$\alpha_s(M_Z)$ FROM UV-SENSITIVE LATTICE OBSERVABLES REVISITED

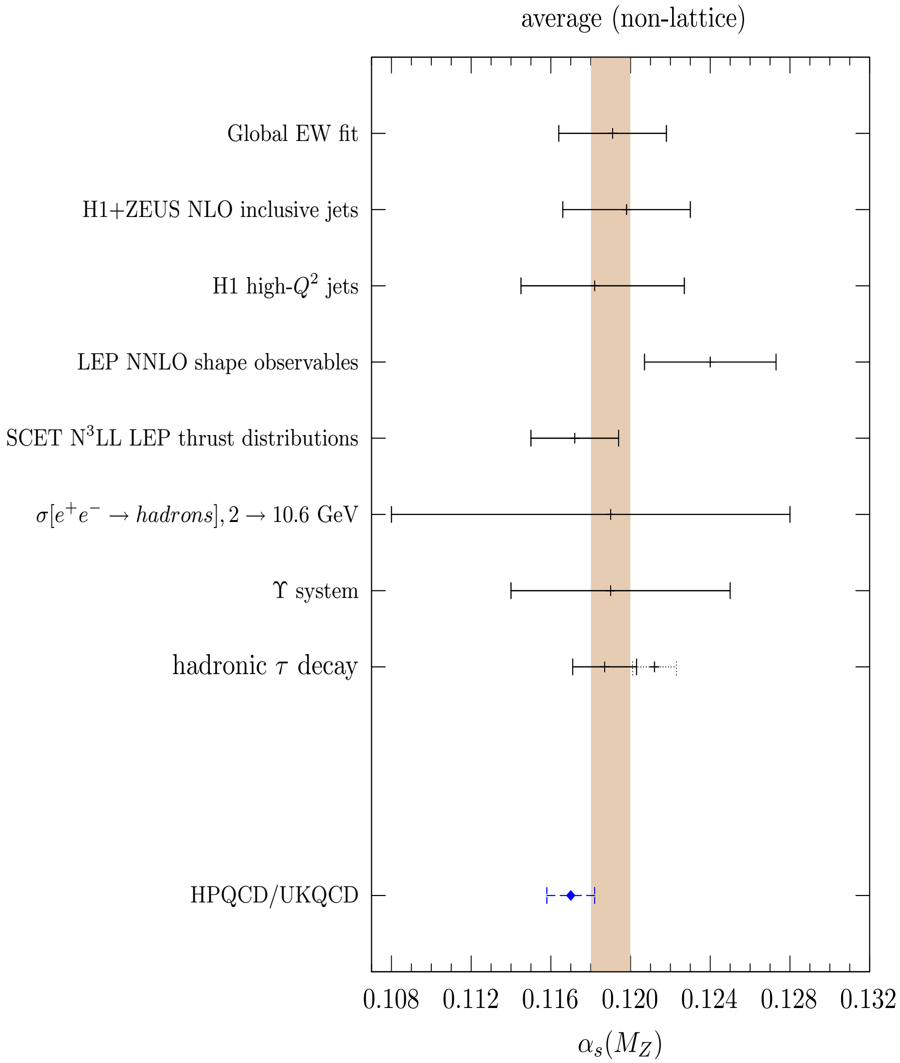
K. Maltman [CSSM, Adelaide and York University]

D. Leinweber, A. Sternbeck, P. Moran [CSSM, Adelaide]

OUTLINE

- *Motivation: other determinations c.f./vs. HPQCD/UKQCD*
- *Some relevant details/issues for HPQCD/UKQCD*
- *Modified analysis and results*

HPQCD/UKQCD c.f. Recent Non-Lattice $\alpha_s(M_Z)$



THE HPQCD/UKQCD ANALYSIS

- Work with
  - UV-sensitive observables $\{O_k\}$ ($log(W_{11})$, $log(W_{12})$, $log(W_{12}/u_0^6)$, $log(W_{13}/W_{22})$ etc.)
  - *DEFINE* $\alpha_V \equiv 3^{rd}$ order $\alpha_s^{\overline{MS}}$ truncation of heavy quark potential coupling [Y. Schroder]

$$\alpha_V(Q) \equiv \alpha_s(Q)\left[1 + a_1\alpha_s(Q) + a_2\alpha_s(Q)^2\right]$$

$\Rightarrow$ $\beta_V(\alpha_V)$ to 4-loops, 4-loop running for α_V

  - Perturbative expansion for O_k

$$O_k = D_k\alpha_{V,k}\left[1 + c_1^{(k)}\alpha_{V,k} + c_2^{(k)}\alpha_{V,k}^2 + c_3^{(k)}\alpha_{V,k}^3 + \cdots\right]$$

* $\alpha_{V,k} \equiv \alpha_V(Q_k)$ ($Q_k = d_k/a$: O_k BLM scale)
* D_k, $c_{1,2}^{(k)}$ from 3-loop LPT [Q. Mason et al.]

– $a \sim 0.09, 0.12, 0.18$ fm MILC $n_f = 2+1$ data

- Find: known $c_{1,2}$ insufficient to fit O_k at all 3 scales

- using data at all 3 scales, 4-loop α_V running, fit $\alpha_V(Q_{ref})$, $c_3(k), \cdots c_{10}^{(k)}$ (priors for the $c_{N>2}$)

$$\Rightarrow \alpha_V^{n_f=3}(7.5 \text{ GeV}) = 0.2082(40)$$

- Running to M_Z:
  - $\alpha_V^{n_f=3}(Q_{ref})$ to chosen $n_f = 3 \to 4$ matching scale ($\bar{m}_c \equiv m_c(m_c) = 1.25$ GeV) with 4-loop V running
  - $\alpha_V^{n_f=3}(\bar{m}_c) \to \alpha_s^{n_f=3}(\bar{m}_c)$ [$V \to \overline{MS}$ conversion scale choice $\mu_{match} = \bar{m}_c$]
  - $n_f = 5$ $\alpha_s(M_Z)$ via $\overline{MS}$ running/matching
    - $n_f = 3 \to 4$ ($4 \to 5$) matching at $\bar{m}_c$ ($\bar{m}_b$)
    - self-consistent combination of 3-loop $n_f = 4, 5$ running/2-loop matching at flavor thresholds yields

$$\alpha_s(M_Z) = 0.1170(12)$$

Some Relevant Numerical Details/Complications

- numerics of the $n_f = 3$ α_V vs α_s expansion:

$$\alpha_V \equiv \alpha_s \left[1 + 0.557\alpha_s + 1.702\alpha_s^2\right]$$

(⇒ much faster running for α_V)

- with $\mu^2 \frac{da}{d\mu^2} \equiv -\sum_{k=0} \beta_k^V a^{k+2}$

k	$\beta_k^{\overline{MS}}$	β_k^V
0	9/4	9/4
1	4	4
2	10.060	33.969
3	47.228	−324.393

- $\Rightarrow$ 4-loop-truncated running breaks down at much higher scales for α_V than for α_s
  - e.g., series for $\mu^2 da_V(\mu^2)/d\mu^2$ at 1.25 GeV

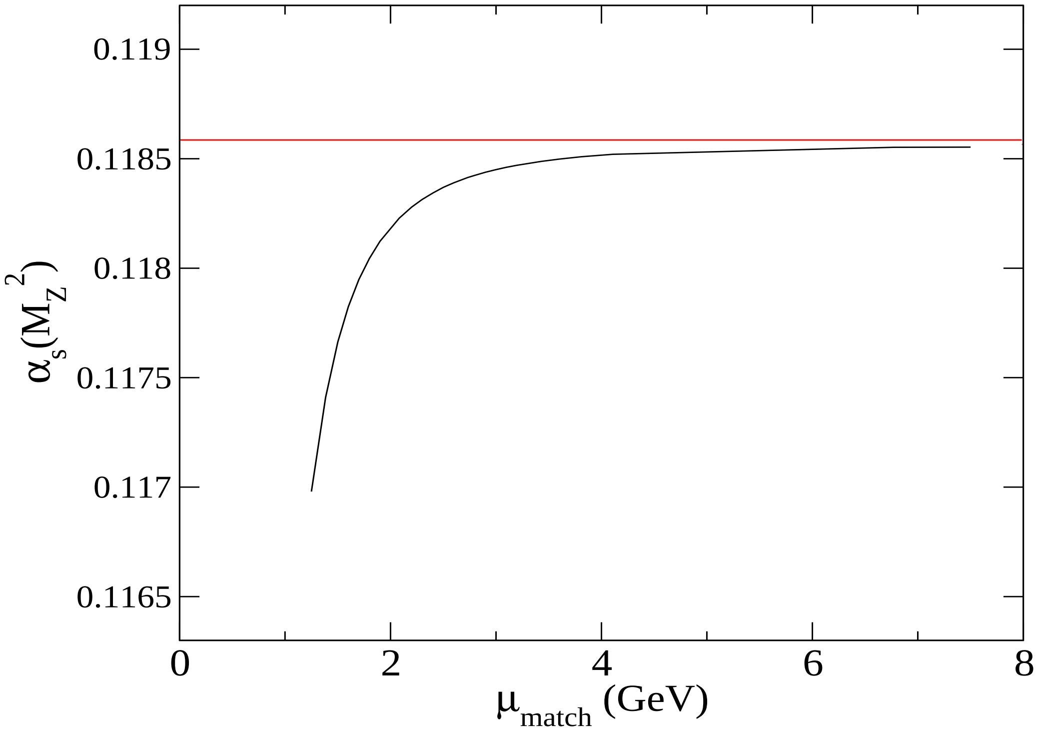
    $\propto \; 1 + 0.3087 + 0.4552 - 0.7549 + \cdots$

    [danger for $\mu = 7.5 \rightarrow 1.25$ GeV $\alpha_V(\mu)$ running]
  - c.f. series for $\mu^2 da_{\overline{MS}}(\mu^2)/d\mu^2$ at 1.25 GeV

    $\propto \; 1 + 0.2128 + 0.0641 + 0.0360$

- impact of μ_{match} choice on $\alpha_s(M_Z)$ [Figure]

Dependence of $\alpha_s(M_Z)$ on $V \to \overline{MS}$ conversion scale



- 4-loop α_V running also a potential problem for fitting $\alpha_V(Q_{ref}) \equiv \alpha_0$, $c_{k>2}$
  - used to run α_0 to $a \sim 0.18$, 0.12, 0.09 fm BLM scales as part of fitting procedure
  - e.g., series for $\frac{-1}{\beta_0 a_V^2}\mu^2\frac{da_V}{d\mu^2}$ at coarsest ($a \sim 0.18$ fm) lattice BLM scales Q_k

Observable	Q_k (GeV)	Series (increasing order)
$log\,(W_{11})$	3.80	$1 + 0.154 + 0.113 - 0.094$
$log\,(W_{12})$	3.43	$1 + 0.162 + 0.125 - 0.108$
$log\left(\frac{W_{12}}{u_0^6}\right)$	2.08	$1 + 0.213 + 0.217 - 0.248$
$log\left(\frac{W_{13}}{W_{22}}\right)$	1.38	$1 + 0.285 + 0.388 - 0.594$

- impact on fitted $c_{k>3}$ of 4-loop-truncated β

  - expand $\alpha_V(Q_k) = \sum_{N=1} c_N(t)\alpha_0^N$ (c_N polynomials in $t = log(Q_k^2/Q_{ref}^2)$, $c_1 = 1$, $\alpha_0 \equiv \alpha_V(Q_k^{max})$)

  - dependence of O_k on unknown $\beta_4^V, \beta_5^V, \cdots, c_3^{(k)}, c_4^{(k)}, \cdots$:

$$\frac{O_k}{D_k} = \cdots \alpha_0^4 \left(c_3^{(k)} \cdots \right) + \alpha_0^5 \left(c_4^{(k)} - 2.87 t c_3^{(k)} \cdots \right)$$
$$+\alpha_0^6 \left(c_5^{(k)} - 0.0033 \beta_4^V t - 3.58 t c_4^{(k)} \right.$$
$$\left. + [5.13 t^2 - 1.62 t] c_3^{(k)} \cdots \right)$$
$$+\alpha_0^7 \left(c_6^{(k)} - 0.0010 \beta_5^V t + [0.0094 t^2 - 0.0065 t c_1^{(k)}] \beta_4^V \right.$$
$$-4.30 t c_5^{(k)} + [7.69 t^2 - 2.03 t] c_4^{(k)}$$
$$\left. + [-7.35 t^3 + 6.39 t^2 - 4.38 t] c_3^{(k)} \cdots \right) + \cdots$$

– Consequences:

* neglect of $\beta_4^V, \beta_5^V, \cdots \Rightarrow$ incorrect scale-dependence without compensating shifts in *at least* $c_4^{(k)}, c_5^{(k)}, \cdots$

* problem minimized by (i) reducing α_0 (choosing O_k with highest maximum BLM scale), and (ii) minimizing maximum t (work with subset of higher-scale lattices)

* $c_3^{(k)}$ also potentially affected since, e.g., compensating shift in $c_4^{(k)}$ at $O(\alpha_0^6)$ no longer fully compensates for missing β_4^V at $O(\alpha_0^7)$ etc.

* $\Rightarrow$ analysis safest for high-scale O_k ($log(W_{11})$), 3-fold (rather than 5-fold) fit

A MODIFIED/UPDATED ANALYSIS

- incorporate new USQCD $a \sim 0.06$ fm (W_{11} and W_{12} only) and MILC $a \sim 0.15$ fm data [plus $a \sim 0.09$, 0.12, 0.18 fm MILC data used by HPQCD/UKQCD]

- 3-loop PT $O(\alpha_V^3)$ expansions as per HPQCD/UKQCD

- all running (between different lattice scales, and to M_Z) through 4-loop-truncated $\overline{MS}$ running

- central fits with $a \sim 0.06, 0.09, 0.12$ fm (same range of relative scales, higher absolute scales c.f. HPQCD/UKQCD)

- extended 5-fold fits including $a \sim 0.15, 0.18$ fm to test consistency, convergence, control of truncation impact

- analyze $log(W_{11})$, $log(W_{12})$ (both "high-scale") and $log(W_{12}/u_0^6)$ ("lower-scale")

- find

  - known terms insufficient for 3-fold fit, despite higher scales

  - $\chi^2/dof < 1$ once $c_3^{(k)}$ included (errors dominated by r_1/a scale uncertainty)

  - $\Rightarrow$ uncertainties do not permit sensible fitting beyond $c_3^{(k)}$

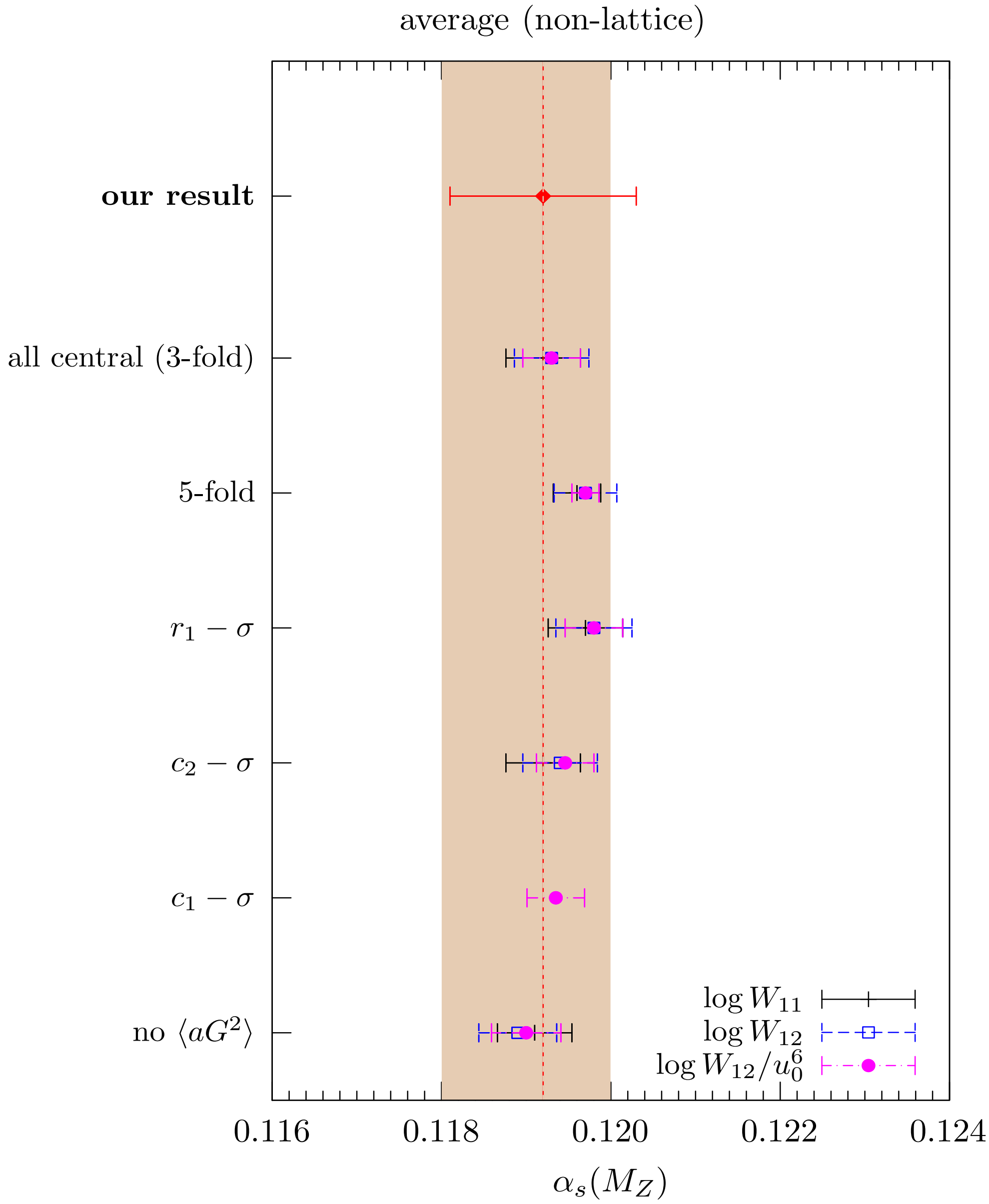
- sources of uncertainty
  - (small) $\delta c_{1,2}^{(k)}$ (numerical integration, 3-loop PT)
  - scale uncertainties (r_1/a) for various am_ℓ/am_s
  - global $r_1 = 0.318(7)$ fm scale uncertainty
  - m_q extrapolation
  - residual NP contribution subtraction (estimated from $\langle aG^2\rangle$ contribution, as per HPQCD/UKQCD, assigned 100% error)
  - any "instability" between the 3-fold and 5-fold fits
  - evolution to M_Z

RESULTS

- $\alpha_s(M_Z)$ values (c.f. HPQCD/UKQCD)

Observable	$\alpha_s(M_Z)$ (our fit)	$\alpha_s(M_Z)$ HPQCD/UKQCD
$log\,(W_{11})$	0.1192(11)	0.1171(12)
$log\,(W_{12})$	0.1193(11)	0.1170(12)
$log\left(\frac{W_{12}}{u_0^6}\right)$	0.1193(10)	0.1162(12)

- error sources
  - $\delta(r_1/a)$, δr_1, δc_2, (δc_1), truncation [Figure]
  - truncation currently only from 3-fold vs 5-fold fit variation (under further investigation)



average (non-lattice)
our result
all central (3-fold)
5-fold
$r_1 - \sigma$
$c_2 - \sigma$
$c_1 - \sigma$
no $\langle aG^2 \rangle$
$\log W_{11}$
$\log W_{12}$
$\log W_{12}/u_0^6$
0.116
0.118
0.120
0.122
0.124
$\alpha_s(M_Z)$

- $c_3^{(k)}$ values (c.f. HPQCD/UKQCD)

Observable	$c_3^{(k)}$ (our fit)	$c_3^{(k)}$ (HPQCD/UKQCD)
$log\,(W_{11})$	−3.8(6)	−5(2)
$log\,(W_{12})$	−4.0(9)	−5(2)
$log\left(\frac{W_{12}}{u_0^6}\right)$	−1.7(8)	−2(1)

- Excellent consistency, similar (so-far-quantified) errors, but safest analysis is one with highest scale ($log(W_{11})$)

$$[\alpha_s(M_Z)]_{lattice} = 0.1192(11)$$

SUMMARY

- high-precision α_s determination using HPQCD/UKQCD approach
- excellent consistency for 3 observables studied
- difficult to improve further without significant improvement in scale determinations [but already most precise of current determinations]
- excellent agreement with independent determinations

- averaging with recent non-lattice determinations yields $\sim 0.6\%$ accuracy determination

$$[\alpha_s(M_Z)]_{ave} = 0.1191(7)$$

for details see arXiv:0807.2020

- Thanks to D. Toussaint and C. De Tar

NOTE ADDED (re Friday HPQCD arXiv:0807.1687)

- re-analysis of 2005 work with 5-fold fits

- average over observables: $\alpha_V(7.5\ \mathrm{GeV}) = 0.2121(25)$

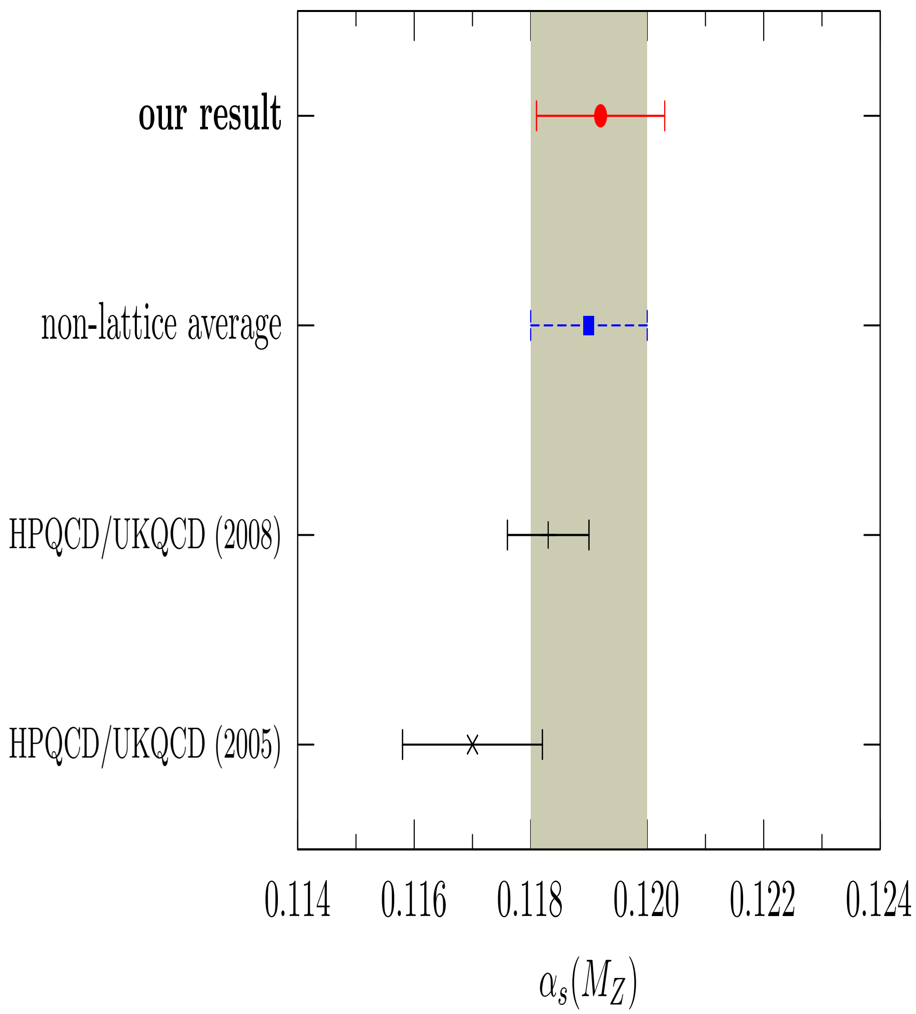
- comparison of $\alpha_V(7.5\ \mathrm{GeV})$ results

Source	3-fold fit	5-fold fit
$log\,(W_{11})$	0.2106	0.2121
$log\,(W_{12})$	0.2112	0.2125
$log\left(\frac{W_{12}}{u_0^6}\right)$	0.2112	0.2124
HPQCD08 (ave)	—	0.2121

- insufficient details on running/matching/conversion to comment/compare in detail BUT if
  - take $\mu_{conv} = 7.5$ GeV [$\Rightarrow \alpha_s(7.5 \text{ GeV}) = 0.1830(18)$]
  - use self-consistent 4-loop running+3-loop matching
  - match at $rm_c(m_c)$, $rm_b(m_b)$ with $m_c(m_c) = 1.286(13)$ GeV, $m_b(m_b) = 4.164(25)$ GeV [Kuhn, Steinhauser, Sturm 2007], $r = 2(1)$

$$\alpha_s(M_Z) = 0.1196(8)(3)_{evol}$$

- final comparison nonetheless



our result
non-lattice average
HPQCD/UKQCD (2008)
HPQCD/UKQCD (2005)
0.114
0.116
0.118
0.120
0.122
0.124
$\alpha_s(M_Z)$