Contributions of the disconnected diagrams in the hyperfine splitting in charmonium in the quenched case

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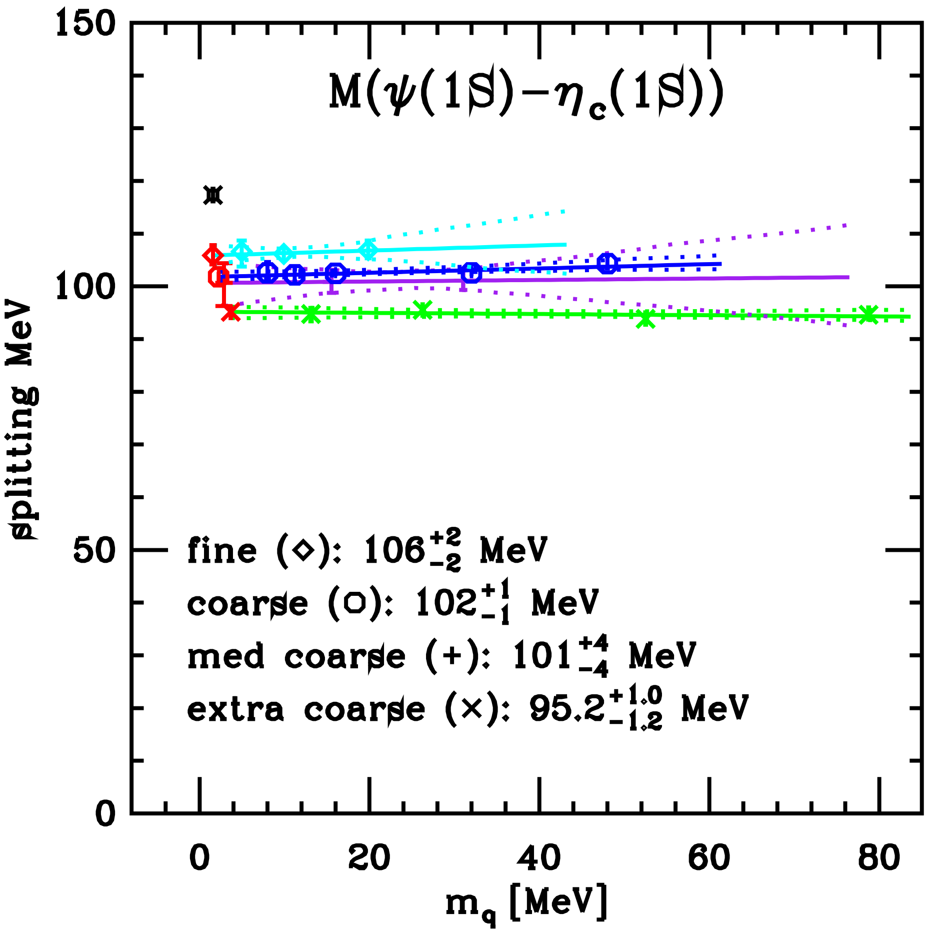
[Lattice 2008, Williamsburg]

Motivation

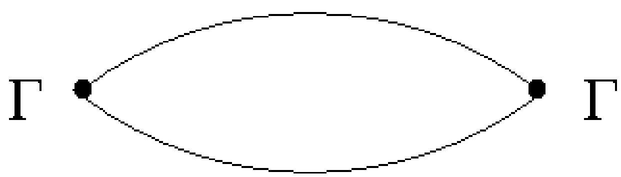


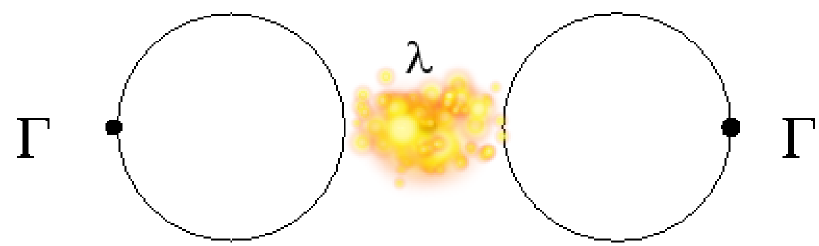
- Lattice calculations of the hyperfine splitting in charmonium show discrepancies with the experimental value of 117 MeV.
- The discrepancy is large (30-40%) in the quenched case. With improved actions it is still around 10 %.
- Possible reasons:
  - Even the current state-of-the-art lattice actions do not reproduce the heavy quark dynamics within the charmonium states well.
  - Neglected contributions of the disconnected diagrams in lattice computations

Diagram contributions to the full propagator

$$F(t) = C(t) + D(t)$$

- Connected and disconnected (singlet) diagrams:





$$\sim \frac{\mathbf{1}}{\mathbf{p^2 + m_c^2}}$$

$$\sim \frac{\mathbf{1}}{\mathbf{p^2 + m_c^2}} \lambda \frac{\mathbf{1}}{\mathbf{p^2 + m_c^2}}$$

- Origins of λ: anomaly, glueball interactions, light modes (dynamical case)

Lattice method for disconnected diagrams

The disconnected part of the correlator is calculated as:

$$D(t) = \langle L(0) L^{\star}(t) \rangle \,, \qquad L(t) = \mathrm{Tr}(\Gamma M^{-1})$$

Previous works explore the ratio:

$$\frac{D(t)}{C(t)} = \frac{F(t)}{C(t)} - 1 = \frac{A_f}{A_c} e^{(m_c - m_{\mathrm{f}})t} - 1.$$

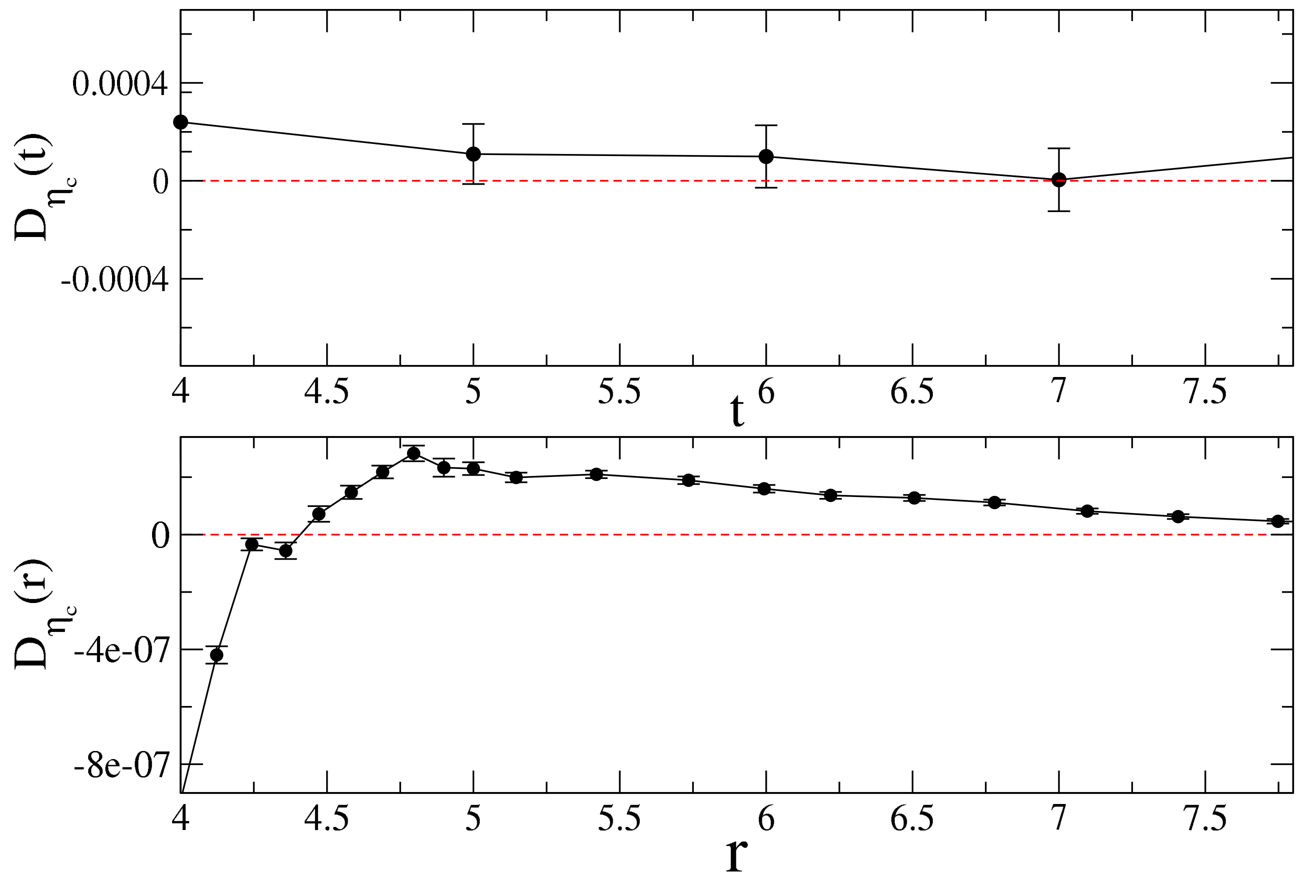
Considering that the available lattices are quenched with respect to the charm quark, an appropriate fitting form would be

$$\frac{D(t)}{C(t)} = (m_c - m_{\mathrm{f}})t + \frac{m_c - m_{\mathrm{f}}}{m_c}$$

if correlators are normalized appropriately.

Our dynamical calculation

- We use 505 Asqtad 2+1 flavor lattices with $V = 40^3 \times 96$ and $a \approx 0.09$ fm. The valence quarks are clover type with tuned $k_c = 0.127$. **Improvements for the stochastic estimation of traces: Unbiased subtraction to $O(3)$**
- Calculating the disconnected point-to-point propagator improves statistics. It has from one to three orders of magnitude smaller relative errors than the time-slice-to-time-slice disconnected propagator in the region where we have a signal.



Asymptotic behavior of the disconnected propagator

- At large distances the dominant behavior of the connected propagator is:

$$C(r) \sim A\frac{e^{-m_c r}}{r^{\frac{3}{2}}},$$

- The disconnected propagator asymptotically will be:

$$D(r) \sim -\frac{d}{dm_c^2}C(r) \sim B\frac{e^{-m_c r}}{r^{\frac{1}{2}}}$$

- Their ratio:

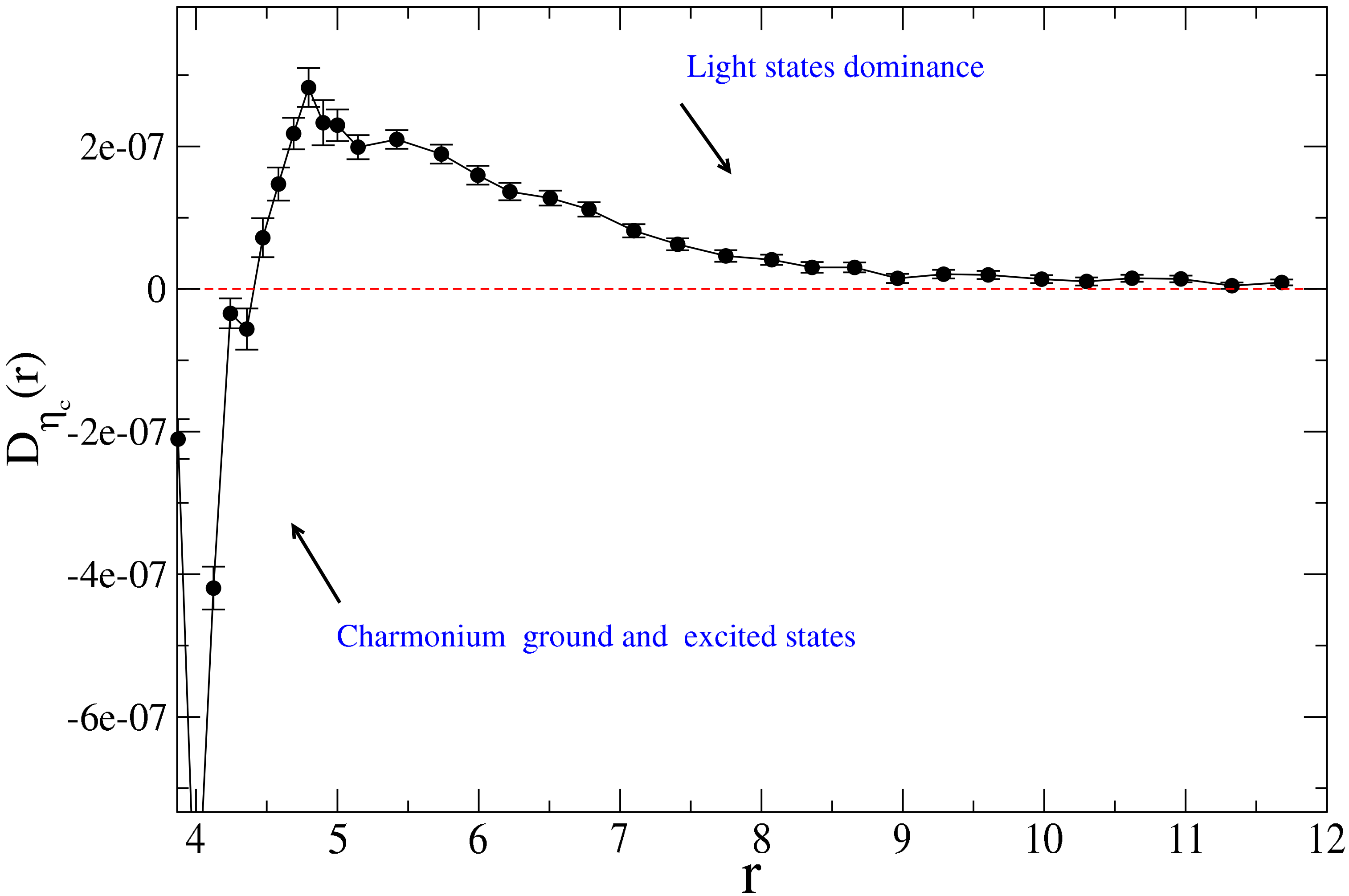
$$\frac{D(r)}{C(r)} \approx \frac{B}{A}r$$

where

$$\frac{B}{A} = m_c - m_f.$$

Extracting the η_c signal from $D(r)$

- $D(r)$ is a sum of ground η_c state, excited states and light states which dominate at large distances (and flip the sign of $D(r)$).



Fitting results for the η_c

$$D^{fit}(r) = \frac{B}{r^{\frac{1}{2}}}(e^{-m_c r} + e^{-m_c^\star r}) + \frac{cB}{r^{\frac{3}{2}}}(e^{-m_c r} - e^{-m_c^\star r}) + \frac{L}{r^{\frac{3}{2}}}e^{-m_l r}$$

- The light mass $m_l = 0.43(1)$ is determined from a single exponential fit from $r = 7 - 12$. In the above fit it is fixed to that value.
- The connected η_c and $\eta_c^\star$ masses, $m_c = 1.1598(7)$ and $m_c^\star = 1.51(5)$, are known from fits to the connected propagator $C(t)$. They are used as constants in the fit as well.
- The constant $c \approx 7$ comes from various assumptions in our model. The fit is not very sensitive to its exact value.
- Results for fitting range $r = 5 - 11$:

$$\frac{B^{fit}}{A^{fit}} = m_c - m_f \in [-4, -1]\,\mathrm{MeV}$$

- Our fit favors disconnected diagram contribution which slightly increases the η_c mass. This is the opposite of the perturbative expectation of ~ 2.4 MeV decrease.
- If the OZI rule for the J/Ψ holds $\Rightarrow$ slight decrease of the hyperfine splitting.

New fitting procedure

- Approximation of the disconnected correlator in momentum space:

$$D(p^2) \sim \underbrace{\left(C + \frac{f}{p^2 + m_l^2}\right)}_{\lambda} \left(\frac{a}{p^2 + m_c^2} + \frac{b}{p^2 + m_c^{\star 2}}\right)^2$$

- Discretized version has $p_\mu^2 = 2(1 - \cos(2\pi/N_\mu))$ and should have the rotation symmetry violations accounted for.Use the Fourier transformed discretized version for fits.

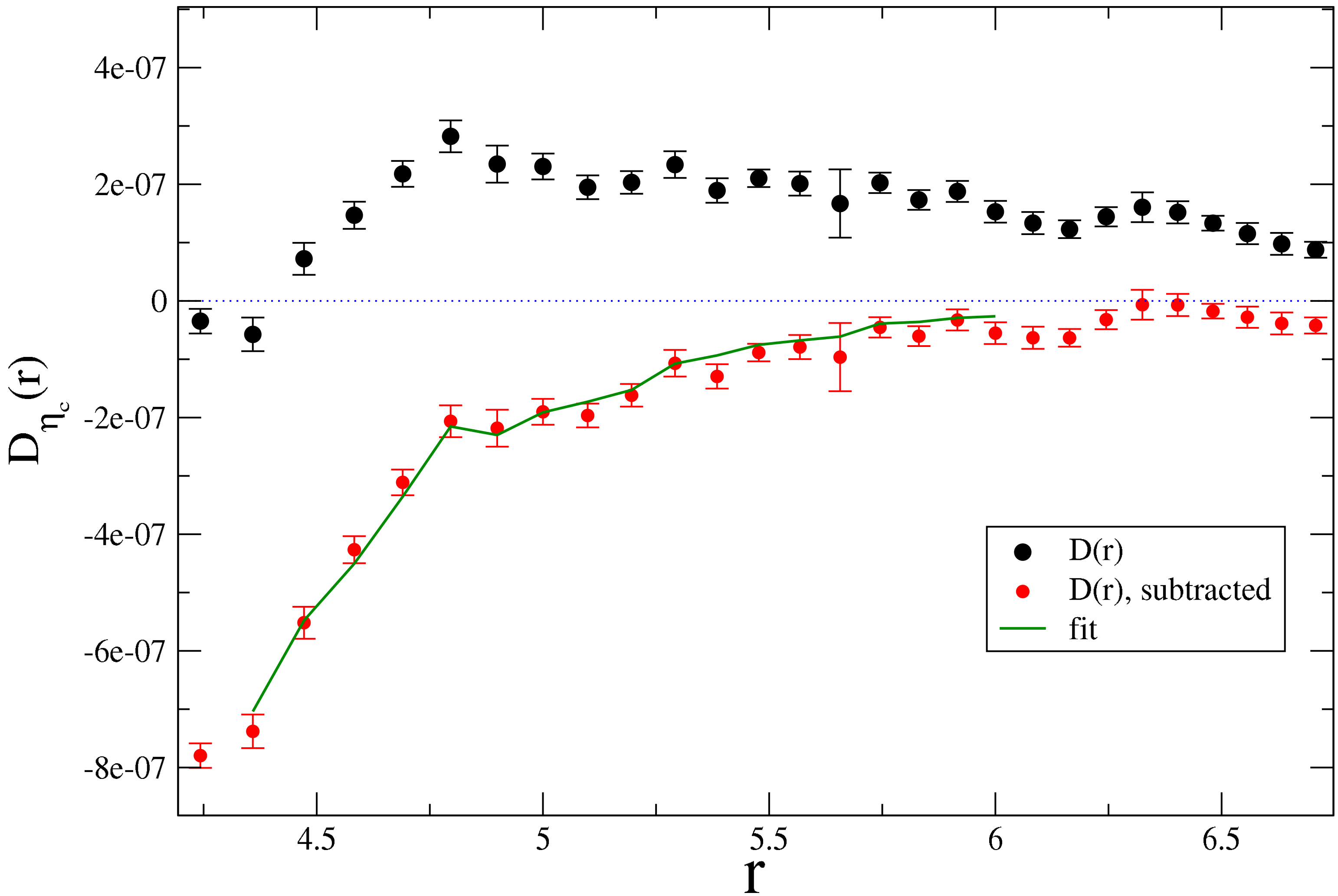
- Applying directly this form to the η_c data doesn't work. Subtract the light mode signal ($\frac{L}{r^{\frac{3}{2}}} e^{-m_l r}$) and the do the fit to a simplified form.

$$D(p^2) \sim C \left(\frac{a}{p^2 + m_c^2} + \frac{b}{p^2 + m_c^{\star 2}}\right)^2$$

- Determine the constants a, b by fits to the coordinate space data for $D(r)$. The amplitude B of the ground disconnected correlator is:

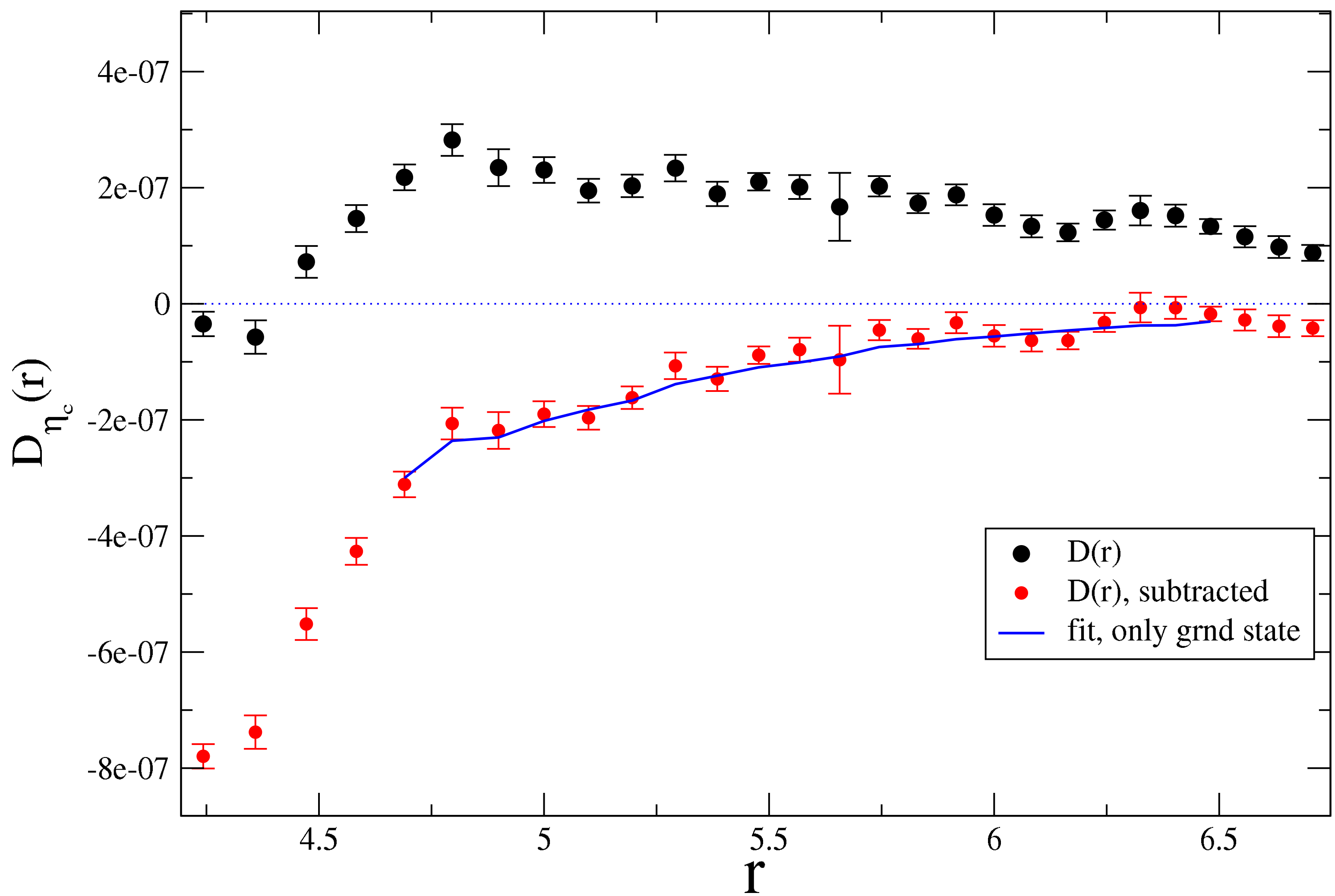
$$B = Ca^2 \left(\frac{1}{128\pi^3 m_c}\right)^{1/2}$$

Results of fitting η_c



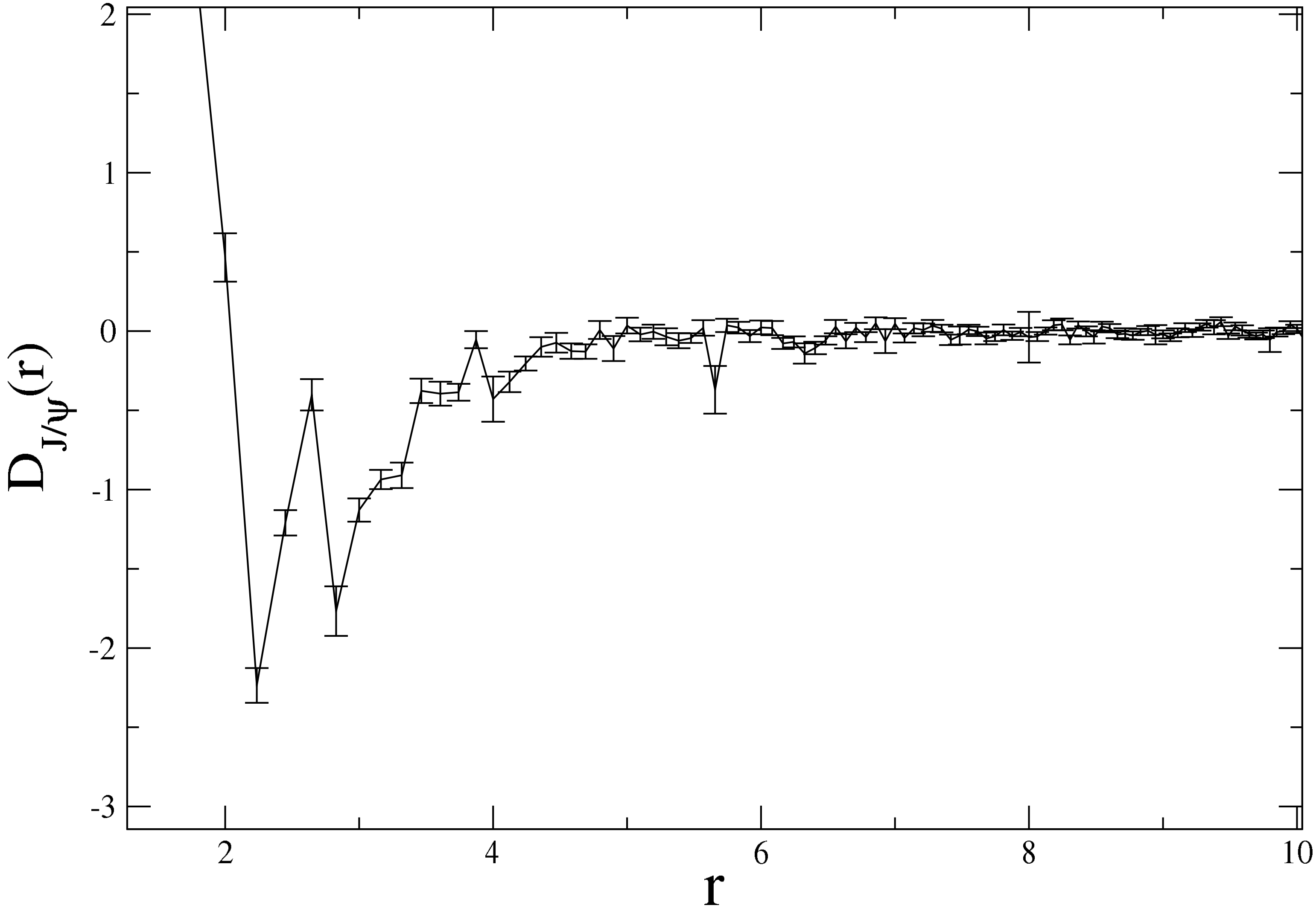
- Fit: $\chi^2 = 15/16$ df, $m_c - m_f = -0.7(5)$ MeV.

Results of fitting η_c



- Fit: $\chi^2 = 21/20$ df, $m_c - m_f = -5.5(4)$ MeV.

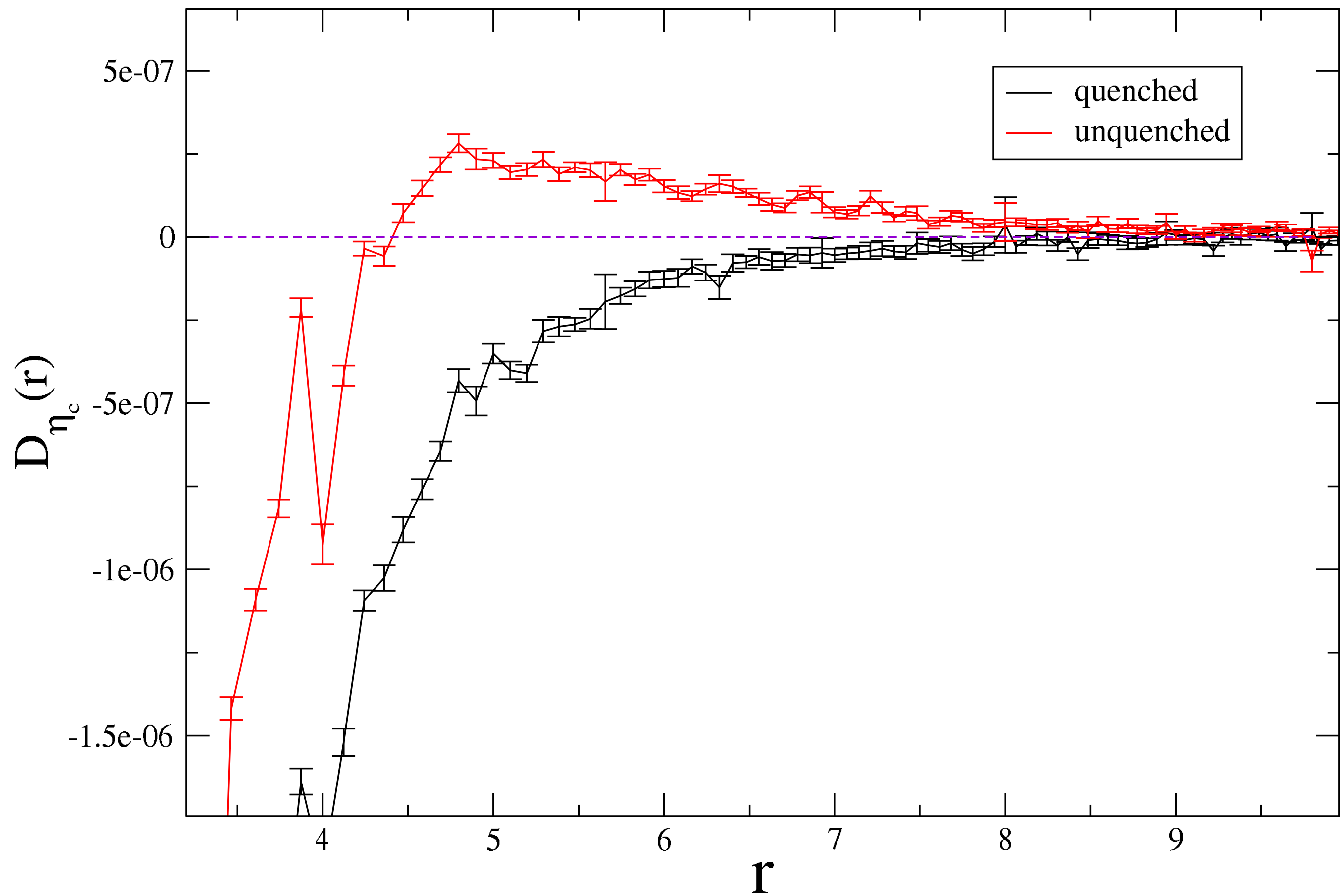
Results of fitting J/Ψ



- Fit: $\chi^2 \approx 1$, $m_c - m_f < 0, |m_c - m_f| < 1$ MeV.

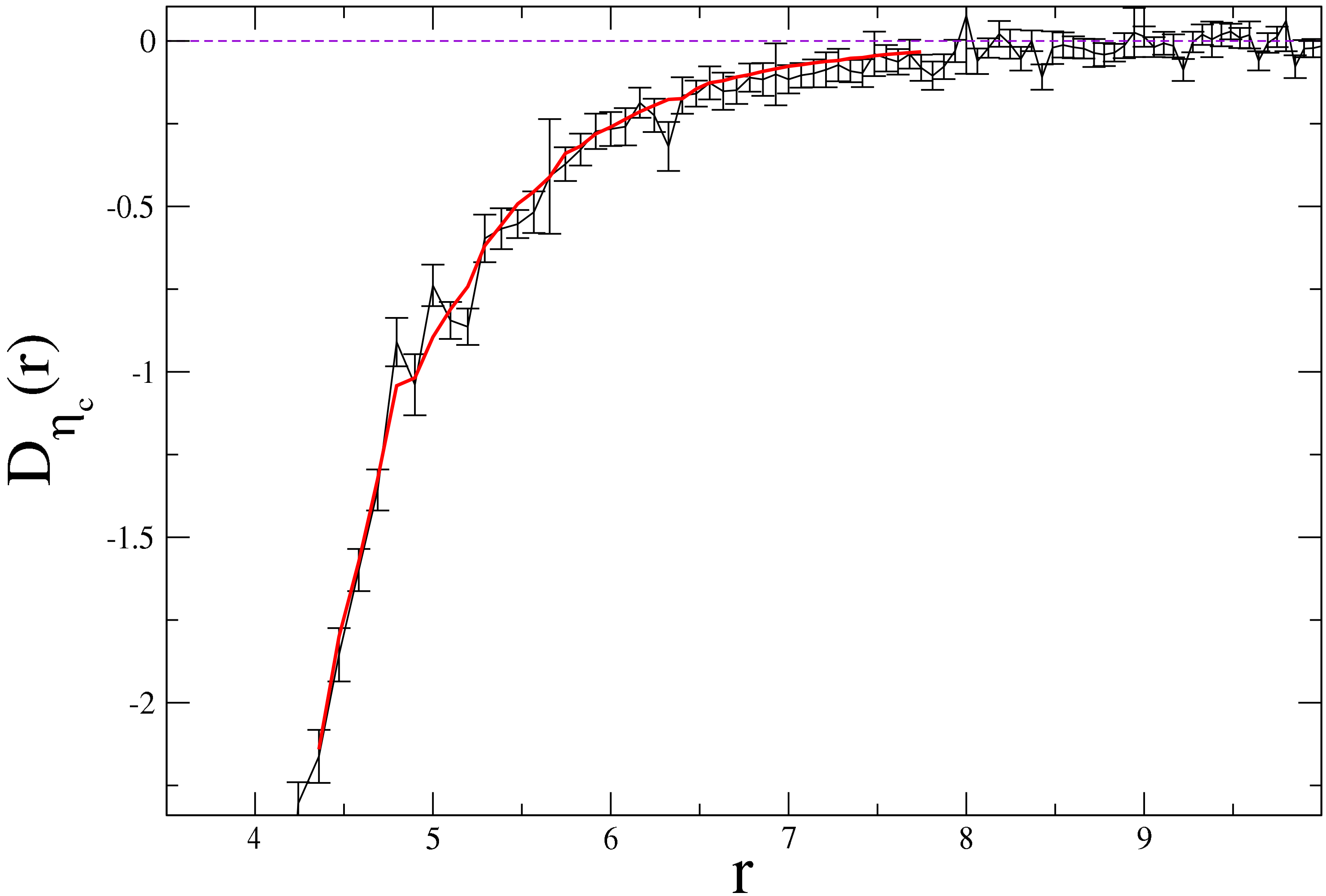
Quenched calculation

- Simplifying the problem: no propagating light modes (if there are no light glueballs)
- Fine lattices: $28^3 \times 96$, $k = 0.127$, 366 configurations, $a \approx 0.09$ fm.
- Superfine lattices: $48^3 \times 144$, $k = 0.130$, 124 configurations, $a \approx 0.063$ fm.
- Are there glue balls in $D_{\eta_c}(r)$? Quenched fine lattices have same lattice spacing as unquenched. Quenched correlator doesn't change sign: small coupling to glueballs



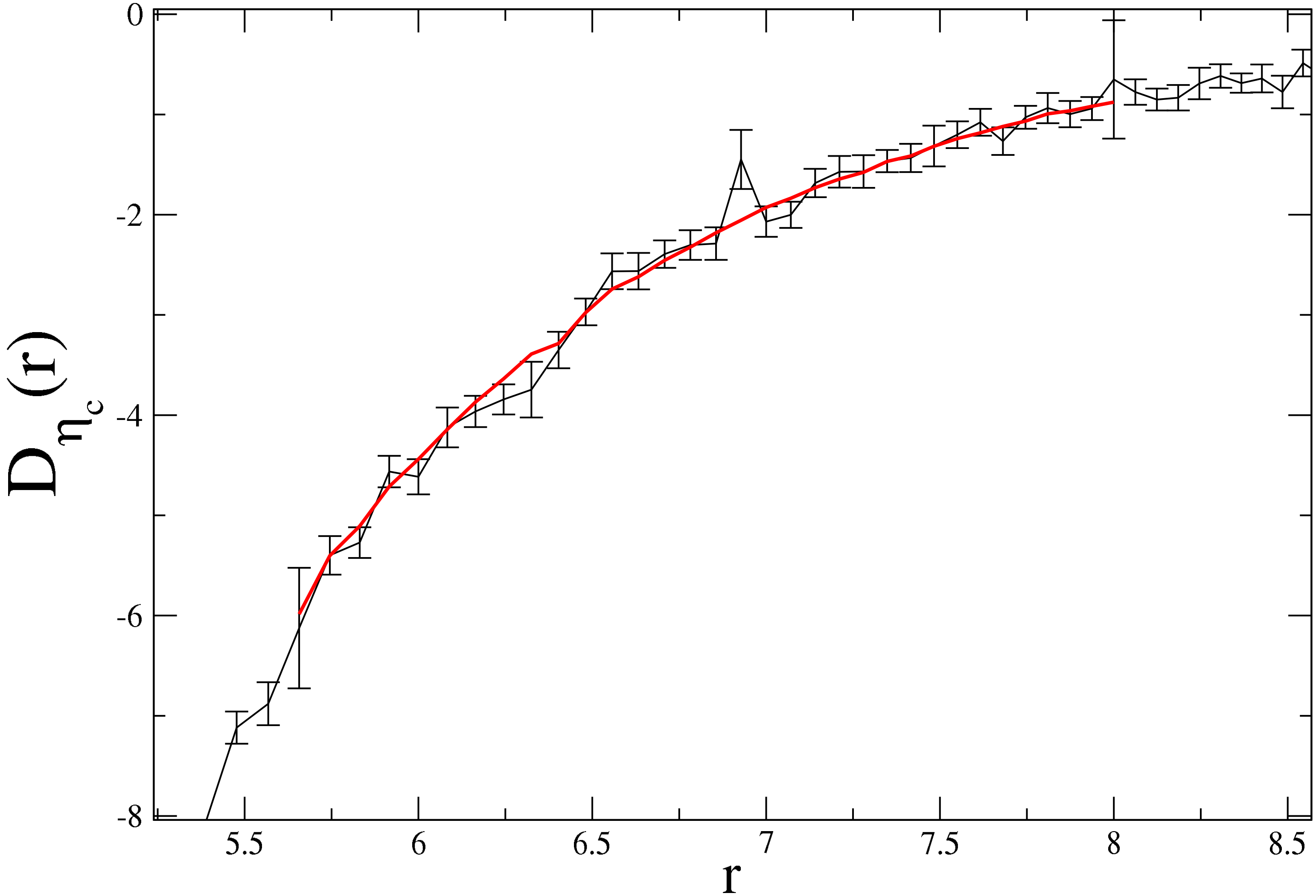
Quenched η_c results on FINE lattices

- The fits to $D(r)$ are done with $m_c = 0.9781$, $m_c^\star = 1.330$. The fitting range is $r = 4.3 - 7.8$, (40 DOF) and the fit has $\chi^2 = 1$. We obtain: $a = 109(15)$, $b = 294(41)$. This means: $m_c - m_f = -3.3(9)\mathrm{MeV}$.



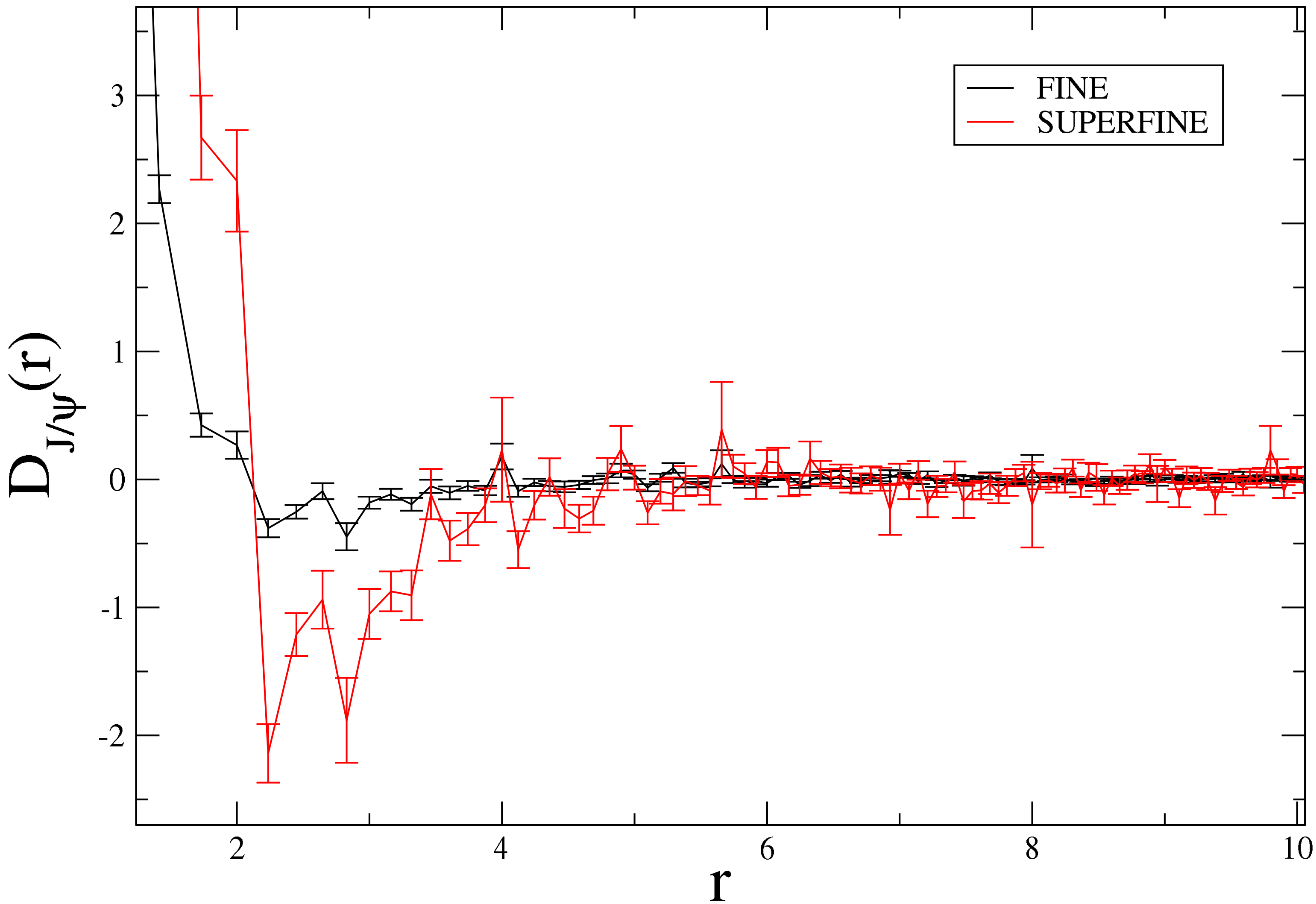
Quenched η_c results on SUPERFINE lattices

- The fits to $D(r)$ are done with $m_c = 0.6509$, $m_c^\star = 0.8606$. The fitting range is $r = 5.6 - 8$, (32 DOF) and the fit has $\chi^2 = 1$. We obtain: $a = 131(17)$, $b = 246(38)$. This means: $m_c - m_f = -3.1(8)\text{MeV}$.



Quenched J/Ψ results on FINE and SUPERFINE lattices

- Estimation: $m_c - m_f < 0$, $|m_c - m_f| < 1$ MeV



Summary and conclusions

- We introduced a new fitting procedure which takes into account rotational symmetry violations. It gives consistent results with our previous fitting method.
- The quenched results for $m_c - m_f$ for the η_c are the same for two lattice spacings 0.09 and 0.06 fm: $-3.3(9)$ and $-3.1(8)$ MeV. This means that the disconnected diagram contributions increase the η_c mass. This conclusion is consistent with the dynamical result estimations.
- We can only estimate that the J/Ψ mass will be increased as well by about 1 MeV. Thus as a whole, the hyperfine splitting is slightly reduced by the disconnected diagrams contributions.
- Our $\eta_c \sim 2000 - 2150$ MeV, physical is 2980 MeV. Will that affect the sign of $m_c - m_f$? We are starting a calculation on superfine lattices at smaller $k = 0.117$.