A new method of calculating the running coupling constant

— theoretical formulation

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Collaborators

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- C.-J. David Lin [c]
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- Hiroshi Ohki [b]
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c : National Chiao-Tung University, and
National Center for Theoretical Sciences

d : KEK

Outline

(1) Introduction

(2) Scheme

(2.1) Perturbative calculation

(2.2) Non-perturbative definition of the running coupling

(3) Steps for numerical study

(4) Summary

1. Introduciton

- Long-term goal

  To study physics of (approximate) conformal gauge theories

  - Theoretical interest
  - Walking Technicolor
  - AdS/CFT

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- Long-term goal

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Question

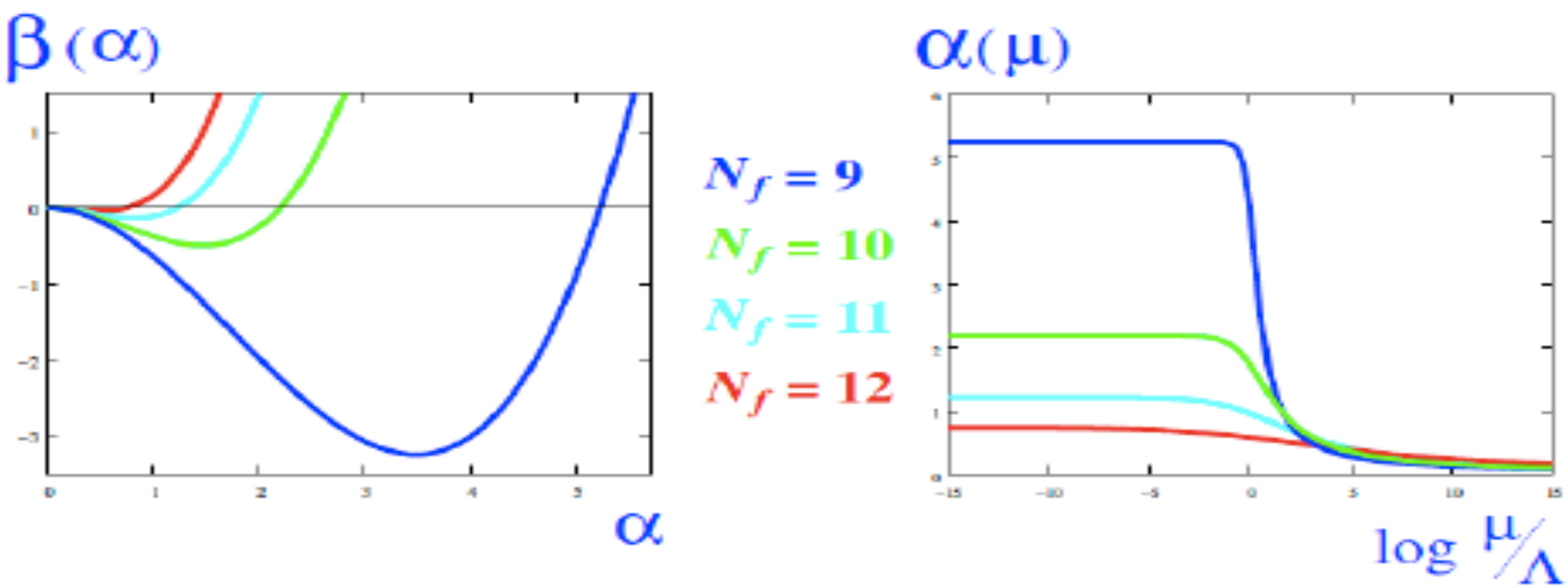
Is there any theory which actually has a conformal nature in a certain energy region?

Large flavor QCD

— a promising candidate for a theory with the IR fixed point

- Two-loop running coupling : $\mu \frac{d}{d\mu} \alpha(\mu) = \beta(\alpha) = -b\, \alpha^2(\mu) - c\, \alpha^3(\mu)$

$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{6\pi}(33 - 2N_f)$	+	+	−
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	+	−	−



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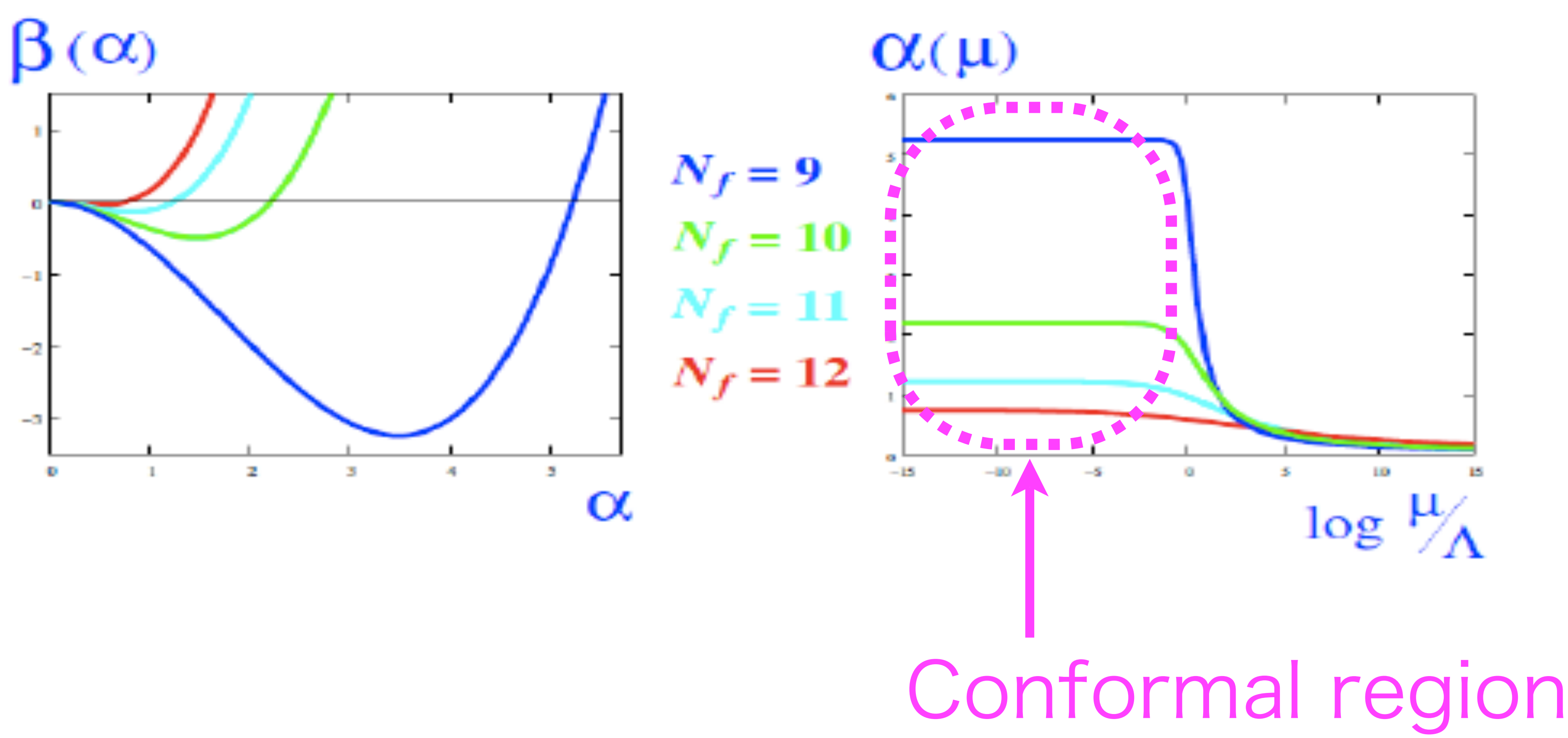

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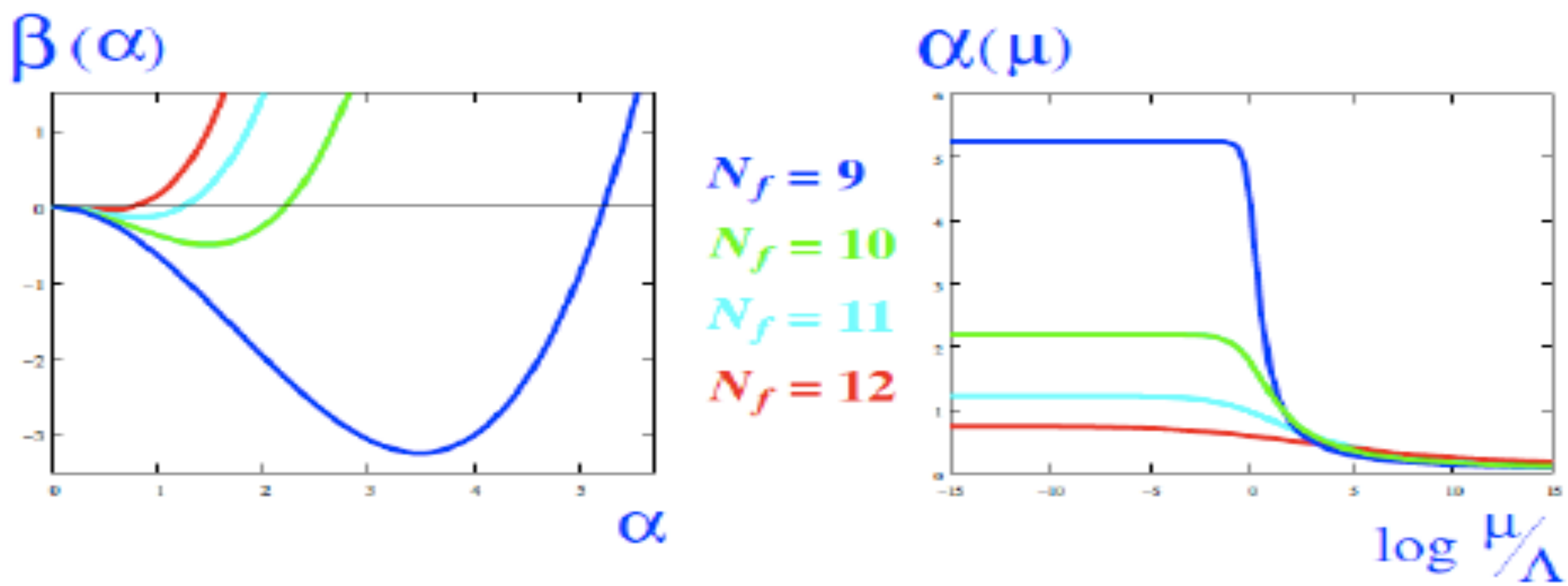


Is this true beyond perturbation?

Evidence from the recent Lattice study

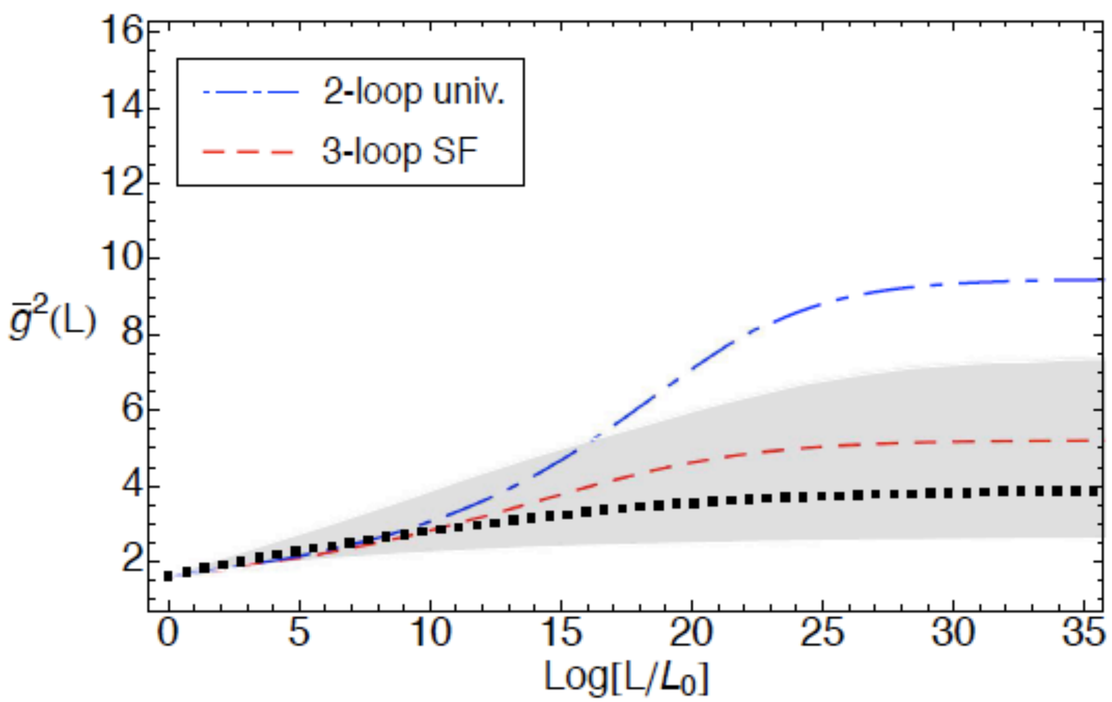
- Running coupling from the Schrödinger functional method

Appelquist, Fleming and Neil, PRL 100, 171607 (2008)

Plenary talk

G. Fleming

Sat. 9:15 am



$\longleftarrow N_f = 12$

FIG. 1: Continuum running coupling from step scaling for $N_f = 12$. The statistical error on each point is smaller than the size of the symbol. Systematic error is shown in the shaded band.

(Pioneering works

Iwasaki, Kanaya, Sakai and Yoshie, PRL 69, 21 (1992)
Iwasaki, Kanaya, Kaya, Sakai and Yoshie, PRD 69, 014507 (2004))

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Each scheme has its own systematic error, for example, SF scheme has O(a) discretization error due to the boundary counter terms

Our method is free from O(a) discretization error

2. Scheme

We extract the running coupling by measuring the finite volume dependence of the Wilson loop : $W(L_0, R, T_0, T, a, g_0)$

L_0 : box size (spatial)

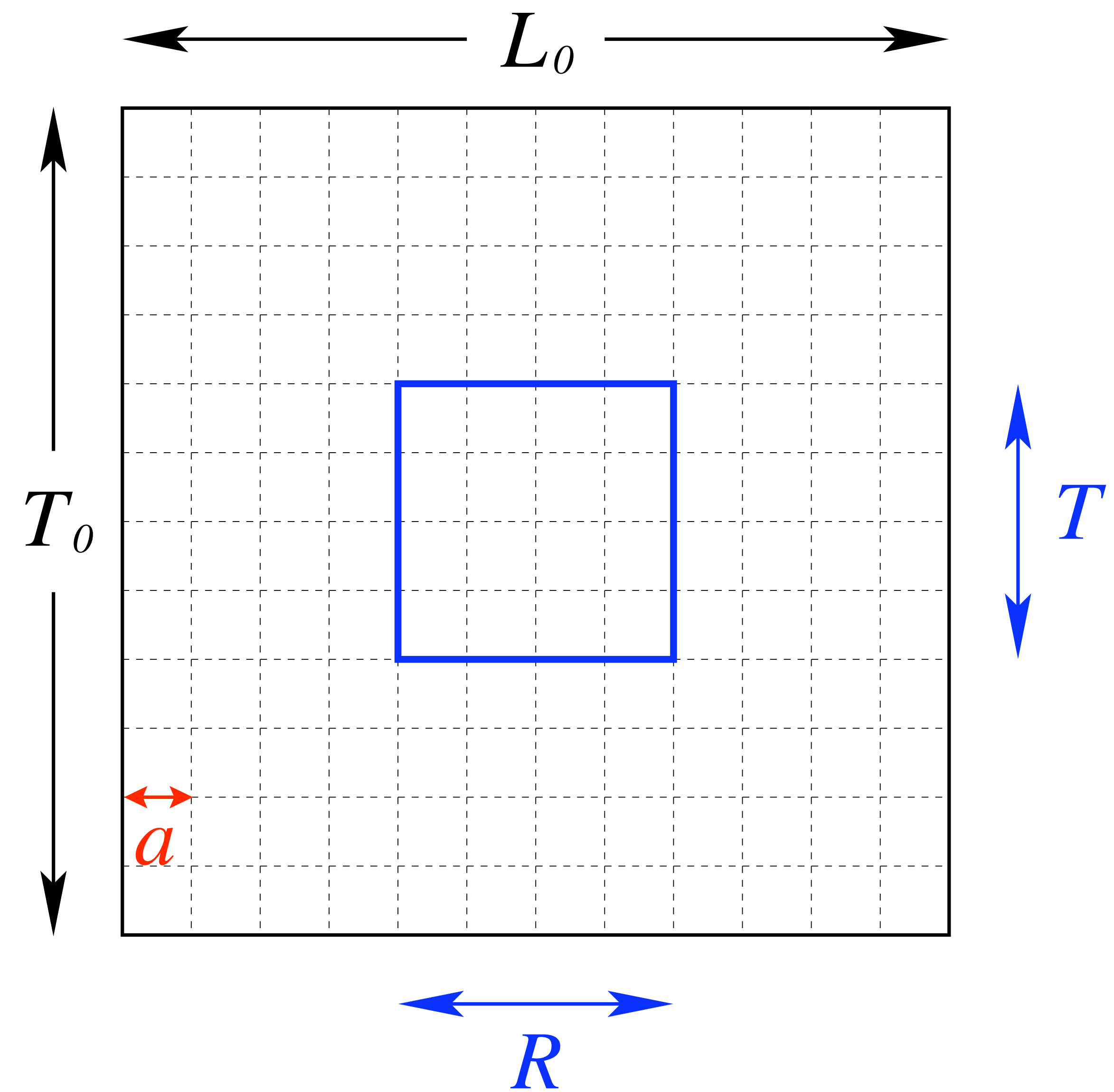
T_0 : box size (temporal)

R : size of the Wilson loop (spatial)

T : size of the Wilson loop (temporal)

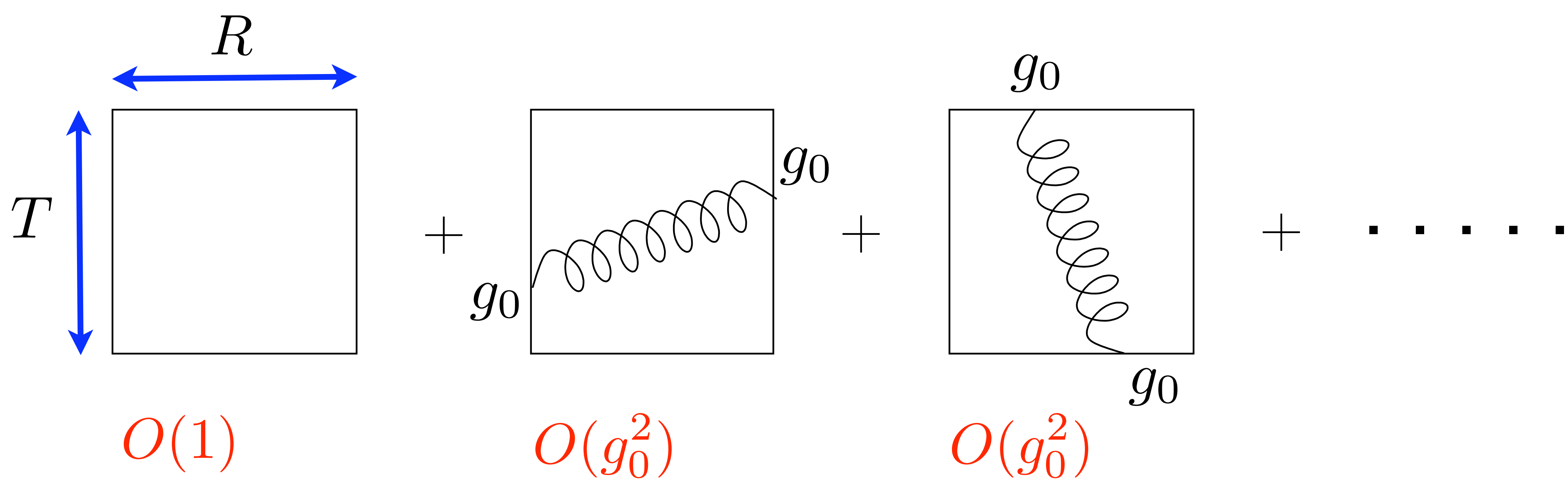
a : lattice spacing

g_0 : bare coupling



2.1 Perturbative calculation

$$W(L_0, R, T_0, T, a, g_0) \;=$$



For simplicity, assume $L_0 = T_0$,

and consider the following quantity :

$$\boxed{-R^2 \; \frac{\partial^2}{\partial R \partial T} \ln\langle W(R,T)\rangle\Big|_{T=R}} \quad \propto \quad g_0^2 \; + \; O(g_0^4)$$

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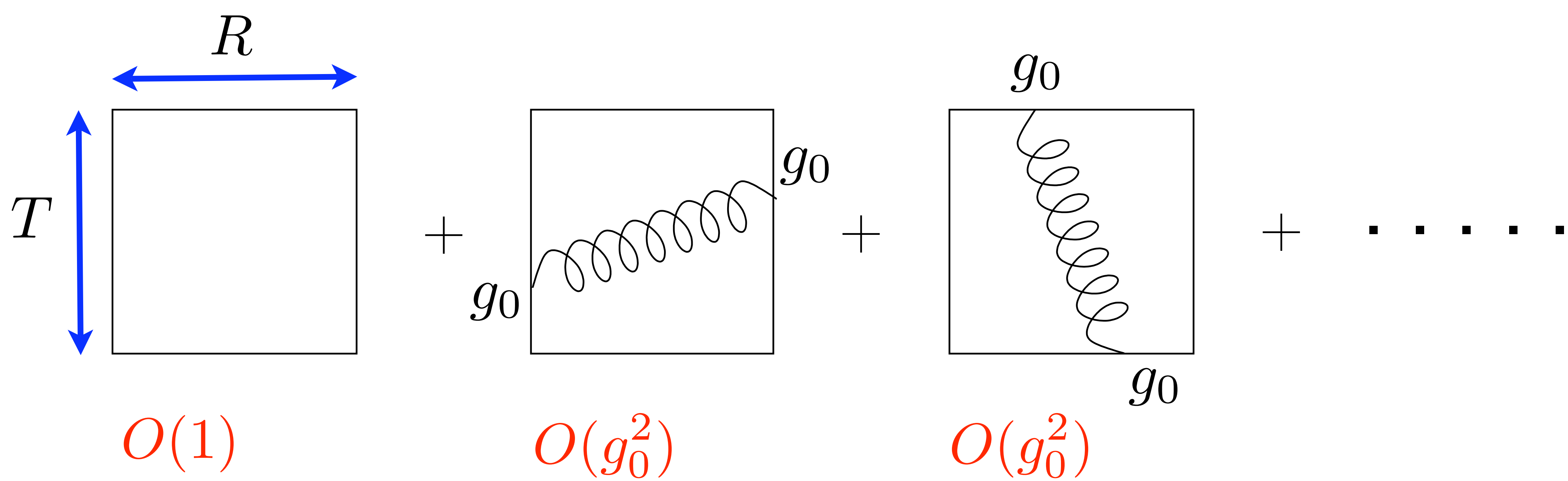


For simplicity, assume $L_0 = T_0$,

and consider the following quantity :

$$-R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle \Big|_{T=R} = k\, g_0^2 \;+\; O(g_0^4)$$

Coefficient k

$$-R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle^{\mathrm{tree}} \Big|_{T=R} = k\, g_0^2$$

Example : Periodic boundary condition

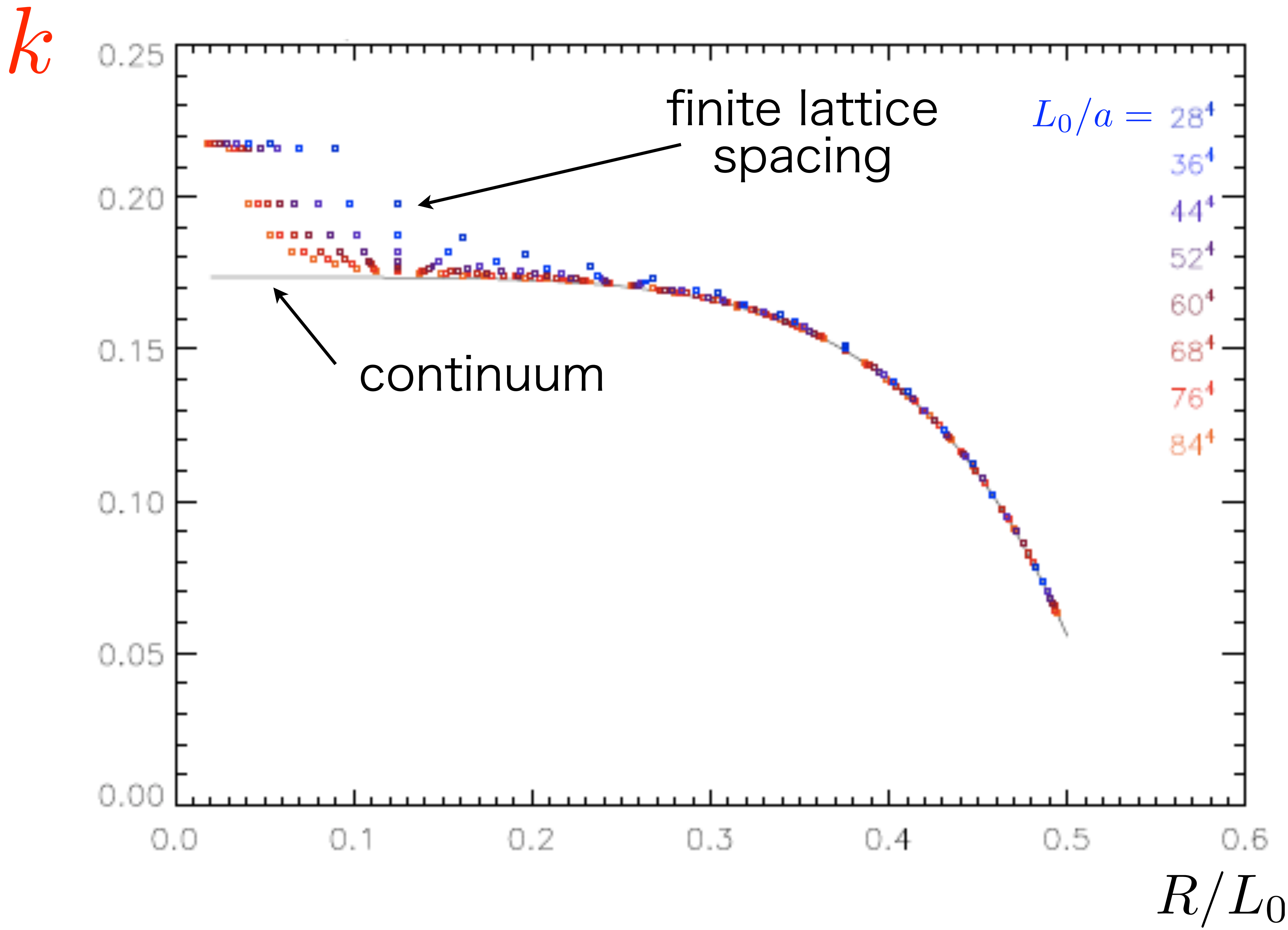
⬇

$$k = -R^2 \frac{\partial^2}{\partial R \partial T} \left[\frac{4}{(2\pi)^4} \sum_{n_0, n_1, n_2, n_3 (\neq 0)} \left(\frac{\sin\left(\frac{\pi n_0 T}{L_0}\right)}{n_0} \right)^2 \frac{e^{i \frac{2\pi n_1 R}{L_0}}}{n_0^2 + \vec{n}^2} \right]_{T=R}$$

$+$ zero-mode contributions (← Coste et.al. NPB262, 67, 1985)

Calculation details (including the case of twisted boundary condition) can be found in our future publication

Coefficient k



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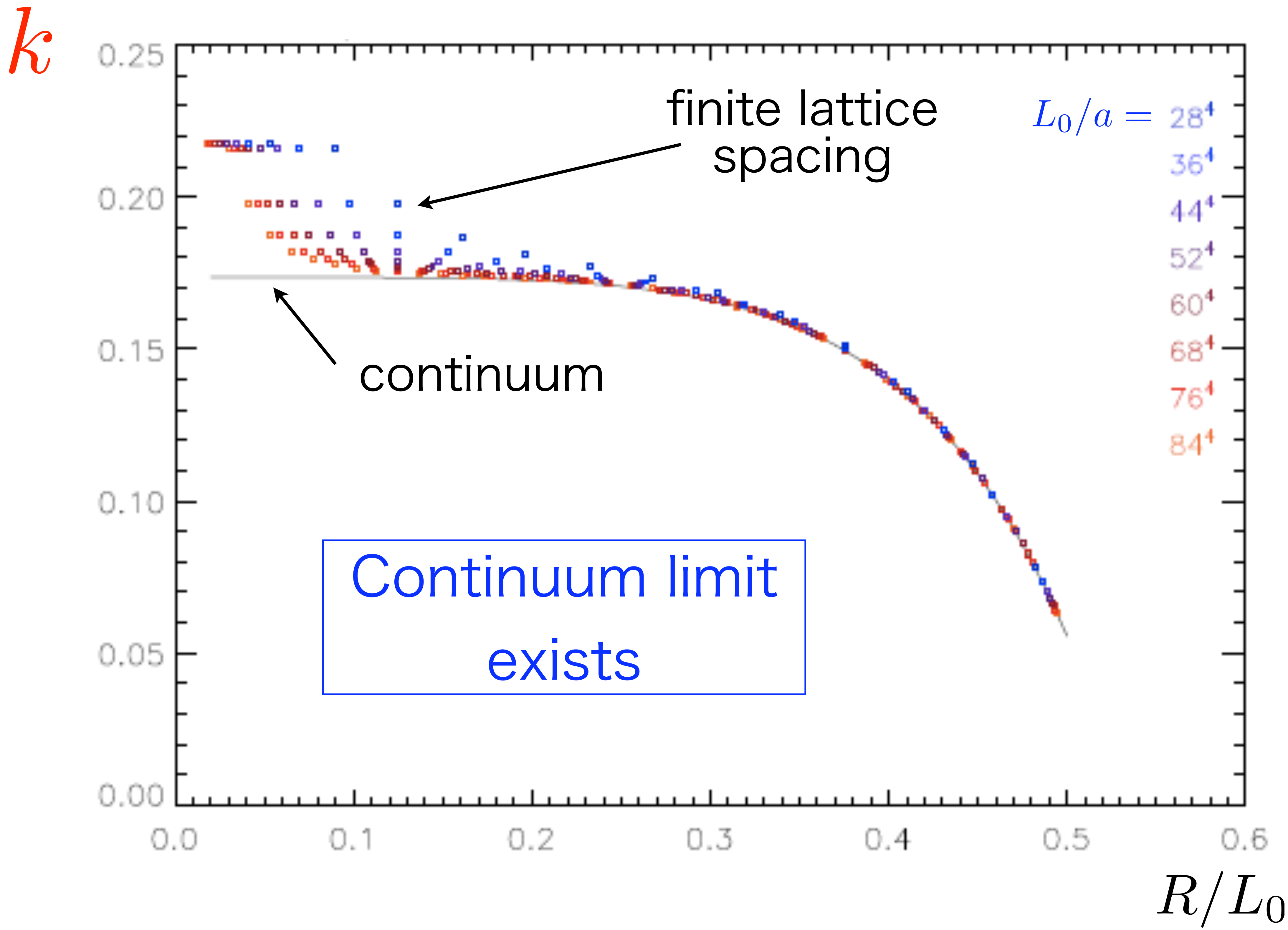

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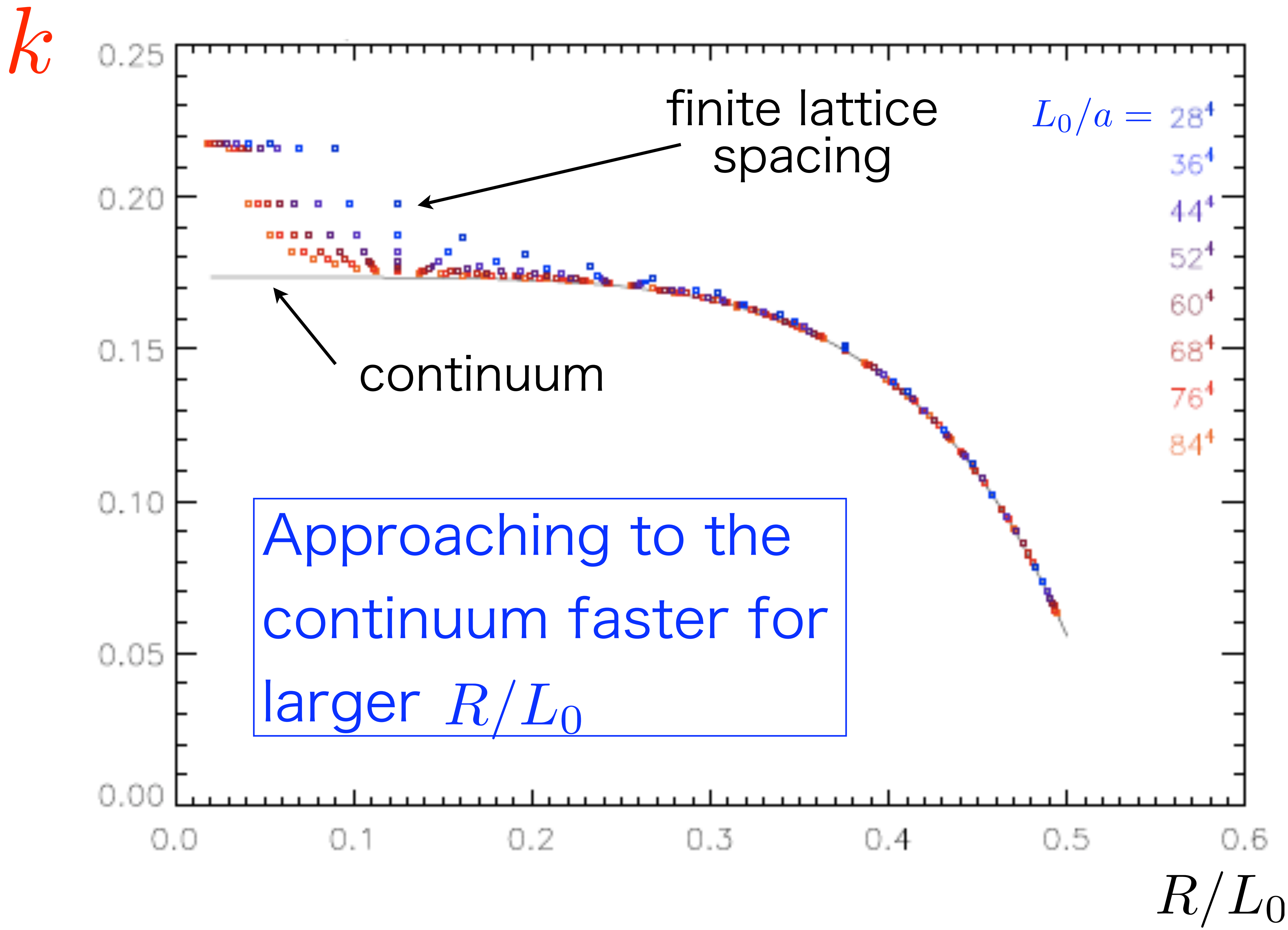

2.2 Non-perturbative definition of the running coupling

$$\left(-R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle^{\text{NP}} \Big|_{T=R} \right)$$

$$\equiv \; Z_g \left(-R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle^{\text{tree}} \Big|_{T=R} \right)$$

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2.2 Non-perturbative definition of the running coupling

Choose a scheme

$$g^2(L_0, R, a) \Longleftrightarrow g^2\left(L_0, \frac{R}{L_0}, \frac{a}{L_0}\right)$$

- Fix the value of $\frac{R}{L_0} \equiv r \ (= 0.3,\ \text{for example})$
- Take the limit of $\frac{a}{L_0} \longrightarrow 0 \quad (\text{continuum limit})$
- L_0 is considered as the scale which defines the running of the coupling g

$$g^2 = -R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle^{\text{NP}} \Big|_{T=R} \Big/ k$$

3. Steps for numerical study

$$g^2 = \left. -R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle^{\text{NP}} \right|_{T=R} \Big/ k$$

$$-\frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T) \rangle$$ is estimated by calculating the Creutz ratio

$$\chi(\hat{R},\hat{T}) = -\ln \left(\frac{W(\hat{R},\hat{T})\ W(\hat{R}-1,\hat{T}-1)}{W(\hat{R},\hat{T}-1)\ W(\hat{R}-1,\hat{T})} \right) \qquad (\hat{R} \equiv R/a,\ \hat{T} \equiv T/a)$$

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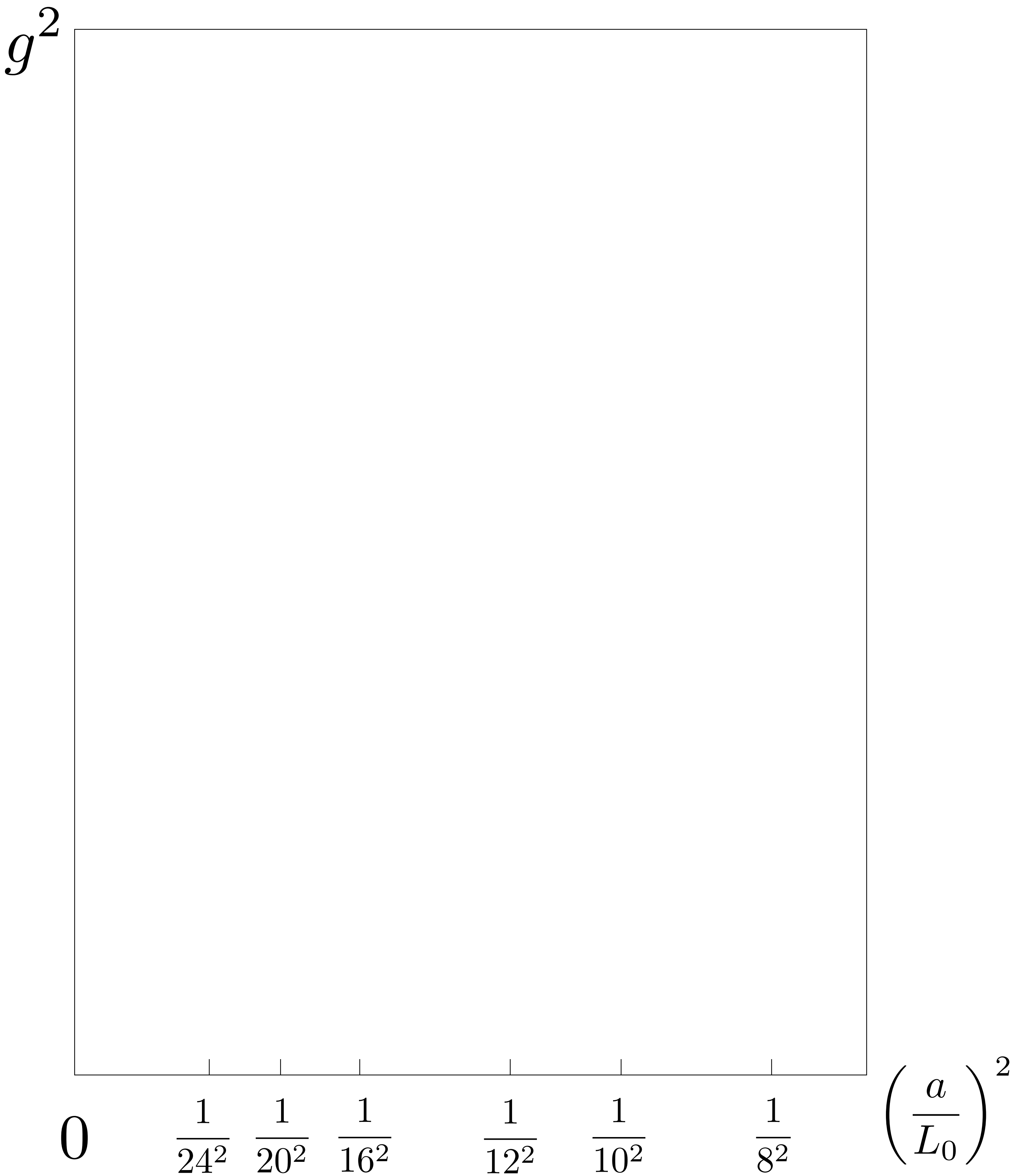
Monte Carlo simulation

The value of g^2 is obtained for given values of

$$\frac{L_0}{a}, \quad r \left(\equiv \frac{R}{L_0} \right), \quad \beta \left(\equiv \frac{6}{g_0^2} \right)$$

Step scaling



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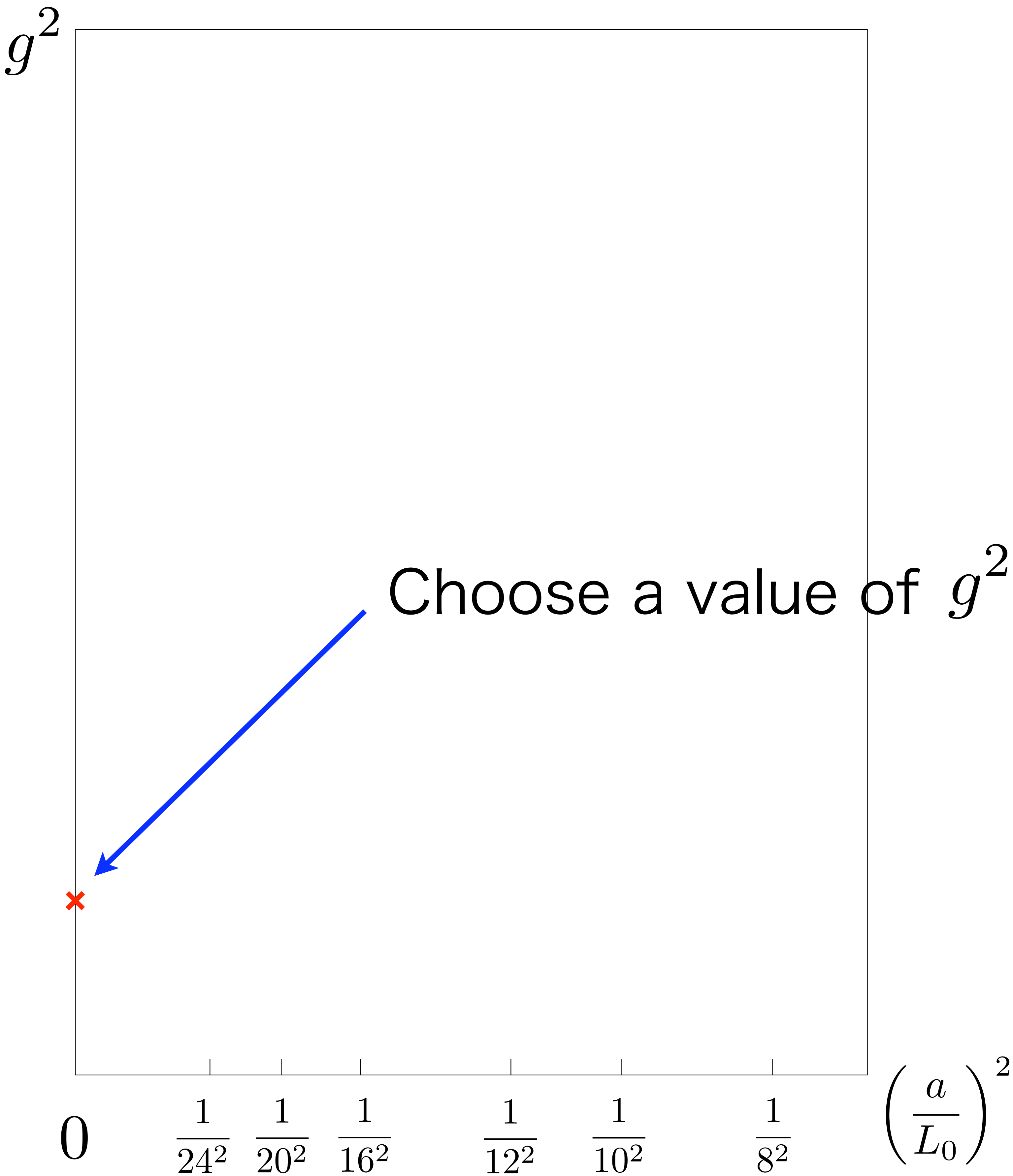

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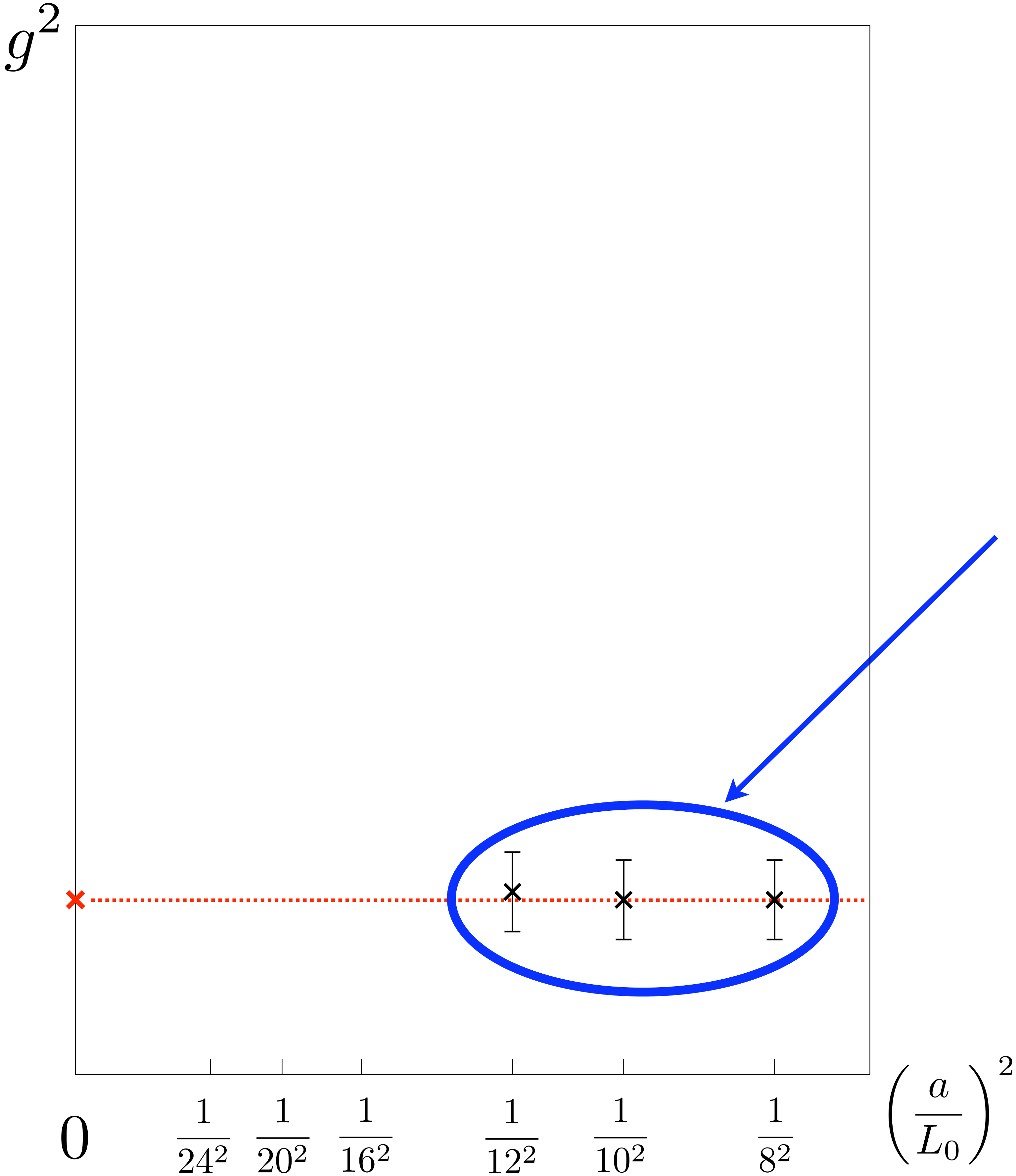
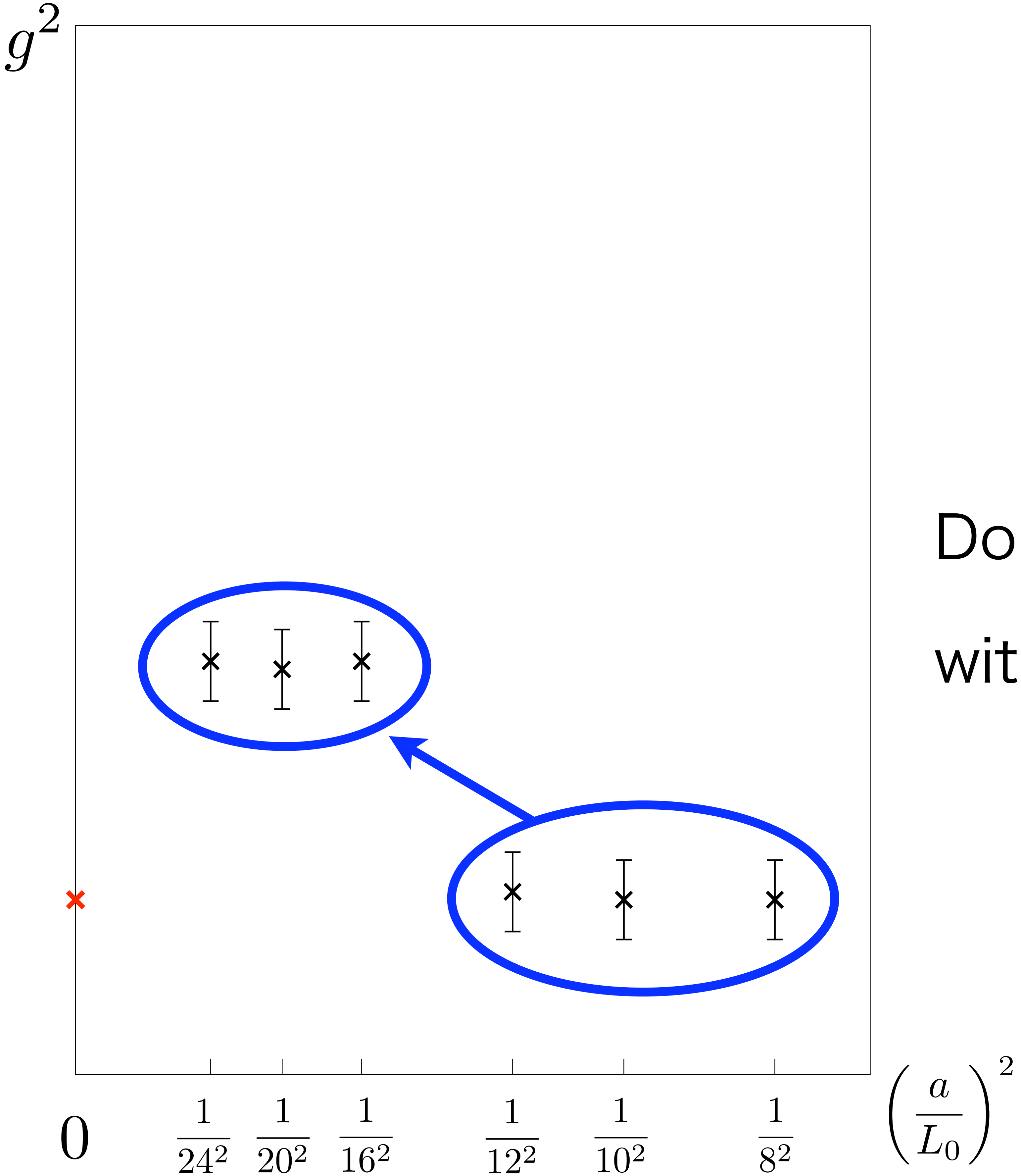


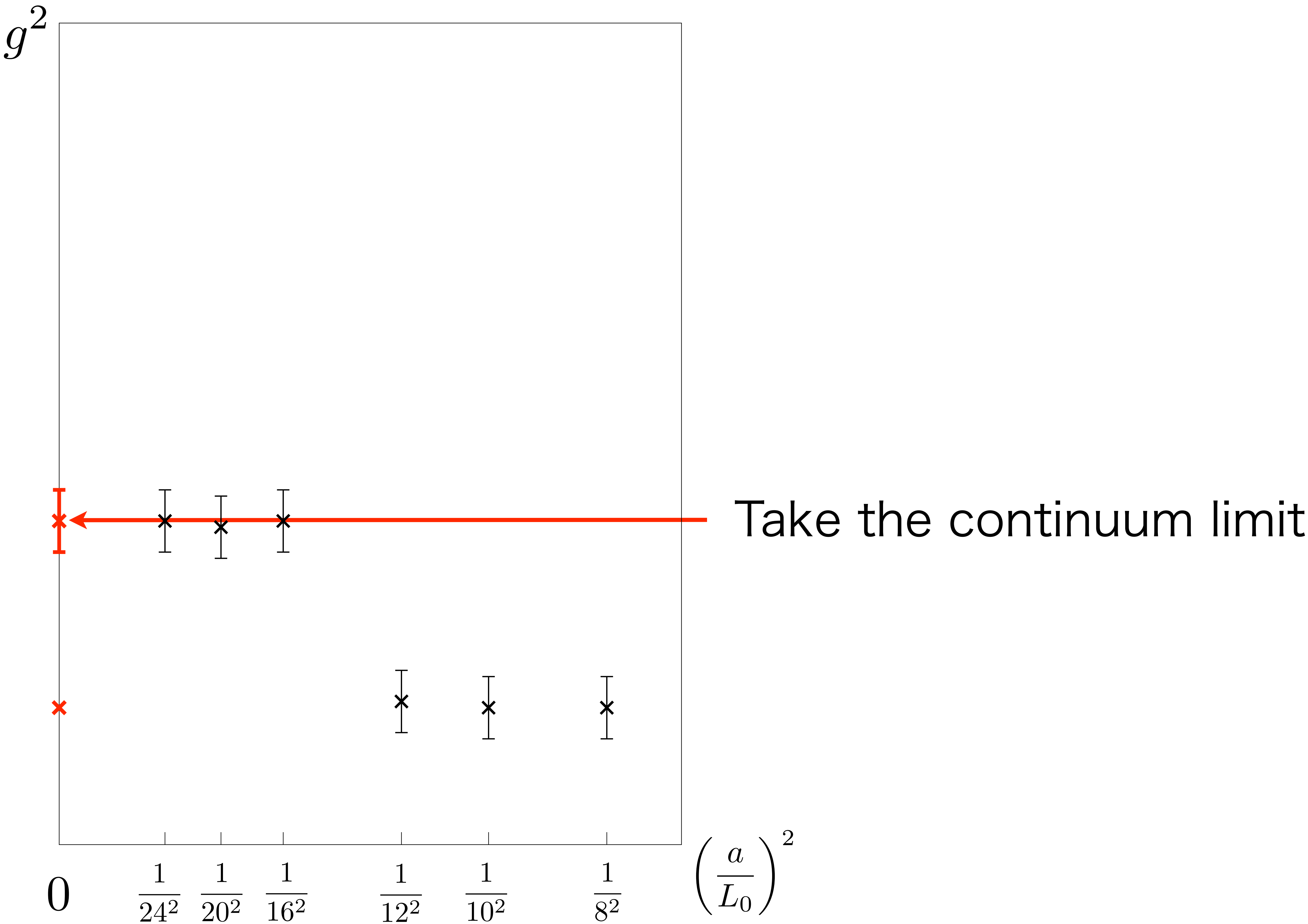
Find these by tuning the value of β

Step scaling



Double the values of $\left(\frac{L_0}{a}\right)$ with the values of β fixed

Step scaling



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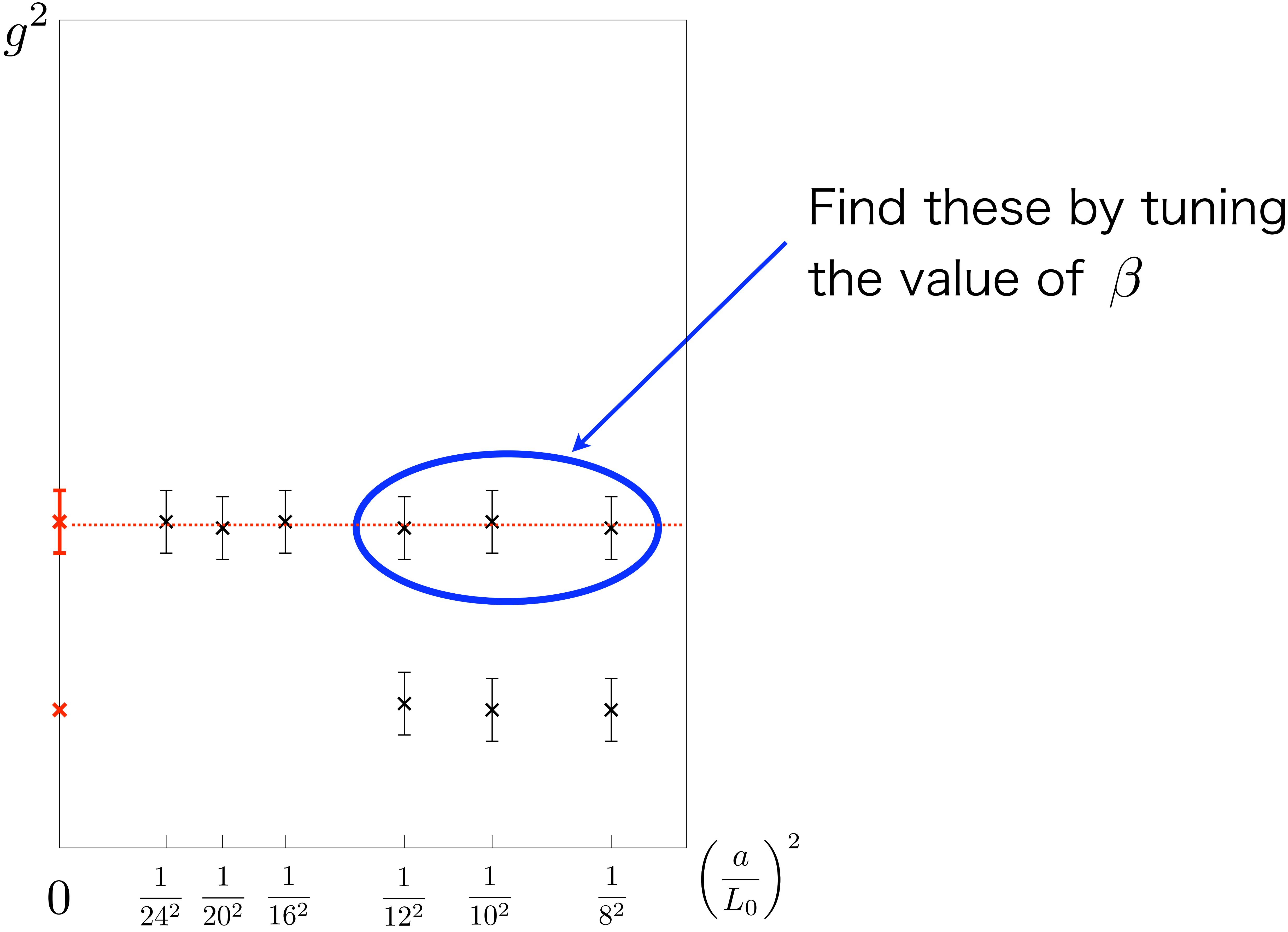

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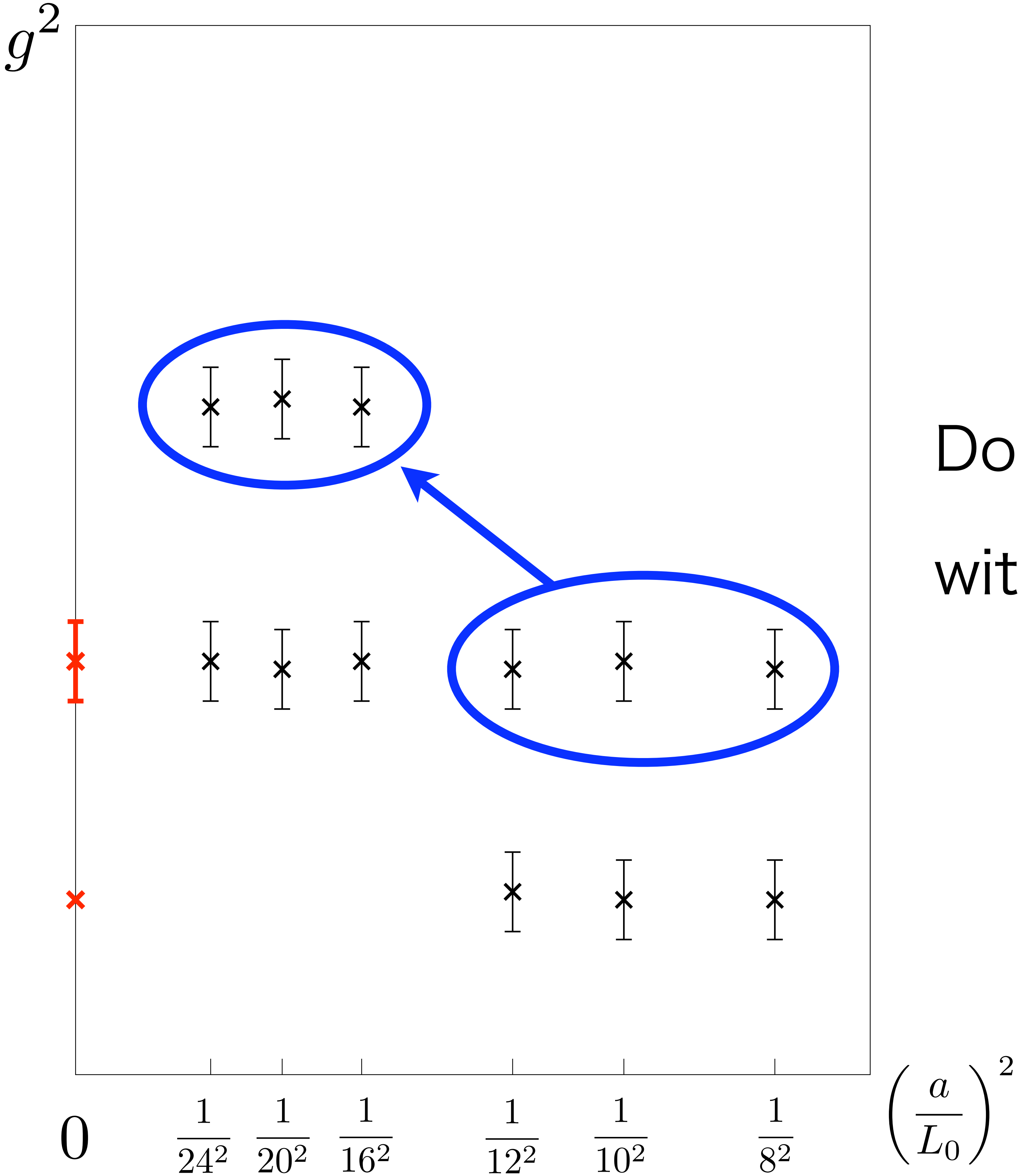

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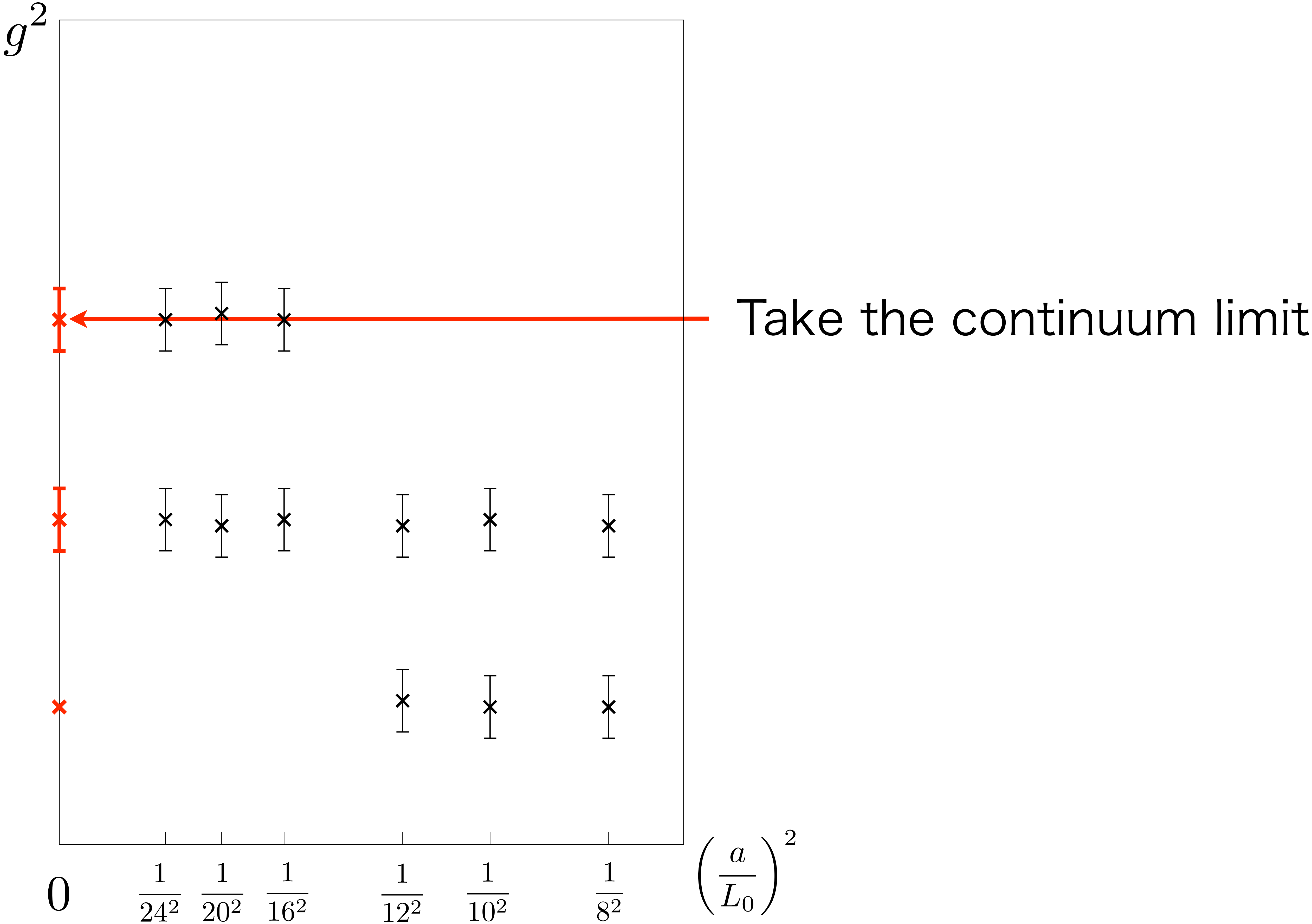

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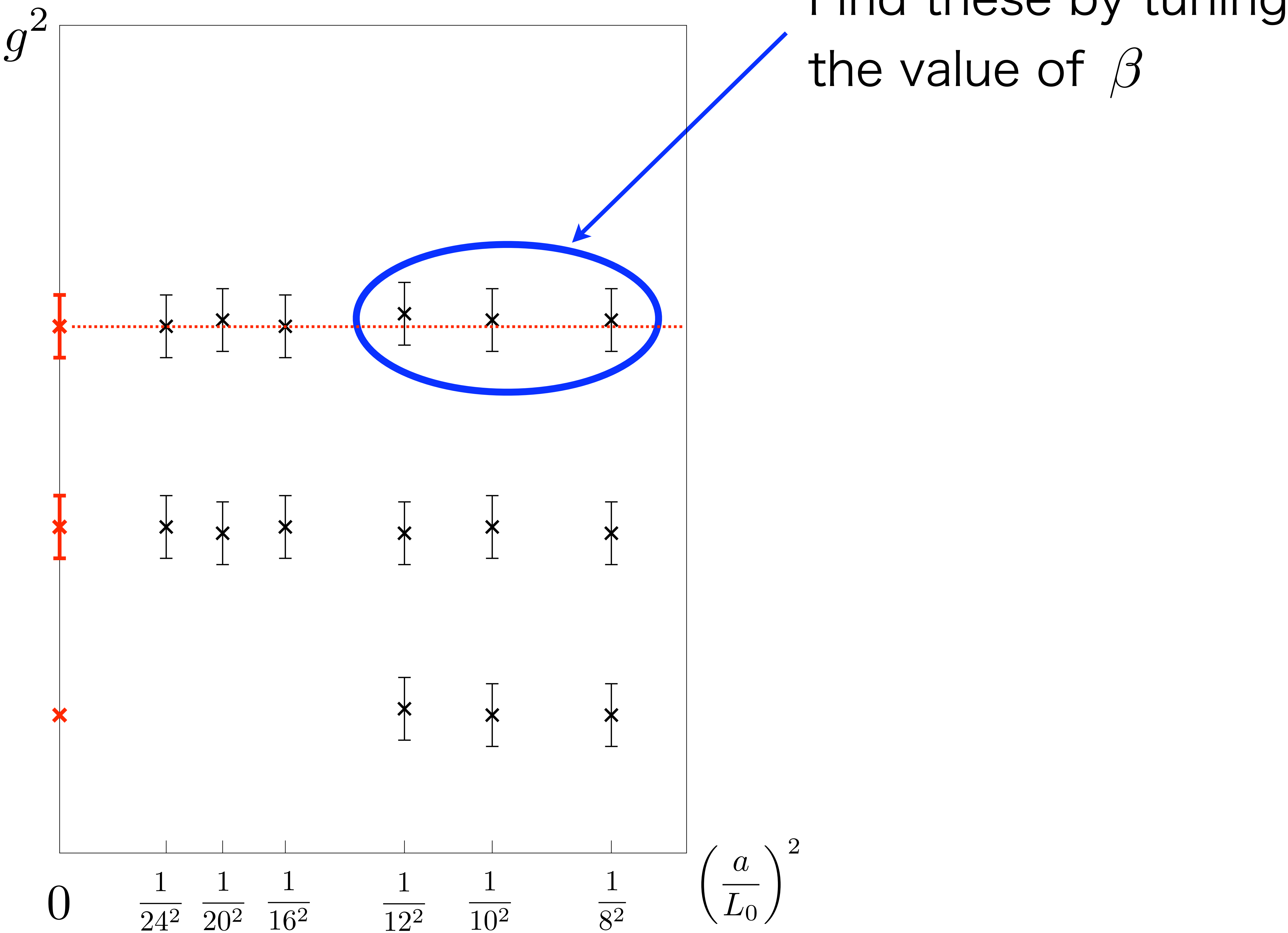

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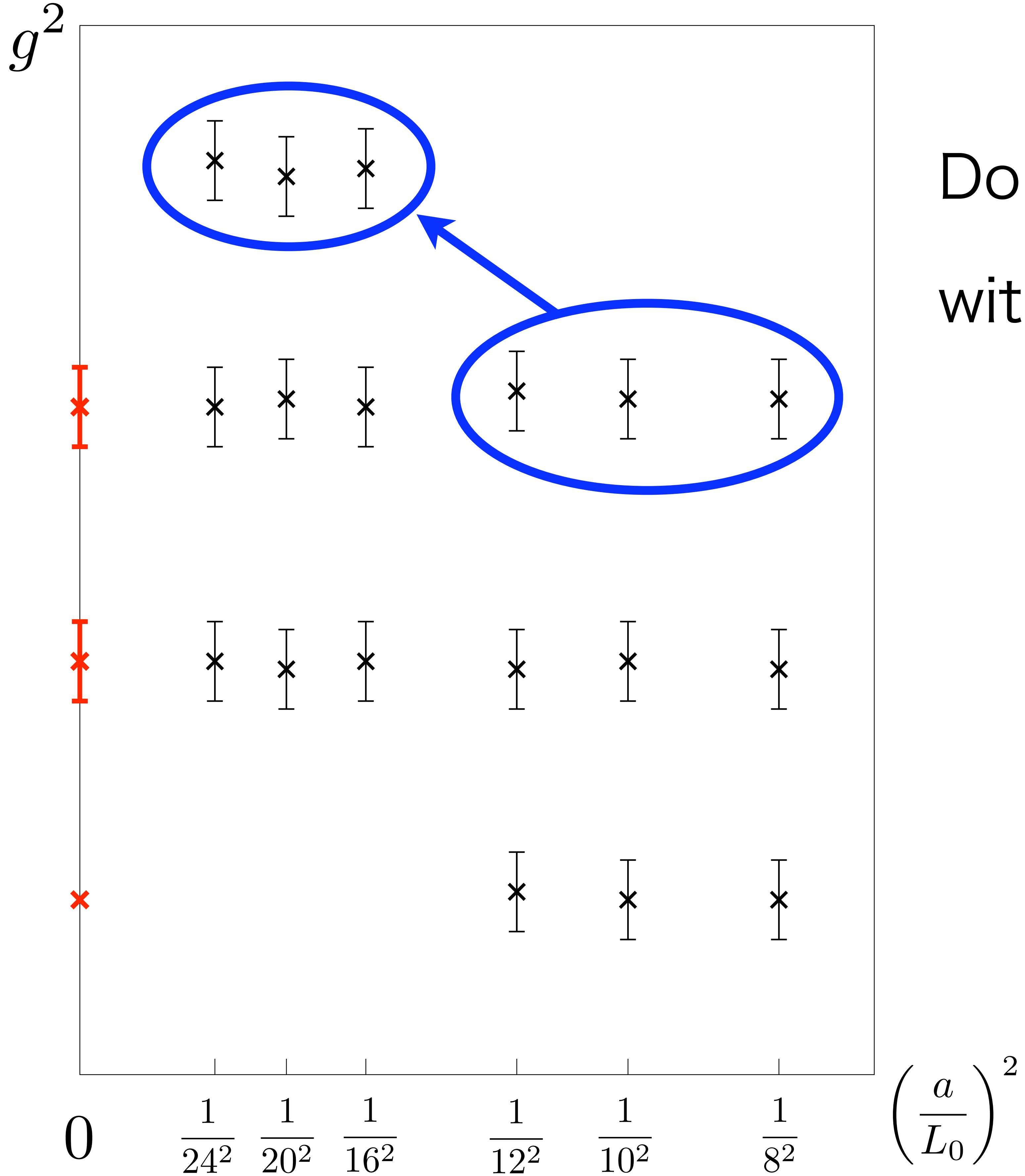

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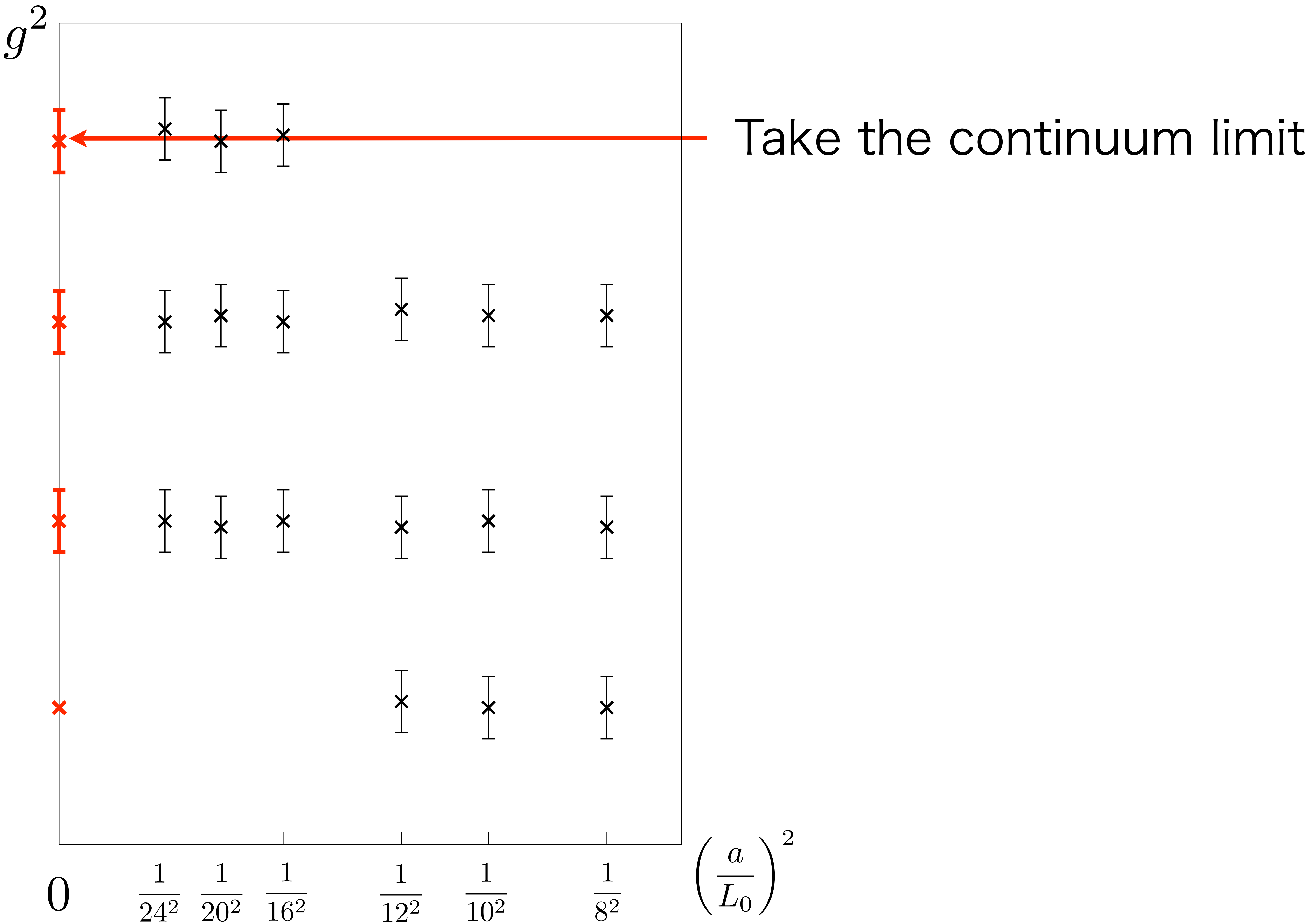

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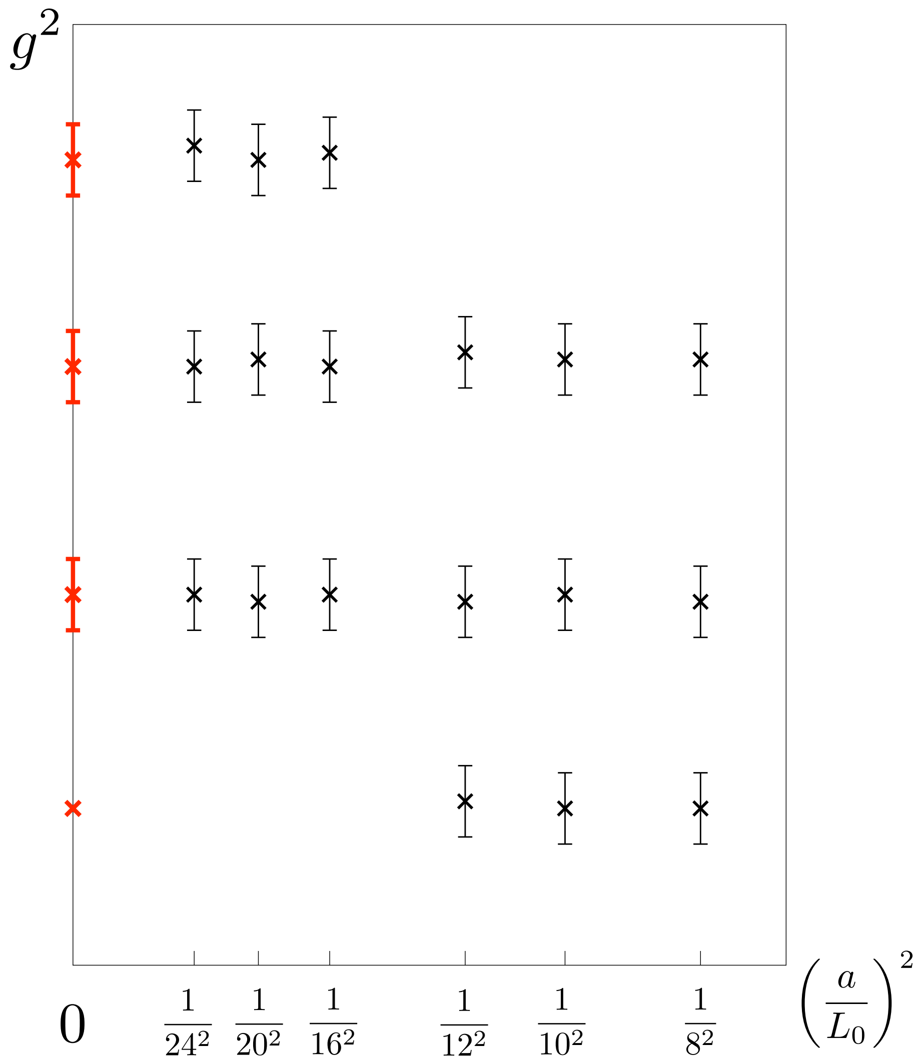
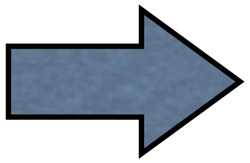
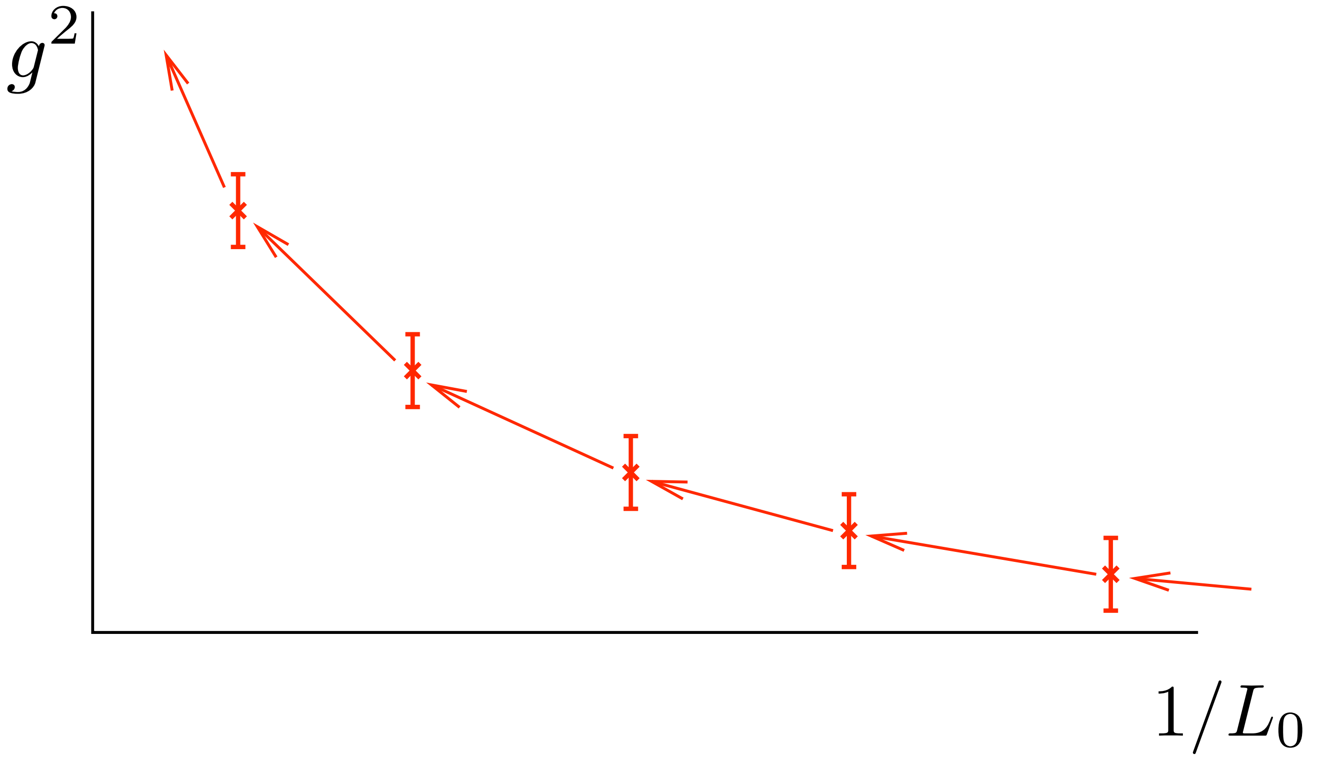


We obtain the scaling of the running coupling





There are studies in which the existence of the IR fixed point is assumed (ex: Schwinger-Dyson, Bethe-Salpeter equation analysis with the improved ladder approximation)

Appelquist, Terning and Wijewardhana, PRL 77, 1214 (1996)
Appelquist, Ratnaweera, Terning and Wijewardhana, PRD 58, 105017 (1998)

- Chiral phase transition at $N_f^{\rm crit} \simeq 4N_c$

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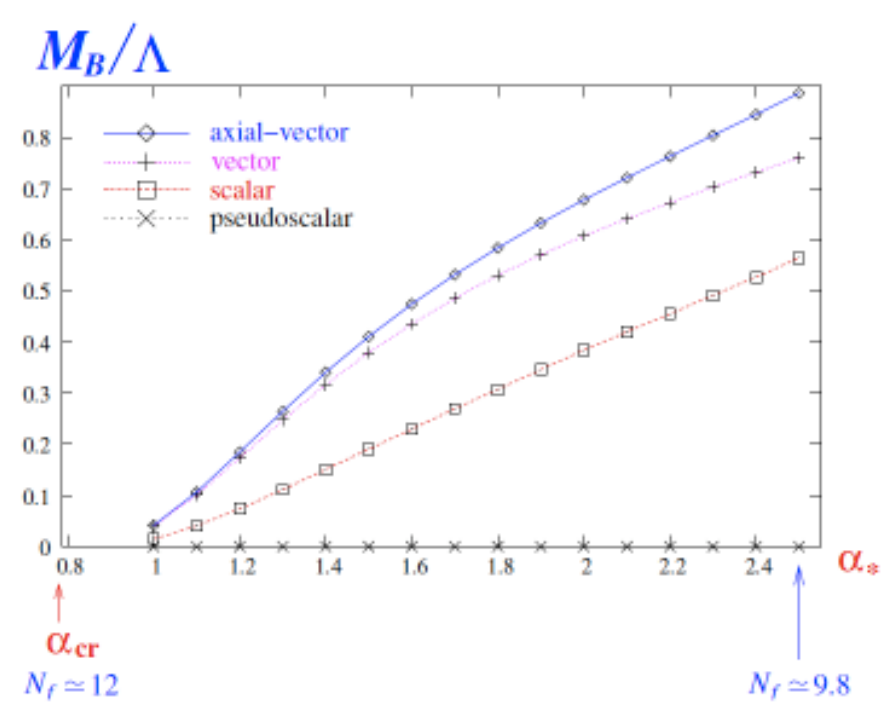
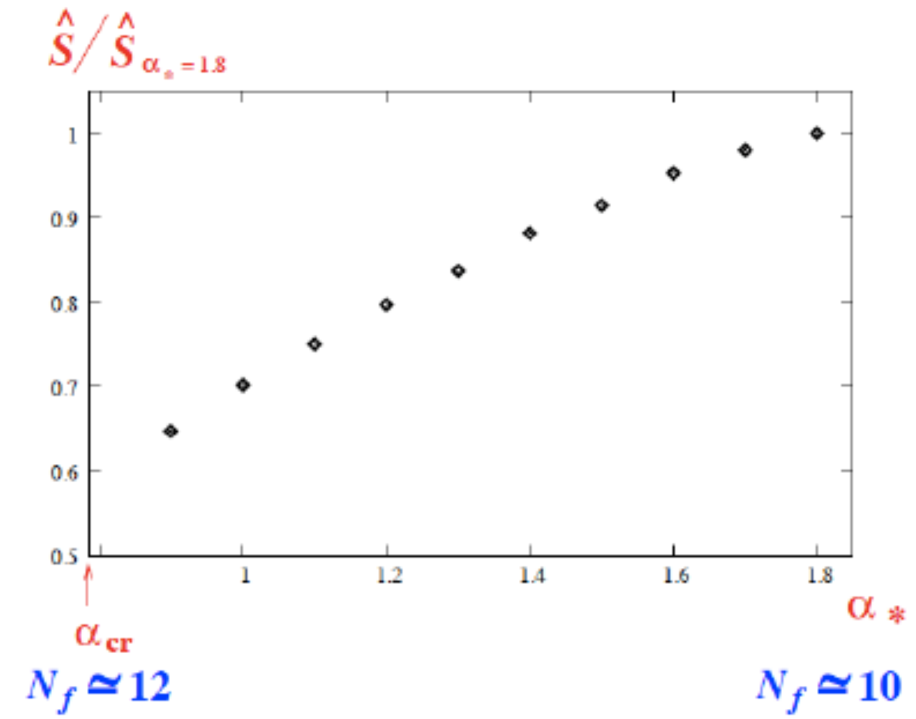
- N_f dependence of Meson masses, S parameter, etc.





Confirmation of the IR fixed point by the Lattice study justify these interesting results

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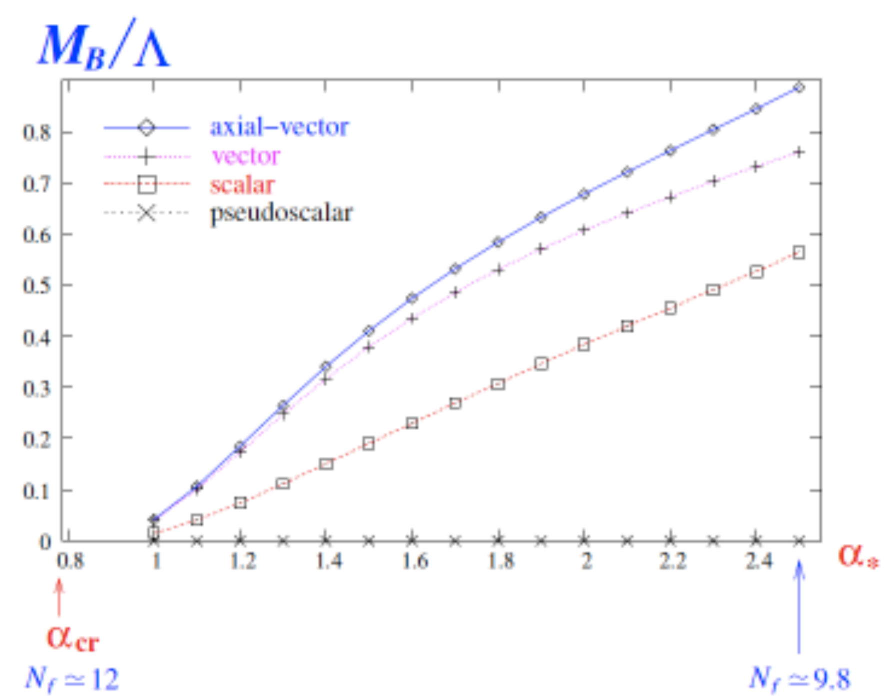

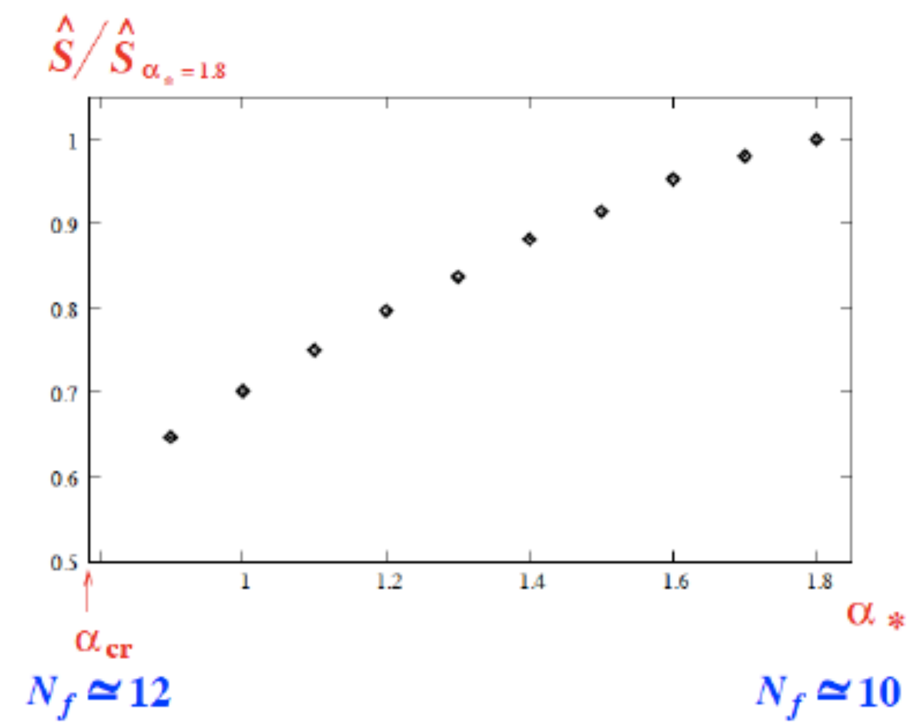

Summary

- We showed a method to extract the scaling of the running coupling from the volume dependence of the Wilson loop
- The method is expected to have small systematic error, and will be a powerful tool for the study of conformal dynamics
- E. Itou will show the numerical results for the application of the method to the quenched QCD as a preliminary test