Tuning improved anisotropic actions in lattice perturbation theory

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Motivation

- Part of the LHPC hadron spectrum effort
- Try to identify all the low-lying excitations predicted by QCD
- Interested in the spectrum beyond ground states
- Strongly constrains the choice of lattice action (well defined single-timeslice transfer operator)
- The use of $3+1$ anisotropic lattices ($a_t < a_s$) greatly assists in the extraction of excited states

Actions

- Use a 3+1 anisotropic Sheikholeslami-Wohlert quark action and a tree-level Symanzik and tadpole-improved gauge action

$$
\begin{aligned}
M_{\text{quark}} &= m_0 + \gamma_t \nabla_t - \frac{a_t}{2} \triangle_t + \nu_s \sum_k \left(\gamma_k \nabla_k - \frac{a_s}{2} \triangle_k \right) \\
&+ \frac{1}{2} \left[c_t a_s \sum_k \sigma_{tk} F_{tk} + c_s a_s \sum_{k<l} \sigma_{kl} F_{kl} \right]
\end{aligned} \quad (1)
$$

$$
S_{\text{gauge}} = -\beta \left[\frac{4\xi_g}{3} \sum_i P_{it} - \frac{\xi_g}{12} \sum_i R_{ti} + \frac{5}{3\xi_g} \sum_{i<j} P_{ij} - \frac{1}{12\xi_g} \sum_{i<j} (R_{ij} + R_{ji}) \right]
$$

- To further reduce lattice artefacts all spatial link variables in the quark action are stout smeared

- To obtain the correct continuum limit, ν_s and ξ_g must be tuned such that measurements of the ratio a_s/a_t using different physical probes agree at a fixed target value
- This can be done non-perturbatively (R. Edwards - this conference)
- However, in principle, new tuning runs are required for each new parameter set
- Lattice perturbation theory can provide precision results at high β
- Real progress is made when both approaches are combined
- The ultimate goal is to combine results from these complementary methods to obtain functional forms for the action parameters which hold over much of parameter space

- The input parameters in this study are the quark mass, the (target) anisotropy and the smearing parameters

The stout smearing algorithm is

$$\begin{aligned} U_\mu^{(n+1)}(x) &= \exp\left(iQ_\mu^{(n)}(x)\right) U_\mu^{(n)}(x) \\ Q_\mu(x) &= \frac{i}{2}\left(\Omega_\mu^\dagger(x) - \Omega_\mu(x)\right) - \frac{i}{2N}\mathrm{Tr}\left(\Omega_\mu^\dagger(x) - \Omega_\mu(x)\right) \\ \Omega_\mu(x) &= \sum_{\nu\neq\mu} \rho_{\mu\nu}\left(U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})\right. \\ &+ \left. U_\nu^\dagger(x-\hat{\nu})U_\mu(x-\hat{\nu})U_\nu(x-\hat{\nu}+\hat{\mu})\right) U_\mu^\dagger(x) \end{aligned} \tag{3}$$

- In our simulations $\rho_{ij} = \rho$ and $\rho_{t\mu} = \rho_{\mu t} = 0$

Quantities of interest

- The parameter ν_s appearing in the quark action is fixed by demanding that the anisotropy measured from the quark dispersion relation takes a predefined target value
- ξ_g is determined from the gluon dispersion relation
- c_t and c_s are tuned by matching lattice scattering amplitudes to their continuum counterparts

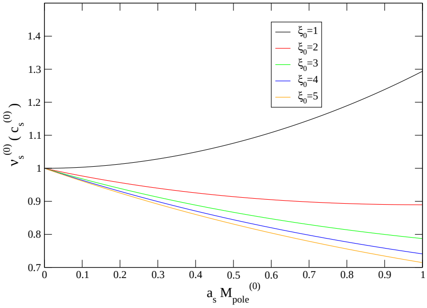
Methodology

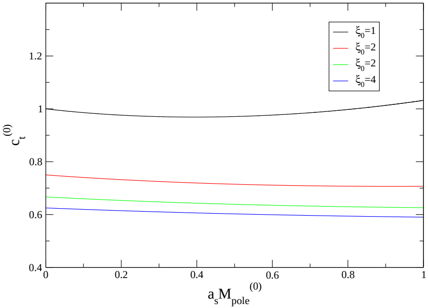
- Smeared vertex functions quickly become complicated
- Automated generation of vertex functions
- HiPPy (Hart et al. [1]) and independent C++ code
- Can handle any number of gluons and any level of smearing (1,2,3,100)
- Suite of C++ code used to evaluate integrands
- All spin manipulations are handled by the code
- Automatic differentiation used to evaluate derivatives with respect to external momenta
- Significantly reduces the chances of human error

[1] J.Comput.Phys.209:340-353,2005

Tree-level values

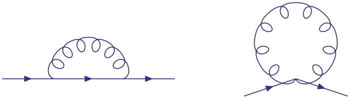
- Already at tree-level the coefficients in the quark action are mass-dependent
- $\mathcal{O}(a_t, a_s)$ improvement requires that $c_s^{(0)} = \nu_s^{(0)}$
- Expressions agree with Fermilab formulae





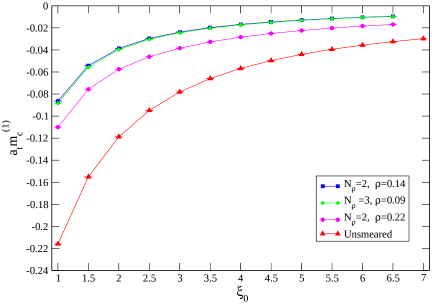
$\nu_s^{(1)}$

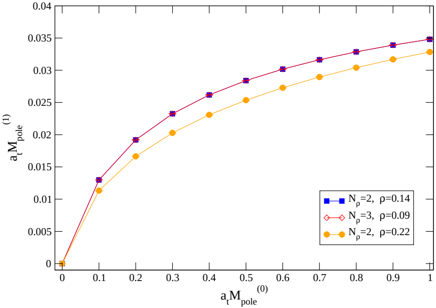
- Get $\nu_s^{(1)}$ (but not $c_s^{(1)}$ or $c_t^{(1)}$) from the one-loop quark propagator
- Solve for the pole and expand the energy in powers of the spatial momentum
- Energy and momenta are measured in lattice units $(1/a_t, 1/a_s)$
- At fixed anisotropy, tune ν_s such that $E^2(\vec{p}) = E^2(\vec{0}) + |\vec{p}|^2$ for small $|\vec{p}|$
- At one loop just two diagrams contribute



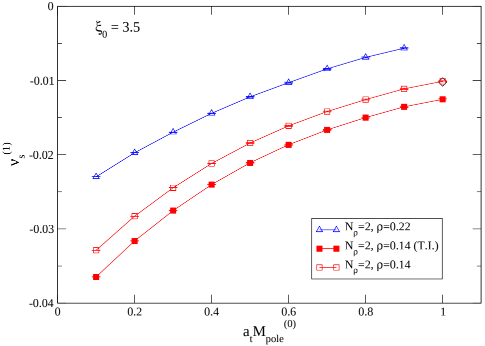
Quark Masses

- Critical quark mass and one-loop rest mass appear in the calculation
- Smearing parameters $N_\rho = 2,\ \rho = 0.14$ minimise the critical quark mass (and maximise the spatial plaquettes) at $\xi = 3.5$





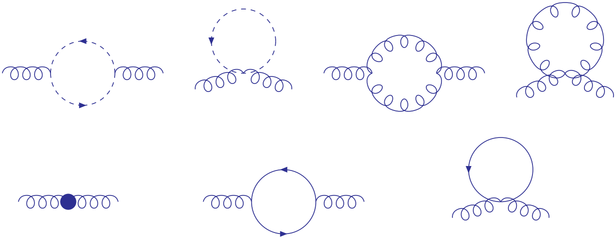
$\nu_s^{(1)}$ *results*



- Corrections to $\nu_s^{(1)}$ are small over a range of quark masses
- The choice of smearing parameters which minimises $a_t m_{\rm crit}^{(1)}$ does not minimise $\nu_s^{(1)}$
- Tadpole improvement has a small effect

Gauge anisotropy

- The gauge anisotropy is fixed by requiring that a gluon obey a relativistic dispersion relation at small momentum
- Determined the gluon propagator to one-loop order and solve for poles
- Seven diagrams contribute to the gluon propagator at one-loop order



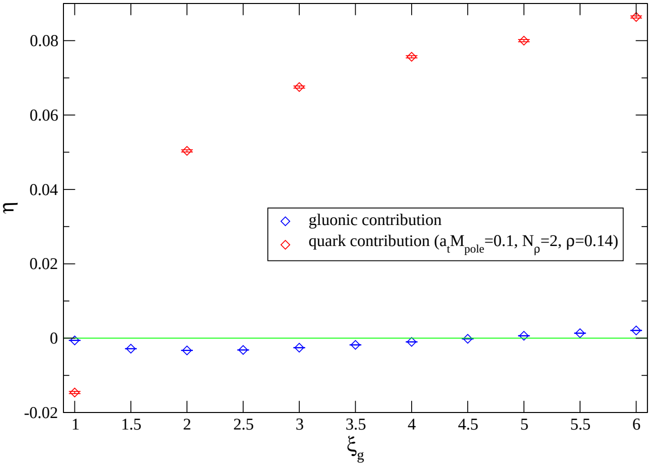
- At one-loop order the measured anisotropy is

$$\begin{aligned} \xi_R &= \left(1 + g_0^2 \eta\right) \xi_g \\ \eta &= \frac{\xi_g^2}{4} \left[\frac{d^2}{d\,(a_s p_i)^2} a_t^2 \Sigma_{jj}^{(1)}(p) \right]_{\vec{p}=(p_i,0,0)} \end{aligned} \tag{4}$$

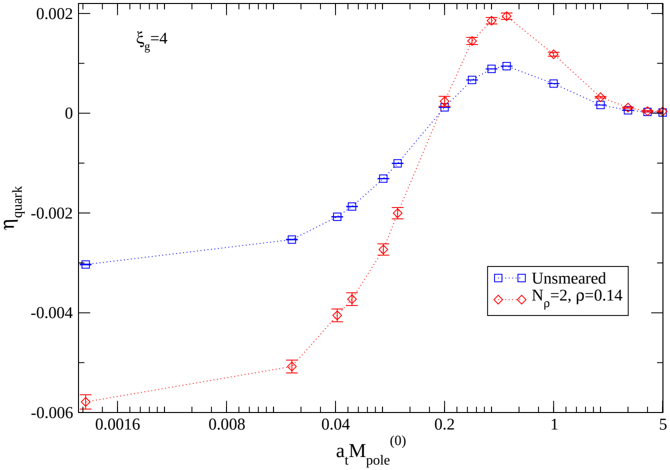
- Corrections to the gauge anisotropy are additive
- At one-loop order the sea quark contribution to the gauge anisotropy is independent of the gauge action
- Pure gluonic part agrees with the calculation Drummond et al. [2]

[2] Phys.Rev.D66:094509,2002

Gauge anisotropy - results



Mass/Smearing dependence



Future directions

- Need to explore parameter space fully
- Monte Carlo comparison
- Use P.T. to guide the choice of smearing parameters

- $\mathcal{O}(\alpha_s a_t, \alpha_s a_s)$ improvement