Vector meson electromagnetic form factors

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Outline

Form factors

- Internal structure of hadrons $\rightarrow$ (generalised) form factors
- low energy quantities $\rightarrow$ lattice
- nucleon, pion, ρ meson
- heavy pions, ρ meson stable
- representative for spin 1 particle

Form factors

interaction hadron - e.m. current

$$\langle p', s' | J^{\alpha} | p, s \rangle = \left(2\sqrt{E_\rho(\vec{p}')E_\rho(\vec{p})} \right)^{-1} \epsilon'^{*}_{\tau}(p', s') J^{\tau\alpha\sigma}(p', p) \epsilon_\sigma(p, s)$$

for spin one particle parametrised by three form factors

$$\begin{aligned} J^{\tau\alpha\sigma}(p', p) = & -G_1\left(Q^2\right) g^{\tau\sigma}\left(p^\alpha + p'^\alpha\right) \\ & -G_2\left(Q^2\right)\left(g^{\alpha\sigma} q^\tau - g^{\alpha\tau} q^\sigma\right) \\ & +G_3\left(Q^2\right)\left(q^\sigma q^\tau \frac{p^\alpha + p'^\alpha}{2m_\rho^2}\right) \end{aligned}$$

momentum transfer $Q^2 = -q^2 = -(p' - p)^2$
polarisation vectors ϵ

Form factors

Sachs form factors

$$G_C\left(Q^2\right) = G_1\left(Q^2\right) + 2/3\,\eta\,G_Q\left(Q^2\right)$$
$$G_M\left(Q^2\right) = G_2\left(Q^2\right)$$
$$G_Q\left(Q^2\right) = G_1\left(Q^2\right) - G_2\left(Q^2\right) + (1+\eta)\,G_3\left(Q^2\right)$$

$\eta = Q^2 / \left(4m_\rho^2\right)$

Interesting static quantities

- charge radius $\langle r^2 \rangle = -6 \frac{\partial G_C}{\partial (Q^2)} |_{Q^2=0}$
- magnetic moment $\mu_M = \frac{e}{2m_\rho} G_M(0)$, aka g factor
- quadrupole moment $\mu_Q = \frac{e}{m_\rho^2} G_Q(0)$

What to expect

- Samsonov *et al*:$\mu_M = 1.8(3)$(QCD sum rules in ext. fields)JHEP 0312:061,2003
- Bhagwat *et al*:$\langle r^2 \rangle = 0.54\ \mathrm{fm}^2$,$\mu_M = 2.01$, $\mu_Q = -0.41\ \mathrm{fm}^2$ (Dyson-Schwinger eqs.) Phys.Rev.C77:025203,2008
- Aliev *et al*:$G_M/G_C > 2$ (light cone sum rules; don't work at small Q^2)Phys.Rev.D70:094007,2004
- Choi *et al*:$\mu_M = 1.92$, $\mu_Q = -0.43\ \mathrm{fm}^2$ (Light front quark model)Phys.Rev.D70:053015,2004
- Alexandrou *et al*:ρ is non-spherical; (density-density correlators, lattice)Phys.Rev.D66:094503,2002
- Hedditch *et al*:$\langle r^2 \rangle \sim 0.6\ \mathrm{fm}^2$, $\mu_M \sim 2.3$, $\mu_Q \sim -0.005\ \mathrm{fm}^2$(quenched lattice simulation, standard 3pt technique)Phys.Rev.D75:094504,2007

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Lattice method

compute three point functions involving $\langle p', s' | J^\alpha | p, s \rangle$
(and two point functions)

system of equations $R^\alpha_{\mu\nu}(p, p') = \sum_i c_i G_i$ for each Q^2

solve numerically (χ^2 minimisation) $\rightsquigarrow G_i(Q^2)$

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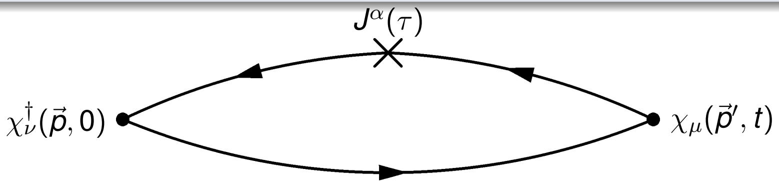
Lattice matrix elements

can be extracted from three point functions

$$G^{\alpha}_{\mu\nu}(t,\tau,\vec{p}',\vec{p}) = \sum_{\vec{x},\vec{\xi}} e^{-i\vec{p}'(\vec{x}-\vec{\xi})} e^{-i\vec{p}\vec{\xi}} \left\langle \Omega \left| \chi_\mu(x) J^\alpha(\xi) \chi^\dagger_\nu(0) \right| \Omega \right\rangle$$



choice of t critical
we will also need the two point functions

$$G_{\mu\nu}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \left\langle \Omega \left| \chi_\mu(x) \chi^\dagger_\nu(0) \right| \Omega \right\rangle$$

transfer matrix formalism; $0 << \tau << t$

$$\lim_{t\to\infty} G_{\mu\nu}(t,\vec{p}) = -\frac{e^{-E_\rho(\vec{p})\,t}}{2E_\rho(\vec{p})}\lambda_\rho(\vec{p})\bar{\lambda}_\rho(\vec{p})\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_\rho^2}\right)$$

$$\lim_{\substack{\tau\to\infty \\ t-\tau\to\infty}} G^{\alpha}_{\mu\nu}(t,\tau,\vec{p}',\vec{p}) = \frac{e^{-E_\rho(\vec{p}')(t-\tau)}e^{-E_\rho(\vec{p})\tau}}{4E_\rho(\vec{p}')E_\rho(\vec{p})}\lambda_\rho(\vec{p}')\bar{\lambda}_\rho(\vec{p}) \times\left(g_{\mu\tau} - \frac{p'_\mu p'_\tau}{m_\rho^2}\right) J^{\tau\alpha\sigma}\left(g_{\sigma\nu} - \frac{p_\sigma p_\nu}{m_\rho^2}\right)$$

$\bar{\lambda}$-overlap of interpolating operator $\chi^\dagger_\mu = \bar{d}\gamma_\mu u$ with ρ

$\langle\Omega\left|\chi_\mu(0)\right|\rho(\vec{p},s)\rangle = \sqrt{2E_\rho(\vec{p})}^{-1}\lambda_\rho(\vec{p})\epsilon_\mu(p,s)$

sum over polarisations using transversality condition $\sum_s \epsilon_\mu(p,s)\epsilon^*_\nu(p,s) = -g_{\mu\nu} + p_\mu p_\nu/m_\rho^2$

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Ratios

$$R^{\alpha}_{\mu\nu}(\tau,\vec{p}',\vec{p}) = \frac{G^{\alpha}_{\mu\nu}(t,\tau,\vec{p}',\vec{p})}{G_{\mu\mu}(t,\vec{p}')}\sqrt{\frac{G_{\nu\nu}(t-\tau,\vec{p})G_{\mu\mu}(\tau,\vec{p}')G_{\mu\mu}(t,\vec{p}')}{G_{\nu\nu}(\tau,\vec{p})G_{\mu\mu}(t-\tau,\vec{p}')G_{\nu\nu}(t,\vec{p})}}$$

is independent of τ
$(\mu,\nu = 1\ldots 3)$
t fixed;potential problems with $\sqrt{}$, argument can be negative

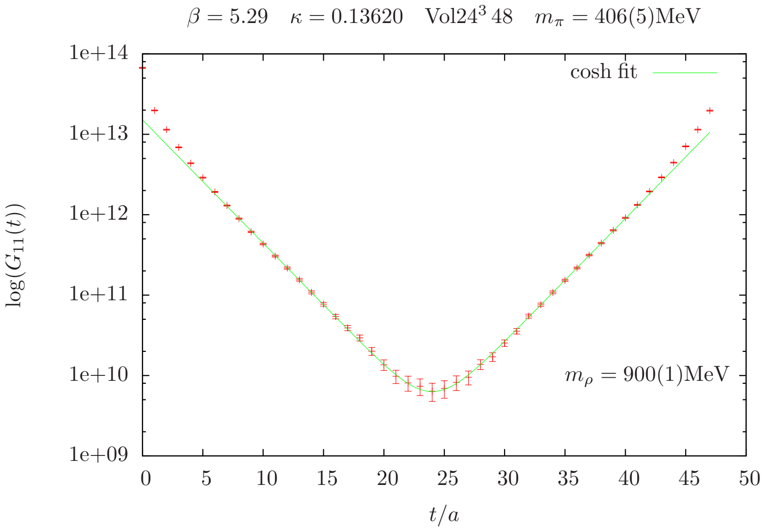
Details of the lattice calculation

- QCDSF-UKQCD configurations
- 2 dynamical flavours of Wilson fermions
- non-perturbatively improved Dirac operator
  $i/4c_{SW}ag^2\bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$, $c_{SW}(g)$ (ALPHA coll.)
- (Jacobi) smeared sources and sinks
- local vector current $\rightsquigarrow$ renormalisation $Z_V = 1/G_1^{\mathrm{unren}}(0)$
- compute for 3 values of $\vec{p}'$ and 17 values of $\vec{p}$ and all polarisation combinations
- no disconnected contribution ($G^{\mathrm{disc}}(U) = -G^{\mathrm{disc}}(U^*)$), both have equal weight

Lattices

Volume	β	κ	m_π[MeV]	a[fm]
16^3 32	5.29	0.13500	929(2)	0.089
16^3 32	5.29	0.13550	784(3)	0.084
24^3 48	5.29	0.13590	591(2)	0.080
24^3 48	5.29	0.13620	406(6)	0.077

two-point function example



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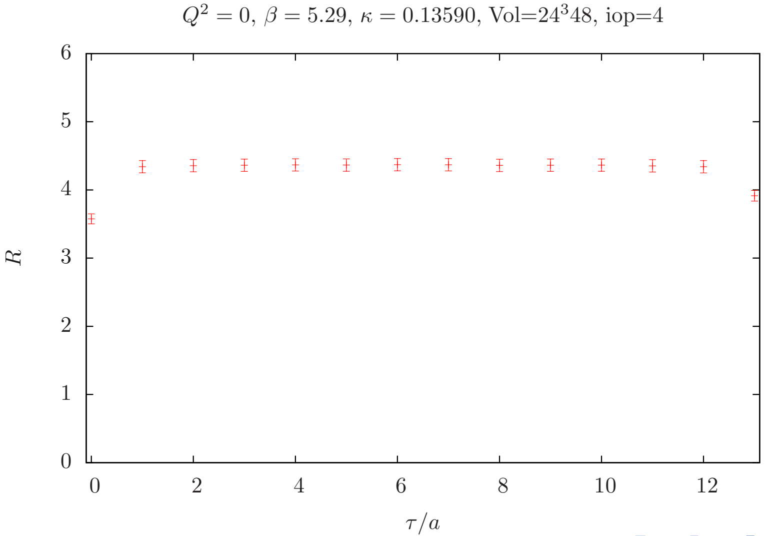
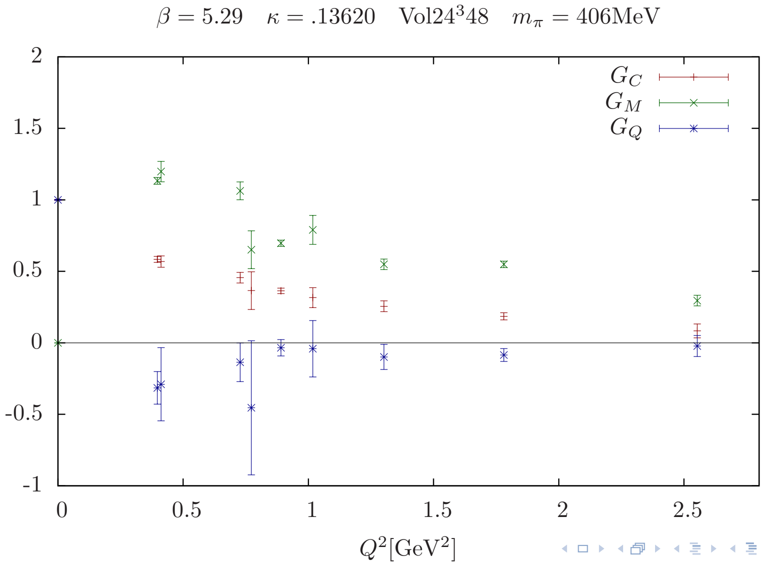
Ratio example



Example for form factor fits



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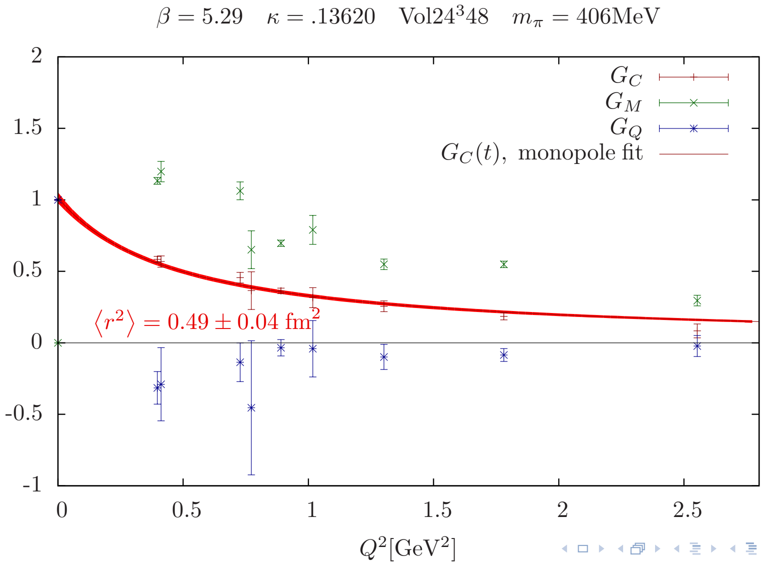
Example for form factor fits



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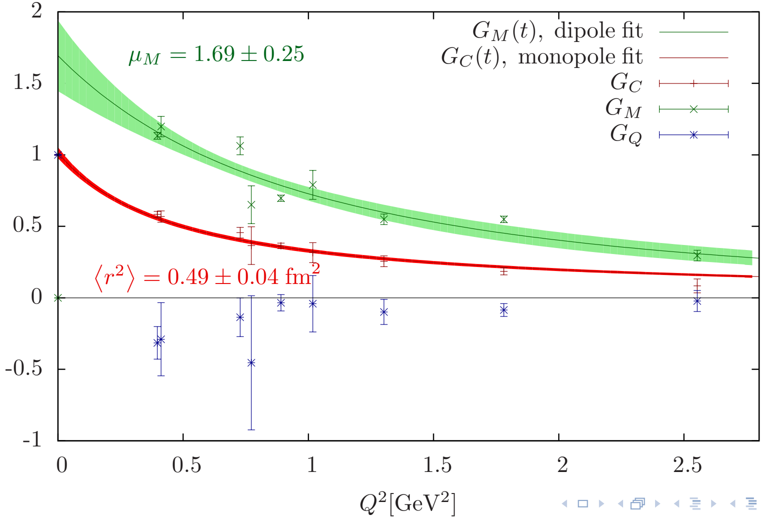
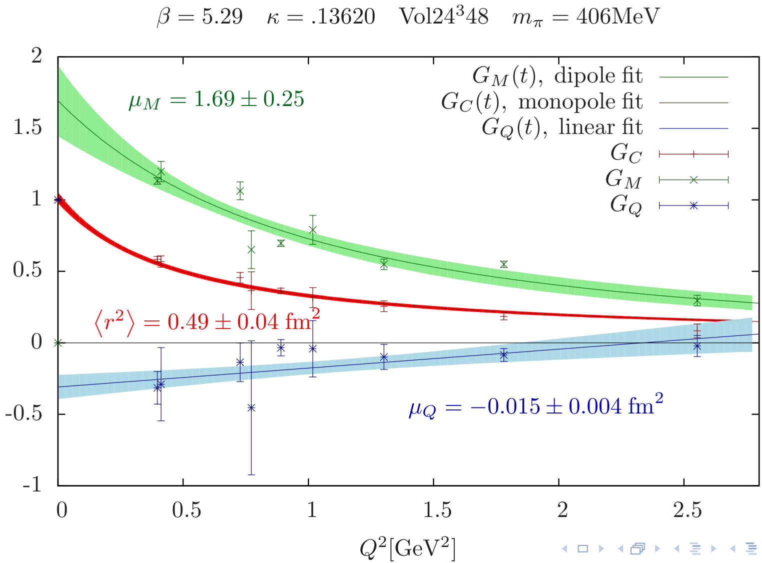
Example for form factor fits

$\beta = 5.29 \quad \kappa = .13620 \quad \mathrm{Vol}24^3 48 \quad m_\pi = 406\mathrm{MeV}$



Example for form factor fits



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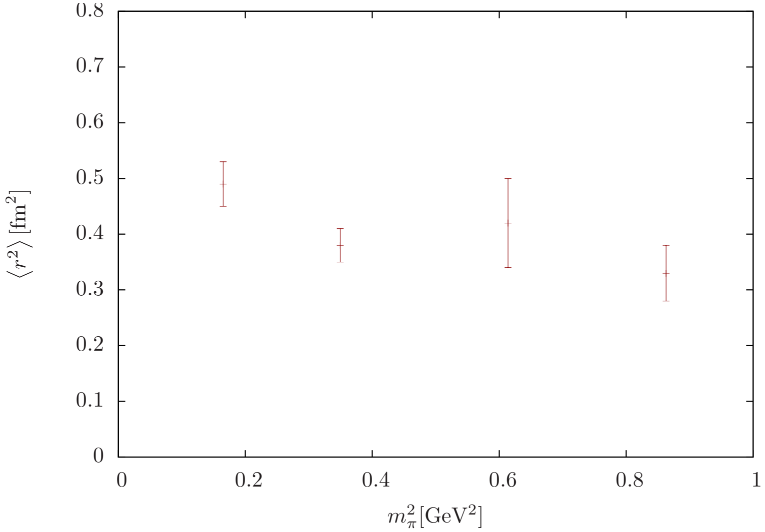
Charge radii



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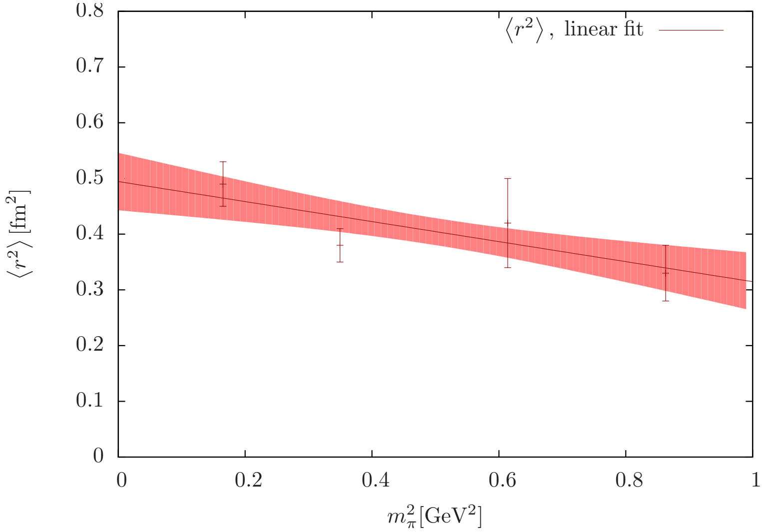
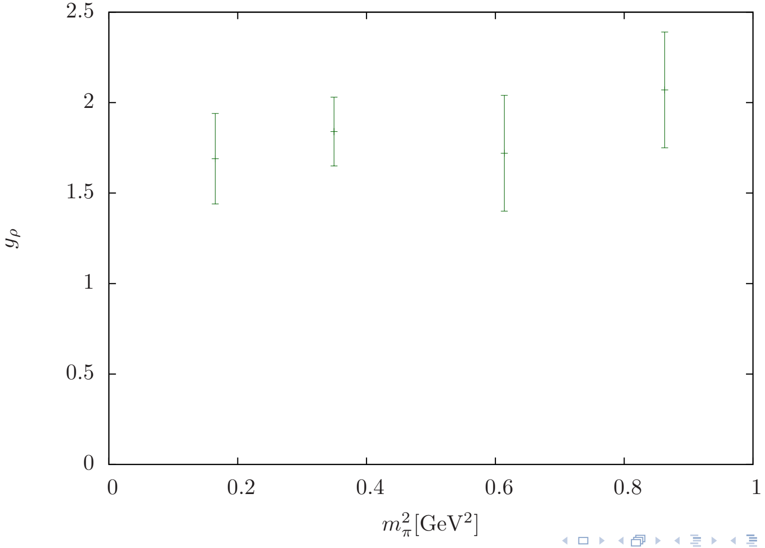
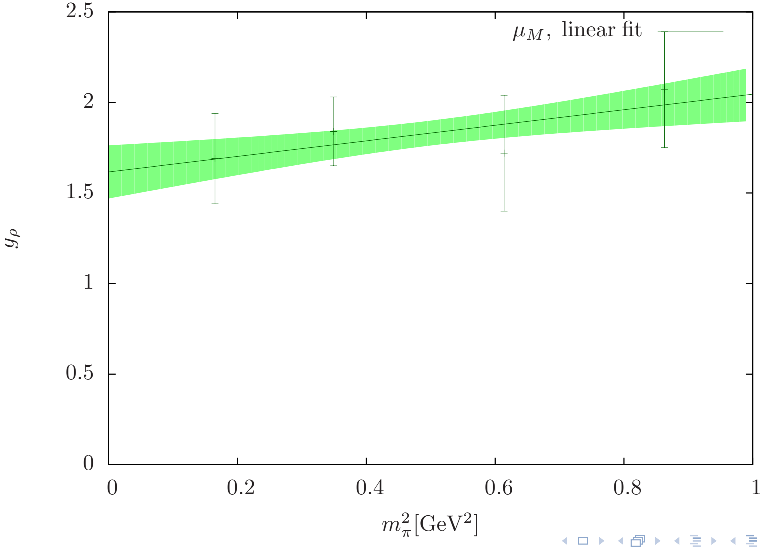
Charge radii



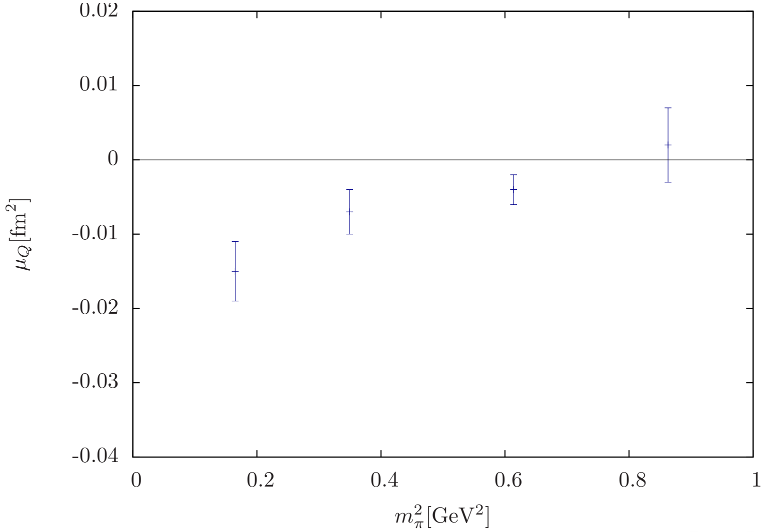
Magnetic moment



Magnetic moment



Quadrupole moment



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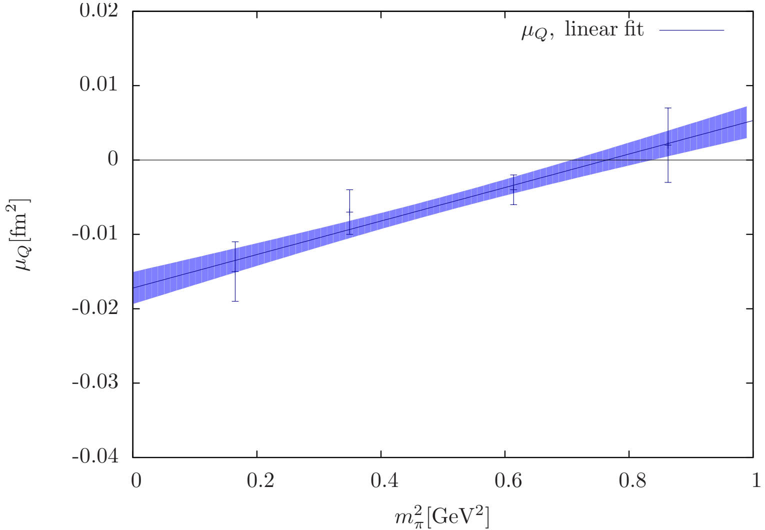
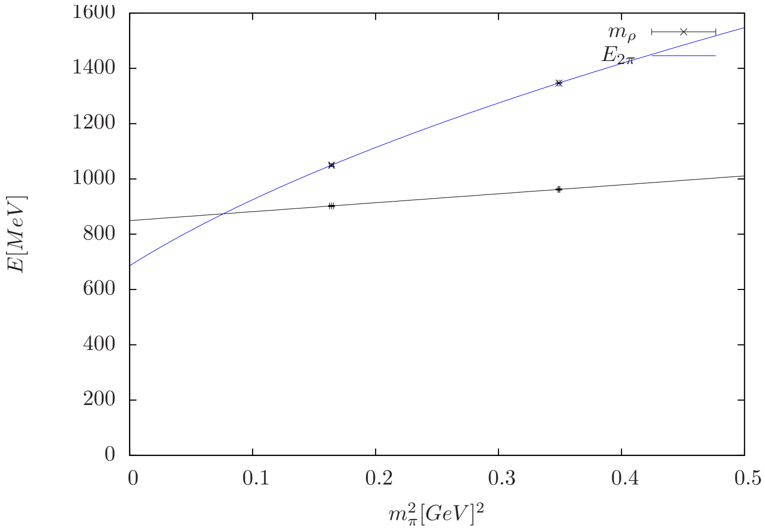
Quadrupole moment



- first unquenched direct computation of the vector meson e.m.form factors
- still preliminary
- Charge radii
  - slightly larger than found by Hedditch et al (larger Q^2 range)
  - growing towards smaller m_q
- g-factor
  - $\sim$ 2; close to quark model expectation
  - chiral curvature?
- quadrupole moment
  - $\sim$ 0 at large pion masses
  - decreasing quark mass: increasingly negative $\rightsquigarrow$ oblate shape
- next: axial/tensor form factors; GFF

When does the ρ decay?



Another ratio example