Geometric effects in lattice QCD thermodynamics

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Outline

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Introduction

- Experiments at temperatures up to a few times T_c show a strongly interacting system **[Arsene *et al.*, 2004]**, **[Back *et al.*, 2004]**, **[Adcox *et al.*, 2004]**, **[Adams *et al.*, 2005]**
- Perturbative approach to the QCD plasma not adequate, and affected by typical difficulties of thermal QFT **[Linde, 1980]**—see also **[Lichtenegger and Zwanziger, 2008]**
- Lattice calculations of the gluon EoS show strong deviations from the Stefan-Boltzmann limit, with a large deficit in the entropy and pressure **[Boyd *et al.*, 1996]**
- Observables related to thermodynamic fluctuations **[Gavai, Gupta and Mukherjee, 2004]** are close to AdS/CFT predictions **[Gubser, Klebanov and Tseytlin, 1998]**
- Similar features also observed in $\mathrm{SU}(N > 3)$ models **[Bringoltz and Teper, 2005]**—see also **[Datta and Gupta, 2008]**

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UV and IR effects on the QCD equation of state — The conventional scenario

UV effects on the lattice QCD equation of state are well understood **[Engels, Karsch and Scheideler, 1990]**:

- The distortion of the continuum dispersion relation induced by the lattice cutoff modifies the normalization of thermodynamic observables
- Effect mitigated by using improved lattice actions
- Overall change in the normalization can be accounted for via a factor that can be evaluated via numerical integration

UV and IR effects on the QCD equation of state — The conventional scenario

IR effects are not expected to play any non-trivial rôle:

- Due to the existence of a chromomagnetic screening length $O(\frac{1}{g^2 T})$, the low-energy plasma dynamics is effectively similar to that of the low-temperature hadronic phase **[DeTar, 1985]**, **[Blaizot and Iancu, 2001]**
- As a consequence, for temperatures of the same order of magnitude as T_c, IR effects are expected to be exponentially suppressed
- Empirically, $N_s/N_t \approx 4$ (or larger) is sufficient to yield complete suppression **[Engels, Fingberg, Karsch, Miller and Weber, 1990]**, **[B. Beinlich, F. Karsch and E. Laermann, 1995]**

Are non-trivial IR effects possible?

It was suggested **[Gliozzi, 2007]** that non-trivial IR effects may still have an impact on the lattice QCD EoS:

- In a finite volume, the Stefan-Boltzmann law for a gas of free gluons (*i.e.* at infinite temperature) gets logarithmic corrections, which depend on the LT aspect ratio
- The logarithm of the partition function is no longer purely extensive:

$$\frac{\log Z}{N^2-1} = \frac{\pi^2}{45}(LT)^3 - \log\sqrt{LT} + O\left(e^{-2\pi LT}\right) \quad (1)$$

- All thermodynamic observables get corrections; in particular: $p \neq -f$

Are non-trivial IR effects possible?

- This effect is due to the existence of periodic boundary conditions: The zero-mode contribution to the functional integral is divergent, and must be regularized
- In principle, the same mechanism may also be at work in the strongly-interacting, $T/T_c = O(1)$ regime—although the effective d.o.f. are completely different
- The effect can only be calculated exactly for the free case, using ζ-function techniques (see, *e.g.*, **[Edery, 2005]**)
- For the interacting case, the free-limit corrections can be considered as an upper bound
- Quantitatively, for typical parameters of finite-temperature lattice simulations—**[Aoki *et al.*, 2005]**, **[Bernard *et al.*, 2006]**, **[Cheng *et al.*, 2007]**—, this value could account for up to about half of the pressure and entropy deficit
- Might be particularly relevant for $\mathrm{SU}(N)$ simulations, where the deconfinement transition can be studied using smaller lattices than for $\mathrm{SU}(3)$ **[Lucini, Teper and Wenger, 2004]**

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Simulation details

- $\mathrm{SU}(N)$ YM simulations on the lattice, combining heat-bath **[Kennedy and Pendleton, 1985]** for $\mathrm{SU}(2)$ subgroups **[Cabibbo and Marinari, 1982]** and full-$\mathrm{SU}(N)$ overrelaxation **[Kiskis, Narayanan and Neuberger, 2003]**, **[de Forcrand and Jahn, 2005]**
- Some $T = 0$ simulations run using Chroma **[Edwards and Joó, 2004]**
- Scale set by measuring and interpolating the string tension—see also **[Lucini, Teper and Wenger, 2004]**
- Equation of state determined by the integral method **[Engels *et al.*, 1990]**
- Normalization due to UV cutoff effects corrected by the N_t-dependent normalization factor **[Engels *et al.*, 2000]**

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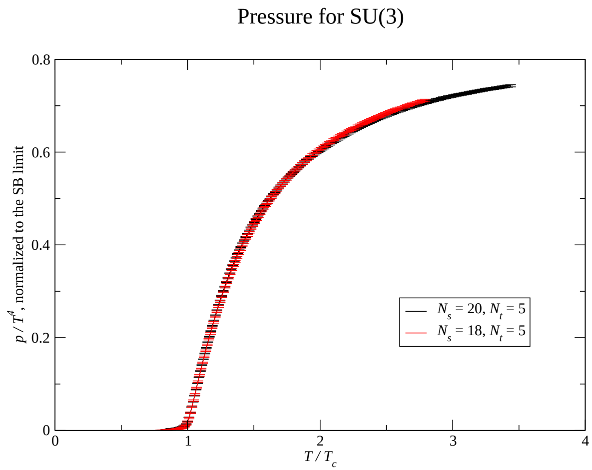
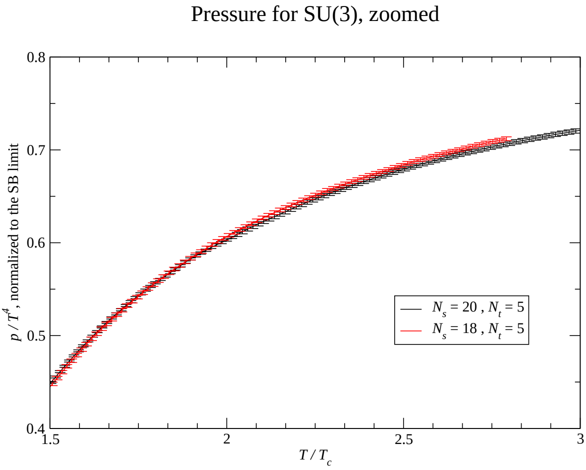
Looking for logarithmic effects in the $\mathrm{SU}(3)$ EoS

To the level of precision obtained, the data do not reveal IR effects, up to about $3T_c$



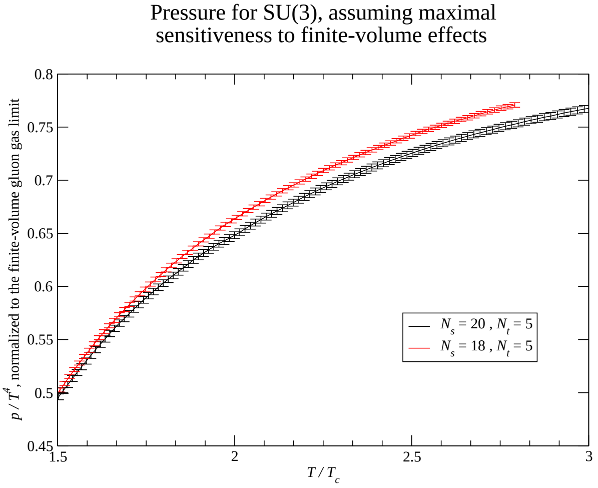
Looking for logarithmic effects in the SU(3) EoS

Deviations compatible with statistical fluctuations



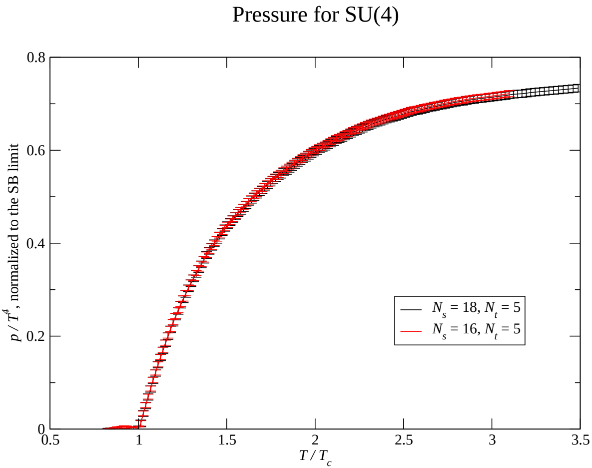
Looking for logarithmic effects in the SU(3) EoS

Assuming maximal sensitiveness to IR effects, the data sets tend to deviate from each other



Looking for logarithmic effects in the EoS for $\mathrm{SU}(N)$

Similar conclusions are found from the analysis of $\mathrm{SU}(4)$ data



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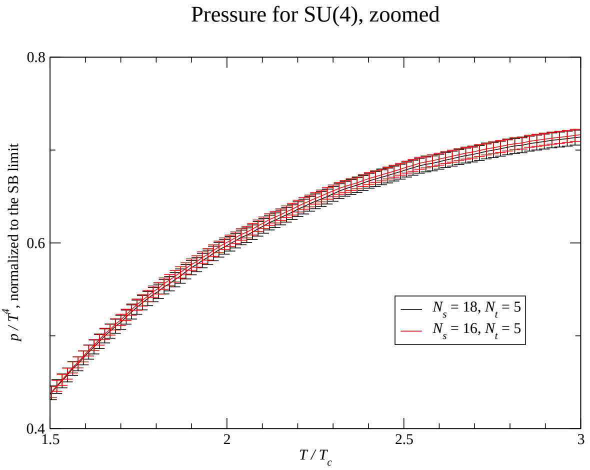

Looking for logarithmic effects in the EoS for $\mathrm{SU}(N)$

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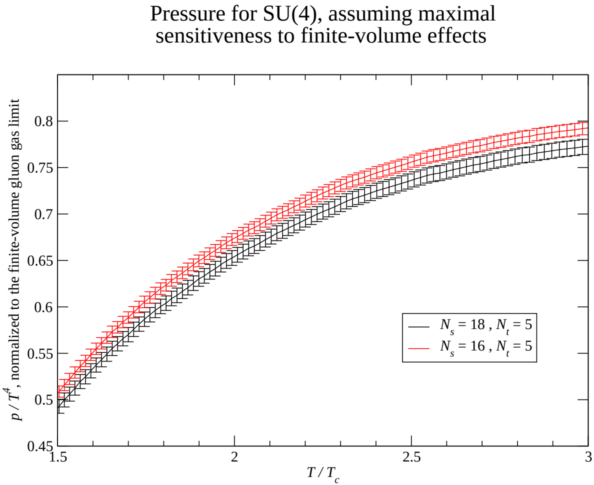
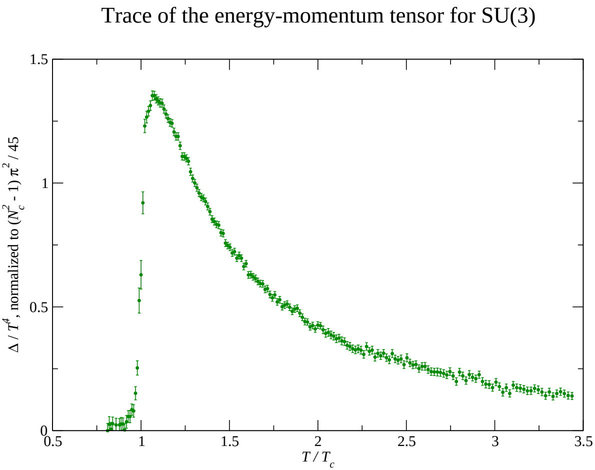


Other thermodynamic observables in SU(3)



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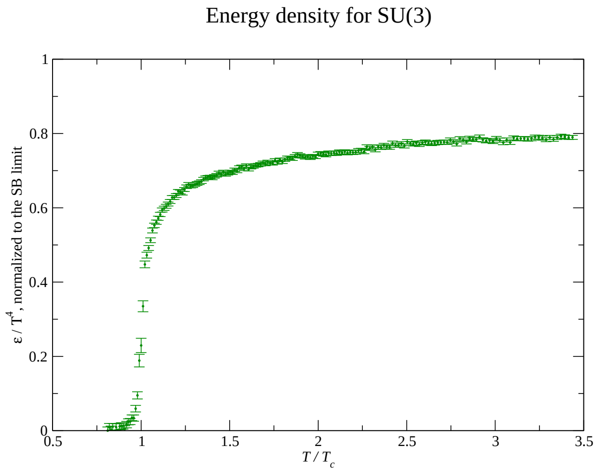
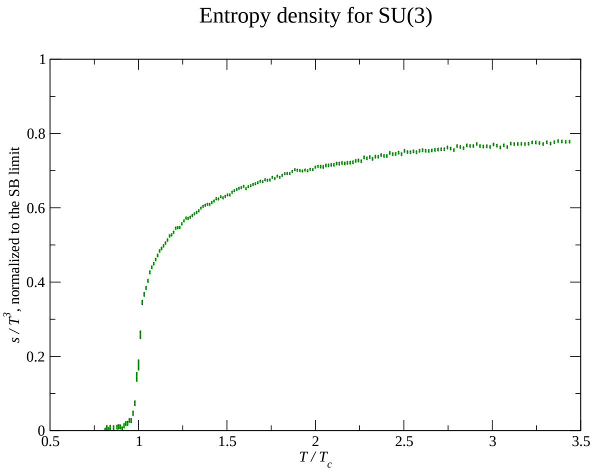
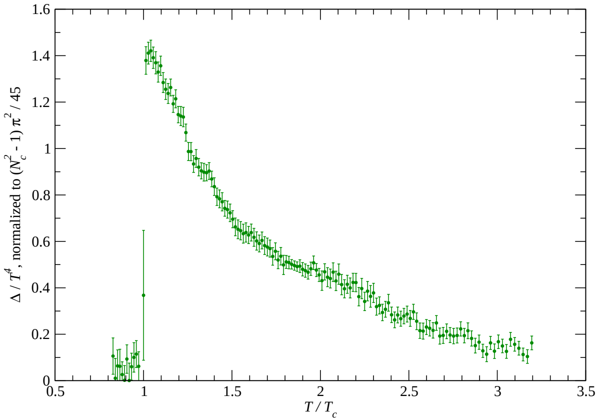


Other thermodynamic observables in $\mathrm{SU}(3)$

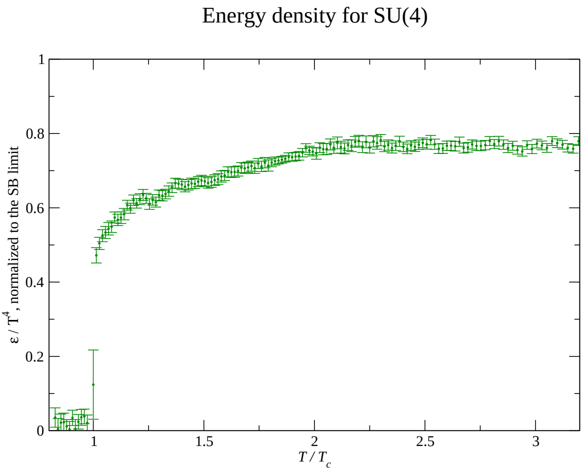


Other thermodynamic observables in SU(4)

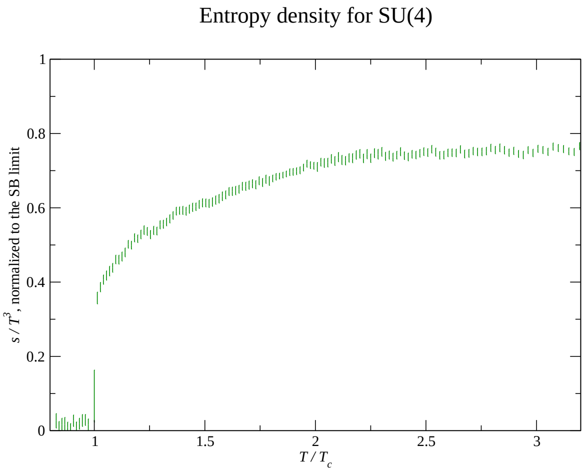
Trace of the energy-momentum tensor for SU(4)



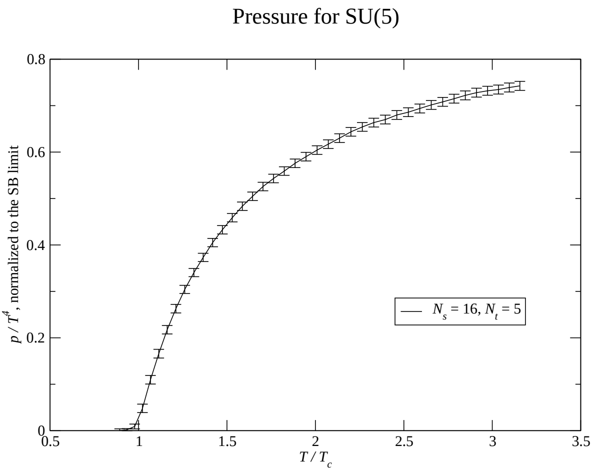
Other thermodynamic observables in $\mathrm{SU}(4)$



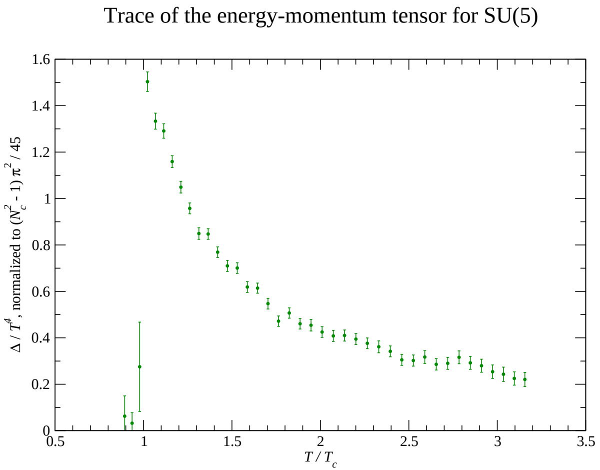
Other thermodynamic observables in SU(4)



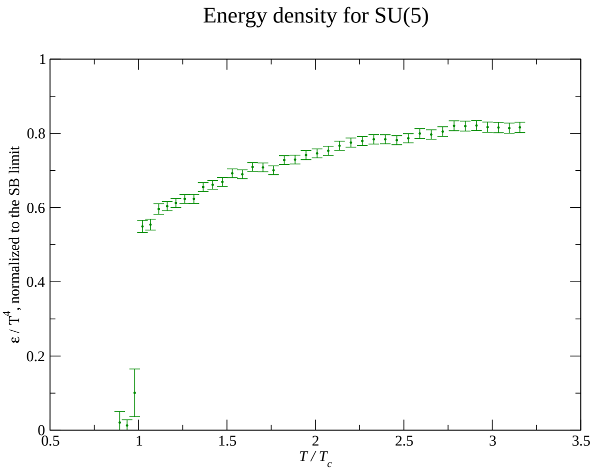
Thermodynamic observables in SU(5)



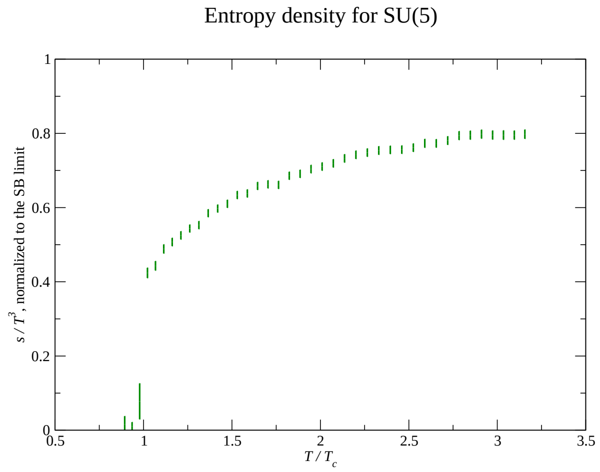
Thermodynamic observables in $\mathrm{SU}(5)$



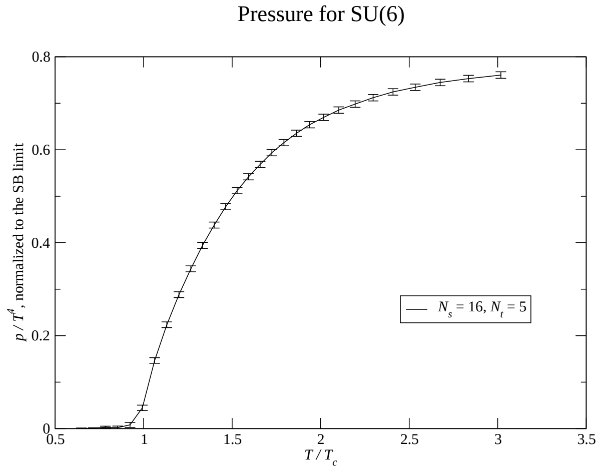
Thermodynamic observables in $\mathrm{SU}(5)$



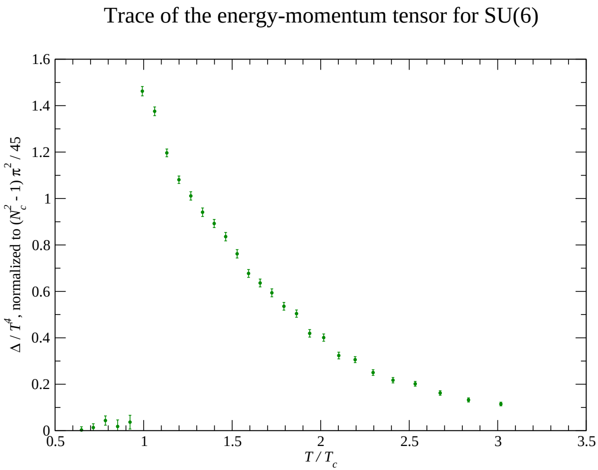
Thermodynamic observables in SU(5)



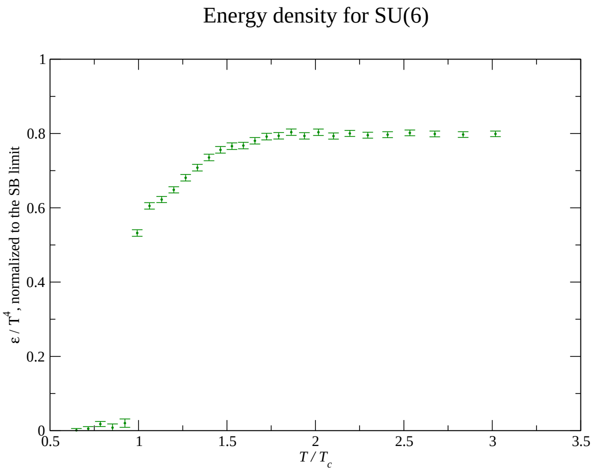
Thermodynamic observables in SU(6)



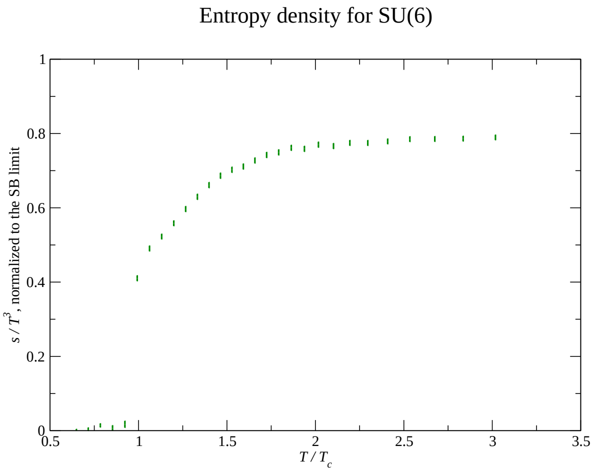
Thermodynamic observables in $\mathrm{SU}(6)$



Thermodynamic observables in $\mathrm{SU}(6)$



Thermodynamic observables in SU(6)



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Conclusions and outlook

- To the level of precision reached, no evidence of logarithmic IR effects at temperatures up to about $3T_c$. However, they may possibly be found at very high temperatures—where lattice investigations are now possible **[Endrődi, Fodor, Katz and Szabó, 2007]**
- Same conclusions seem to hold for $\mathrm{SU}(4)$ (albeit the data precision is lower)
- In progress: complete study of thermodynamic observables for $\mathrm{SU}(3)$ and $\mathrm{SU}(N)$ lattice gauge theories