Non-Commutative Product Formulation of ***Exact Lattice SUSY at Large N***

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Based on “Exact Lattice Supersymmetry at Large N”
arXiv:0805.4235 [hep-lat]

Introduction

SUSY on a Lattice

a challenging subject..

- G. Bergner, F. Bruckmann and J. M.Pawlowski, arXiv:0807.1110 [hep-lat].
- S. Arianos, A. D'Adda, A. Feo, N. Kawamoto and J. Saito, arXiv:0806.0686.
- J. W. Elliott, J. Giedt and G. D. Moore, arXiv:0806.0013 [hep-lat].
- M. Kato, M. Sakamoto and H. So, JHEP 0805 (2008) 057.
- S. Catterall and A. Joseph, Phys. Rev. D 77, 094504 (2008).
- S. Catterall, JHEP 0801 (2008) 048.
- I. Kanamori, F. Sugino and H. Suzuki, Prog. Theor. Phys. 119 (2008) 797; Phys.Rev.D77:091502,2008.
- S. Matsuura, JHEP 0712 (2007) 048; arXiv:0805.4491.
- P. H. Damgaard and S. Matsuura, JHEP 0709 (2007) 097; Phys. Lett. B661 52-56,2008.

⋮

Motivations

- Constructive formulation of SUSY models
- Non-perturbative dynamics
- Fermions v.s. Regularization
- AdS/CFT, etc...

Obstacles

- Lattice symm. group < Cont. symm. group
- Lattice Leibniz rule problem : $\{Q_A, Q_B\} \sim \Delta_{\pm\mu}$

$$\Delta_{\pm\mu} f(x)g(x) = (\Delta_{\pm\mu} f(x))g(x) + f(x \pm n_\mu)(\Delta_{\pm\mu} g(x))$$

- Fermion doubling, etc...

Previous works

- A. D'Adda, I. Kanamori, N. Kawamoto, KN, Nucl. Phys. B 707 (2005) 100-144;
  Phys. Lett. B 633 (2006) 645-652; Nucl. Phys. B 798 (2008) 168-183.
- KN, JHEP 0801 (2008) 041.

Lattice Leibniz rule problem

Fermion doubling

(Dirac-Kähler) Twisted SUSY

$$\begin{cases} \mathcal{N}=2\ D=2 & \text{(DKKN, 2005, 2006)} \\ \mathcal{N}=4\ D=3 & \text{(DKKN, 2008, KN 2008)} \\ \mathcal{N}=4\ D=4 & \text{(DKKN, 2006)} \end{cases}$$

can preserve Lattice Leibniz rule

Staggered Fermions

$$\begin{cases} N_f=2\ D=2 \\ N_f=4\ D=3 \\ N_f=4\ D=4 \end{cases}$$

“Mild” Non-commutativity
in Superspace :

$$[x_\mu, \theta_A] = 2(a_A)_\mu \theta_A$$

where $a_A \sim \mathcal{O}(lat.\ const.)$

Manifestly SUSY invariant models (Wess-Zumino, SYM)
w.r.t. all the supercharges at tree level

What about the quantum corrections ?

In this talk

(KN, arXiv:0805.4235 [hep-lat])

(1) A novel “star” product honestly representing the “mild” NC :

$$[x_\mu, \theta_A]_* \equiv x_\mu * \theta_A - \theta_A * x_\mu = 2(a_A)_\mu \theta_A$$

$$\Phi_1(x, \theta_A) * \Phi_2(x, \theta_B) \neq \Phi_2(x, \theta_B) * \Phi_1(x, \theta_A)$$

(2) Perturbative corrections for mass and coupling const. in

global $U(N)$ Wess-Zumino model

with $\mathcal{N} = 2 \; D = 2$ Twisted SUSY

- ***Planar diagrams*** strictly respect exact Lattice SUSY
  Non-planar diagrams spoil Lattice SUSY $\sim \mathcal{O}(a^2/N)$

- Exact Lattice SUSY realization w.r.t. all the supercharges
  in the 't Hooft large-N limit

Introduce a "star" product

Superspace : (x_μ, θ_A) $\begin{matrix} \mu = 1 \sim D \\ A = 1 \sim r \end{matrix}$ superfields : $\Phi_1(x,\theta_A), \Phi_2(x,\theta_B), \ldots$

$$\Phi_1(x,\theta_A) * \Phi_2(x,\theta_B) \equiv \mu(\mathcal{F}_L^{-1}\mathcal{F}_R \, \Phi_1(x,\theta_A) \otimes \Phi_2(x,\theta_B))$$

- Twist elements : $\begin{cases} \mathcal{F}_L = e^{\sum_\rho \sum_A (a_A)_\rho \theta_A \frac{\partial}{\partial\theta_A} \otimes \frac{\partial}{\partial x_\rho}} \\ \mathcal{F}_R = e^{\sum_\rho \sum_A \frac{\partial}{\partial x_\rho} \otimes (a_A)_\rho \theta_A \frac{\partial}{\partial\theta_A}} \end{cases}$
- Multiplication map : $\mu(f \otimes g) = fg$
- Associative : $(\Phi_1 * \Phi_2) * \Phi_3 = \Phi_1 * (\Phi_2 * \Phi_3)$

satisfies :

$$\theta_A * f(x) = \theta_A f(x - a_A)$$
$$f(x) * \theta_A = f(x + a_A)\theta_A$$

$\Downarrow$ $f(x) = x$

$$[x_\mu, \theta_A]_* = 2(a_A)_\mu \theta_A$$

"Mild" non-comm. relation

For derivative :

$$[x_\mu, \frac{\partial}{\partial\theta_A}]_* = -2(a_A)_\mu \frac{\partial}{\partial\theta_A}$$

Grassmann parameters : ξ_A

$$\theta_A \frac{\partial}{\partial\theta_A} \Rightarrow \theta_A \frac{\partial}{\partial\theta_A} + \xi_A \frac{\partial}{\partial\xi_A}$$

$$[x_\mu, \xi_A]_* = 2(a_A)_\mu \xi_A$$

Lattice SUSY Algebra & transformation

$$\boxed{\{Q_A, Q_B\} = d_\mu}$$

$Q_{A,B}$: supercharges

d_μ : "formal" difference operator

$$\boxed{\delta_A \Phi(x,\theta_C) \equiv [\xi_A Q_A, \Phi(x,\theta_C)]_*}$$

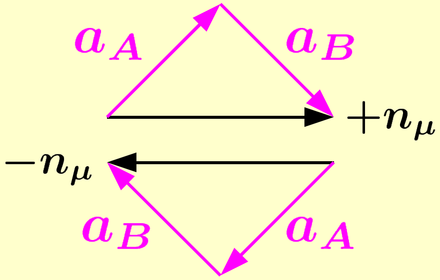
$$\begin{aligned}[\delta_A, \delta_B]\Phi(x,\theta_C) &= -\xi_A \xi_B d_\mu [\Phi(x - a_A - a_B, \theta_C) \\ &\qquad - \Phi(x + a_A + a_B, \theta_C)]\end{aligned}$$

cont. limit

$$\Rightarrow -\xi_A \xi_B \partial_\mu \Phi(x,\theta_C)$$

Lattice Leibniz rule conditions

$$\boxed{\begin{array}{lll} a_A + a_B = +n_\mu & \textit{and} & d_\mu = -\frac{1}{2} \\ a_A + a_B = -n_\mu & \textit{and} & d_\mu = +\frac{1}{2} \end{array}}$$



satisfied for Dirac-Kähler Twisted Algebra of

$$\left\{\begin{array}{l} \mathcal{N} = 2 \ D = 2 \\ \mathcal{N} = 4 \ D = 3 \\ \mathcal{N} = 4 \ D = 4 \end{array}\right.$$

(DKKN 2005, 2006)

(DKKN 2008, KN 2008)

(DKKN 2006)

$\vdots$

N=D=2 Twisted SUSY Algebra on Lattice

$$\{Q_{\alpha i}, Q_{\beta j}\} = 2i\delta_{ij}(\gamma_\mu)_{\alpha\beta}\partial_\mu$$

- $\mu = 1, 2$: $2D$ Euclidean
- α, β : spinor indices
- i, j : internal indices
- $\gamma_1 = \sigma_3, \gamma_2 = \sigma_1, \gamma_5 = \gamma_1\gamma_2$

Dirac-Kähler expansion

$$Q_{\alpha i} = (1Q + \gamma_\mu Q_\mu + \gamma_5 \tilde{Q})_{\alpha i}$$

$$\{Q, Q_\mu\} = i\partial_\mu, \quad \{\tilde{Q}, Q_\mu\} = -i\epsilon_{\mu\nu}\partial_\nu$$

Lattice

$$\{Q, Q_\mu\} = id^+_\mu, \quad \{\tilde{Q}, Q_\mu\} = -i\epsilon_{\mu\nu}d^-_\nu$$

$$a + a_\mu = +n_\mu \qquad \tilde{a} + a_\mu = -|\epsilon_{\mu\nu}|n_\nu$$

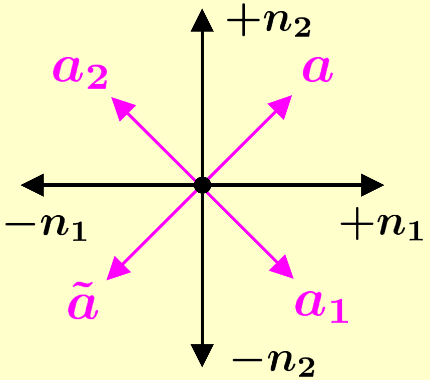
$$d^+_\mu = -\frac{1}{2} \qquad d^-_\mu = +\frac{1}{2}$$

$$Q = \frac{\partial}{\partial\theta} + \frac{i}{2}\theta_\mu d^+_\mu$$

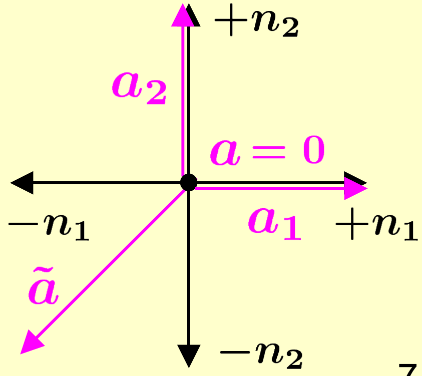
$$Q_\mu = \frac{\partial}{\partial\theta_\mu} + \frac{i}{2}\theta d^+_\mu - \frac{i}{2}\tilde{\theta}\epsilon_{\mu\nu}d^-_\nu$$

$$\tilde{Q} = \frac{\partial}{\partial\tilde{\theta}} - \frac{i}{2}\theta_\mu\epsilon_{\mu\nu}d^-_\nu$$

Symm. choice



Asymm. choice



N=D=2 Twisted SUSY inv. action at tree level

$$S=\sum_x\Big[\int d^4\theta K_*(\overline{\Phi},\Phi)+\int d^2\theta F_*(\Phi)+\int d^2\overline{\theta}\,\overline{F}_*(\overline{\Phi})\Big]$$

$$d^4\theta=d\theta d\tilde{\theta}d\theta_1 d\theta_2,\quad d^2\theta=d\theta_2 d\theta_1,\quad d^2\overline{\theta}=d\tilde{\theta}d\theta$$

Chiral superfield : $[\xi D,\Phi]_*=[\tilde{\xi}\tilde{D},\Phi]_*=0,$

Anti-chiral superfield : $[\xi_\mu D_\mu,\overline{\Phi}]_*=0,\quad (\mu : \text{no sum})$

$$K_*=\mathrm{tr}\,\overline{\Phi}*\Phi \qquad \Phi,\overline{\Phi}\in u(N)$$

$$F_*=\mathrm{tr}[\tfrac{1}{2}m\Phi*\Phi+\tfrac{g}{3}\Phi*\Phi*\Phi]$$

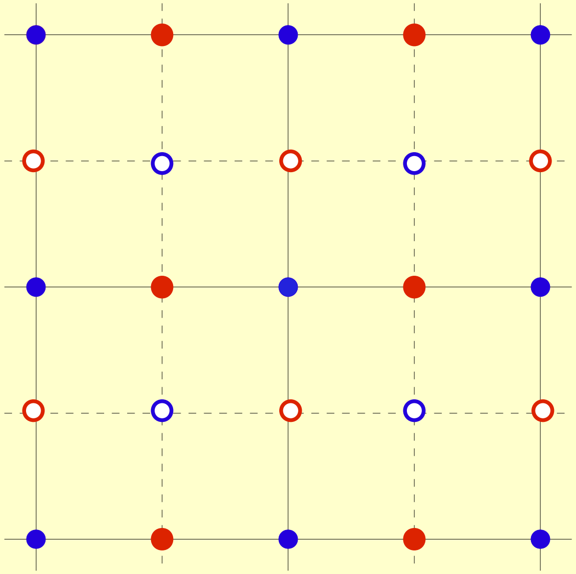
$$\overline{F}_*=\mathrm{tr}[\tfrac{1}{2}m\overline{\Phi}*\overline{\Phi}+\tfrac{g}{3}\overline{\Phi}*\overline{\Phi}*\overline{\Phi}]$$

For symm. choice :

$$\sum_x=\sum_{\bullet\,(m_1,m_2)}+\sum_{\circ\,(m_1+\frac{1}{2},m_2+\frac{1}{2})}+\sum_{\bullet\,(m_1+\frac{1}{2},m_2)}+\sum_{\circ\,(m_1,m_2+\frac{1}{2})}$$

SUSY trans.

SUSY trans.



Superfield Propagators

$$\underset{(x^{(1)},\theta^{(1)}_A)}{\Phi'^{j_1}_{i_1}} \dashrightarrow \underset{(x^{(2)},\theta^{(2)}_A)}{\Phi'^{j_2}_{i_2}} = \delta^{j_2}_{i_1}\delta^{j_1}_{i_2}\int_{-2\pi}^{2\pi}\frac{d^2p}{(4\pi)^2}e^{ip(x^{(1)}-x^{(2)})}\frac{-m}{\sin^2 p_\mu + m^2}\delta^2(\theta^{(1)}-\theta^{(2)})$$

$$\underset{(x^{(1)},\theta^{(1)}_A)}{\Phi'^{j_1}_{i_1}} \dashrightarrow \underset{(x^{(2)},\overline{\theta}^{(2)}_A)}{\overline{\Phi}'^{j_2}_{i_2}} = \delta^{j_2}_{i_1}\delta^{j_1}_{i_2}\int_{-2\pi}^{2\pi}\frac{d^2p}{(4\pi)^2}e^{ip(x^{(1)}-x^{(2)})}\frac{e^{+E^{(21)}_\mu}\sin p_\mu}{\sin^2 p_\mu + m^2}$$

$$\Phi' \equiv U * \Phi * U^{-1}, \qquad \theta_A = (\theta_1, \theta_2) \qquad E^{(ij)}_\mu \equiv \theta^{(i)}\theta^{(j)}_\mu + \epsilon_{\mu\nu}\tilde{\theta}^{(i)}\theta^{(j)}_\nu$$

$$\overline{\Phi}' \equiv U^{-1} * \overline{\Phi} * U, \qquad \overline{\theta}_A = (\theta, \tilde{\theta})$$

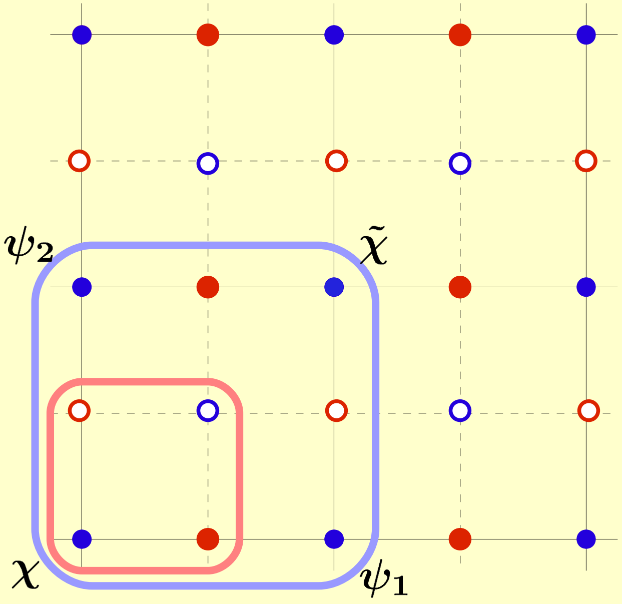
$$U \equiv e_*^{-\frac{i}{2}(\theta\theta_\mu d^+_\mu - \epsilon_{\mu\nu}\tilde{\theta}\theta_\mu d^-_\nu)}$$

Spectrum doubling in two-fold

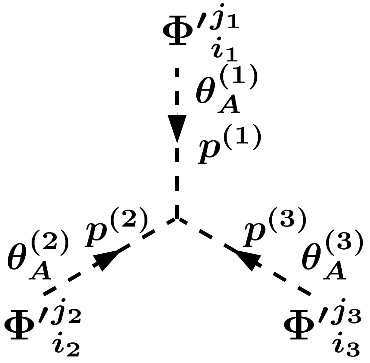
$$\int_{-2\pi}^{2\pi}\frac{d^2p}{(4\pi)^2} = 4(\text{copies}) \times \int_{-\pi}^{\pi}\frac{d^2p}{(4\pi)^2}$$

$$= \underline{4(\text{copies})} \times \underset{\substack{\downarrow \\ 2(\text{spinor}) \times 2(\mathcal{N}=2)}}{\underline{4(\text{Dirac-Kähler})}} \times \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{d^2p}{(4\pi)^2}$$

$$\Psi_{\alpha i}(x) = (1\chi(x) + \gamma_\mu\psi_\mu(x+n_\mu) + \gamma_5\tilde{\chi}(x+n_1+n_2))_{\alpha i},$$



Vertex functions at tree level



$$\delta^2(p^{(1)}+p^{(2)}+p^{(3)})$$
$$\times\frac{g}{2}\Big[\delta^{j_3}_{i_1}\delta^{j_1}_{i_2}\delta^{j_2}_{i_3}\prod_A(\theta_A^{(1)p^{(3)}}-\theta_A^{(2)})(\theta_A^{(1)-p^{(2)}}-\theta_A^{(3)})$$
$$+\delta^{j_2}_{i_1}\delta^{j_1}_{i_3}\delta^{j_3}_{i_2}\prod_A(\theta_A^{(1)-p^{(3)}}-\theta_A^{(2)})(\theta_A^{(1)p^{(2)}}-\theta_A^{(3)})\Big]$$

$$\theta_A^{(k)p^{(l)}}\equiv\theta_A^{(k)}e^{i\sum_\mu(a_A)_\mu p_\mu^{(l)}}$$

$$[x_\mu,\theta_A]_*=2(a_A)_\mu\theta_A$$
$$a_A\sim\mathcal{O}(lat.const.)$$

Note : $\Phi_1*\Phi_2\neq\Phi_2*\Phi_1$

⇩

$$\left.\begin{array}{l}\bullet\ \prod_A(\theta_A^{(1)p^{(3)}}-\theta_A^{(2)})(\theta_A^{(1)-p^{(2)}}-\theta_A^{(3)})\\ \bullet\ \prod_A(\theta_A^{(1)-p^{(3)}}-\theta_A^{(2)})(\theta_A^{(1)p^{(2)}}-\theta_A^{(3)})\end{array}\right\}\xrightarrow{\text{cont. limit}}\ \Rightarrow\ \delta^2(\theta^{(1)}-\theta^{(2)})\delta^2(\theta^{(1)}-\theta^{(3)})$$

On the lattice

Planar diagrams $\neq$ ***Non-planar diagrams***

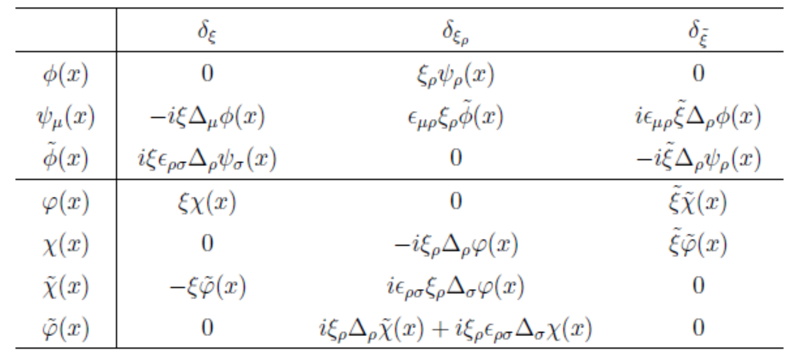
Issue of “proper” ordering v.s. Superfields

- F. Bruckmann, M. de Kok, Phys.Rev.D 73, 074511 (2006)
- F. Bruckmann, S. Catterall, M. de Kok, Phys.Rev.D 75, 045016 (2007)
- A. D'Adda, I. Kanamori, N. Kawamoto, KN, Nucl. Phys. B 798 (2008) 168-183
- S. Arianos, A. D'Adda, N. Kawamoto, J. Saito, PoS LATTICE2007:259,2007.
- S. Arianos, A. D'Adda, A. Feo, N. Kawamoto, J. Saito, arXiv:0806.0686 [hep-lat]

$$\Phi_1 = \phi_1(x) + \theta_A \psi_1(x) + \ldots$$

$$\Phi_2 = \phi_2(x) + \theta_A \psi_2(x) + \ldots$$

$$\Phi_1 * \Phi_2 \neq \Phi_2 * \Phi_1$$



	δ_ξ	δ_{ξ_ρ}	$\delta_{\tilde{\xi}}$
$\phi(x)$	0	$\xi_\rho \psi_\rho(x)$	0
$\psi_\mu(x)$	$-i\xi\Delta_\mu\phi(x)$	$\epsilon_{\mu\rho}\xi_\rho\tilde{\phi}(x)$	$i\epsilon_{\mu\rho}\tilde{\xi}\Delta_\rho\phi(x)$
$\tilde{\phi}(x)$	$i\xi\epsilon_{\rho\sigma}\Delta_\rho\psi_\sigma(x)$	0	$-i\tilde{\xi}\Delta_\rho\psi_\rho(x)$
$\varphi(x)$	$\xi\chi(x)$	0	$\tilde{\xi}\tilde{\chi}(x)$
$\chi(x)$	0	$-i\xi_\rho\Delta_\rho\varphi(x)$	$\tilde{\xi}\tilde{\varphi}(x)$
$\tilde{\chi}(x)$	$-\xi\tilde{\varphi}(x)$	$i\epsilon_{\rho\sigma}\xi_\rho\Delta_\sigma\varphi(x)$	0
$\tilde{\varphi}(x)$	0	$i\xi_\rho\Delta_\rho\tilde{\chi}(x) + i\xi_\rho\epsilon_{\rho\sigma}\Delta_\sigma\chi(x)$	0

$$\delta_A(\phi_1(x)\phi_2(x)) \neq \delta_A(\phi_2(x)\phi_1(x))$$

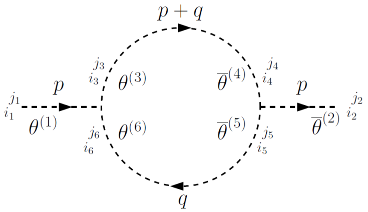
Component fields should be “properly” ordered.

Nevertheless :

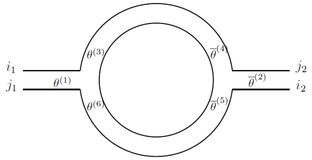
(⇨ Large-N feature of this formulation)

- ***Superfields automatically accommodate the “proper” ordering.***
- ***No ordering ambiguity in superfield calculations.***

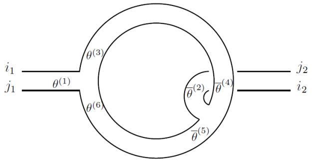
One-loop correction to $< \Phi' \overline{\Phi}' >$



$$\begin{aligned}
\sim\ & g^2 \sum_{i_3\sim i_6} \sum_{j_3\sim j_6} \int dq \int d^2\theta^{(3)} d^2\overline{\theta}^{(4)} d^2\overline{\theta}^{(5)} d^2\theta^{(6)} \\
& \times \frac{1}{2}\left[\delta^{j_3}_{i_1}\,\delta^{j_1}_{i_6}\,\delta^{j_6}_{i_3}\ \prod_A (\theta_A^{(1)-(p+q)} - \theta_A^{(6)})(\theta_A^{(1)-q} - \theta_A^{(3)}) +\ \delta^{j_6}_{i_1}\,\delta^{j_1}_{i_3}\,\delta^{j_3}_{i_6}\ \prod_A (\theta_A^{(1)p+q} - \theta_A^{(6)})(\theta_A^{(1)q} - \theta_A^{(3)})\right] \\
& \times \frac{1}{2}\left[\delta^{j_5}_{i_2}\,\delta^{j_2}_{i_4}\,\delta^{j_4}_{i_5}\ \prod_B (\overline{\theta}_B^{(2)-q} - \overline{\theta}_B^{(4)})(\overline{\theta}_B^{(2)-(p+q)} - \overline{\theta}_B^{(5)}) +\ \delta^{j_4}_{i_2}\,\delta^{j_2}_{i_5}\,\delta^{j_5}_{i_4}\ \prod_B (\overline{\theta}_B^{(2)q} - \overline{\theta}_B^{(4)})(\overline{\theta}_B^{(2)p+q} - \overline{\theta}_B^{(5)})\right] \\
& \times \delta^{j_4}_{i_3}\,\delta^{j_3}_{i_4}\,\frac{e^{+E_\mu^{(43)}\sin(p+q)_\mu}}{\sin^2(p+q)_\mu + m^2}\ \delta^{j_6}_{i_5}\,\delta^{j_5}_{i_6}\,\frac{e^{-E_\mu^{(56)}\sin q_\mu}}{\sin^2 q_\mu + m^2} \qquad (E_\mu^{(ij)} \equiv \theta^{(i)}\theta_\mu^{(j)} + \epsilon_{\mu\nu}\tilde{\theta}^{(i)}\theta_\nu^{(j)}\ )
\end{aligned}$$



$$= \frac{g^2 N}{2}\delta^{j_2}_{i_1}\delta^{j_1}_{i_2}\ \underbrace{e^{+E_\mu^{(21)}\sin p_\mu}}_{\textbf{Exact SUSY}}\ \underbrace{\int_{-2\pi}^{2\pi}\frac{d^2q}{(4\pi)^2}\frac{1}{\sin^2(p+q)_\mu + m^2}\frac{1}{\sin^2 q_\mu + m^2}}_{\text{wave function renormalization}}.$$

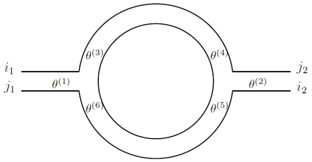


$$= \frac{g^2}{2}\delta^{j_1}_{i_1}\delta^{j_2}_{i_2} e^{+E_\mu^{(21)}\sin p_\mu}\int_{-2\pi}^{2\pi}\frac{d^2q}{(4\pi)^2}\frac{1}{\sin^2(p+q)_\mu + m^2}\frac{1}{\sin^2 q_\mu + m^2} + \mathcal{O}(a^2)$$

SUSY spoiling contribution suppressed by $\mathcal{O}(\frac{a^2}{N})$

in the 't Hooft large-N limit : $N \to \infty, \quad \lambda \equiv g^2 N\ :\ \text{fixed}$

One-loop correction to $< \Phi'\Phi' >, < \Phi'^3 >$

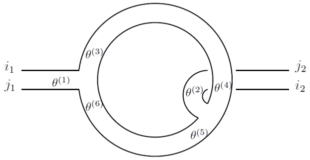


$$\propto \; g^2 N \Big[\prod_A (\theta_A^{(1)-(p+q)} - \theta_A^{(2)-(p+q)})(\theta_A^{(1)-q} - \theta_A^{(2)-q}) + \prod_A (\theta_A^{(1)p+q} - \theta_A^{(2)p+q})(\theta_A^{(1)q} - \theta_A^{(2)q}) \Big]$$

$$= \; g^2 N \Big[\prod_A (\theta_A^{(1)} - \theta_A^{(2)})(\theta_A^{(1)} - \theta_A^{(2)}) + \prod_A (\theta_A^{(1)} - \theta_A^{(2)})(\theta_A^{(1)} - \theta_A^{(2)}) \Big]$$

$$= 0$$

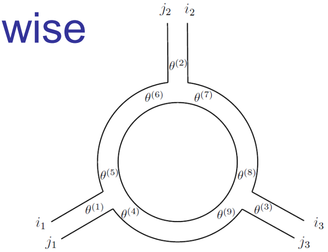
Exact SUSY



$$\propto \; g^2 \Big[\prod_A (\theta_A^{(1)-(p+q)} - \theta_A^{(2)p+q})(\theta_A^{(1)-q} - \theta_A^{(2)q}) + \prod_A (\theta_A^{(1)p+q} - \theta_A^{(2)-(p+q)})(\theta_A^{(1)q} - \theta_A^{(2)-q}) \Big]$$

$$= \; 8\, g^2 \theta_1^{(1)} \theta_1^{(2)} \theta_2^{(1)} \theta_2^{(2)} \sin(a_1 \cdot p)\, \sin(a_2 \cdot p) \; \sim \; \mathcal{O}(a^2/N),$$

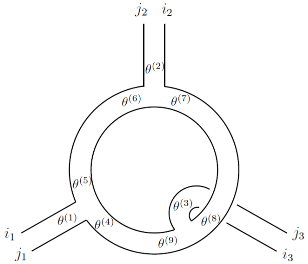
Likewise



$$= 0$$

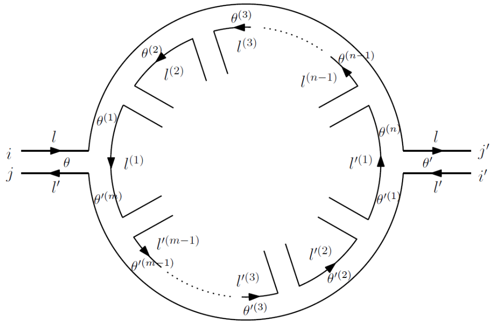
Exact SUSY

Mass and coupling const. are not renormalized in the planar diagrams



$$+ \text{ cyclic permutation of } (i_1, j_1), (i_2, j_2), (i_3, j_3) \qquad \sim \mathcal{O}(a^2/N^{\frac{3}{2}})$$

Any-loop correction to $< \Phi'\Phi' >, < \Phi'^3 >$



$$\propto \int d^2\theta^{(1)} d^2\theta^{(2)} \cdots d^2\theta^{(n)}\; d^2\theta'^{(1)} d^2\theta'^{(2)} \cdots d^2\theta'^{(m)}$$

$$\times \Bigg[\prod_A (\theta_A^{l^{(1)}} - \theta_A^{(1)l'})(\theta_A^{(1)l^{(2)}} - \theta_A^{(2)l^{(1)}})(\theta_A^{(2)l^{(3)}} - \theta_A^{(3)l^{(2)}}) \times \cdots \times (\theta_A^{(n-1)l'^{(1)}} - \theta_A^{(n)l^{(n-1)}})(\theta_A^{(n)l'} - \theta'^{l'^{(1)}}_A)$$

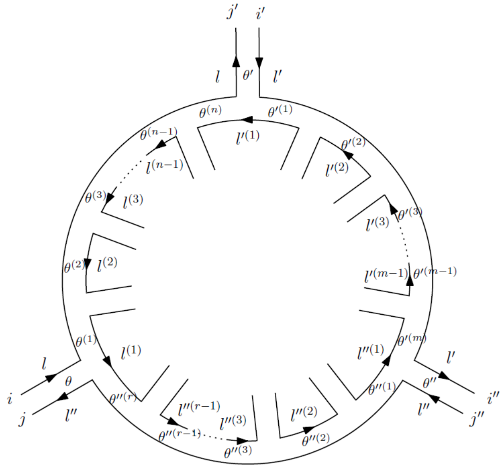
$$\times \prod_B (\theta'^{l'^{(1)}}_B - \theta'^{(1)l}_B)(\theta'^{(1)l'^{(2)}}_B - \theta'^{(2)l'^{(1)}}_B)(\theta'^{(2)l'^{(3)}}_B - \theta'^{(3)l'^{(2)}}_B) \times \cdots \times (\theta'^{(m-1)l^{(1)}}_B - \theta'^{(m)l'^{(m-1)}}_B)(\theta'^{(m)l}_B - \theta_B^{l^{(1)}})$$

$$+ (\text{the term with the opposite phase}) \Bigg]$$

$$= 2 \prod_A (\theta_A - \theta'_A) \prod_B (\theta_B - \theta'_B)$$

$$= 0$$

Exact SUSY



$$\propto \int d^2\theta^{(1)} d^2\theta^{(2)} \cdots d^2\theta^{(n)}\; d^2\theta'^{(1)} d^2\theta'^{(2)} \cdots d^2\theta'^{(m)}\; d^2\theta''^{(1)} d^2\theta''^{(2)} \cdots d^2\theta''^{(r)}$$

$$\times \Bigg[\prod_A (\theta_A^{l^{(1)}} - \theta_A^{(1)l''})(\theta_A^{(1)l^{(2)}} - \theta_A^{(2)l^{(1)}})(\theta_A^{(2)l^{(3)}} - \theta_A^{(3)l^{(2)}}) \times \cdots \times (\theta_A^{(n-1)l'^{(1)}} - \theta_A^{(n)l^{(n-1)}})(\theta_A^{(n)l'} - \theta'^{l'^{(1)}}_A)$$

$$\times \prod_B (\theta'^{l'^{(1)}}_B - \theta'^{(1)l}_B)(\theta'^{(1)l'^{(2)}}_B - \theta'^{(2)l'^{(1)}}_B)(\theta'^{(2)l'^{(3)}}_B - \theta'^{(3)l'^{(2)}}_B) \times \cdots \times (\theta'^{(m-1)l''^{(1)}}_B - \theta'^{(m)l'^{(m-1)}}_B)(\theta'^{(m)l''}_B - \theta''^{l''^{(1)}}_B)$$

$$\times \prod_C (\theta''^{l''^{(1)}}_C - \theta''^{(1)l'}_C)(\theta''^{(1)l''^{(2)}}_C - \theta''^{(2)l''^{(1)}}_C)(\theta''^{(2)l''^{(3)}}_C - \theta''^{(3)l''^{(2)}}_C) \times \cdots \times (\theta''^{(r-1)l^{(1)}}_C - \theta''^{(r)l''^{(r-1)}}_C)(\theta''^{(r)l}_C - \theta_C^{l^{(1)}})$$

$$+ (\text{the term with the opposite phase}) \Bigg]$$

$$= 2 \prod_A (\theta_A^{l'} - \theta'^{l''}_A) \prod_B (\theta'^{l''}_B - \theta''^{l}_B) \prod_C (\theta''^{l}_C - \theta_C^{l'})$$

$$= 0$$

Exact SUSY

Chiral sectors are strictly protected in planar diagrams.

⇨ ***Manifestation of non-renormalization theorem on the lattice***

Summary & Discussions

- ***“Mild” Non-Commutative formulation of Lattice SUSY***
  - ⇒ ***Exactly realized in the 't Hooft large-N limit***
  - Why large-N limit ?
    - ⇒ *“Proper” ordering = Planarity in QFT language*
  - Large-N reduction point of view
    - ⇒ *Lattice SUSY ~ Supersymmetrizing* $\boldsymbol{\Gamma}_\mu \boldsymbol{\Gamma}_\nu = Z_{\nu\mu} \boldsymbol{\Gamma}_\nu \boldsymbol{\Gamma}_\mu$ *(TEK)*
  - Non-perturbative with exact SUSY ?
    - ⇒ *Non-commutative probability theory* *(e.g. Gopakumar-Gross '94 etc.)*
  - Proving Non-renormalization theorem on the lattice
    - ⇒ *Grisaru-Rocek-Siegel formulation on lattice*
  - Application to super Yang-Mills ?
    - ⇒ *Lattice gauge cov. = “star” gauge cov. in “mild” NC superspace*