Renormalized Polyakov loops in various Representations in finite Temperature SU(2) gauge theory

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Introduction

- Polyakov loop is the order parameter of the deconfinement transition in SU(N_c) theories
- needs renormalisation due to UV-divergencies
- higher representations of Polyakov loop probe sensitivity to breakdown of center symmetry
- check Casimir scaling, $L_D^{C_2(D')} = L_{D'}^{C_2(D)}$
- has been done in SU(3) pure gauge theory
  - S. Gupta, K. H. and O. Kaczmarek, Phys. Rev. D **77**, 034503 (2008) [arXiv:0711.2251 [hep-lat]].
- today: renormalize Polyakov loop in 3 lowest irreducable reps in SU(2)

Set up

$n/2$	$D(n)$	N_c-ality	$C_2(n)$	d_n	
$1/2$	2	1	$3/4$	1	fundamental
1	3	0	2	$8/3$	adjoint
$3/2$	4	1	$15/4$	5	

- Polyakov loop in fundamental representation with $n = 1$ (spin $1/2$) and recursion formula for higher representations with spin $n/2$ at each $\mathbf{x}$

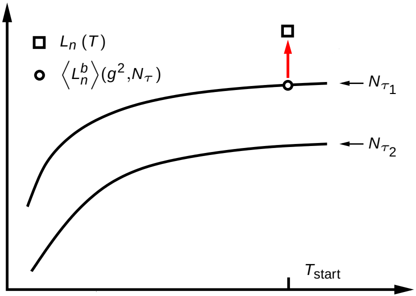
$$l_1(\mathbf{x}) = \mathrm{Tr} \prod_{x_0=0}^{N_\tau-1} U_4(\mathbf{x}, x_0) \qquad \text{and} \qquad l_{n+1} = l_n l_1 - l_{n-1}, \quad l_0 = 1$$

- renormalised Polyakov loop in representation with spin $n/2$

$$L_n(T) = Z_n(g^2)^{d_n N_\tau} \left\langle L_n^b \right\rangle (g^2, N_\tau) \quad \text{where} \quad L_n^b = \frac{\sum_{\mathbf{x}} l_n(\mathbf{x})}{D(n)V} \quad \text{and} \quad T = \left(a(g^2)N_\tau\right)^{-1}$$

- $d_n = \frac{C_2(n)}{C_2(n{=}1)}$, where $C_2(n=1) = \frac{N_c^2-1}{2N_c} = \frac{3}{4}$
- SU(2) standard Wilson action, $N_\tau = 4, 5, 6, 8, 12$ and $N_\sigma/N_\tau = 4$ and some $N_\sigma/N_\tau = 8$, statistics up to $O(10^5)$ close to T_c

Renormalization



- Choose $Z_n(g^2_{start})$ and $N_{\tau_1} < N_{\tau_2}$
- $T_{start} = (a(g^2_{start})N_{\tau_1})^{-1}$
- $L_n(T_{start}) = Z_D(g^2_{start})^{d_n N_{\tau_1}} \langle L_n^b \rangle (g^2_{start}, N_{\tau_1})$
- $T_1 = (a(g^2_{start})N_{\tau_2})^{-1}$
- $L_n(T_1)$
- determine g_1^2
- $Z_n(g_1^2) = \left(\frac{L_n(T_1)}{\langle L_n^b \rangle (g_1^2, N_{\tau_1})} \right)^{1/d_n N_{\tau_1}}$
- iterate

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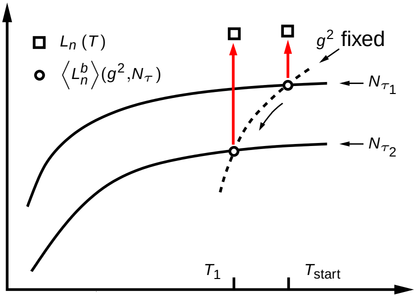
Renormalization



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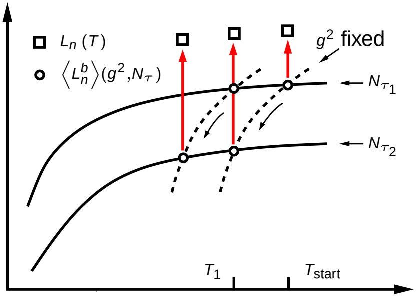

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Seed Renormalization Constants

β	V_0	Z_1	Ref
2.5115	0.537(4)	1.308(26)	[1]
2.74	0.482(3)	1.2725(19)	[1]
2.96	0.4334(9)	1.2419(6)	[2]

- use self-energy contributions in $T = 0$-potential

$$V(R) = V_0 + \frac{a}{R} + \sigma R$$

- V_0 connected to renormalisation constant

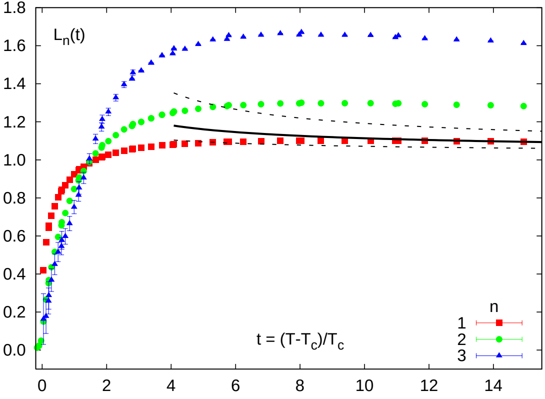
$$Z_1(g^2) = e^{V_0(g^2)/2}$$

- interpolate between 2 largest couplings, check consistency later
- assume $Z_n(g^2_{\text{start}}) = Z_1(g^2_{\text{start}})$ for all $n = 1, 2, 3$

[1] G. S. Bali, J. Fingberg, U. M. Heller, F. Karsch and K. Schilling, Phys. Rev. Lett. **71**, 3059 (1993) [arXiv:hep-lat/9306024].

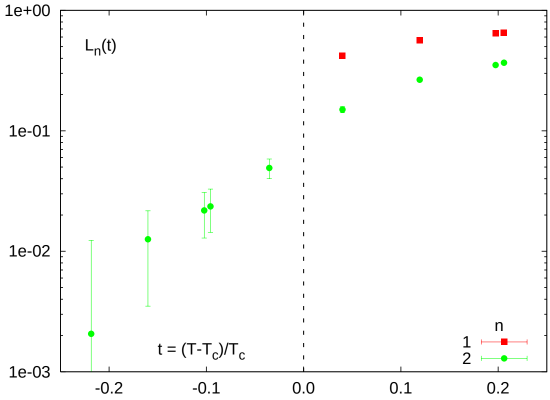
[2] G. S. Bali, K. Schilling and A. Wachter, Phys. Rev. D **55**, 5309 (1997) [arXiv:hep-lat/9611025].

Renormalized Polyakov Loops



HTL-result for high T: E. Gava and R. Jengo, Phys. Lett. B **105**, 285 (1981).

Renormalized Polyakov Loops



Fits

- 3d Ising universality class: $\beta = 0.3265(3)$, $\omega = 0.84(4)$, $\nu = 0.6301(1)$.
  $\Delta = \omega\nu = 0.530(16)$
  - A. Pelissetto and E. Vicari, Phys. Rept. **368**, 549 (2002) [arXiv:cond-mat/0012164].
- leading order behavior:

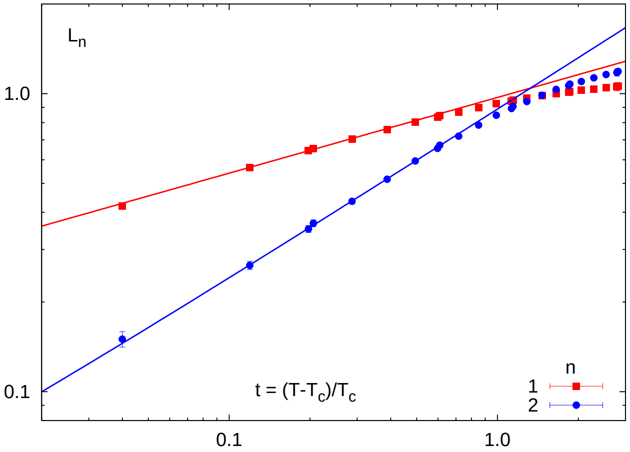
$$L_n = A_n t^{n\beta} + c_n \delta_{d,0}, \quad \text{where} \quad d: N_c\text{-ality}$$

- observed for bare loops (without c_n-term):
  - P. H. Damgaard, Phys. Lett. B **194**, 107 (1987).
  - K. Redlich and H. Satz, Phys. Lett. B **213**, 191 (1988).
  - J. E. Kiskis, Phys. Rev. D **41**, 3204 (1990).
- NLO-behavior:

$$L_n = A_n t^{n\beta}(1 + B_n t^{\Delta}) + c_n \delta_{d0}$$

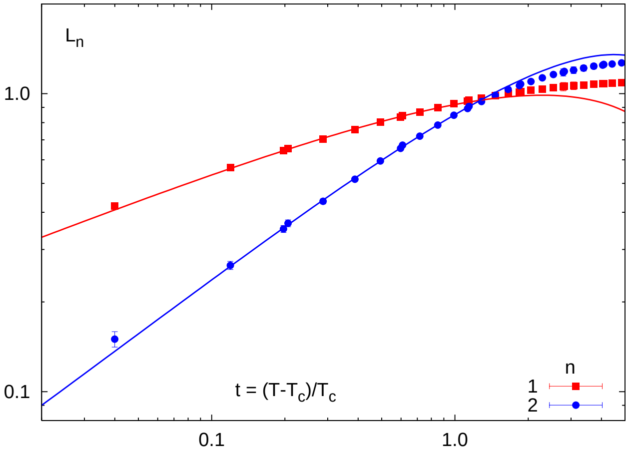
Fits II

n	Interval	A_n	$n\beta$	c_n
1	[0.01 : 0.25]	0.9952(89)	0.2666(46)	-
2	[0.1; 0.5]	0.881(24)	0.576(56)	0.007(40)



Fits II

n	Interval	A_n	B_n	c_n
1	[0.01 : 1.0]	1.2203(69)	−0.2464(65)	-
2	[0.1 : 1.0]	1.126(40)	−0.249(20)	0.005(10)



Renormalization Constants

- Z_n independent of n
- consistent with initial interpolation and V_0-data
- also observed in SU(3)

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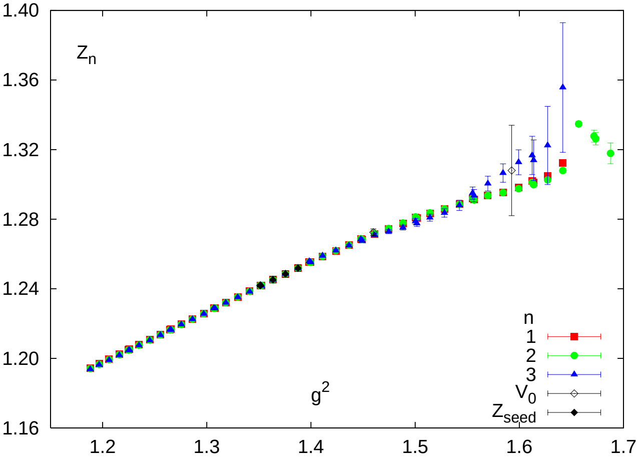
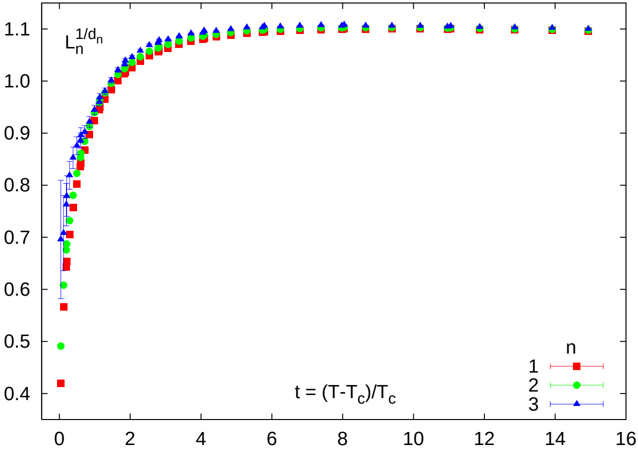


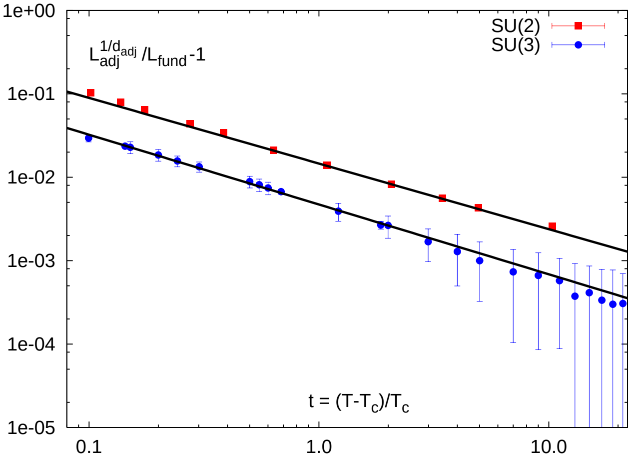
Casimir Scaling

- $L_n = L_1^{d_n} \quad \longrightarrow \quad L_n^{1/d_n}$ independent of n



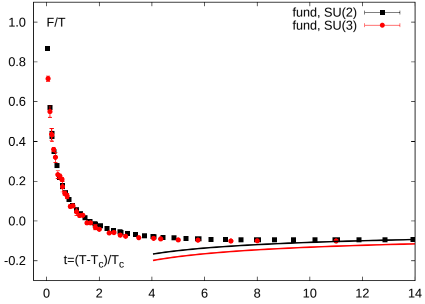
Casimir Scaling II

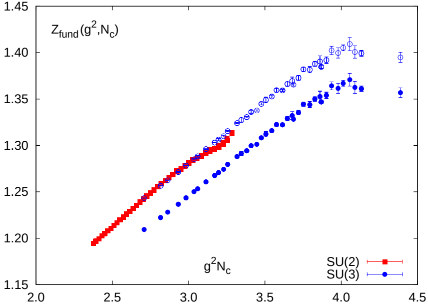
- use bare loops because of noise
- fit power law At^x in interval [0.1 : 10.]
- SU(2): $x = -0.7870(64)$ and SU(3): $x = -0.8366(91)$



SU(2) and SU(3)

- use $F = -T \ln L$
- factor $c = 1.028$, due to g^4 corrections?





Conclusions and Outlook

- we have computed renormalized Polyakov loops in the three lowest irreducable representations in finite temperature SU(2) gauge theory
- we found renormalization constants to be independent of the representation
- Casimir scaling is realized for the Polyakov loops at high temperatures, deviations diverge when approaching T_c following a power law
- renormalized fundamental Polyakov loops in SU(2) and SU(3) agree very well for a large interval of temperatures
- do renormalization of L in SU(4) and possibly beyond