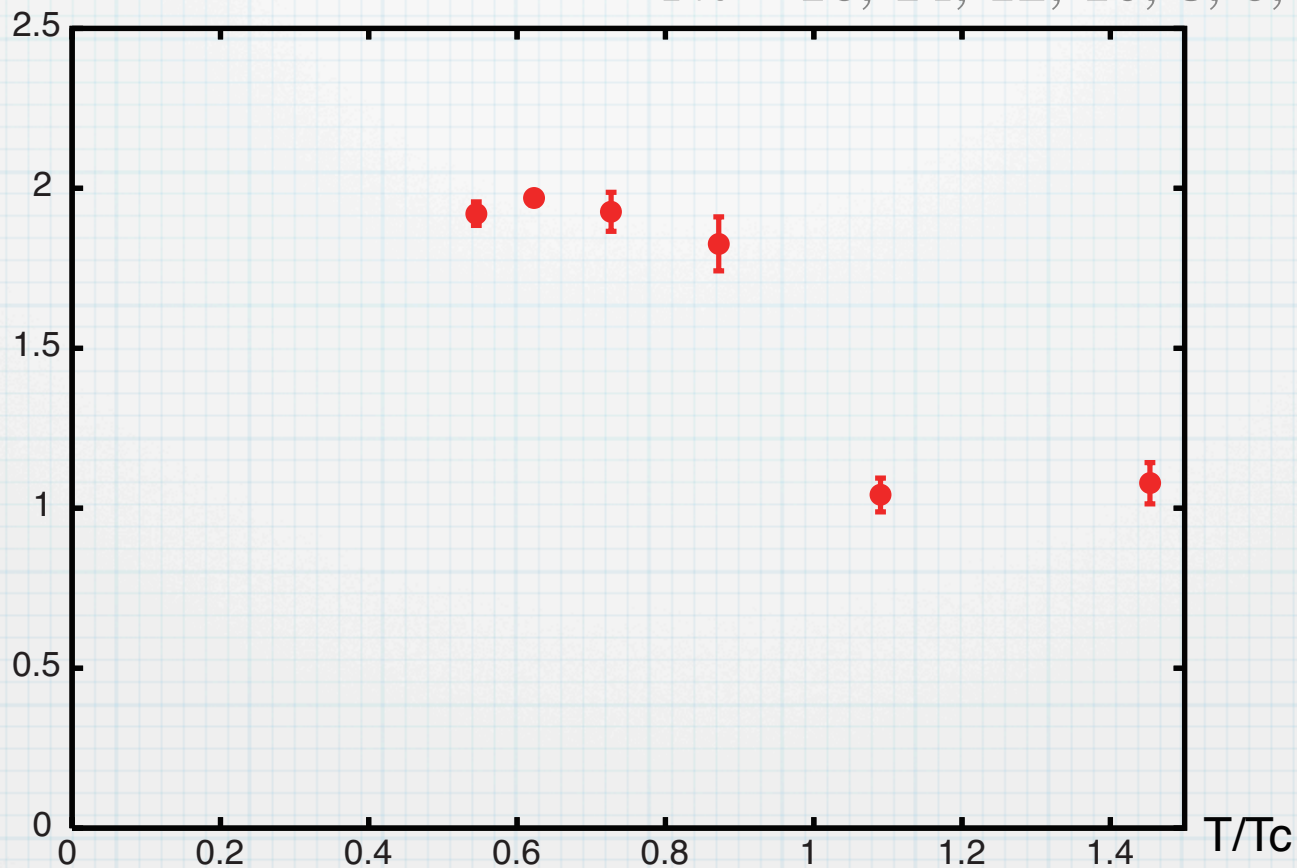


\*  $\beta = 6.10$ ,  $\kappa = 0.1345559$

# Numerical Result 6

Temperature dependence of quark mass

$m/T_c$  \*  $N_t = 16, 14, 12, 10, 8, 6, N_s = 32$



\*  $\beta = 6.10, \kappa = 0.1345559$

# Summary

- \* Effective mass shows that quark propagators include negative norm state.
- \* This feature remains at deconfinement phase.
- \* We can not fit the scalar part of time-time correlation functions.
- \* These behavior do not depend quark mass or spatial volume.
- \* Effective mass at confinement phase twice larger than it at deconfinement phase.